Chapter 5

The Operational Amplifier

Text: *Electric Circuits, 10*th Edition, by J. Nilsson and S. Riedel
Prentice Hall

Engr 17 Introductory Circuit Analysis
Instructor: Russ Tatro

Chapter 5 Overview

The *Op Amp* is introduced at this time in order to motivate the use of the circuit analysis tools developed so far in this course.

The real opamp uses transistors, capacitors, resistors and partially acts as a dependent source.

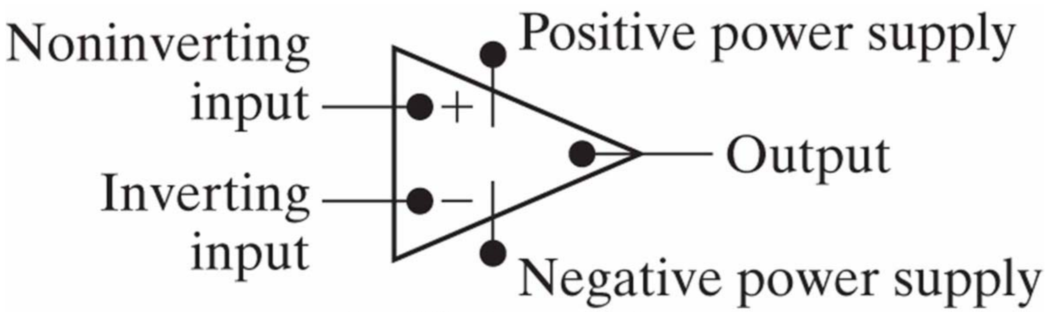
Instead of directly analyzing the real world internal device complexity, we will use the simpler ideal opamp model.

This chapter is not about memorizing the results! It is about using the tool set.

Section 5.1
Operational Amplifier Terminals

The OpAmp

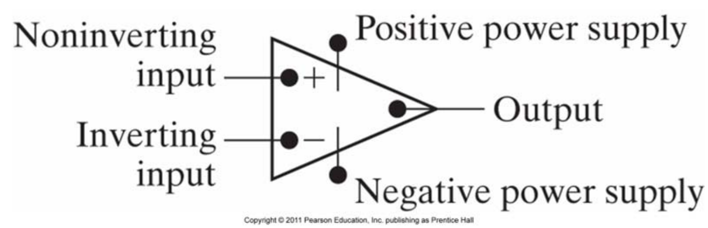
The opamp uses the standard amplifier symbol.



The diagram lists 5 named terminals and assumes that the circuit has an external, but not shown, reference node.

The OpAmp

The five terminals are:



Noninverting input – a *signal* applied to this terminal generates an output that follows the level of the input signal.

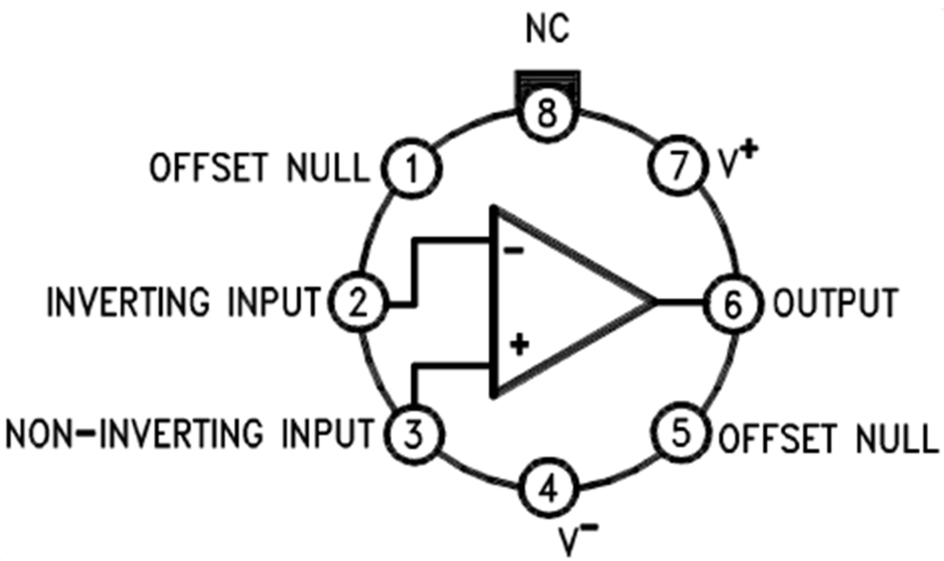
Inverting input – a *signal* applied to this terminal generates an output that *negates* (inverts) the level of the input signal.

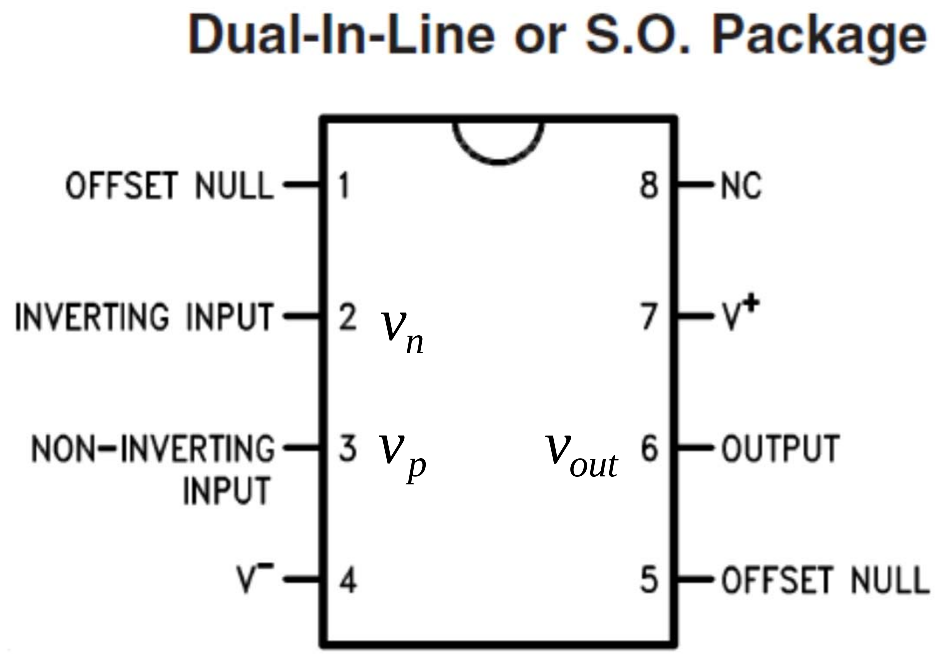
Output – the terminal that is taken as the opamp output. This is usually a voltage signal w.r.t the reference node that allows up to some current level.

