Mathematics for Decisions

Worksheet 1

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Overview

Swimming team (I)

A medley relay swimming team consists of four swimmers.
The first member of the team swims one length of backstroke, then the second person swims a length of breaststroke, then the next a length of butterfly and finally the fourth person a length of crawl. Each member of the team must swim just one length. All the team members could swim any of the lengths, but some members of the team are faster at one or two particular strokes.

The table shows the time, in seconds, each member of the team took to swim each length using each type of stroke during the last training session:

	Back	**Breast**	**Butterfly**	**Crawl**
Jack	18	20	19	14
Kyle	19	21	19	14
Liam	17	20	20	16
Mike	20	21	20	15

Swimming team (II)

1. State the best time in which a team could complete the race;
2. Show that there is more than one way of allocating the team so that they can achieve this best time.

Induction meeting (I)

Talkalot College holds an induction meeting for new students. The meeting consists of four talks: I (Welcome), II (Options and Facilities), III (Study Tips) and IV (Planning for Success). The four department heads, Clive, Julie, Nicky and Steve, deliver one of these talks each. The talks are delivered consecutively and there are no breaks between talks. The meeting starts at 10 a.m. and ends when all four talks have been delivered. The time, in minutes, each department head takes to deliver each talk is given in the table below:

	Talk I	**Talk II**	**Talk III**	**Talk IV**
Clive	12	34	28	16
Julie	13	32	36	12
Nicky	15	32	32	14
Steve	11	33	36	10

Induction meeting (II)

1. Find the earliest time that the meeting could end and the final allocation;
2. Find the latest time that the meeting could end.

Hospital shifts

The nurse manager of Borgo Roma Hospital wants to organize the nurses' shifts. Each nurse works 5 consecutive days – regardless of the starting day within the week – and is entitled to two days off. Service requirements for the various days of the week require the presence of the following minimum number of nurses:

Day	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**S**
Number	17	13	15	19	14	

She wants to organize the service in such a way so as to minimize the total number of nurses.

Radio production

An electronic company needs to produce at least 20,000 radios in a 4-week period. The unit revenues from the sale of the radios each week are as follows:

Week	**1**	**2**	**3**	**4**
Revenue (Euro)	20	18	16	14

The company initially has 40 workers, each producing 50 radios per week. It is also possible to hire apprentice workers, three of whom can be trained by a worker within a 1-week period. During the training period the teacher worker does not produce any radio. A worker costs 200 Euros per week, while an apprentice costs 100 Euros for the apprenticeship week. Each radio needs basic components for a cost of 2 Euros. The goal is to maximize the overall profit.

Phone production

A company produces phones, each of which needs a series of processing steps linked by precedence relations. Each processing has a weight and a duration. The first processing step conventionally begins at $t = 0$. The aim is to determine the start times of each processing step so as to minimize the weighted average. The problem data is summarized in the following table:

Processing Step	Weight	Duration	Previous Steps
1	0	10	-
2	2	23	1
3	5	12	1
4	7	10	2
5	10	3	3, 4
6	12	11	2
7	17	5	2
8	21	4	7
9	26	10	5, 6

For instance, processing step 5 cannot begin before the end of steps 3 and 4, has a weight equal to 10, and a duration equal to 3.

Boat production

Constructing a boat requires the completion of the following operations (the table also gives the number of days needed for each operation):

Processing Step	Duration	Previous Steps
1	2	-
2	4	1
3	2	1
4	5	1
5	3	2, 3
6	3	5
7	2	5
8	7	4, 5, 7
9	4	6, 7

Some of the operations are alternative to each other: only one of B and C needs to be executed, the same for F and G. Moreover, if both C and G are executed, the duration of I increases by 2 days. Consider also the precedences for each operation (e.g., H can start only after the end of E, D and G, if G will be executed). Which operations should be executed in order to minimize the total duration of the construction of the boat?

The monkey and the coconuts

"Five men and a monkey were shipwrecked on an island. They spent the first night gathering coconuts. During the night, one man woke up and decided to take his share of the coconuts. He divided them into five piles. One coconut was left over so he gave it to the monkey, then hid his share, put the rest back together, and went back to sleep. Soon a second man woke up and did the same thing. After dividing the coconuts into five piles, one coconut was left over which he gave to the monkey. He then hid his share, put the rest back together, and went back to bed. The third, fourth, and fifth man followed exactly the same procedure. The next morning, after they all woke up, they divided the remaining coconuts into five equal shares. This time no coconuts were left over.
How many coconuts were there in the original pile?"

Archimedes' cattle problem

"The sun god had a herd of cattle consisting of bulls and cows, one part of which was white, a second black, a third spotted, and a fourth brown. Among the bulls, the number of white ones was one half plus one third the number of the black greater than the brown; the number of the black, one quarter plus one fifth the number of the spotted greater than the brown; the number of the spotted, one sixth and one seventh the number of the white greater than the brown. Among the cows, the number of white ones was one third plus one quarter of the total black cattle; the number of the black, one quarter plus one fifth the total of the spotted cattle; the number of spotted, one fifth plus one sixth the total of the brown cattle; the number of the brown, one sixth plus one seventh the total of the white cattle. What was the composition of the herd?"

The cannonball problem

"*Find a way to stack a square of cannonballs laid out on the ground into a square pyramid (i.e., find a square number which is also pyramidal).*"

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