

For generating a structured mesh in a rectangular domain $\Omega := (x_0, x_1) \times (y_0, y_1)$, the following command is available in my code:

(a)

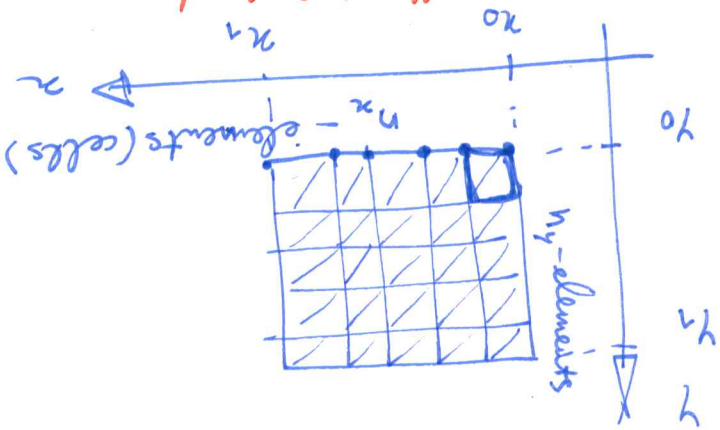
$$\text{mesh}(n_x, n_y, x_0 + (x_1 - x_0)x, y_0 + (y_1 - y_0)y).$$

With a prescribed number of elements n_x & n_y in corresponding directions, it produces the mesh with a constant mesh size, and it can be guessed & checked directly that, in fact, e.g.

$$y_0 + (y_1 - y_0) \cdot \frac{t}{n_y} = y_0 + (y_1 - y_0) \cdot \frac{t}{n_y}$$

how it's arranged

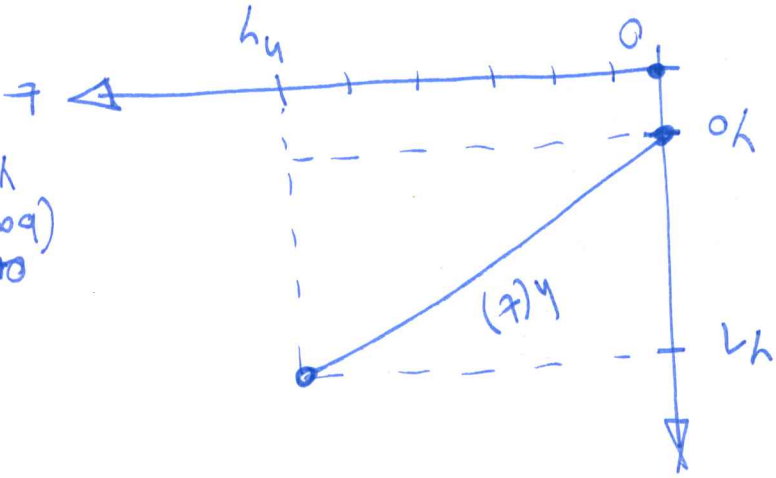
what actually is intended with $t \in [0, n_y]$,



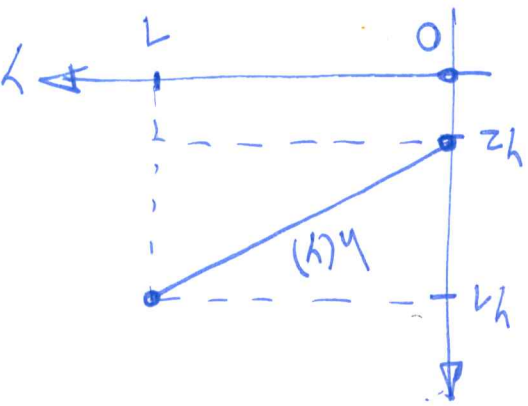
Recall, that normally in this direction (y) we need a "graded" grid, that would account for a support of a PF variable adequately. \Rightarrow modification of $h(t)$ (or $h(y)$) is required.

That is $\frac{y_1 - y_0}{n_y}$ actually $\in [0, 1]$ defines the mesh size (unit cell size) in a direction y .