Positive and Negative Power supply – these two terminals provide the power to the opamp. Not all opamps require both a positive and negative power supply – so called *unirail* opamps.

The 741 OpAmp

A common opamp is the venerable 741 in use since 1968.

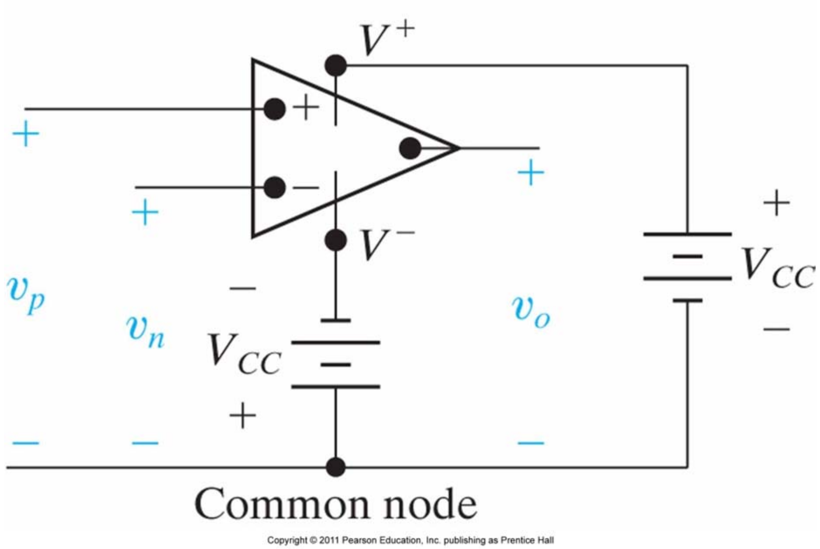




The 741 opamp was not the first opamp. Vacuum tube opamps operating at high voltages existed since the late 1930's. But the 741 became the first "all-in-one" integrated circuit design to reach the market. See Bob Widlar and Fairchild Semiconductor.

Section 5.2
Terminal Voltages and Currents

Opamp voltages and currents

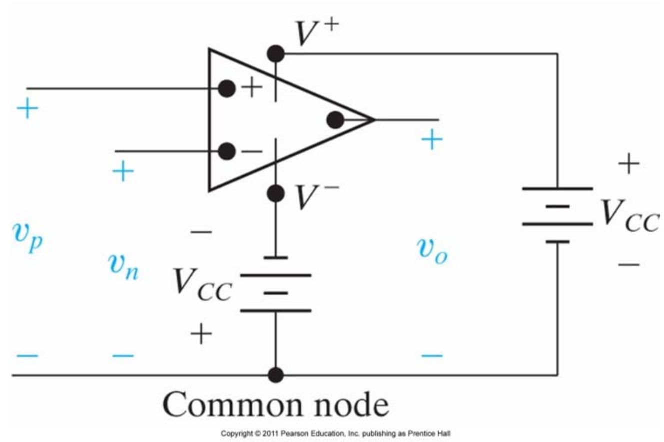


This figure is our reference circuit for the discussion in this chapter.

Note that we have defined a reference node and labeled the node *Common node* in the figure.

We have also used the standard symbol for a battery which evolved from the "pile of plates" used by early electricity researchers.

Opamp voltages and currents



The opamp is considered a *differential amplifier* with the output a linear function of the difference in voltage $v_p - v_n$.

However there is a constraint on the difference voltage – the resulting output voltage must be equal to or less than the power supply voltages.

In the linear region the output voltage can be a scaled function ("A" in the equation below) of the difference of the two input voltages.

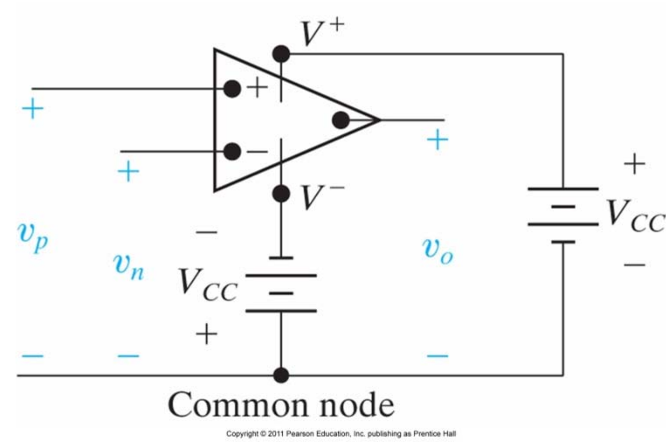
$$v_0 = A(v_p - v_n)$$

The constraint insists that the output be

$$-V_{CC} \leq A(v_p - v_n) \leq +V_{CC}$$

Opamp voltages and currents

So the discussion implies we have three regions of operation.



