IFEM - getting started

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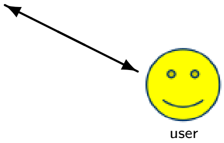
IFEM module overview

GEOM – Geometry Modeller

- Edit g2-files (GoTools) by hand
- Using Rhino and small C++ tools
- Coming: Interactive volumetric modeler



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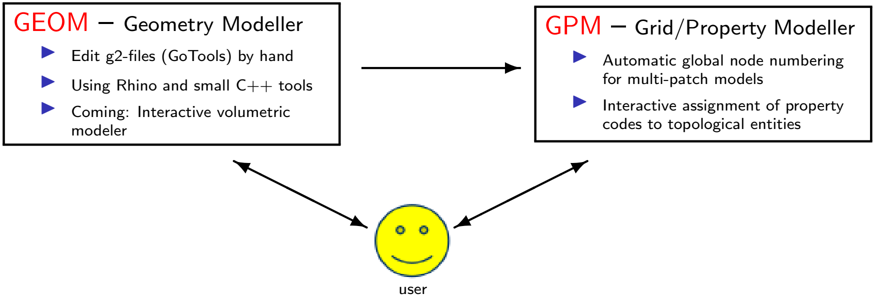

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GPM – Grid/Property Modeller

- Automatic global node numbering for multi-patch models
- Interactive assignment of property codes to topological entities

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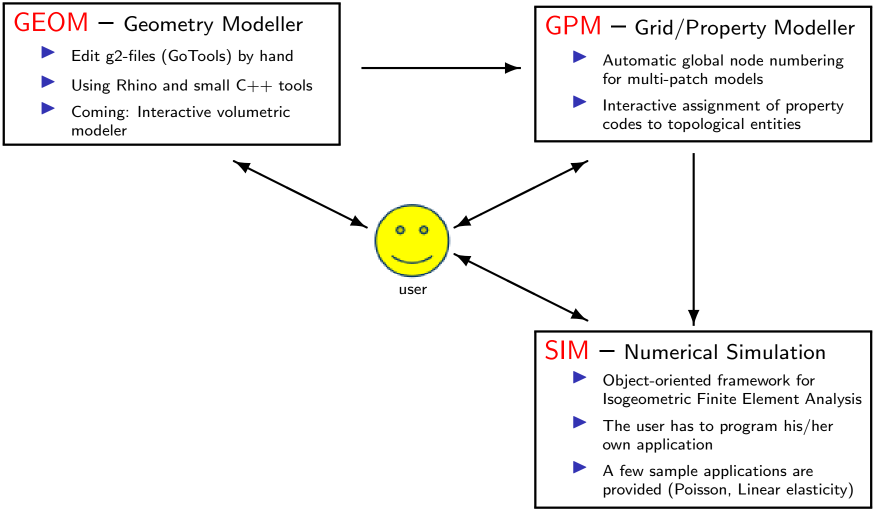

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SIM – Numerical Simulation

- Object-oriented framework for Isogeometric Finite Element Analysis
- The user has to program his/her own application
- A few sample applications are provided (Poisson, Linear elasticity)

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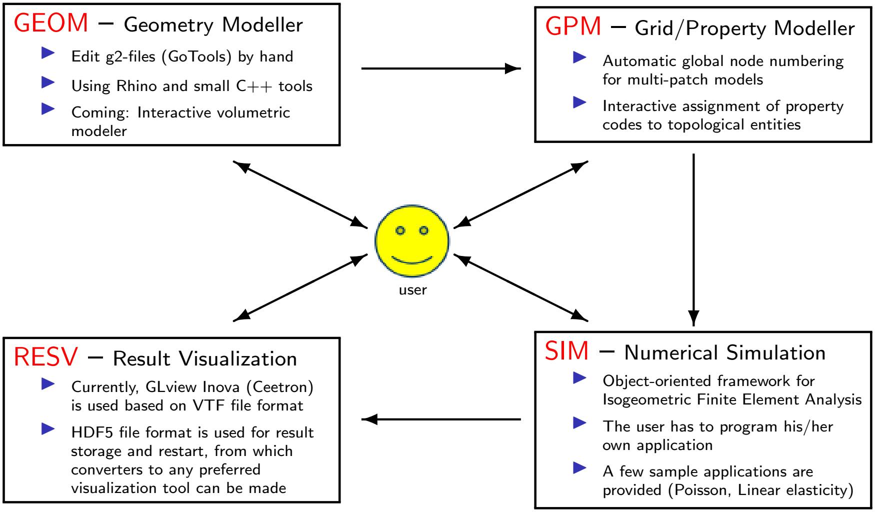

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RESV – Result Visualization

- Currently, GLview Inova (Ceetron) is used based on VTF file format
- HDF5 file format is used for result storage and restart, from which converters to any preferred visualization tool can be made

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- The user has to program his/her own application
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ASM class hierarchy - *the Isogeometry level*

ASMbase

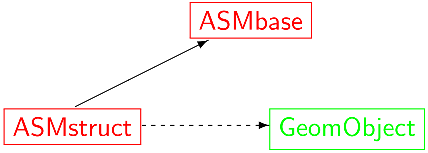
$\longrightarrow$ 'is-a' relationship

$\dashrightarrow$ 'has-a' relationship

IFEM classes

GoTools classes

ASM class hierarchy - *the Isogeometry level*



$\longrightarrow$ 'is-a' relationship
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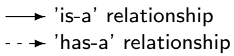
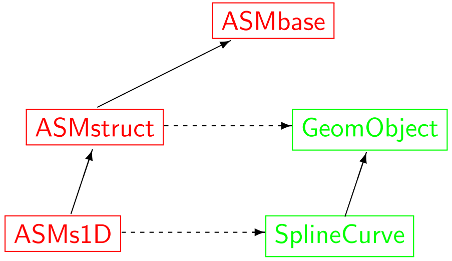
IFEM classes
GoTools classes

