Definition: Let X and Y be topological spaces. Two continuous maps $f, g : X \to Y$ are said to be *homotopic* if there exists a continuous map

$$H : X \times [0, 1] \to Y$$

such that for all $x \in X$,

$$H(x, 0) = f(x) \quad \text{and} \quad H(x, 1) = g(x).$$

The map H is called a *homotopy* between f and g. If such a homotopy exists, we say that f and g are *homotopic*, denoted by $f \simeq g$.