

# Chapter 6

## Inductance and Capacitance

Text: *Electric Circuits* by J. Nilsson and S. Riedel  
Prentice Hall

Engr 17 Introductory Circuit Analysis  
Instructor: Russ Tatro

## Chapter 6 Overview

The separation of charge used to define *Voltage* which is also used to define the concept of capacitance.

$$i_c = C \frac{dv}{dt}$$

The movement of charge is used to define *Current* which is also used to define the concept of inductance.

$$v_L = L \frac{di}{dt}$$

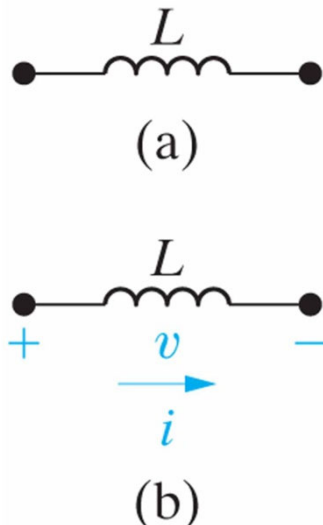
Sections 6.4 and 6.5 will be deferred until the discussion of ideal transformers in Chapter 9.

# Section 6.1

## The Inductor

## The Inductor

The inductance – symbol  $L$  - is the consequence of a conductor carrying a current which is linked to a magnetic field.



(a)

$$v_L = L \frac{di}{dt}$$

(b)

$$i_L(t) = \frac{1}{L} \int_{t_0}^t v_L dt + i_L(t_0)$$

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Note that the voltage across an inductor is written in accordance with the passive sign convention!

The unit of inductance is the henry H.

## The Inductor

The behavior of the inductor's voltage/current relationship is thus different from the resistor.

It is the slope of a constant change in current that creates a constant voltage across the inductor.

$$v_L = L \frac{di}{dt}$$

If the current through an inductor is constant, then the voltage across the inductor is ZERO.

## Power and Energy in the Inductor

Recall that power is given by:  $p = (\pm)vi$

And the voltage across an inductor is  $v_L = L \frac{di}{dt}$

Then the power stored in an inductor's magnetic field is

$$p_L = (\pm)vi = (\pm) \left( L \frac{di}{dt} \right) i = (\pm) Li \frac{di}{dt}$$

The energy  $w$  that an inductor can deliver to a load is (see text for derivation)

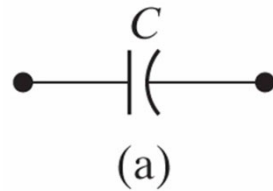
$$w = \frac{1}{2} Li^2$$

# Section 6.2

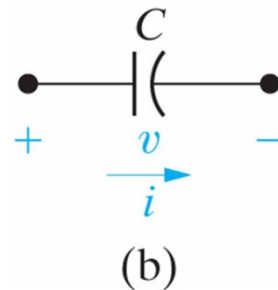
## The Capacitor

## The Capacitor

The capacitor – symbol  $C$  – is the consequence charge separated by a *dielectric* (insulator) which is linked to an electric field.



$$i_C = C \frac{dv}{dt}$$



$$v_C(t) = \frac{1}{C} \int_{t_0}^t i_C dt + v(t_0)$$

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Note that the charge separation is due to a *displacement current* due to a time varying electric field.

Note that the voltage across a capacitor is written in accordance with the passive sign convention!

The unit of capacitance is the farad F.



## The Capacitor

The capacitor acts like the opposite twin of the inductor.

It is the slope of a constant change in voltage that creates a constant displacement current around the capacitor!

$$i_C = C \frac{dv}{dt}$$

If the voltage across a capacitor is constant then the change in current displaced around a capacitor is ZERO.