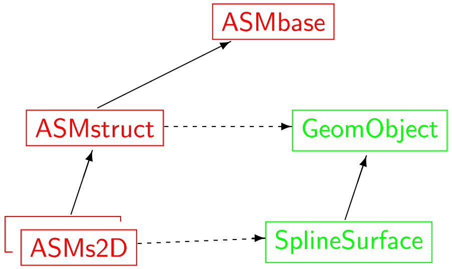
ASM class hierarchy - *the Isogeometry level*




$$\begin{tikzcd}
 & \boxed{\text{ASMbase}} & \\
\boxed{\text{ASMstruct}} \arrow[ur] \arrow[rr, dashed] & & \boxed{\text{GeomObject}} \\
\boxed{\text{ASMs1D}} \arrow[u] \arrow[rr, dashed] & & \boxed{\text{SplineCurve}} \arrow[u]
\end{tikzcd}$$

IFEM classes
GoTools classes

ASM class hierarchy - *the Isogeometry level*



→ 'is-a' relationship

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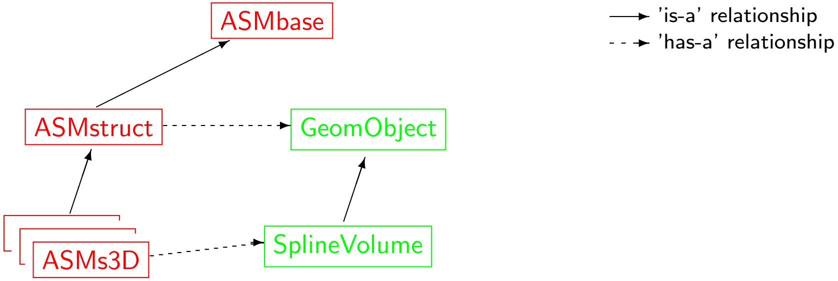

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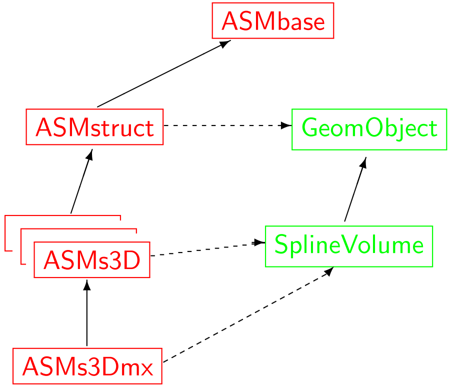

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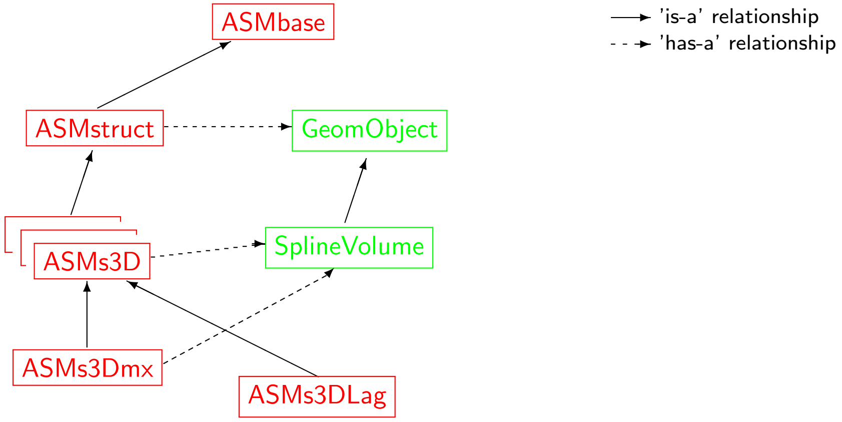

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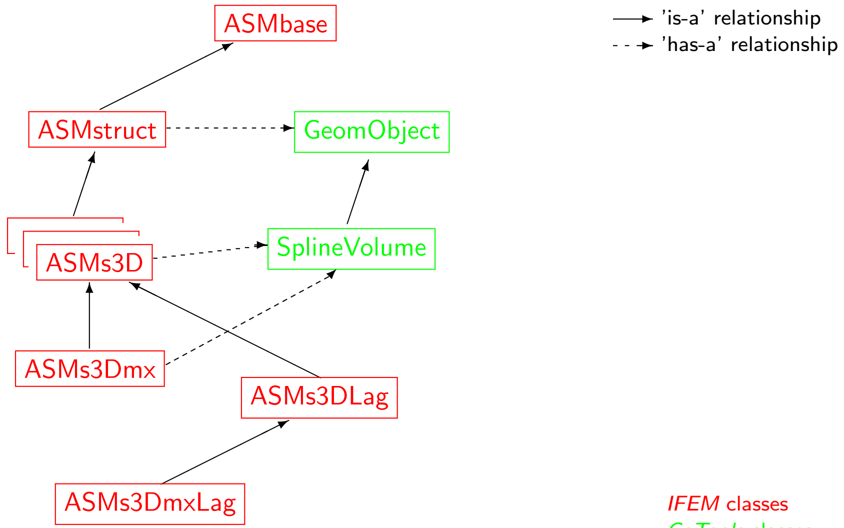