The opamp can be pushed to the negative power supply voltage (or nearly to that voltage).

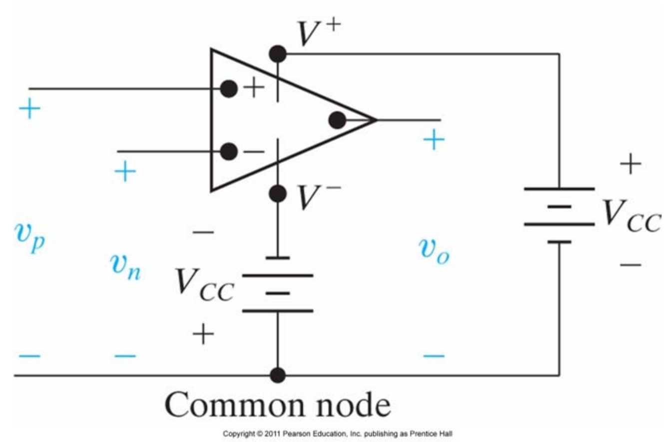
The opamp can be pushed to the positive power supply voltage (or nearly to that voltage).

The opamp can output a linear signal between the power supply voltages that is scaled w.r.t the input voltages.

We can state the three regions compactly by the following equation.

$$v_0 = \begin{cases} -V_{CC} \\ A(\mathrm{v}_p - v_n) & \quad \textit{Linear region } -V_{CC} \le A(\mathrm{v}_p - v_n) \le +V_{CC} \\ +V_{CC} \end{cases}$$

Opamp voltages and currents



The scaled value “A” is called the *gain* of the amplifier.

Gain is usually considered from two views.

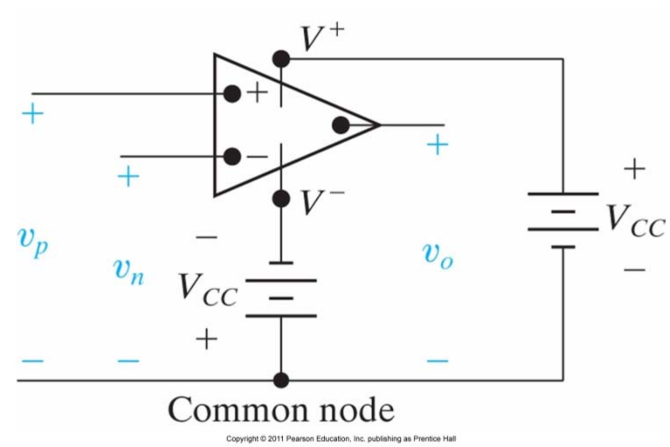
The *open loop gain* is the response of the system without some external control of the system (opamp).

Opamps have very large open loop gain – often 10^4 and greater.

Closed loop gain is when we add circuitry to reduce the internal gain of the system (opamp).

Reducing the gain with an external mechanism is called *negative feedback.*

Opamp voltages and currents



A consequence of the large open loop gain is that the difference signal $v_p - v_n$ must be small for linear operation.

We call this condition the *virtual short.*

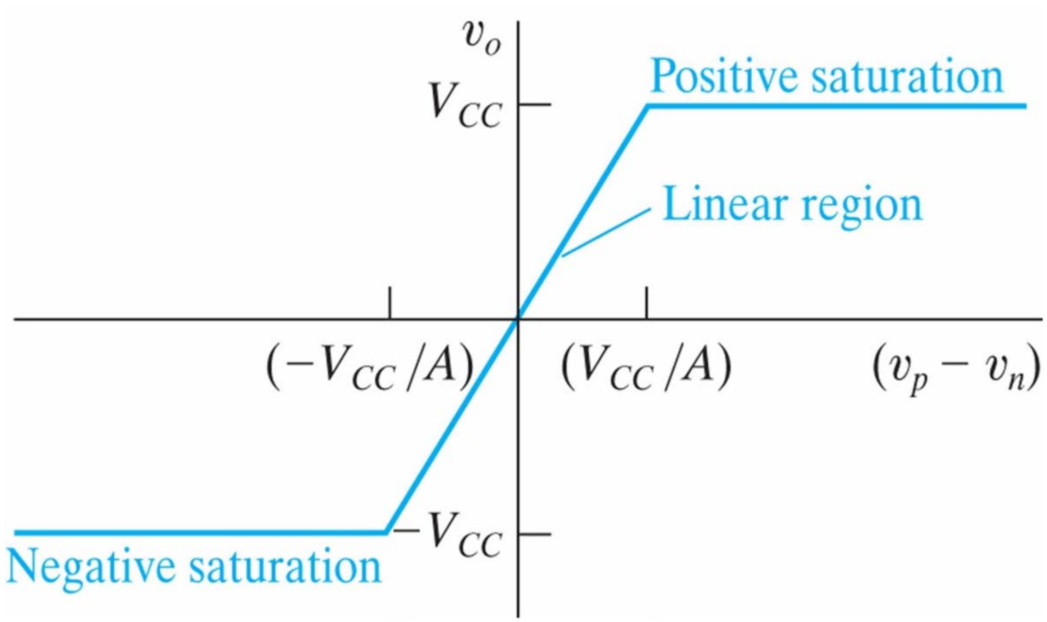
The difference in voltage between v_p and v_n will be simplified as equal to zero just like a short circuit.

$$v_p - v_n \equiv 0$$

But we say *virtual short* because we do NOT actually short (inside the opamp) the two inputs together!

