The Dynamics of Applied Stock-Flow Consistent Models

Introduction to Goodwin's Class Struggle Model

Giovanni Longo

March 7, 2023

Abstract

The stock-flow consistent model (SFC) has become an increasingly popular temporal model to explain the relationship between various economic factors. While the standard dynamic stochastic general equilibrium model (DSGE) is still more common in practice, recent SFC adaptations have produced results that align more accurately with observed data. One such model that utilizes the core assumptions of the SFC is the Goodwin model, which describes the economy as a system of equations reliant on employment and worker product share. This paper discusses Goodwin's intuition on modeling the economy after the Lotka-Volterra model by introducing relevant terminology and describing how Goodwin's model aligns with heterodox economic theory. From here, possible future research is outlined and modifications are suggested that could address some of the model's shortcomings.

1 Financial Terminology

1.1 Introduction to the DSGE Model

In modern theory, the most common model for macroeconomic analysis is the dynamic stochastic general equilibrium (DSGE). This model focuses on overall economic performance, attempting to predict general economic shifts over time through indicators such as inflation, GDP fluctuation, unemployment, and government policy. Additionally, randomness is added through agent-based behavioral assumptions, considering the actions and choices of consumers, firms, and government entities to explain economic trends. Though the DSGE has proven useful in areas such as forecasting, policy analysis, and financial modeling, its overall popularity does not imply perfect accuracy. Several underlying principles of modern macroeconomic theory assume unrealistic or incomplete conditions and cause DSGE estimates to vary drastically from actual data. As a result, professionals consistently formulate new systems of describing economic movement, with one popular alternative being the stock-flow consistent model (SFC). Before going into explicit details about the SFC, we first discuss the underpinnings of heterodox economic theory to formalize the foundational differences between the DSGE and SFC.

1.2 A Different Economic Viewpoint

In essence, the SFC emphasizes accounting transactions as the basis for macroeconomic understanding, dynamically combining financial entities (e.g. assets, equity) with economic flows (e.g. income, savings). As such, the SFC is reliant on four key insights, the first being the concept of "money" as a hierarchical structure. When thinking about the term itself, many misinterpret "money" to be synonymous to currency; however, there are various financial instruments that ensure this currency holds significance in the actual economy. When a gold standard is present, securities and bank deposits (two of these financial instruments) act as promises for currency and give the dollar a feasible backing should the consumer ever want to exchange their currency for gold[2]. Because these systems rely on consumer behavior, current reserves, and macroeconomic factors, they are often seen as "credits" for currency. Despite this common misunderstanding, the level of value for securities and deposits are not static for different groups; even though the typical consumer cannot exchange these items for other goods and services, the federal government would find such instruments necessary when conducting foreign trade and negotiating massive deals with the private sector. Therefore, "money" becomes a broadly applied term and is structured by the level of confidence in an underlying asset. The specifics of the hierarchy are not important for our purposes, but it is crucial to understand that this concept makes the SFC dynamic since at any time scale there are now fluctuations in the quantity and usefulness of credit.

Once money has been established as a hierarchy, it can be easily understood that this supply arises endogenously from the bank. Specifically, banks can directly influence the creation and destruction of money through loans, making them an integral part of money circulation[3]. In addition, this premise gives the central bank control over the money supply through implicit methods, as adjusted interest rates directly influence the consumer demand for debt and the amount of available loans. This assumption adds another layer of dynamics to the system due human speculation, as companies will attempt to predict the policy decisions mandated by the central bank.

When money is internally generated and fluctuates in value depending on the credit landscape, private debt begins to hold relevance in predicting overall market trends. Unlike the government, consumers and companies begin to take on additional debt when loans are "cheap" (e.g. when interest rates are low, when the market is undervaluing assets, etc)[4]. This constraint can signal much about the overall health of an economy, as periods of growth would be marked with rapid amounts of private debt and recessions with decreasing or stagnant debt levels. National debt is not constrained in this same way, so a complete macroeconomic analysis must consider the rate at which the private sector feels comfortable accepting more debt.