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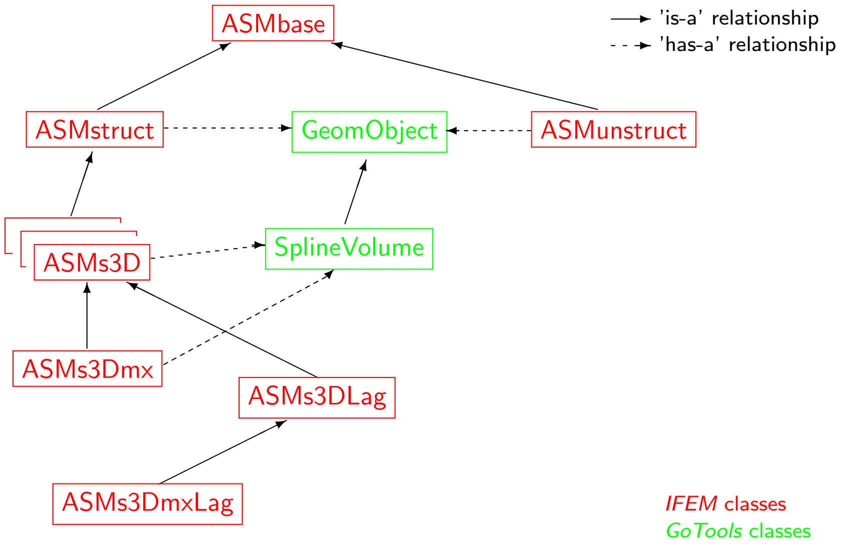

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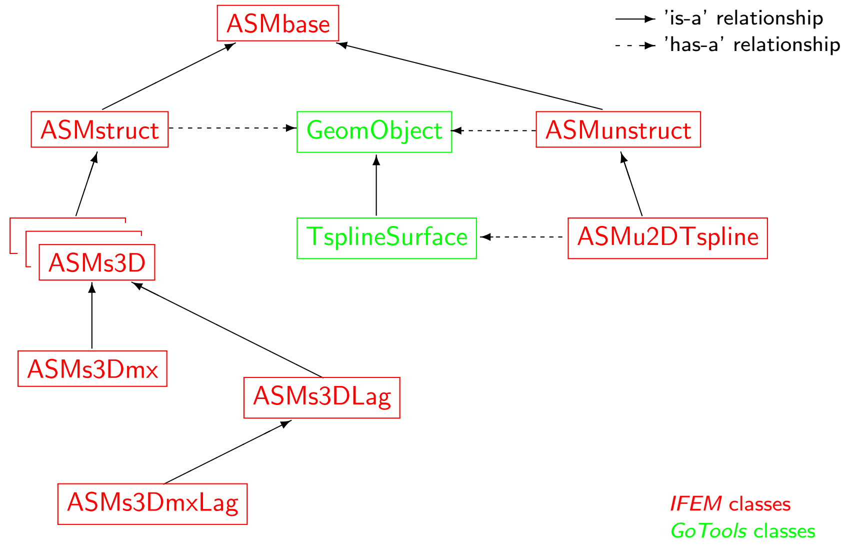

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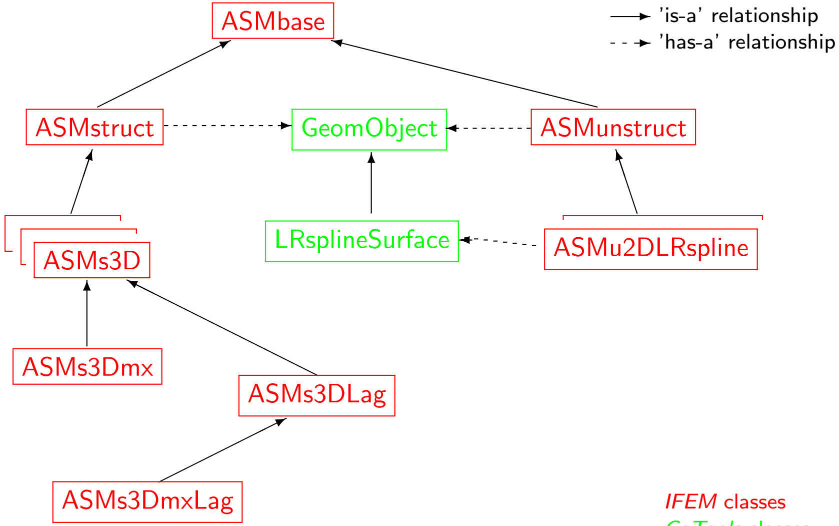

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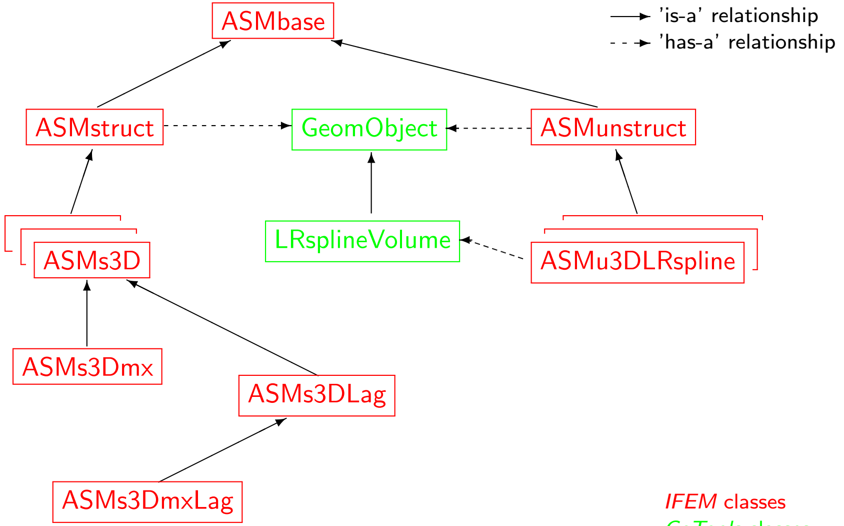

