

## 1 About

*This* will be my first proof done in Latex!

## 2 The proof to be done

Proof that  $(\mathbb{Q}, +, \cdot)$  is a Field

## 3 Axioms

The set of integers  $\mathbb{Z}$  forms a group under addition, denoted as  $(\mathbb{Z}, +)$ . It is also closed under multiplication.

## 4 The proof itself:

To prove that  $(\mathbb{Q}, +, \cdot)$  is a field, we will verify that the set of rational numbers  $\mathbb{Q}$  satisfies the field axioms under the operations of addition  $+$  and multiplication  $\cdot$ , and has an identity  $e$ .

### 1. Closure

Let  $a, b \in \mathbb{Q}$ . Then  $a = \frac{m}{n}$  and  $b = \frac{p}{q}$ , for some numbers  $p, q, m, n \in \mathbb{Z}$ :

**Addition:**

$$a + b = \frac{m}{n} + \frac{p}{q} = \frac{mq + np}{nq} \in \mathbb{Q}$$

Seeing as  $(\cdot, +)$  are closed under  $\mathbb{Z}$ . And  $n, q \neq 0$

**Multiplication:**

Trivially the same.

## 5 trivial stuff, gonna skip

## 6 Distributivity

### 6.1 Addition

$+$  Distributes over  $\cdot$  :

$$\frac{a}{b} \left( \frac{x_1}{y_1} + \frac{x_2}{y_2} \right) = \frac{a}{b} \frac{x_1 y_2 + x_2 y_1}{y_1 y_2} = \frac{a(x_1 y_2 + x_2 y_1)}{b y_1 y_2}$$

## 6.2 Multiplication

$$\frac{a}{b} \frac{x_1}{y_1} + \frac{a}{b} \frac{x_2}{y_2} = \frac{ax_1}{by_1} + \frac{ax_2}{by_2} = \frac{ax_1by_2 + by_1ax_2}{b^2y_1y_2} = \frac{ax_1y_2 + ay_1x_2}{by_1y_2}$$

## 7 maybe not so trivial

All nonzero fractions have inverses under multiplication since if  $\frac{a}{b}$  is not 0, then it has an inverse  $\frac{b}{a}$ . Which exists since  $a \neq 0$ .

Overall this was really awful to write. Writing latex feels slow as shit and I never want to do it again. Looks pretty though.  $\frac{a}{b}$