Strong Controllability of Disjunctive Temporal Problems with Uncertainty

Bart Peintner[1], Kristen Brent Venable[2], and Neil Yorke-Smith[1]

[1] Artificial Intelligence Center, SRI International
{peintner,nysmith}@ai.sri.com
[2] University of Padova, Italy
kvenable@math.unipd.it

Abstract. The Disjunctive Temporal Problem with Uncertainty (DTPU) is an extension of the Disjunctive Temporal Problem (DTP) that accounts for events not under the control of the executing agent. We investigate the semantics of DTPU constraints, refining the existing notion that they are simply disjunctions of STPU constraints. We then develop the first sound and complete algorithm to determine whether Strong Controllability holds for a DTPU. We analyze the complexity of our algorithm with respect to the number of constraints in different classes, showing that, for several common subclasses of DTPUs, determining Strong Controllability has the same complexity as solving DTPs.

1 Introduction

The Simple Temporal Problem (STP) [1] is a temporal constraint formalism widely used for modeling and solving real-world planning and scheduling problems. Several extensions of this framework have been proposed in the literature. The Disjunctive Temporal Problem (DTP) [2] allows for non-convex and non-binary constraints by introducing disjunctions of STP constraints. The *Simple Temporal Problem with Uncertainty* (STPU) [3] extends the STP by allowing two classes of events, *controllable* and *uncontrollable*. Uncontrollable events are controlled by exogenous factors, often referred to as 'Nature'. The concept of *consistency* of an STP is replaced by varying notions of *controllability* of an STPU. The level of controllability for a problem describes the conditions under which an executor can guarantee all constraints will be satisfied, w.r.t. Nature's behavior. In problems that exhibit Strong Controllability (SC), there exists a time assignment to all events that ensures all constraints will be satisfied whatever Nature's realisation of the uncontrollable events.

The recently introduced *Disjunctive Temporal Problem with Uncertainty* (DTPU) [4] allows for both disjunctive constraints and contingent events. Such a coexistence is intrinsic to many real-world planning and scheduling problems (e.g., [5]). In this paper we focus on Strong Controllability of DTPUs, which provides an appropriate notion of solution for application domains such as production planning, and in situations where the entire schedule of actions must be known in advance. We present a sound and complete algorithm to determine whether Strong Controllability of a DTPU holds. We then analyze the complexity of the algorithm with respect to the quantity of different constraint types and we show that for several common subclasses of DTPUs, determining SC has the same complexity as solving a classical DTP without uncertainty.

C. Bessiere (Ed.): CP 2007, LNCS 4741, pp. 856–863, 2007.

2 Background

Temporal Problems. A *Simple Temporal Problem* [1] is defined by a set of time-point variables X, which represent instantaneous events, and a set of quantitative constraints $\mathcal{C}$, which restrict the temporal distance between events. STP constraints are binary and convex, taking the form $X_j - X_i \in [a_{ij}, b_{ij}]$. A distinguished event, denoted *TR*, marks the start of time. Unary domain constraints thus can be modeled as binary relations to *TR*. Solving an STP equates to deciding consistency and deriving its minimal network. An STP is consistent iff it has a *solution*: an assignment to all variables such that all constraints are satisfied. Consistency can be tested with an All-Pairs Shortest Path algorithm, requiring $\mathcal{O}(n^3)$ time for n variables [1]. The *minimal network* is the tightest representation of the constraints that includes all solutions to the problem.

The *Disjunctive Temporal Problem* [2] generalizes the STP by admitting non-binary temporal relations. DTP constraints consist of disjunctions of STP constraints: $X_{j1} - X_{i1} \in [a_{i1j1}, b_{i1j1}] \vee X_{j2} - X_{i2} \in [a_{i2j2}, b_{i2j2}] \vee \cdots \vee X_{j\ell} - X_{i\ell} \in [a_{i\ell j\ell}, b_{i\ell j\ell}]$. Note that a variable may appear in more than one disjunct. While the worst-case complexity of solving a DTP is **NP**-hard [2], in practice efficient solving techniques have been developed (e.g., [6]), and tractability results are known for some classes of DTPs (e.g., [7]). A simple way to solve a DTP is to consider the *component STPs* obtained by selecting one disjunct from each constraint. A DTP is consistent iff it contains a consistent component STP. A search through the meta space of component STPs is the heart of most constraint-based DTP solvers (e.g., [6]); these solvers have been shown to be very efficient. The time complexity of the search depends on the number of component STPs. In the worst case, for m constraints with a maximum of k disjuncts each, there are $\mathcal{O}(k^m)$ component STPs, and the complexity is $\mathcal{O}(n^2 m k^m)$ [6].

Temporal Problem with Uncertainty. The STP and DTP formalisms assume that all events are under the complete control of the execution agent. Recognizing that this assumption is often not valid, the *Simple Temporal Problem with Uncertainty* (STPU) [3] distinguishes two classes of variables, *controllable* V_c and *uncontrollable* V_u. The values of controllable variables are chosen by the execution agent and correspond to events in standard STPs. The values of uncontrollable variables, by contrast, are determined by exogenous factors ('Nature'); such a *realisation* is observed but cannot be controlled by the execution agent. The only information known prior to observation of an uncontrollable variable λ is that Nature will ensure that its value respects a single *contingent constraint* $\lambda - \mathbf{X} \in [\mathbf{a}, \mathbf{b}]$, with $a \geq 0$. Contingent constraints are assumed independent, and Nature is assumed to be consistent. Besides contingent constraints, which we distinguish by using a bold typeface, all other constraints in an STPU are *executable*.

The semantics of STPU constraints can be summarized as follows:

1. ***Contingent STPU constraints*** (S) model a priori information the agent is given about when an event controlled by Nature can occur (e.g., "An experiment will end (uncontrollable) between 5 and 10 minutes after it begins (controllable)").
2. ***Executable STPU constraints*** (S_e) model requirements the agent has between variables it controls and those controlled by Nature (e.g., "Data cannot be sent (controllable) until 3 minutes after the experiment ends (uncontrollable)").