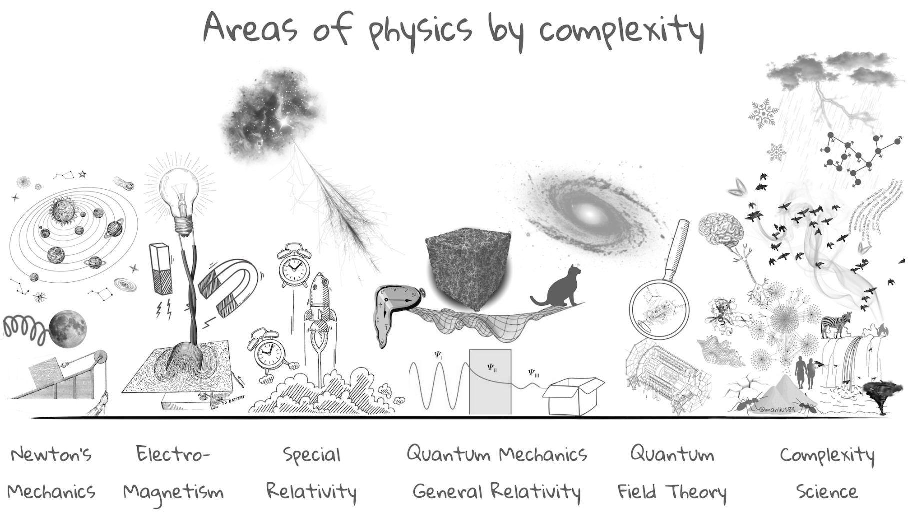
Final Report

Physics of Complex Networks: Structure and Dynamics

Università degli Studi di Padova



Project 27: Robustness of Noisy Quantum Networks

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1 | ER random graphs

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1.1 | Introduction to the ER study

The aim here is to replicate the results obtained via simulations of the graphs in figure 3 of the original paper. The study is of both the diameter of an ER network for different wiring probabilities p and the size of its LCC. Finally, we find the size of the LCC for different values of the percolation probability p_{ext} by obtaining the relative optimal value of the probability p_{op} that a link has enough entangled pairs to perform a functional connection. This pairs of values (p_{ext}, p_{op}) are the ones satisfying the condition $l(p_{op}) = D(p_{op}p_{ext})$ and were obtained from the simulation results.

1.2 | Hysteresis in the first order phase transition

The first step is to obtain the minimum value of p_{op} for a given value of p_{ext}. In the paper an exponential distribution of qbit pairs is assumed, $g(n) = \exp(-n)$ such that the functional dependence of l_{op} on p_{op} is:

$$l_{op} = n_{op}^{1/\alpha} \tag{1.1}$$

$$p_{op} = \int_{n_{op}}^{+\infty} dnA \exp\left(-n/\langle n\rangle\right) dn \tag{1.2}$$

$$= -A\langle n\rangle \left[0 - \exp\left(-n_{op}/\langle n\rangle\right)\right] \tag{1.3}$$

$$l_{op} = \left[-\langle n\rangle \log\left(\frac{p_{op}}{A\langle n\rangle}\right)\right]^{1/\alpha} \tag{1.4}$$

$$A: \quad \int_{0}^{+\infty} dnA \exp\left(-n/\langle n\rangle\right) dn = 1 \tag{1.5}$$

$$A = \frac{1}{\langle n\rangle} \tag{1.6}$$

Since the condition is $l_{op}(p_{op}) = D(p_{op}p_{ext})$, for a given value of p_{ext} we can find p_{op}.

So the idea is:

- Find the average diameter of the ER graphs of some finite ensemble for varying p
- Get the corresponding critical value for p_{op} (more than one for the hysteresis region)
- Calculate the average backbone for all pairs (p_{ext}, p_{op})

But we encounter right away the first problem: the histeresis region appears only with really high numbers of nodes $N \sim 10^5$, which is far too much for my laptop since a fundamental part of the analysis consists in calculating over and over the diameter of a huge network (and multiple times). I wasn't able to replicate the results in time and it seems that the transition to the coexistance of phases is quite sharp, since even at $5 \cdot 10^4$ nodes it wasn't showing up. In order to widen the coexistance region, the following results were calculated for $\alpha = 2$.