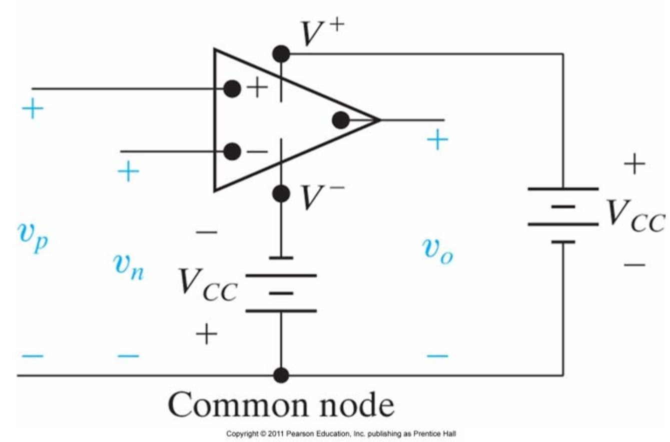
Opamp voltages and currents

We are now in the position to look at the low frequency response of the opamp.



The figure shows the three regions of operation for the opamp.

Opamp voltages and currents



One final opamp consideration is the current into the input nodes.

You must take it on faith for now, but the signal input nodes (v_p and v_n) have very large impedances.

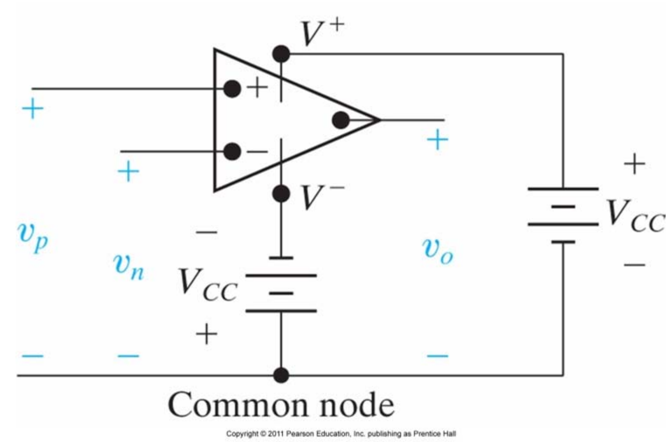
This means that the current into v_p and v_n is essentially equal to zero.

$$i_p = i_n \cong 0$$

For simplification in this course, we will say that the currents are exactly zero.

$$i_p = i_n \equiv 0$$

Opamp analysis rules



So our ideal opamp rules are:

1. $v_p = v_n$

2. $i_n = i_p = 0$

3. Open loop gain $A = \infty$
This allows negative feedback to control the circuit in a linear manner.

4. The opamp output varies between the power supply *rails*.

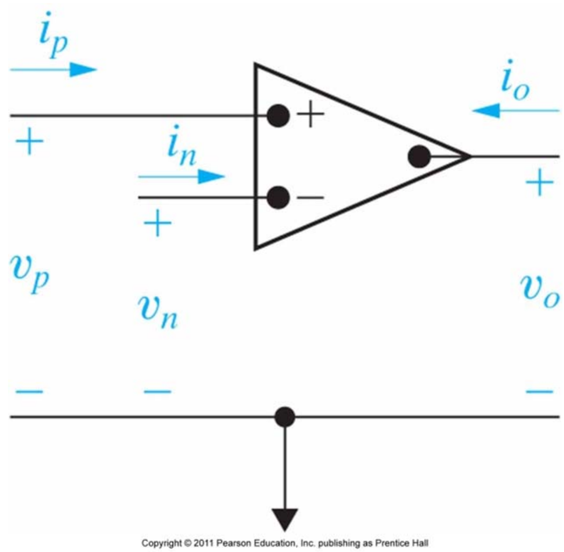
$$-V_{CC} \leq v_0 \leq +V_{CC}$$

Ideal Opamp

We will usually use the following figure to represent our opamp.

$$i_p = i_n = 0$$

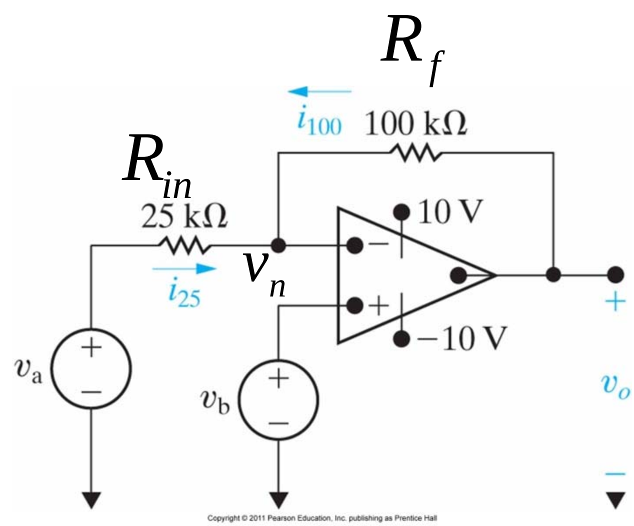
$$v_p = v_n$$



$$-V_{CC} \leq v_0 \leq +V_{CC}$$

While the power supplies are not explicitly shown, the positive and negative supplies must still be part of the circuit analysis.

Analyzing an Opamp – Example 5.1



We will analyze this opamp algebraically first.