## Power and Energy in the Capacitor

Recall that power is given by:  $p = (\pm)vi$

And the displacement current around a capacitor is  $i_c = C \frac{dv}{dt}$

Then the power stored in a capacitor's electric field is

$$p_c = (\pm)vi = (\pm)v \left( C \frac{dv}{dt} \right) = (\pm)Cv \frac{dv}{dt}$$

The energy  $w$  that a capacitor can deliver to a load is (see text for derivation)

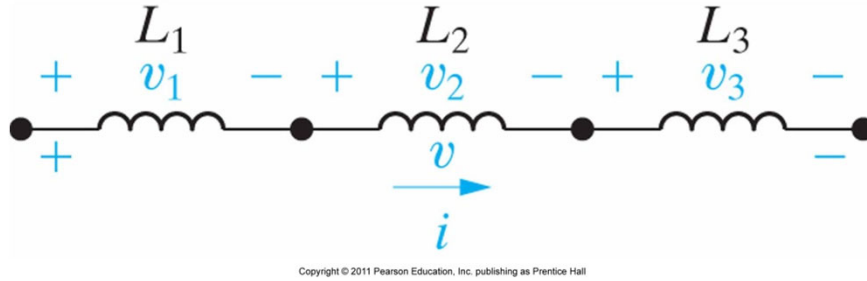
$$w = \frac{1}{2} Cv^2$$

## Section 6.3

# Series-Parallel Combinations of Inductance and Capacitance

## Series Inductors

Recall: a series connection has the same current through the series elements.



$$v_1 = L_1 \frac{di}{dt} \quad v_2 = L_2 \frac{di}{dt} \quad v_3 = L_3 \frac{di}{dt}$$

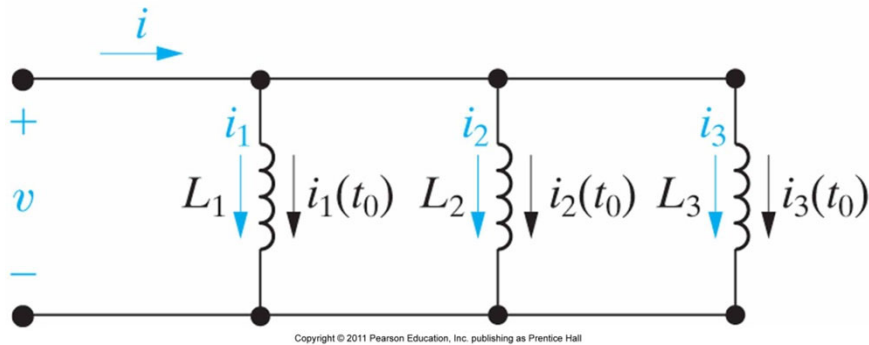
$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3) \frac{di}{dt} = L_{Eq} \frac{di}{dt}$$

Thus we see that the series inductors behave the “resistor in series” rule.

$$\text{Series } L_{Eq} = L_1 + L_2 + L_3$$

## Parallel Inductors

Recall: a parallel connection has the same voltage across the parallel elements.



