1 Definition of a Matrix

A **matrix** is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns. It is typically denoted as $A = [a_{ij}]$, where a_{ij} represents the element in the i-th row and j-th column.

1.1 Size of a matrix

A matrix's size is denoted in terms of its rows and columsn. So:
$M_{m \times n}$

1.2 Diagonal in a Matrix

In a matrix $A = [a_{ij}]$, the **diagonal** refers to the elements a_{ii} where the row index equals the column index. For example, in a square matrix, the diagonal runs from the top-left to the bottom-right of the matrix:

$$\text{Diagonal elements: } a_{11}, a_{22}, a_{33}, \ldots, a_{nn}.$$

1.3 Square Matrix

A **square matrix** is a matrix with the same number of rows and columns, i.e., if the matrix A is of order $n \times n$. For example:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

is a square matrix of order 3×3.

2 Different Matrices

2.1 Identity Matrix

An **identity matrix** is a square matrix I_n of order n with ones on the main diagonal and zeros elsewhere:

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}.$$

2.2 Zero Matrix

A **zero matrix** is a matrix in which all elements are zero. For a matrix of size $m \times n$, it is denoted as $O_{m \times n}$:

$$O_{m\times n} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}.$$

2.3 Scalar Matrix

A **scalar matrix** is a square matrix in which all the diagonal elements are equal and all off-diagonal elements are zero. For a scalar c, it is represented as:

$$S = cI_n = \begin{bmatrix} c & 0 & 0 & \cdots & 0 \\ 0 & c & 0 & \cdots & 0 \\ 0 & 0 & c & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & c \end{bmatrix}.$$

2.4 Transpose of a Matrix

The **transpose** of a matrix A, denoted $A^\top$, is obtained by interchanging its rows and columns. For $A = [a_{ij}]$, the transpose $A^\top = [a_{ji}]$. For example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad A^\top = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}.$$

2.5 Symmetric Matrix

A **symmetric matrix** is a square matrix A that satisfies $A^\top = A$, where $A^\top$ is the transpose of A. For example:

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, \quad \text{where } A^\top = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}.$$

So, for a matrix to be symmetric, it must be a square matrix, and must also meet the condition that: $\forall i, j : a_{ij} = a_{ji}$

2.6 Skew-Symmetric Matrix

A **skew-symmetric matrix** is a square matrix A that satisfies $A^\top = -A$. Additionally, all diagonal elements of a skew-symmetric matrix must be zero.

For example:

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}.$$

similarily, here it must meet the condition $\forall i, j : a_{ij} = -a_{ji}$

2.7 Upper Triangular Matrix

An **upper triangular matrix** is a square matrix in which all elements below the main diagonal are zero. For example:

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a_{22} & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn} \end{bmatrix}.$$

Therefore it must be that: $\forall i, j, i > j, a_{ij} = 0$

2.7.1 Lower Triangular Matrix

A **lower triangular matrix** is a square matrix in which all elements above the main diagonal are zero. For example:

$$L = \begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & 0 & \cdots & 0 \\ a_{31} & a_{32} & a_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}.$$

3 Operations on matrices

3.1 Addition

3.1.1 Subtraction

Similarily,

3.2 Multiplication