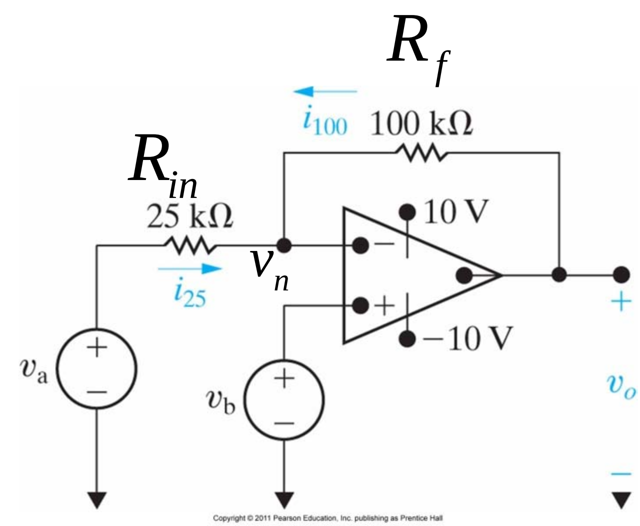
At node v_n and assume all currents leave the node
(i.e. ignore the figure's current directions for a moment).

$$\frac{v_n - v_a}{R_{in}} + i_n + \frac{v_n - v_o}{R_f} = 0$$

From the ideal opamp rules $v_p = v_n$ thus $v_n = v_b$

$$\frac{v_b - v_a}{R_{in}} + i_n + \frac{v_b - v_o}{R_f} = 0$$

Analyzing an Opamp – Example 5.1



$$\frac{v_b - v_a}{R_{in}} + i_n + \frac{v_b - v_o}{R_f} = 0$$

Now solve for v_o

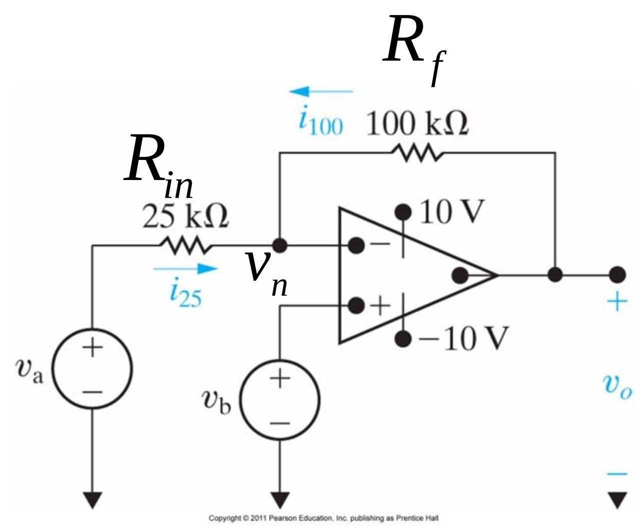
$$\frac{v_o}{R_f} = \frac{v_b - v_a}{R_{in}} + i_n + \frac{v_b}{R_f}$$

Recall that $i_n = 0$

$$v_o = (v_b - v_a)\frac{R_f}{R_{in}} + v_b$$

$$or\ equally\ \ v_o = v_b\left(1 + \frac{R_f}{R_{in}}\right) + v_a\left(-\frac{R_f}{R_{in}}\right)$$

Analyzing an Opamp – Example 5.1



Given $v_a = 1$ V and $v_b = 2$ V and the resistor values shown on the figure.

Solve for v_o.

$$v_o = v_b\left(1+\frac{R_f}{R_{in}}\right)+v_a\left(-\frac{R_f}{R_{in}}\right)$$

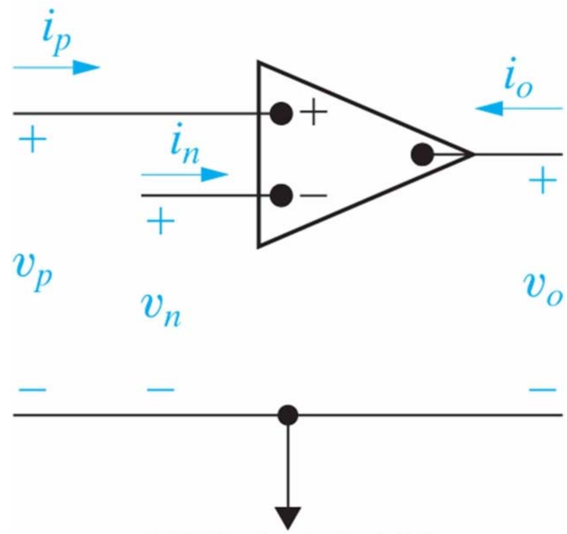
$$= 2V\left(1+\frac{100k\Omega}{25k\Omega}\right)+1V\left(-\frac{100k\Omega}{25k\Omega}\right)$$

$$= 2\,\mathrm{V}(5)+1V(-4) \quad = 6V$$

We see that each input voltage is scaled at the output v_o.
In the case of v_b the input is scaled by +5.
For v_a the input is scaled by -4.

General comments

There is very little reason to memorize an opamp configuration just to avoid deriving the input/output relationships.



The remainder of this chapter will look at typical opamp configurations but each configuration is readily found from solving the node equations for the opamp.

The risk in memorizing the opamp configurations is that subtleties abound and your memorized version may not really apply in a specific circuit.

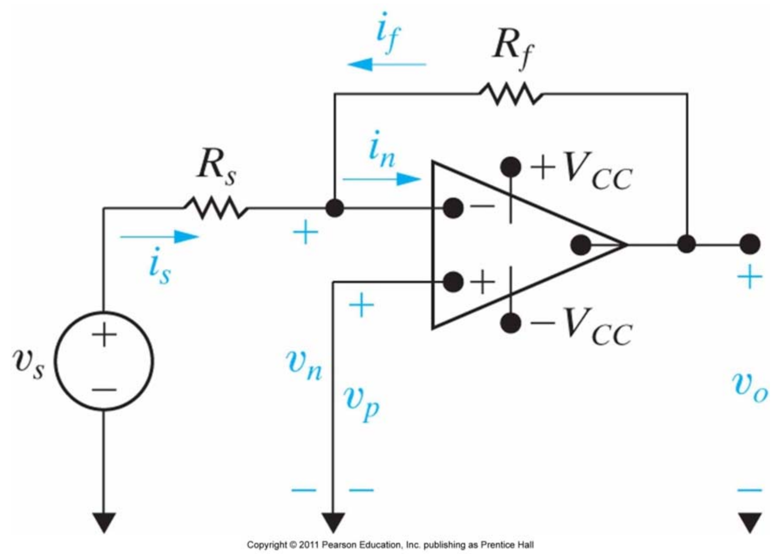
Just solve the specific circuit with node analysis!

