1 About

This will be my first proof done in Latex!

2 The proof to be done

Proof that $(\mathbb{Q}, +, \cdot)$ is a Field

3 Axioms

The set of integers $\mathbb{Z}$ forms a group under addition, denoted as $(\mathbb{Z}, +)$. It is also closed under multiplication.

4 The proof itself:

To prove that $(\mathbb{Q}, +, \cdot)$ is a field, we will verify that the set of rational numbers $\mathbb{Q}$ satisfies the field axioms under the operations of addition $+$ and multiplication $\cdot$, and has an identity e.

1. Closure

Let $a, b \in \mathbb{Q}$. Then $a = \frac{m}{n}$ and $b = \frac{p}{q}$, for some numbers $p, q, m, n \in \mathbb{Z}$:

Addition:

$$a + b = \frac{m}{n} + \frac{p}{q} = \frac{mq + np}{nq} \in \mathbb{Q}$$

Seeing as $(\cdot, +)$ are closed under $\mathbb{Z}$. And $n, q \neq 0$

Multiplication:

Trivially the same.

5 trivial stuff, gonna skip

6 Distributivity

6.1 Addition

$+$ Distirbutes over $\cdot$:
$\frac{a}{b}\left(\frac{x_1}{y_1} + \frac{x_2}{y_2}\right) = \frac{a}{b}\frac{x_1y_2+x_2y_1}{y_1y_2} = \frac{a(x_1y_2+x_2y_1)}{by_1y_2}$

6.2 Multiplication

$\frac{a}{b}\frac{x_1}{y_1} + \frac{a}{b}\frac{x_2}{y_2} = \frac{ax_1}{by_1} + \frac{ax_2}{by_2} = \frac{ax_1by_2+by_1ax_2}{b^2y_1y_2} = \frac{ax_1y_2+ay_1x_2}{by_1y_2}$

7 maybe not so trivial

All nonzero fractions have inverses under multiplication since if $\frac{a}{b}$ is not 0, then it has an inverse $\frac{b}{a}$. Which exists since $a \neq 0$.

Overall this was really awful to write. Writing latex feels slow as shit and I never want to do it again. Looks pretty though. $\frac{a}{b}$

$\mathbb{Z} = \{a \mid a \in \mathbb{N}\} \cup \{a^-1 \mid a \in \mathbb{N}\} \cup \{0\}$