Finally, the SFC draws on the idea that the financial sector does more than just intermediate transactions. A realistic interpretation must account for the fact that when such institutions facilitate the exchange of economic instruments, they are implicitly accepting the risk of these underlying promises[1]. For example, equities traded on an open exchange can depreciate in value, so a consumer may become interested in "shorting" (hedging against) that specific equity. If a company allows customers to trade financial instruments based on market valuation, there will always be uncertainties and risks that they must accept. In this way, there is a range of acceptable losses, and a company's willingness to take on the risk of distributing financial instruments defines the real size of an economy[5].

1.3 Why the SFC?

With the above assumptions, it is now sensible to define the economy in terms of sector transactions. The SFC aims to model macroeconomic activity through an aggregate balance sheet, creating a system that concretely defines "economic flow" as the exchange of assets and liabilities between companies[1]. With this framework, accounting principles govern economic movement and predictions are based on more tangible measures of fluctuation. This presents the SFC with several advantages over the DSGE. Firstly, because behavioral assumptions are relaxed under the SFC, there is presumably less importance placed on individual preference or emotion. This not only reduces the uncertainty of outcomes, as defining the motivation of actions is both difficult and time-consuming, but also leads to more straightforward interpretations since results can be expressed through accounting terminology. The second major benefit of the SFC is that both real and monetary factors are utilized simultaneously. Here, "real" factors refer to physical aspects of the economy (e.g. labor, PP&E), and "monetary" factors are defined as those concerned with the exchange of money (e.g. inflation, interest rates). Considering real and monetary changes in tandem produces more extensive and credible outcomes that can be traced directly to interactions among sectors. Lastly, the SFC relies on "consistent flows", meaning that each "stock" (i.e. asset or liability) is attached to an investment or loan: more simply put, stocks will always have a corresponding transaction of exchange. This allows for easier assessments of macroeconomic imbalances and financial stability, as the SFC captures the interdependence of different economic agents through these tangible monetary flows.

2 Goodwin's Model: A Predator-Prey Interpretation of the SFC

NOTE: The remaining material is drawn from Reference [6].

2.1 The Philip's Curve

The Philip's curve is the nonlinear, inverse relationship between annual average unemployment and inflation as expressed by the consumer price index (CPI). More formally, with $\frac{\dot{w}}{w}$ defined as the rate of change of the nominal wage rate for labor and U as unemployment, Phillip's concluded that there exists some monotonically decreasing f such that

$$\frac{\dot{w}}{w} = f(U).$$

Though this relationship proves inaccurate over long time frames, Philip's curve was the first model to analyze inflation as a product of unemployment, effectively stating that employment policy changes have consequences to short term inflation. From this underlying principle, Goodwin posited that economic growth and business cycles could be connected through Philip's curve, transforming the basic labor share model into a dynamic interpretation of investors and workers.

2.2 Goodwin's Model Derivations

Though many economists have tried to explain the business cycle through nonlinear models, one of the most successful attempts was produced by Richard Goodwin, who applied the predator-prey system to explain business cycles. His fundamental claim was that capitalist economies must collapse near full employment, implying the existence of an employment-product equilibrium where economic trends fluctuate between growth and recession. Whereas the standard interpretation of the business cycle only accounts for exogenous economic factors, Goodwin was able to frame his model so that endogenous stimuli could also influence economic activity.

The Goodwin model consists of two governing equations, described by

$$\frac{\dot{v}}{v} = \frac{\dot{l}}{l} - \frac{\dot{n}}{n} = \frac{\dot{q}}{q} - \alpha - \beta$$
$$\frac{\dot{u}}{u} = \frac{\dot{w}}{w} - \alpha.$$

Below is a relevant list of the definitions of each parameter to reference for later use:

(i) q output,
(ii) k capital,
(iii) w wage,
(iv) $a = a_0 e^{\alpha t}$ labour productivity, where α is the growth parameter,
(v) $s = q/k = 1/\sigma$ capital productivity,
(vi) $k/q = \sigma$ capital-output ratio,
(vii) $u = w/a$ workers' share of product,
(viii) $(1 - w/a)$ capitalists' share of product,
(ix) $(1 - w/a)q = \dot{k}$ surplus = profit = savings = investments,
(x) $\dot{k}/k = \dot{q}/q = (1 - w/a)/\sigma$ profit rate,
(xi) $n = n_0 e^{\beta t}$ labour supply, where β is the growth parameter,
(xii) $l = q/a$ employment,
(xiii) $v = l/n$ employment rate.