Section 5.3
The Inverting-Amplifier Circuit

Inverting OpAmp

This circuit is the well-known inverting opamp.

Note:



$v_p = 0$ (“grounded”)

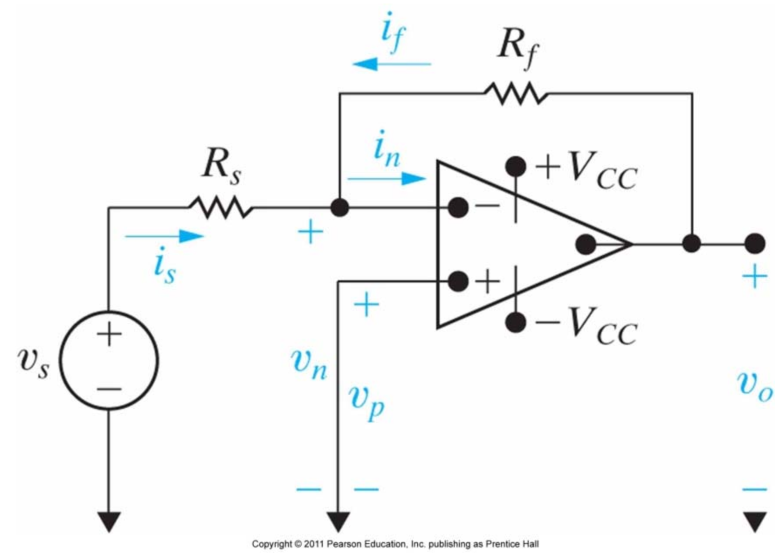
Thus $v_n = v_p = 0$

By the ideal opamp rules $i_n = i_p = 0$

By node analysis at node v_n and writing the currents <u>with</u> the defined directions in the circuit above:

$$-\frac{v_s - v_n}{R_s} + \underbrace{i_n}_{=0} - \frac{v_o - v_n}{R_f} = 0$$

Inverting OpAmp



$$-\frac{v_s - v_n}{R_s} + \underbrace{i_n}_{=0} - \frac{v_o - v_n}{R_f} = 0$$

With $v_n = v_p = 0$, solve for v_o

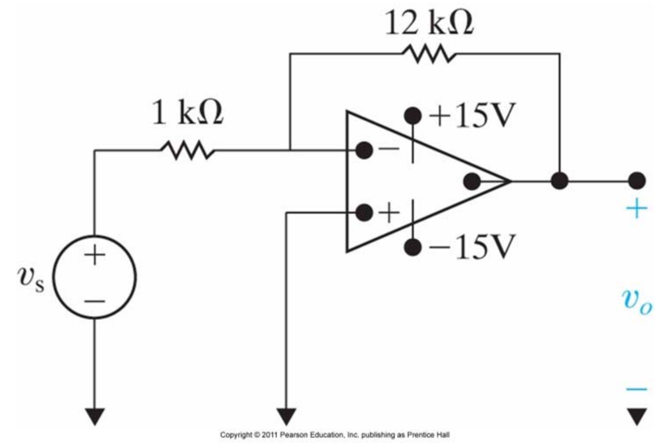
$$-\frac{v_s}{R_s} - \frac{v_o}{R_f} = 0$$

$$v_o = v_s\left(-\frac{R_f}{R_s}\right)$$ The output is scaled by the ratio of R_f to R_s.

It is called inverting since a positive input results in a negative output voltage.

When we look at periodic and time varying signals, we see this inverting result is the same as a phase delay of 180°.

Example 5.2 - Inverting OpAmp



Find the range of v_s that keeps the opamp output in the linear region.

$$v_o = v_s\left(-\frac{R_f}{R_s}\right) = v_s\left(-\frac{12k\Omega}{1k\Omega}\right) = v_s(-12)$$

The linear region is $-V_{CC} \le v_o \le +V_{CC}$

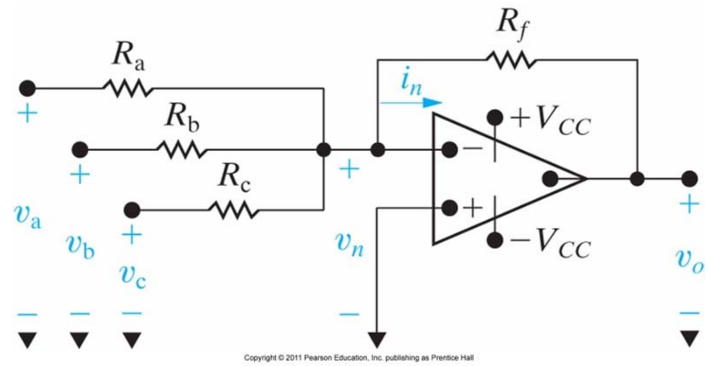
Substitute in for the input and the power supply voltages and we have

$$-15V \le -12v_s \le +15V \quad \Rightarrow \frac{-15V}{-12} \ge v_s \ge \frac{+15V}{-12}$$

$$1.25 \ge v_s \ge -1.25 \qquad or\ equally\ \ -1.25 \le v_s \le 1.25$$

Section 5.4
The Summing-Amplifier Circuit

Summing Amplifier



This circuit will sum the inputs and perform scaling to the output.

