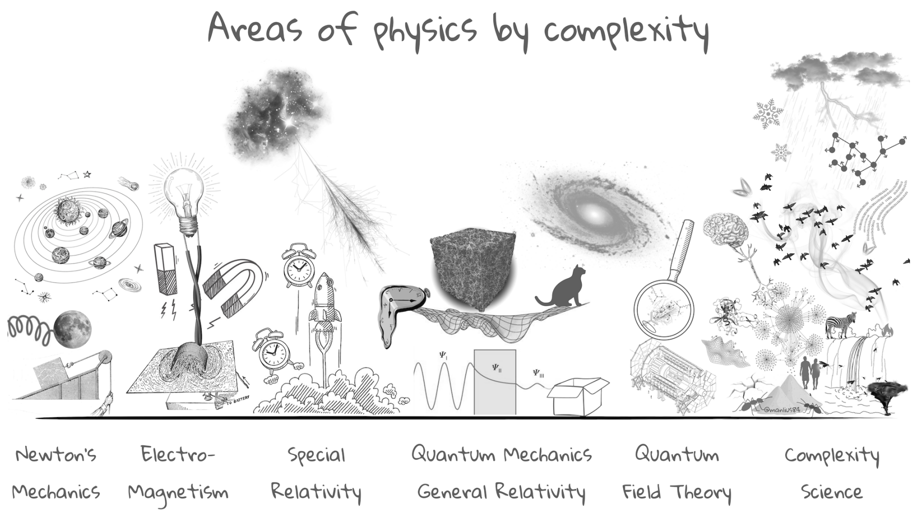
Final Report

Physics of Complex Networks: Structure and Dynamics

Università degli Studi di Padova



Project 27: Robustness of Noisy Quantum Networks

Vicentini Gioele

Last update: September 16, 2024

Contents

1 | ER random graphs

Task leader(s): *Vicentini Gioele*

1.1 | Introduction to the ER study

The aim here is to replicate the results obtained via simulations of the graphs in figure 3 of the original paper. The study is of both the diameter of an ER network for different wiring probabilities p and the size of its LCC. Finally, we find the size of the LCC for different values of the percolation probability p_{ext} by obtaining the relative optimal value of the probability p_{op} that a link has enough entangled pairs to perform a functional connection. This pairs of values (p_{ext}, p_{op}) are the ones satisfying the condition $l(p_{op}) = D(p_{op}p_{ext})$ and were obtained from the simulation results.

1.2 | Hysteresis in the first order phase transition

The first step is to obtain the minimum value of p_{op} for a given value of p_{ext}. In the paper an exponential distribution of qbit pairs is assumed, $g(n) = \exp(-n)$ such that the functional dependence of l_{op} on p_{op} is:

$$l_{op} = n_{op}^{1/\alpha} \tag{1.1}$$

$$p_{op} = \int_{n_{op}}^{+\infty} dnA \exp\left(-n/\langle n\rangle\right) dn \tag{1.2}$$

$$= -A\langle n\rangle \left[0 - \exp\left(-n_{op}/\langle n\rangle\right)\right] \tag{1.3}$$

$$l_{op} = \left[-\langle n\rangle \log\left(\frac{p_{op}}{A\langle n\rangle}\right)\right]^{1/\alpha} \tag{1.4}$$

$$A: \quad \int_{0}^{+\infty} dnA \exp\left(-n/\langle n\rangle\right) dn = 1 \tag{1.5}$$

$$A = \frac{1}{\langle n\rangle} \tag{1.6}$$

Since the condition is $l_{op}(p_{op}) = D(p_{op}p_{ext})$, for a given value of p_{ext} we can find p_{op}.

So the idea is:

- Find the average diameter of the ER graphs of some finite ensemble for varying p
- Get the corresponding critical value for p_{op} (more than one for the hysteresis region)
- Calculate the average backbone for all pairs (p_{ext}, p_{op})

But we encounter right away the first problem: the histeresis region appears only with really high numbers of nodes $N \sim 10^5$, which is far too much for my laptop since a fundamental part of the analysis consists in calculating over and over the diameter of a huge network (and multiple times). I wasn't able to replicate the results in time and it seems that the transition to the coexistance of phases is quite sharp, since even at $5 \cdot 10^4$ nodes it wasn't showing up. In order to widen the coexistance region, the following results were calculated for $\alpha = 2$.

The results are in graphs:

2 | Barabasi Albert

Task leader(s): *Vicentini Gioele*

2.1 | Introduction to the BA study

The procedure is the same as for the ER graphs, but this time there is a single critical value p_{op} for a given p_{ext} as it can be seen from the graphs. As seen in the paper (and as expected from the general percolation of links), the scale free model is more robust against random links failure.

Results in graphs:

3 | Task title...

Task leader(s): *Author name(s)...*

Structure as[1]*:*

- *A short (max 1 page) explanation of the task, including references. Include mathematical concepts.*
- *Max 2 pages for the whole task (including figures)*
- *It is possible to use appendices for supplementary material, at the end of the report. Max 5 pages per task*

A total of 3 pages + 5 supplementary pages per task

3.1 | A section...

Reference examples: book [**?**], article [**?**], website [**?**]

3.2 | Another section...

[1]Remove this part from the report

4 | Task title...

Task leader(s): *Author name(s)...*

Structure as[1]*:*

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[1] Remove this part from the report