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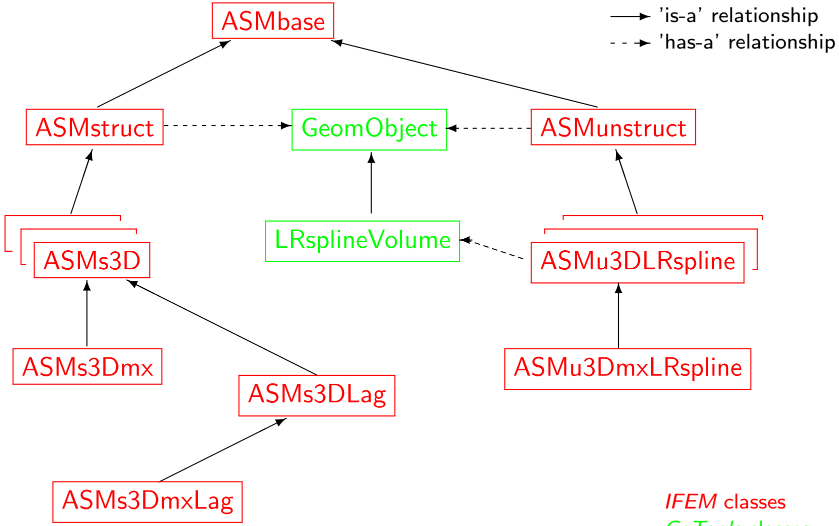

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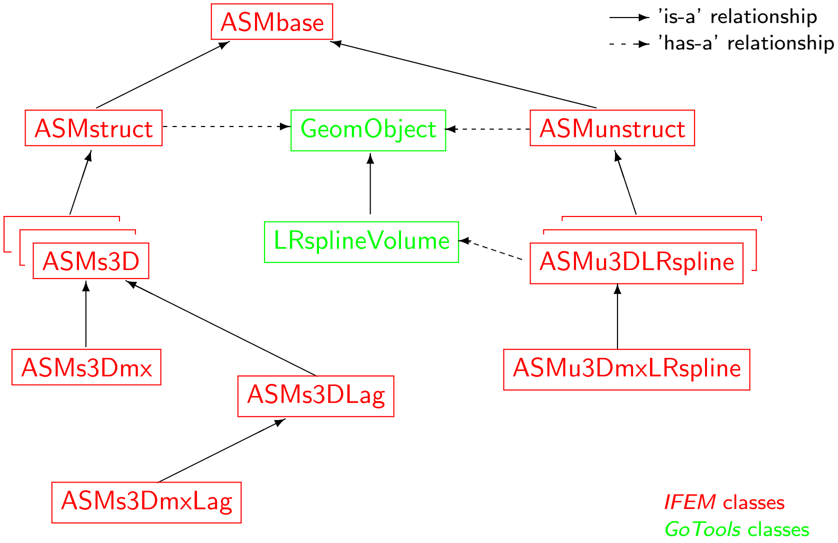

ASMbase is the interface between the isogeometric FE procedures, and the solution algorithms (from above) and the physical problem to be solved (from below).

The main ASM methods

```
typedef std::vector<LocalIntegral*> LintegralVec; //!< Local integral container

class ASMbase
{
public:
  //! \brief Evaluates an integral over the interior patch domain.
  //! \param integrand Object with problem-specific data and methods
  //! \param glbInt The integrated quantity
  //! \param[in] time Parameters for nonlinear/time-dependent simulations
  //! \param locInt Vector of element-wise contributions to \a glbInt
  virtual bool integrate(Integrand& integrand,
                         GlobalIntegral& glbInt, const TimeDomain& time,
                         const LintegralVec& locInt = LintegralVec()) = 0;

  //! \brief Evaluates a boundary integral over a patch face/edge.
  //! \param integrand Object with problem-specific data and methods
  //! \param[in] lIndex Local index of the boundary face/edge
  //! \param glbInt The integrated quantity
  //! \param[in] time Parameters for nonlinear/time-dependent simulations
  //! \param locInt Vector of element-wise contributions to \a glbInt
  virtual bool integrate(Integrand& integrand, int lIndex,
                         GlobalIntegral& glbInt, const TimeDomain& time,
                         const LintegralVec& locInt = LintegralVec()) = 0;
};
```

LocalIntegral and GlobalIntegral are interfaces to the element-level and system-level matrices of the FE problem. TimeDomain contains the integration parameters needed for nonlinear and/or time-dependent simulations.