Here is the plot of a function $h(t) := y_0 + (y_1 - y_0) \cdot \frac{t}{n_y}$.



or $y = \frac{t}{n_y}$ (back in)



The proposed modification:

$$f(y) = Ay^5 + By^4 + Cy^3 + Ey^2 + Fy + G, \quad (2)$$

such that

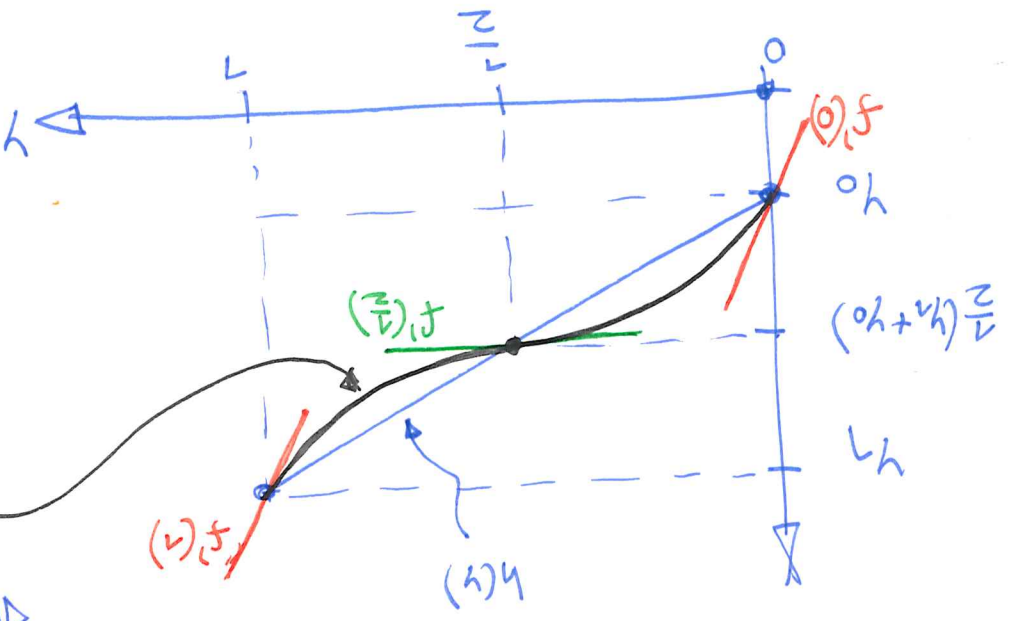
$$\begin{aligned} f(0) &= y_0 \\ f\left(\frac{1}{2}\right) &= \frac{1}{2}(y_1 + y_0) \\ f(1) &= y_1 \end{aligned}$$

and

$$\begin{aligned} f'(0) &= f'(1) = \chi_2(y_1 - y_0), \chi_2 \in (1, +\infty) \\ f'\left(\frac{1}{2}\right) &= \chi_1(y_1 - y_0), \chi_1 \in (0, 1) \end{aligned}$$

where two parameters χ_1 & χ_2 control the slopes of f at (and hence the density of the mesh size) at/near the corresponding regions: $y = \frac{1}{2}$ is where the support of d is "expected", $y \in \{0, 1\}$ the lower & upper edges of Ω .

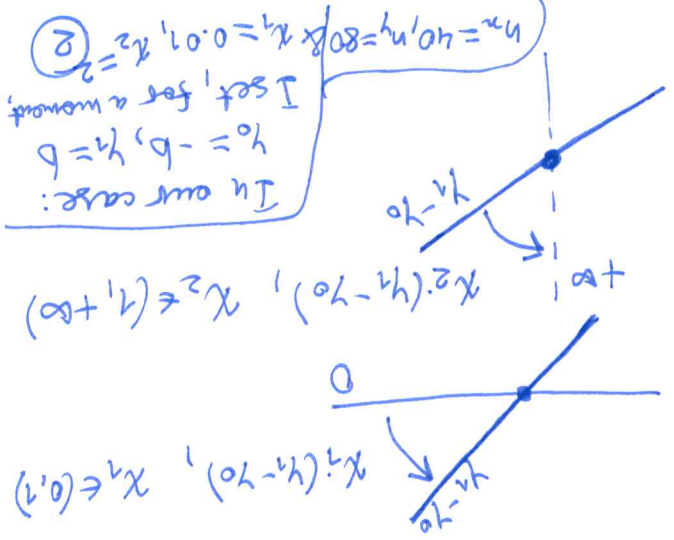
Note, that $\chi_1 = 1$ and $\chi_2 = 1$ would turn f into h , since the slope of h is nothing but $y_1 - y_0$:



the limiting cases:

$$\begin{cases} A = 8(2\chi_1 + \chi_2 - 3)(y_1 - y_0) \\ B = -20(2\chi_1 + \chi_2 - 3)(y_1 - y_0) \\ C = 2(16\chi_1 + 9\chi_2 - 25)(y_1 - y_0) \\ E = -(8\chi_1 + 7\chi_2 - 15)(y_1 - y_0) \\ F = \chi_2(y_1 - y_0) \\ G = y_0 \end{cases}$$

(2) \rightarrow (1) yields:



In our case:
 $y_0 = -b, y_1 = b$
 I set, for a moment,
 $\chi_1 = 40, \chi_2 = 80$
 $\chi_1 = 0.01, \chi_2 = 2$