$$i_1 = \frac{1}{L_1} \int_{t_0}^t v \, d\tau + i_1(t_0)$$

$$i_2 = \frac{1}{L_2} \int_{t_0}^t v \, d\tau + i_2(t_0)$$

$$i_3 = \frac{1}{L_3} \int_{t_0}^t v \, d\tau + i_3(t_0)$$

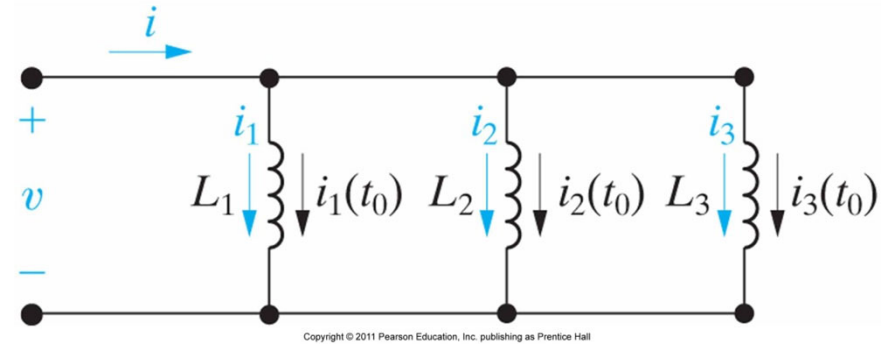
$$i = i_1 + i_2 + i_3$$

$$= \left[ \left( \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right) \int_{t_0}^t v \, d\tau \right] + \underbrace{i_1(t_0) + i_2(t_0) + i_3(t_0)}_{\text{initial conditions}}$$

$$= \left[ \frac{1}{L_{Eq}} \int_{t_0}^t v \, d\tau \right] + i_1(t_0) + i_2(t_0) + i_3(t_0)$$

## Parallel Inductors

$$i = \frac{1}{L_{Eq}} \int_{t_0}^t v \, d\tau + [i_1(t_0) + i_2(t_0) + i_3(t_0)]$$



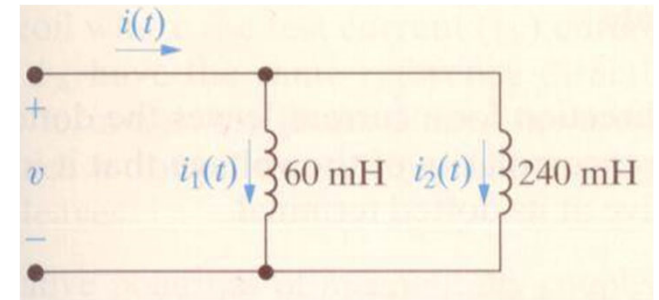
Thus we see that the parallel inductors behave the “resistors in parallel” rule.

$$\text{Parallel} \quad \frac{1}{L_{Eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

### AP6.4 Inductors in Parallel

Given:  $i_1(t_0) = 3\text{A}$        $i_2(t_0) = -5\text{A}$

$V = -30e^{-5t} \text{ mV}$  for  $t \geq 0$

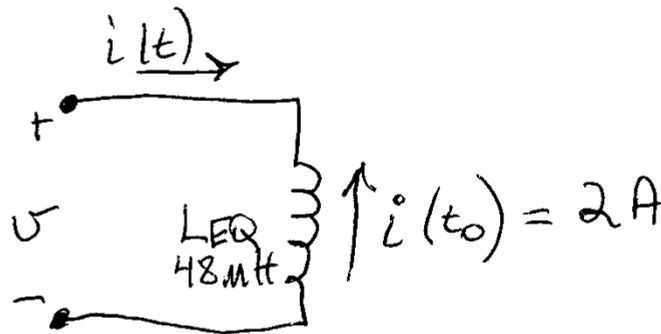


a) What is the  $L_{Eq}$ ?

$$L_{Eq} = \left[ \frac{1}{L_1} + \frac{1}{L_2} \right]^{-1} = \left[ \frac{1}{60\text{mH}} + \frac{1}{240\text{mH}} \right]^{-1} = 48\text{mH}$$

b) What is the initial current and its reference direction in the equivalent inductor?

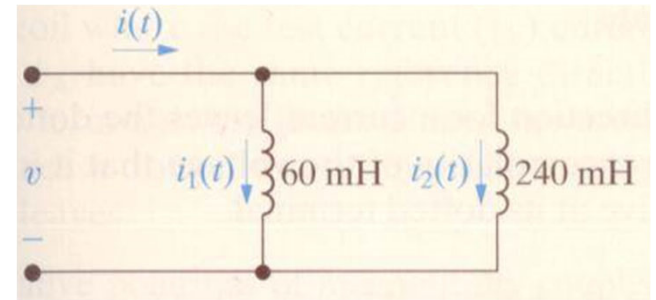
$$i(t_0) = 3 - 5 = -2\text{A}$$



### AP6.4 Inductors in Parallel

Given:  $V = -30e^{-5t}$  mV for  $t \geq 0$

c) Find  $i(t)$ .