Numerical integration method for a 3D spline patch

```
bool ASMs3D::integrate (Integrand& integrand, GlobalIntegral& glInt,
                        const TimeDomain& time, const LintegralVec& locInt)
{
























 }
```

Numerical integration method for a 3D spline patch

```
bool ASMs3D::integrate (Integrand& integrand, GlobalIntegral& glInt,
                        const TimeDomain& time, const LintegralVec& locInt)
{
  Compute parameter values (u,v,w) of all integration points within the patch
  basis->SplineVolume::computeBasisGrid(u,v,w,splineData);
    splineData contains derivatives w.r.t. u,v,w of all basis functions
    at all integration points and the function values themselves









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  Loop over elements (knot-spans); do iel=0,nel-1

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        Fetch data from splineData belonging to current integration point; N, dN/du




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        Fetch data from splineData belonging to current integration point; N, dN/du
        Compute Cartesian coordinates and Jacobian; X = N*Xnod, J = dN/du*Xnod
        and the gradient; dN/dX = dN/du * J^{-1}

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        Compute Cartesian coordinates and Jacobian; X = N*Xnod, J = dN/du*Xnod
        and the gradient; dN/dX = dN/du * $J^{-1}$
        integrand.evalInt(locInt[iel], time, detJ*weight, N, dN/dX, X);
      end do i, j, k


    end if
  end do iel
 }
```

Numerical integration method for a 3D spline patch

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bool ASMs3D::integrate (Integrand& integrand, GlobalIntegral& glInt,
                        const TimeDomain& time, const LintegralVec& locInt)
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        integrand.evalInt(locInt[iel], time, detJ*weight, N, dN/dX, X);
      end do i, j, k
      integrand.finalizeElement(locInt[iel]);

    end if
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        integrand.evalInt(locInt[iel], time, detJ*weight, N, dN/dX, X);
      end do i, j, k
      integrand.finalizeElement(locInt[iel]);
      glInt.assemble(locInt[iel], MGEL[iel]);
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Numerical integration method for a 3D spline patch

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        integrand.evalInt(locInt[iel], time, detJ*weight, N, dN/dX, X);
      end do i, j, k
      integrand.finalizeElement(locInt[iel]);
      glInt.assemble(locInt[iel], MGEL[iel]);
    end if
  end do iel
}
```

The “user” interface ...

```
class Integrand
{
  //! \brief Evaluates the integrand at an interior point.
  //! \param elmInt The local integral object to receive the contributions
  //! \param[in] fe Finite element data of current integration point
  //! \param[in] time Parameters for nonlinear and time-dependent simulations
  //! \param[in] X Cartesian coordinates of current integration point
  //!
  //! \details The default implementation forwards to the stationary version.
  //! Reimplement this method for time-dependent or non-linear problems.
  virtual bool evalInt(LocalIntegral*& elmInt, const FiniteElement& fe,
                       const TimeDomain& time, const Vec3& X) const;


  //! \brief Evaluates the integrand at a boundary point.
  //! \param elmInt The local integral object to receive the contributions
  //! \param[in] fe Finite element data of current integration point
  //! \param[in] time Parameters for nonlinear and time-dependent simulations
  //! \param[in] X Cartesian coordinates of current integration point
  //! \param[in] normal Boundary normal vector at current integration point
  //!
  //! \details The default implementation forwards to the stationary version.
  //! Reimplement this method for time-dependent or non-linear problems.
  virtual bool evalBou(LocalIntegral*& elmInt, const FiniteElement& fe,
                       const TimeDomain& time,
                       const Vec3& X, const Vec3& normal) const;

  . . .
};
```

Overloaded versions of these method interfaces exist without the `TimeDomain` argument, for stationary/linear problems.

Finite element data at integration point level

```
class FiniteElement
{
public:
  int       iel;    //!< Element identifier
  double    u;      //!< First parameter of current point
  double    v;      //!< Second parameter of current point
  double    w;      //!< Third parameter of current point
  double    h;      //!< Characteristic element size
  Vector    N;      //!< Basis function values
  Vector    Navg;   //!< Volume-averaged basis function values
  Matrix    dNdX;   //!< First derivatives (gradient) of the basis functions
  Matrix3D d2NdX2; //!< Second derivatives of the basis functions
  double    detJxW; //!< Weighted determinant of the coordinate mapping
};
```

An object of this class is used to transport all integration point quantities to the application-dependent integrands.

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  - But, we only get $\mathcal{C}^{p-2}$ continuity in the highest-order solution field (p being the polynomial order of the first basis), and $\mathcal{C}^{p-1}$ continuity in the other field.
- TODO: Manage bases with $\mathcal{C}^{p-1}$ continuity in both fields.

Numerical integration for the two-field mixed method

```
bool ASMs3Dmx::integrate (Integrand& integrand, GlobalIntegral& glInt,
                          const TimeDomain& time, const LintegralVec& locInt)
{



 }
```

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bool ASMs3Dmx::integrate (Integrand& integrand, GlobalIntegral& glInt,
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```

  Compute parameter values (u,v,w) of all integration points within the patch

```
  basis1->SplineVolume::computeBasisGrid(u,v,w,splineData1);
  basis2->SplineVolume::computeBasisGrid(u,v,w,splineData2);
```

```
 }
```

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  Loop over elements (knot-spans); do iel=0,nel-1
    If current knot span is non-zero in all three directions then








    end if
  end do iel
}
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      Initialize for numerical integration over the element
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      Loop over integration points; do i=1,nGauss, j=1,nGauss, k=1,nGauss






      end do i, j, k


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        Fetch data from splineData[12]; $N_1$, $dN_1/du$, $N_2$, $dN_2/du$





      end do i, j, k


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        Fetch data from splineData[12]; $N_1$, $dN_1/du$, $N_2$, $dN_2/du$
        Compute Cartesian coordinates and Jacobian; $X = N_2 * Xnod$, $J = dN_2/du * Xnod$
        and the gradients; $dN_1/dX = dN_1/du * J^{-1}$, $dN_2/dX = dN_2/du * J^{-1}$,


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        and the gradients; $dN_1/dX = dN_1/du * J^{-1}$, $dN_2/dX = dN_2/du * J^{-1}$,
        integrand.evalInt(locInt[iel], time, detJ*weight,
                          $N_1$, $dN_1/dX$, $N_2$, $dN_2/dX$, X);
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        integrand.evalInt(locInt[iel], time, detJ*weight,
                          $N_1$, $dN_1/dX$, $N_2$, $dN_2/dX$, X);
      end do i, j, k
      integrand.finalizeElement(locInt[iel]);
      glInt.assemble(locInt[iel], MGEL[iel]);
    end if
  end do iel
}
```