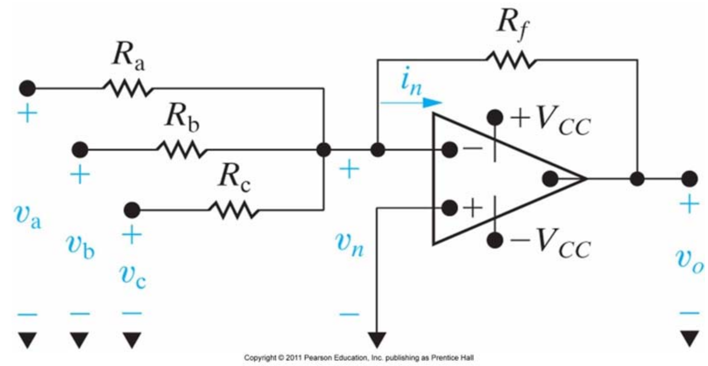
Use the ideal opamp rules and find the input/output relationship.

At node v_n and note that $v_p = 0 = v_n$.

$$\frac{v_n - v_a}{R_a} + \frac{v_n - v_b}{R_b} + \frac{v_n - v_c}{R_c} + \underbrace{i_n}_{=0} + \frac{v_n - v_o}{R_f} = 0$$

$$\frac{-v_a}{R_a} + \frac{-v_b}{R_b} + \frac{-v_c}{R_c} + \frac{-v_o}{R_f} = 0$$

Summing Amplifier



$$\frac{-v_a}{R_a}+\frac{-v_b}{R_b}+\frac{-v_c}{R_c}+\frac{-v_o}{R_f}=0$$

Now solve for v_o.

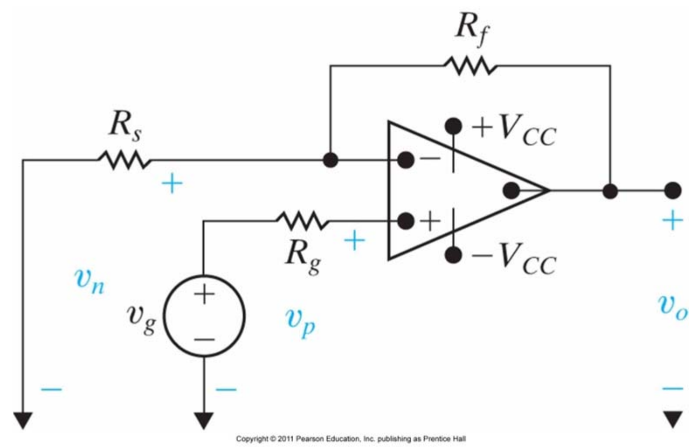
$$v_o = v_a\left(-\frac{R_f}{R_a}\right)+v_b\left(-\frac{R_f}{R_b}\right)+v_c\left(-\frac{R_f}{R_c}\right)$$

The last result shows that the circuit clearly performs a scaled summation, albeit with an negative scaling.

A net positive summation can quickly be achieved by passing the first circuit's output through another inverting amplifier with a gain of -1!

Section 5.5
The Noninverting-Amplifier Circuit

Noninverting Amplifier



Now let us explore a slightly different circuit.

Use the ideal opamp rules and find the input/output relationship.

At node v_n and note that $v_p = v_g = v_n$ since $i_n = 0$ and there is no voltage drop across R_g.

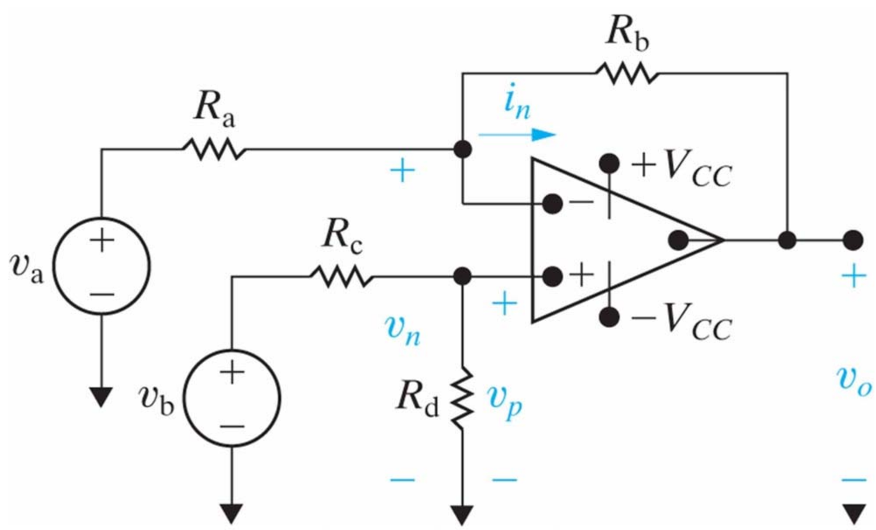
$$\frac{v_n - 0}{R_s} + \underbrace{i_n}_{=0} + \frac{v_n - v_o}{R_f} = 0 \quad \Rightarrow \frac{v_g - 0}{R_s} + \frac{v_g - v_o}{R_f} = 0$$

$$v_o = v_g\left(1 + \frac{R_f}{R_s}\right)$$ Which shows scaled and noninverting output.