Additionally, the parameter $\frac{\dot{w}}{w} = \rho v - \gamma$ denotes the linearized Philips curve for some ρ and γ. Rearranging our equations, we see that

$$\frac{\dot{v}}{v} = \frac{\dot{q}}{q} - \alpha - \beta \implies \dot{v} = \left[\frac{\dot{q}}{q} - \alpha - \beta\right] v = \left[\frac{(1 - \frac{w}{a})}{\sigma} - \alpha - \beta\right] v = \left[\frac{(1 - u)}{\sigma} - \alpha - \beta\right] v$$

and

$$\frac{\dot{u}}{u} = \frac{\dot{w}}{w} - \alpha \implies \dot{u} = \left[\frac{\dot{w}}{w} - \alpha\right] u = \left[\frac{\dot{w}}{w} - \alpha\right] u = [\rho v - \gamma - \alpha]\, u.$$

To reduce the number of parameters, denote $\alpha + \beta = \delta$ and $\gamma + \alpha = \varepsilon$. Then our system becomes

$$\dot{v} = \left[\frac{(1-u)}{\sigma} - \delta\right] v = [s(1-u) - \delta]v$$
$$\dot{u} = [\rho v - \varepsilon]\, u = [\rho v - \varepsilon]u.$$

In financial terms, we see that the rate of change of the employment rate is related to the the some proportion of capital productivity (determined by the product share of the workers and parameters α, β) times the employment rate, and the rate of change of the product share is some linearized Philip's curve times the current product share. This may be difficult to conceptualize in application, but examining the predator-prey model and utilizing our previous knowledge about general SFC models can help simplify the dynamics that these equations describe.

2.3 Interpretting Goodwin's Model

Recall that the Lotka-Volterra (LV) model consists of two groups, often denoted as "predators" and "prey". The LV describes the rate of change in the number of predators and prey over time, and therefore can be modeled as a dynamical system given by

$$\dot{x} = \alpha x - \beta xy = [\alpha - \beta y]x$$
$$\dot{y} = \delta xy - \gamma y = [\delta x - \gamma]y.$$

Immediately, we can see how this system is the basis for the Goodwin model, differing only in the terminology behind the actual parameters. Whereas the standard predator-prey model does not clearly define a specific "predator" or "prey", Goodwin applied the model to economic theory and created a dynamic flow of employment and workers share. Namely, the Goodwin model considers two actors, with capitalists filling in for "predators" and workers taking the place of "prey". The reason capitalists are analogous to predators, in Goodwin's view, is because low labor demand decreases real wages, and a reduction in wages inherently means that profit shares for the capitalists increases. We can then see that rapid increases to the employment rate would be preferable to the capitalist, as greater employment for the same labor supply must lead to a decrease in the labor demand. Then capitalists enjoy positive values of $\dot{v}$ in the Goodwin model, and as a consequence must also desire low or negative values of $\dot{u}$. Intuitively, this makes sense because a decreasing rate of the worker's product share must lead to an increase in investment potential for the capitalist. Note that many of these conclusions arise from consequences of the Philip's curve and SFC assumptions; increased employment means (assumedly) higher levels of inflation, which then leads to decreases in private debt. We know from insight three of SFC economics that decreased levels of private debt signify a recession, and this is good for the capitalist who can restore his profit share by investing more capital in the bottomed economy.