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- Projection-based, decoupled Navier–Stokes solvers (Chorin method)
  - Mixed formulation (work in progress)

Implementational issues (integration point level)

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- Using splines as basis function, especially the higher-order ones, the "elements" become large (in terms of nodal connectivities) $\Rightarrow$ large, dense element matrices
- Element-level linear algebra: Use machine-optimized **BLAS** rather than inline C++ code
- Important to express the nonlinear FE formulation on *matrix* form (Voigt notation) — not *tensor* form

System-level linear algebra – equation solving

- Interfaced through classes `SystemMatrix` and `SystemVector` with sub-classes for particular solvers.
- Current available linear equation solvers:
  - LAPACK `DGESV` (dense matrices, small problems only)
  - SuperLU (direct methods) `http://crd.lbl.gov/~xiaoye/SuperLU`
  - PETSc (iterative methods) `http://www.mcs.anl.gov/petsc`
  - Parallelization in progress (based on PETSc and MPI)

Detailed source code documentation

See the doxygen-generated html-pages `../html/index.html`

Tutorial: Poisson equation in R^2

Given a heat source function $f(x, y)$ defined over a domain $\Omega \in R^2$, a flux function $h(x, y)$ defined over the boundary $\partial\Omega_h$, and a function $g(x, y)$ defined over the boundary $\partial\Omega_g = \partial\Omega \setminus \partial\Omega_h$, find the scalar function $u(x, y)$ satisfying

$$\left.\begin{aligned} q_{i,i} &= f \\ q_i &= -\kappa_{ij} u_{,j} \end{aligned}\right\} \quad \forall \quad \{x, y\} \in \overline{\Omega} \tag{1}$$

$$q_i n_i = h \quad \forall \quad \{x, y\} \in \partial\Omega_h \tag{2}$$

$$u = g \quad \forall \quad \{x, y\} \in \partial\Omega_g \tag{3}$$

where κ_{ij} is the conductivity tensor and n_i defines the outward-directed unit normal vector on $\partial\Omega_h$.

Tutorial: Poisson equation

```
class Poisson : public Integrand
{
protected:
  // Physical properties
  double    kappa;   //!< Conductivity (constant)
  VecFunc*  fluxFld; //!< Boundary heat flux field
  RealFunc* heatSrc; //!< Interior heat source field

  // Finite element quantities
  Matrix* eM; //!< Element coefficient matrix
  Vector* eS; //!< Element right-hand-side vector
  Vector* eV; //!< Element solution vector

  mutable ElmMats myMats; //!< Local element matrices
```

Define the class `Poisson` as an `Integrand` subclass, containing data and methods that are specific to the 2D Poisson problem (assuming constant conductivity).

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```

Define the class `Poisson` as an `Integrand` subclass, containing data and methods that are specific to the 2D Poisson problem (assuming constant conductivity).

```
public:
  Poisson() : kappa(1.0), fluxFld(0), heatSrc(0)
  {
    primsol.resize(1);
    myMats.A.resize(1);
    myMats.b.resize(2);
    eM = &myMats.A[0];
    eS = &myMats.b[0];
    eV = &myMats.b[1];
  }
  virtual ~Poisson() {}
```

Define the class constructor and destructor. The constructor `Poisson()` initializes the data members.

Tutorial: Poisson equation

```
void setMaterial(double K) { kappa = K; }
void setFlux(VecFunc* tf) { fluxFld = tf; }
void setSource(RealFunc* src) { heatSrc = src; }
```

Initialization of physical properties.

Tutorial: Poisson equation

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Initialization of physical properties.

```
virtual bool initElement(const std::vector<int>& MNPC);
virtual bool initElementBou(const std::vector<int>& MNPC);
```

Virtual methods for element initialization during the numerical integration.

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virtual bool initElement(const std::vector<int>& MNPC);
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Virtual methods for element initialization during the numerical integration.

```
virtual bool evalInt(LocalIntegral*& elmInt,
                     const FiniteElement& fe,
                     const Vec3& X) const;
virtual bool evalBou(LocalIntegral*& elmInt,
                     const FiniteElement& fe,
                     const Vec3& X,
                     const Vec3& normal) const;
virtual bool evalSol(Vector& s,
                     const Vector& N,
                     const Matrix& dNdX,
                     const Vec3& X,
                     const std::vector<int>& MNPC) const;
virtual bool evalSol(Vector& s,
                     const VecFunc& asol,
                     const Vec3& X) const;
```

Virtual methods for integrand and solution field evaluation.

Tutorial: Poisson equation

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    void setMaterial(double K) { kappa = K; }
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    virtual bool evalSol(Vector& s,
                         const VecFunc& asol,
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```

Virtual methods for integrand and solution field evaluation.

```
    virtual NormBase* getNormIntegrand(AnaSol* asol = 0) const;

    bool evalSol(Vector& s,
                 const Matrix& dNdX,
                 const Vec3& X) const;
    bool formCmatrix(Matrix& C, const Vec3& X,
                     bool invers = false) const;
  };
```

Methods for solution norm integration.