The results are in graphs Figure 1.1.

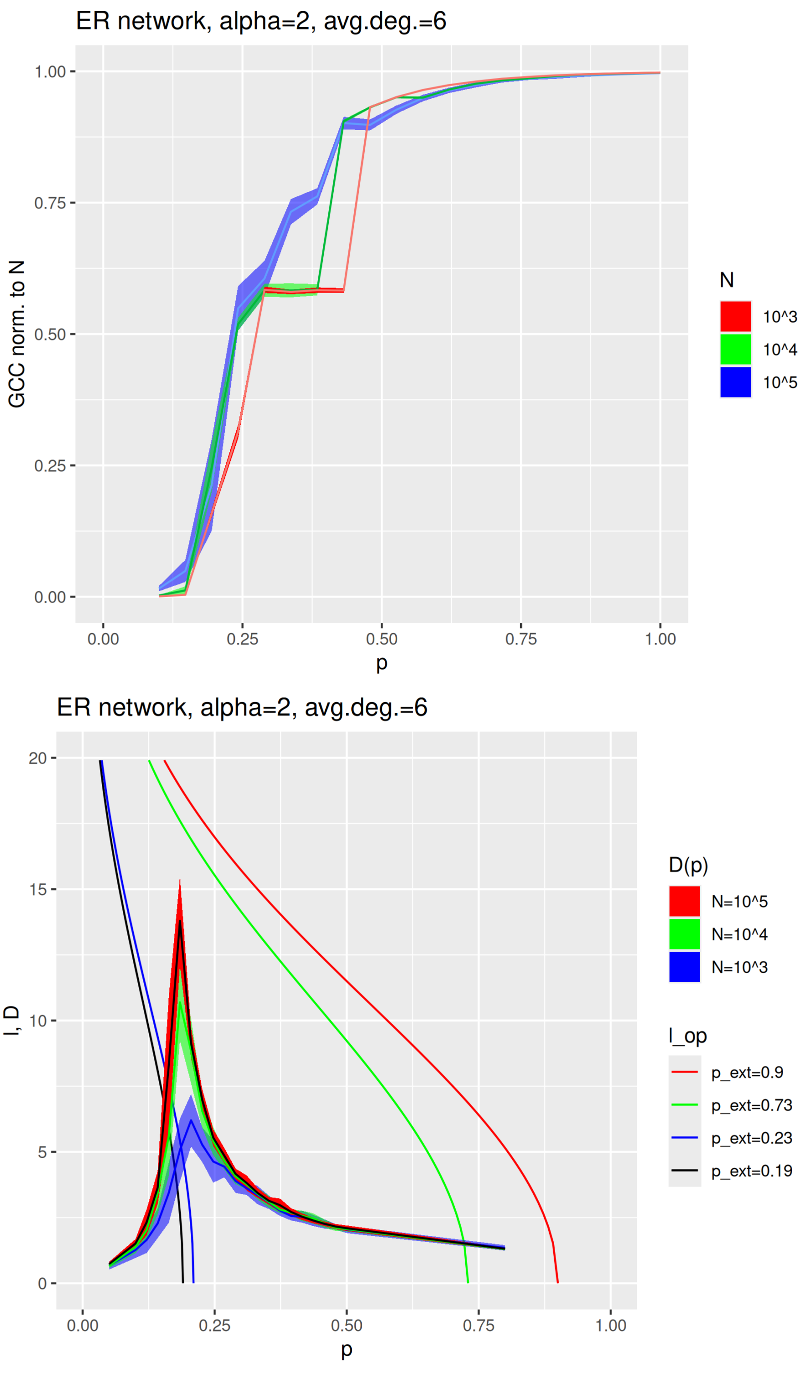

Figure 1.1: Results on ER networks: average diameter under percolation and l_{op}, size of the GCC. No hysteresis can be seen as the critical condition $l_{op}(p_{op}) = D(p)$ is hardly ever met if $N < 10^5$.