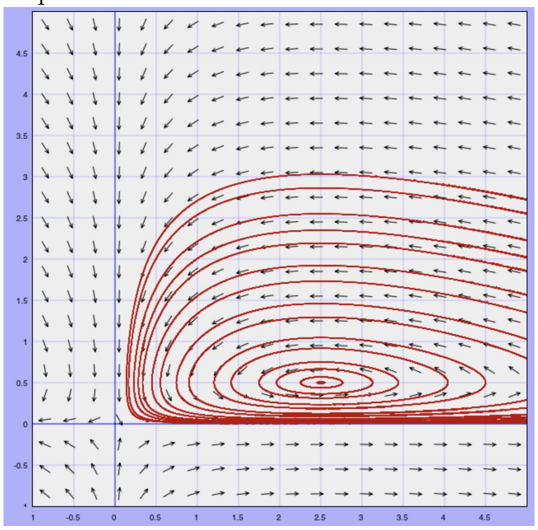
With the capitalists preferences known, the predator-prey framework of the model necessarily implies that workers then want higher values of $\dot{u}$ and low values of $\dot{v}$. This is a sensible conclusion since workers will always want a greater share of their output and will be unhappy when their labor is in low demand (i.e. when $\dot{v}$ is high). Now that the Goodwin model has been interpretted through the perspective of Lotka-Volterra, the previous system can be more easily understood. Avoiding the confusion created by the financial terminology, Goodwin essentially describes macroeconomic activity as the competition between capitalists and workers. Capitalists are always trying to increase their profits through investments and workers are always trying to gain a greater product share. Therefore, an equilibrium occurs when the economy settles around the employment rate and worker product share that maximizes profits for both the capitalist and worker.

2.4 Stability of the Goodwin Model

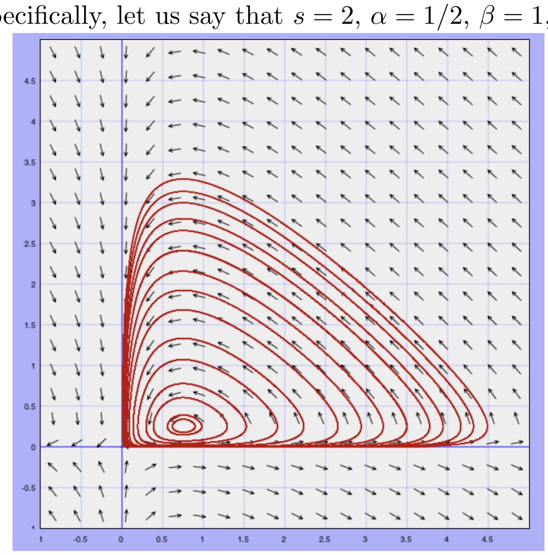
Before analyzing the equilibria and stability of the Goodwin model, let us first qualitatively analyze our system. Since our system of equations is written in terms of the rate of change of u and v, we will explore how different values in the other variables affect the equilibria. Remember that

$$\dot{v} = [s(1-u) - \delta]v \text{ and } \dot{u} = [\rho v - \varepsilon]u.$$

To make our system sensible, we must have that $u, v > 0$, thereby implying that $s > \alpha + \beta$. Let us say that s is 3, $\alpha = 1/2$, $\beta = \rho = 1$, and $\gamma = 2$. Then $\delta = 1.5$ and $\varepsilon = 2.5$, which gives us the following phase portrait:



This portrait seems to follow trajectories moving away from the origin and cycling around another part of the portrait, forming closed loops very similar to those of the standard LV model. To get a better sense of the equilibrium they orbit, let us examine one more phase portrait. Here, we will simply change the value of ρ, s, and γ to get a better sense of how α and β play a role in our fixed point. Specifically, let us say that $s = 2$, $\alpha = 1/2$, $\beta = 1$, $\rho = 2$, and $\gamma = 1$, which yields



With a decrease in s, we see that our second fixed point has gotten closer to $y = 0$, and with an increase in ρ and decrease in γ, we see that it has also gotten closer to $x = 0$. Then ρ is inversely proportional to v^*, γ is proportional to v^*, and s is proportional to u^*. Explicit computations for (v^*, u^*) have been completed in "Nonlinearites in Economics", and are given by

$$v^* = \frac{\gamma + \alpha}{\rho} \text{ and } u^* = 1 - \frac{\alpha + \beta}{s}.$$