Tutorial: Poisson equation

```
bool Poisson::initElement (const std::vector<int>& MNPC)
{
  const size_t nen = MNPC.size();

  eM->resize(nen,nen,true);
  eS->resize(nen,true);

  int ierr = 0;
  if (!primsol.front().empty())
    if ((ierr = utl::gather(MNPC,1,primsol.front(),*eV)))
      std::cerr <<" *** Poisson::initElement: Detected "
                << ierr <<" node numbers out of range."
                << std::endl;

  myMats.withLHS = true;
  return ierr == 0;
}


bool Poisson::initElementBou (const std::vector<int>& MNPC)
{
  eS->resize(MNPC.size(),true);

  myMats.withLHS = false;
  return true;
}
```

Element initialization:
Set the size of the element matrices based on the number of element nodes.
Extract the element solution vector from the global (patch-level) vector.
Indicate whether the left-hand-side matrices are to be integrated or not.

Tutorial: Poisson equation

```
bool Poisson::evalInt (LocalIntegral*& elmInt,
                       const FiniteElement& fe,
                       const Vec3& X) const
{
  elmInt = &myMats;

  Matrix C; C.diag(kappa,2); // Diagonal constitutive matrix

  Matrix CB;
  CB.multiply(C,fe.dNdX,false,true).multiply(fe.detJxW);
  eM->multiply(fe.dNdX,CB,false,false,true);

  if (heatSrc)
    eS->add(fe.N,(*heatSrc)(X)*fe.detJxW);

  return true;
}
```

Integrand evaluations:
Assuming here $\kappa_{ij} = \kappa\delta_{ij}$

$$[CB] = [C]\cdot[\partial N/\partial \mathbf{X}]^T|J|w$$

$$[eM] = \sum([\partial N/\partial \mathbf{X}]\cdot[CB])$$

$$\{eS\} = \sum(h\{N\}|J|w)$$

```
bool Poisson::evalBou (LocalIntegral*& elmInt,
                       const FiniteElement& fe,
                       const Vec3& X, const Vec3& normal) const
{
  elmInt = &myMats;
  if (!fluxFld) return false;

  Vec3 q = (*fluxFld)(X); // heat flux at point X

  double flux = -(q*normal);
  eS->add(fe.N,flux*fe.detJxW);

  return true;
}
```

$$\{eS\} = \sum(-(\mathbf{q}\cdot\mathbf{n})\{N\}|J|w)$$

Tutorial: Poisson equation

```
bool Poisson::evalSol (Vector& q, const Vector&,
                       const Matrix& dNdX, const Vec3& X,
                       const std::vector<int>& MNPC) const
{
  if (primsol.front().empty()) return false;

  Matrix C;
  this->formCmatrix(C,X);

  Vector Dtmp;
  int ierr = utl::gather(MNPC,1,primsol.front(),Dtmp);
  if (ierr > 0) return false;

  // Evaluate the heat flux vector
  Matrix CB;
  CB.multiply(C,dNdX,false,true).multiply(Dtmp,q);
  q *= -1.0;

  return true;
}
```

Secondary solution evaluation:

$$\mathbf{q} = [C] \cdot [\partial N/\partial \mathbf{X}]^T \cdot \{Dtmp\}$$

```
bool Poisson::evalSol (Vector& s, const VecFunc& asol,
                       const Vec3& X) const
{
  s = Vector(asol(X).ptr(),2);
  return true;
}
```

Analytical solution

Tutorial: Poisson equation

```
NormBase* Poisson::getNormIntegrand (AnaSol* asol) const
{
  if (asol)
    return new PoissonNorm(*const_cast<Poisson*>(this),
                           asol->getScalarSecSol());
  else
    return new PoissonNorm(*const_cast<Poisson*>(this));
}


class PoissonNorm : public NormBase
{
  Poisson& problem; //!< The problem-specific data
  VecFunc* anasol;  //!< Analytical heat flux

public:
  PoissonNorm(Poisson& p, VecFunc* a = 0)
  : problem(p), anasol(a) {}
  virtual ~PoissonNorm() {}

  virtual bool initElement(const std::vector<int>& MNPC)
  {
    return problem.initElement(MNPC);
  }

  virtual bool evalInt(LocalIntegral*& elmInt,
                       const FiniteElement& fe,
                       const Vec3& X) const;
};
```

Accompanying class for solution norm integration
`NormBase` is a sub-class of `Integrand` with a couple of added methods common to all norm classes.

Tutorial: Poisson equation

```
bool PoissonNorm::evalInt (LocalIntegral*& elmInt,
                           const FiniteElement& fe,
                           const Vec3& X) const
{
  ElmNorm* eNorm = dynamic_cast<ElmNorm*>(elmInt);
  if (!eNorm) return false;

  // Evaluate the inverse constitutive matrix at this point
  Matrix Cinv;
  if (!problem.formCmatrix(Cinv,X,true)) return false;

  // Evaluate the finite element heat flux field
  Vector sigmah;
  if (!problem.evalSol(sigmah,fe.dNdX,X)) return false;

  // Integrate the energy norm a(u^h,u^h)
  ElmNorm& pnorm = *eNorm;
  pnorm[0] += sigmah.dot(Cinv*sigmah)*fe.detJxW;
  if (anasol)
  {
    // Evaluate the analytical heat flux
    Vector sigma((*anasol)(X).ptr(),sigmah.size());
    // Integrate the energy norm a(u,u)
    pnorm[1] += sigma.dot(Cinv*sigma)*fe.detJxW;
    // Integrate the error in energy norm a(u-u^h,u-u^h)
    sigma -= sigmah;
    pnorm[2] += sigma.dot(Cinv*sigma)*fe.detJxW;
  }

  return true;
}
```

