1 Contents

I will prove the sum of the Nth roots of unity is 0 for any n.

2 Proof

So we will say that ζ_1 is the first Nth root of unity if and only if: $\zeta_1^n = 1$.

It is known that for any complex number z, there are n Nth roots. Therefore, this also holds for 1.

So there are n Nth roots of unity. We will try to show that for the roots of unity their sum is always 0. We can find the Nth roots of a complex number Z by this formula:

$\sqrt[n]{z} = \sqrt[n]{r} \cdot \operatorname{cis}\left(\frac{\theta+2k\pi}{n}\right)$ The polar represenation of 1 is $1cis(0)$. So this simplifies to:

$\sqrt[n]{1} = 1 \cdot \operatorname{cis}\left(\frac{2k\pi}{n}\right)$

It is proven that $\frac{cis(a)}{cis(b)} = cis(a-b)$. Therefore:

$\frac{cis(\frac{2k\pi}{n})}{cis(\frac{2(k+1)\pi}{n})} = cis(\frac{2k\pi}{n} - (\frac{(2k+2)\pi}{n})) = cis(\frac{2\pi}{n})$

So in general the w Nth root of unity is:

$a_w = cis(0) \cdot cis(\frac{2\pi}{n})^{w-1}$