Clearly the origin presents a fixed point, which is unstable since all trajectories are repelled from (0,0). Analyzing the fixed point (v^*, u^*), we note any small perturbation leads to a closed trajectory. As such, we know that each $\mathrm{Re}(\lambda_j)$ must be equal to 0, meaning (v^*, u^*) is NOT hyberbolic. Since the Hartman-Grobman theorem is violated, there is no guarantee that linearization is appropriate for analysis (which is why we have not expressed the Jacobian for this point), however, our qualitative portraits reveal that (v^*, u^*) is in fact a center. Then the Goodwin model emits one center and one source; in financial language, this means that when there is no employment and no worker product share, the economy will eventually shift onto a closed orbit such that employment rate and worker share are non-zero. Additionally, for any trajectory sufficiently close to (v^*, u^*), we see that a cycle will emerge around the equilibrium. These closed loops are directly related to the RBC insofar as values close to (v^*, u^*) will emit shorter economic cycles. Then the cyclic nature of the predator-prey model is qualitatively aligned with economic expectations, i.e. Goodwin's model can provide a good starting point for examining dynamic financial models.

3 Conclusion

While Goodwin's model has yielded accurate predictions in many cases, there are several issues present within the model's underlying assumptions. His seven original core principles are given as:

(a) steady technical progress (disembodied),
(b) steady growth in the labour force,
(c) only two factors of production, labour and "capital" (plant and equipment), both homogeneous and non-specific,
(d) all quantities real and net,
(e) all wages consumed, all profits saved and invested,
(f) a constant capital-output ratio,
(g) a real wage rate that rises in the neighbourhood of full employment.

3.1 Goodwin Model Adaptations

Though (a)-(d) present reasonable underpinnings, problems begin to arise starting at (e). Namely, even accounting for the caveat that this assumption can be transformed into constant proportional savings, the premise is that workers consume their wages and capitalists invest their profits. Both profits and wages can be saved for future use through savings- albeit at a discounted rate due to inflation- and the modern era allows open equity exchange to almost all consumer. Additionally, (f) claims that the ratio of capital input to production output remains constant, which should intuitively be false when simultaneously assuming (a) as true. Though varying from this assumption would indeed complicate the model, advancements on the technological front have historically proven to both reduce costs and increase total quantity of produced goods.

3.2 Future Research

So why are these pedantic discrepancies relevant to dynamics? First, from a general level we know that all mathematical models need reasonable and encompassing assumptions, as otherwise the resulting outputs will not adequately model actual data. Second, and more importantly, the issues above lead to different conclusions from those that the Goodwin model proposes. If we assume that wages and investments are open to all modeled agents (or that savings are relevant), then both the capitalist and the worker can simultaneously invest and work. As such, each agent is now apart of some homogeneous "citizen" group, where product share benefits from a decreasing capital-output ratio (COR) and increasing wage rate. Automation and the COR are inversely proportional since technology substitutes (leading to lower costs per item). Such advancements require capital investments, but simultaneously also decrease the overall real wage rate. This in turn leads to less investments in automation, and we begin to see a hypothetical cycle emerge. We can reason then that some equilibrium in this new model will converge to a point (x^*, y^*) where automation and human labor coexist in a manner to maximize product share to our citizen. Research proceeding this paper would concretely define the new system in terms of automated and human labor, thereby adhering to the Lotka-Volterra theme while also addressing a fundamentally different question about macroeconomics.

References

1. Grasselli, M. R. (2023, March). An introduction to stock-flow consistent models in macroeconomics. Reading.

2. Mehrling, P. (2012). (tech.). The Inherent Hierarchy of Money.

3. Palley, T. I. (2001). (rep.). Endogenous Money: What it is and Why it Matters. Washington D.C., Maryland.

4. International Monetary Fund. (2022). (rep.). Private Sector Debt and the Global Recovery (pp. 45–66). Washington D.C., Maryland.

5. Carr, P., Geman, H., amp; Maden, D. (2001). Pricing and hedging in incomplete markets. Journal of Financial Economics, 62, 131–167.

6. Orlando, G., amp; Pisarchik, A. N. (2022). Growth and Cycles as a Struggle: Lotka-Volterra, Goodwin and Phillips. In R. Stoop (Ed.), Nonlinearities in economics: An interdisciplinary approach to economic dynamics, growth and cycles (Vol. 29, pp. 191–209). essay, Springer.