Norm integrand evaluation

Tutorial: Poisson equation

```
class SIMPoisson2D : public SIM2D
{
  Poisson   prob; //!< Poisson data and methods
  RealArray mVec; //!< Material data

public:
  SIMPoisson2D() : SIM2D(1), prob(2)
  { myProblem = &prob; }

  virtual ~SIMPoisson2D()
  { myProblem = 0; }

protected:
  virtual bool parse(char* keyWord, std::istream& is);
  virtual bool initMaterial(size_t propInd);
  virtual bool initNeumann(size_t propInd);
};


bool SIMPoisson2D::initMaterial (size_t propInd)
{
  if (propInd >= mVec.size()) return false;
  prob.setMaterial(mVec[propInd]);
  return true;
}

bool SIMPoisson2D::initNeumann (size_t propInd)
{
  VecFuncMap::const_iterator tit = myVectors.find(propInd);
  if (tit == myVectors.end()) return false;
  prob.setTraction(tit->second);
  return true;
}
```

Simulation driver class

Tutorial: Poisson equation

```
bool SIMPoisson2D::parse (char* keyWord, std::istream& is)
{
  char* cline = 0;
  if (!strncasecmp(keyWord,"ISOTROPIC",9))
  {
    int nmat = atoi(keyWord+10);
    std::cout <<"\nNumber of isotropic materials: "<< nmat << std::endl;
    for (int i = 0; i < nmat && (cline = utl::readLine(is)); i++)
    {
      int    code  = atoi(strtok(cline," "));
      double kappa = atof(strtok(NULL," "));
      std::cout <<"\tMaterial code "<< code <<": "<< kappa << std::endl;
      if (code == 0)
        prob.setMaterial(kappa);
      else if (this->setPropertyType(code,Property::MATERIAL,mVec.size()))
        mVec.push_back(kappa);
    }
  }
  else if (!strncasecmp(keyWord,"SOURCE",6))
  {
    cline = strtok(keyWord+6," ");
    if (!strncasecmp(cline,"SQUARE",6))
    {
      double L = atof(strtok(NULL," "));
      std::cout <<"\nHeat source function: Square L="<< L << std::endl;
      prob.setSource(new Square2DHeat(L));
    }
    else
      std::cerr <<"  ** SIMPoisson2D::parse: Unknown source function "
                << cline << std::endl;
  }
```

Tutorial: Poisson equation

```
    else if (!strncasecmp(keyWord,"ANASOL",6))
    {
      cline = strtok(keyWord+6," ");
      if (!strncasecmp(cline,"SQUARE",6))
      {
        double L = atof(strtok(NULL," "));
        std::cout <<"\nAnalytical solution: Square L="<< L << std::endl;
        mySol = new AnaSol(NULL,new Square2D(L));
      }
      else if (!strncasecmp(cline,"LSHAPE",6))
      {
        mySol = new AnaSol(NULL,new LshapePoisson());
        std::cout <<"\nAnalytical solution: Lshape"<< std::endl;
      }
      else
      {
        std::cerr <<"  ** SIMPoisson2D::parse: Unknown analytical solution "
                  << cline <<" (ignored)"<< std::endl;
        return true;
      }

      // Define the analytical boundary traction field
      int code = (cline = strtok(NULL," ")) ? atoi(cline) : 0;
      if (code > 0 && mySol->getScalarSecSol())
      {
        this->setPropertyType(code,Property::NEUMANN);
        myVectors[code] = mySol->getScalarSecSol();
      }
    }
    else
      return this->SIM2D::parse(keyWord,is);

    return true;
  }
```

Tutorial: Poisson equation

```
int main (int argc, char** argv)
{
  // (Lots of initialisations skipped here...)

  // Read in model definitions and establish the FE data structures
  SIMbase* model = new SIMPoisson2D();
  if (!model->read(infile))
    return 1;
  if (!model->preprocess(ignoredPatches,fixDup))
    return 1;

  model->setQuadratureRule(nGauss);

  Matrix eNorm;
  Vector gNorm, sol;

  model->initSystem(solver,1,1);
  model->setAssociatedRHS(0,0);
  if (!model->assembleSystem())
    return 2;

  // Solve the linear system of equations
  if (!model->solveSystem(sol,1))
    return 3;

  // Evaluate solution norms
  if (!model->solutionNorms(Vectors(1,sol),eNorm,gNorm))
    return 4;

  // Print output to terminal and VTF, etc.
}
```

Core parts of the main program

Tutorial: Poisson equation, sample input files

```
PATCHFILE    lshape2d.g2
PROPERTYFILE lshape2d.prc

RAISEORDER 1
# patch ru rv
  1     2  2

REFINE 1
# patch ru rv
  1     7  7

DIRICHLET 1
# code
  1

# Analytical solution
# Specifier   code
ANASOL Lshape 2
```

lshape2d.prc:

```
1 0 1 2
2 0 1 0
2 0 1 1
2 0 1 3
```

lshape2d.g2:

```
200 1 0 0
3 0
7 4
0 0 0 0 1 1 1 2 2 2 2
4 4
0 0 0 0 1 1 1 1

0.000000 -1.000000 0
0.000000 -0.666667 0
0.000000 -0.333333 0
0.000000 -0.000000 0
0.333333 -0.000000 0
0.666667 -0.000000 0
1.000000 -0.000000 0
-0.333333 -1.000000 0
-0.333333 -0.555556 0
-0.333333 -0.111111 0
-0.333333 0.333333 0
0.111111 0.333333 0
0.555556 0.333333 0
1.000000 0.333333 0
-0.666667 -1.000000 0
-0.666667 -0.444444 0
-0.666667 0.111111 0
-0.666667 0.666667 0
```

+ 10 more lines