Section 5.6
The Difference-Amplifier Circuit

Difference-Amplifier

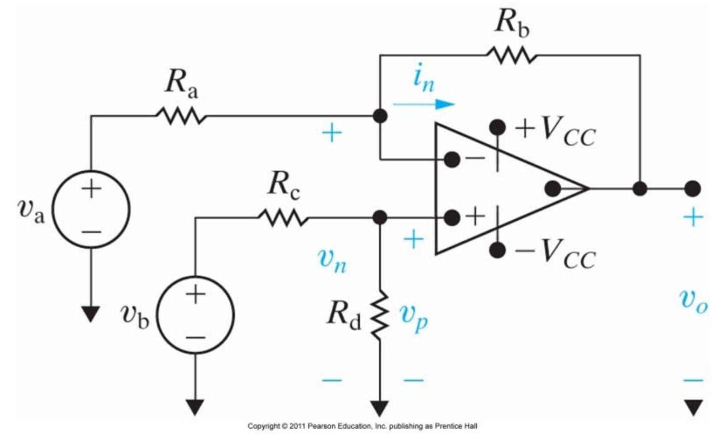
The output of a difference amplifier is proportional to the difference between the two input voltages as shown in this circuit.



With the ideal opamp rules that $i_p = 0$, we have

$$v_p = v_b \frac{R_d}{R_d + R_c}$$

Difference-Amplifier



We proceed with node analysis at node v_n.

$$\frac{v_n - v_a}{R_a} + \underbrace{i_n}_{=0} + \frac{v_n - v_o}{R_b} = 0$$

$$\frac{v_o}{R_b} = \frac{v_n - v_a}{R_a} + \frac{v_n}{R_b}$$

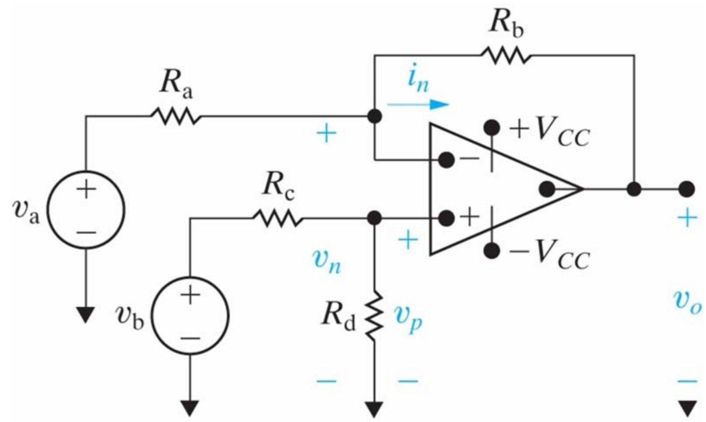
$$v_o = \frac{R_b}{R_a}\left(v_n - v_a\right) + v_n \qquad = v_n\left(1 + \frac{R_b}{R_a}\right) + v_a\left(-\frac{R_b}{R_a}\right)$$

Now substitute in for our $v_n = v_p$ result.

$$v_0 = v_b\left(\frac{R_d}{R_d + R_c}\right)\left(1 + \frac{R_b}{R_a}\right) + v_a\left(-\frac{R_b}{R_a}\right)$$

Difference-Amplifier

$$v_0 = v_b\left(\frac{R_d}{R_d+R_c}\right)\left(1+\frac{R_b}{R_a}\right)+v_a\left(-\frac{R_b}{R_a}\right)$$



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The next insight is to make the result be in the form $v_o = A(v_b - v_a)$.

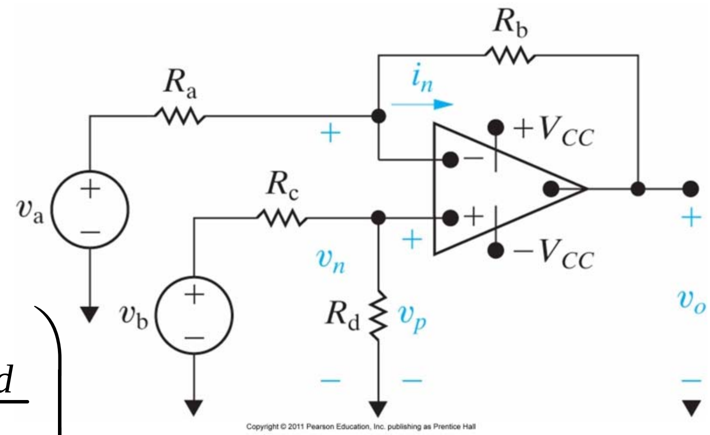
This will happen if we let $\dfrac{R_b}{R_a} = \dfrac{R_d}{R_c}$

$$= v_b\left(\frac{R_d}{R_d+R_c}\right)\left(1+\frac{R_d}{R_c}\right)+v_a\left(-\frac{R_d}{R_c}\right)$$

$$= v_b\left(\frac{R_d}{R_d+R_c}\right)\left(\frac{R_c+R_d}{R_c}\right)+v_a\left(-\frac{R_d}{R_c}\right)$$

Difference-Amplifier

Continuing with the derivation:



$$v_o = v_b\left(\frac{R_d}{R_d + R_c}\right)\left(\frac{R_c + R_d}{R_c}\right) + v_a\left(-\frac{R_d}{R_c}\right)$$

$$= v_b\left(\frac{R_d}{R_c}\right) + v_a\left(-\frac{R_d}{R_c}\right)$$

$$= \frac{R_d}{R_c}(v_b - v_a) \qquad where \quad \frac{R_b}{R_a} = \frac{R_d}{R_c}$$

$$\text{Thus } v_0 = \frac{R_b}{R_a}(v_b - v_a)$$

With a clever choice of resistors, this circuit performs the function of a scaled difference amplifier.

Chapter 5

The Operational Amplifier

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