$$i(t) = \left[ \frac{1}{L_{Eq}} \int_{0^+}^t (-.03e^{-5x}) dx \right] - 2A$$

$$\text{use } \int e^{au} du = \frac{1}{a} e^{au} + C$$

$$= \left[ \frac{-.03}{.048} \int_{0^+}^t e^{-5x} dx \right] - 2A = \left[ -0.625 \left( \frac{-1}{5} e^{-5x} \Big|_0^t \right) \right] - 2A$$

$$= \left[ \frac{-0.625}{-5} (e^{-5t} - e^0) \right] - 2A = [0.125e^{-5t} - 0.125(1)] - 2A$$

$$= 0.125e^{-5t} - 2.125A \text{ for } t \geq 0$$

Note  $i(t = 0) = -2A$ !

$$i(t = 0) = 0.125e^0 - 2.125 = -2A$$

Answer checks at  $t = 0$



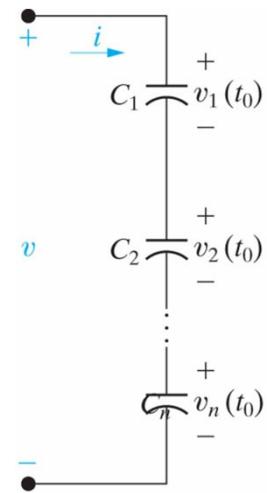
## Series Capacitors

$$v = v_1 + v_2 + v_3$$

$$= \frac{1}{C_1} \int_{t_0}^t i \, d\tau + v_1(t_0) + \frac{1}{C_2} \int_{t_0}^t i \, d\tau + v_2(t_0) + \frac{1}{C_3} \int_{t_0}^t i \, d\tau + v_3(t_0)$$

$$= \left[ \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \int_{t_0}^t i \, d\tau \right] + \underbrace{v_1(t_0) + v_2(t_0) + v_3(t_0)}_{\text{initial conditions}}$$

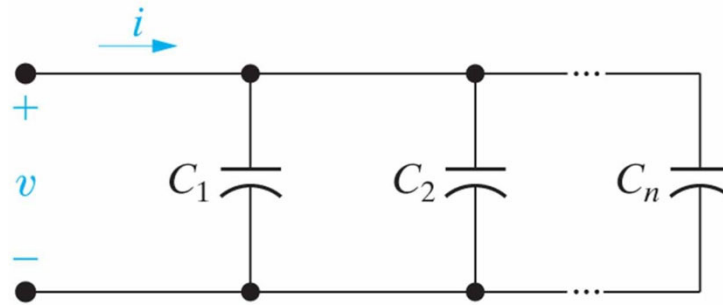
$$= \left[ \frac{1}{C_{Eq}} \int_{t_0}^t i \, d\tau \right] + v_1(t_0) + v_2(t_0) + v_3(t_0)$$



Thus we see that the series capacitors behave the “resistor in parallel” rule.

$$\text{Series } \frac{1}{C_{Eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

## Parallel Capacitors



$$i = i_1 + i_2 + i_3 = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt}$$

$$= (C_1 + C_2 + C_3) \frac{dv}{dt} = C_{Eq} \frac{dv}{dt}$$

Thus we see that the parallel capacitors follow the “resistors in series” rule.

$$C_{Eq} = C_1 + C_2 + C_3$$

### AP6.5 Capacitors in Series

Given:  $v_1(t_0) = -10V$        $v_2(t_0) = -5V$

$$i = 240e^{-10t} \mu A \text{ for } t \geq 0$$

a) Find  $C_{Eq}$ .

$$C_{Eq} = \left[ \frac{1}{C_1} + \frac{1}{C_2} \right]^{-1} = \left[ \frac{1}{2\mu F} + \frac{1}{8\mu F} \right]^{-1} = 1.6\mu F$$

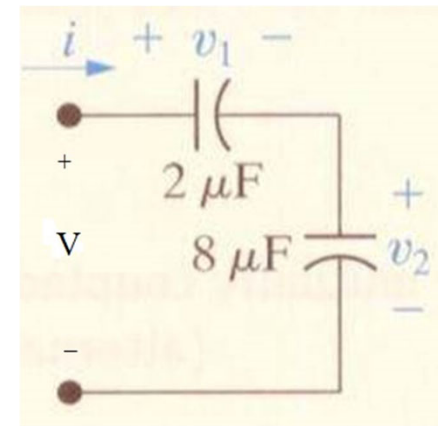
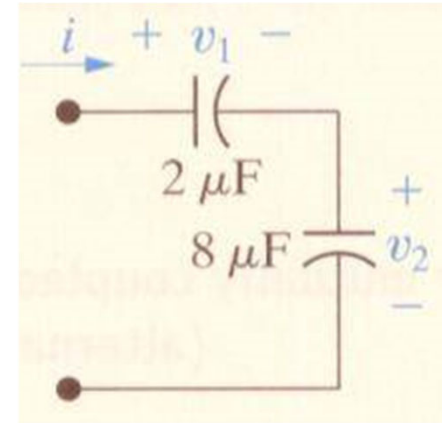
b) What is the initial voltage across  $C_{eq}$ ?

First define the polarity across the capacitors.

Then write a KVL equation:

$$-V - 10 - 5 = 0$$

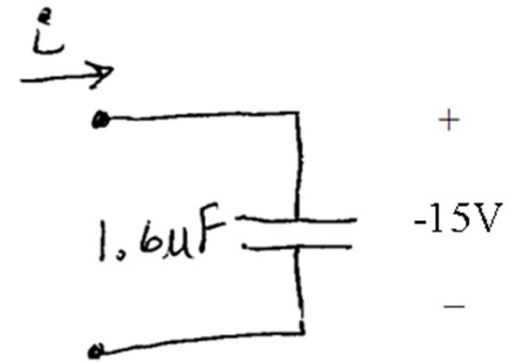
$$V = -10 - 5 = -15V$$



## AP6.5 Capacitors in Series

Given:  $i = 240e^{-10t} \mu\text{A}$  for  $t \geq 0$

$$v(t) = \left[ \frac{1}{C_{Eq}} \int_{t_0}^t i d\tau \right] + v(t_0) = \left[ \frac{1}{1.6\mu\text{F}} \int_{t_0}^t i d\tau \right] - 15\text{V}$$



c) What is the voltage across  $C_{Eq}$  as  $t \rightarrow \infty$ ?

$$\begin{aligned} v(t \rightarrow \infty) &= \left[ \frac{1}{1.6 \times 10^{-6} \text{ F}} \int_{0^+}^{\infty} (240 \times 10^{-6}) e^{-10x} dx \right] - 15\text{V} \\ &= \left[ \frac{240 \times 10^{-6}}{1.6 \times 10^{-6}} \int_{0^+}^{\infty} e^{-10x} dx \right] - 15\text{V} = \left[ 150 \int_{0^+}^{\infty} e^{-10x} dx \right] - 15\text{V} \end{aligned}$$

$$\text{use } \int e^{au} du = \frac{1}{a} e^{au} + C$$

$$= \left[ \frac{150}{-10} \left( e^{-10x} \Big|_0^{\infty} \right) \right] - 15\text{V} = \left[ -15 \left( e^{-\infty} - e^0 \right) \right] - 15\text{V}$$

$$= \left[ -15(0 - 1) \right] - 15\text{V} = 15\text{V} - 15\text{V} = 0\text{V}$$

# Chapter 6

## Inductance and Capacitance

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