

~> Lista 1: Intervalos Nublados

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1. a) $\bar{A} = A$ (Inclusão)

$$\mu_{\bar{A}} = 1 - \mu_A(x)$$

$$\mu_{\bar{A}} = 1 - \mu_{\bar{A}}(x) = 1 - (1 - \mu_A(x)) = 0 \quad \mu_{\bar{A}}(x) = \cancel{1} + \mu_A(x) = 0 \quad \mu_{\bar{A}}(x) = \mu_A(x)$$

b) $A \cup (A \cap B) = A$ (Absorção)

$$A \cap B: \mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$$

$$A \cup (A \cap B): \mu_{A \cup (A \cap B)}(x) = \max[\mu_A(x), \min[\mu_A(x), \mu_B(x)]]$$

O resultado dessa expressão pode gerar dois resultados:

1- Caso $\min[\mu_A(x), \mu_B(x)] = \mu_B$, temos μ_A como valor maior. Assim, ao calcular o valor de $\max[\mu_A(x), \mu_B(x)] = \mu_A$

2- No caso de $\min[\mu_A(x), \mu_B(x)] = \mu_A$, calcular $\max[\mu_A(x), \mu_B(x)]$, logo, $\mu_{A \cup (A \cap B)}(x) = \mu_A(x)$

c) $A \cap \bar{A} = \emptyset$ (Contradição)

$$\mu_{\bar{A}} = 1 - \mu_A \quad \mu_{\emptyset} = 0$$

$$\mu_{A \cap \bar{A}}(x) = \min(\mu_A(x), \mu_{\bar{A}}(x)) = \min(\mu_A(x), (1 - \mu_A(x)))$$

$$\begin{array}{cc|l} 0 & 1 = 0 & \text{Se } \mu_A(x) = 0 \rightarrow \min(0, 1) = 0 \\ 1 & 0 = 0 & \text{Se } \mu_A(x) = 1 \rightarrow \min(1, 0) = 0 \end{array}$$

$$\mu_{A \cap \bar{A}}(x) = 0 = \mu_{\emptyset}(x)$$

d) $\overline{A \cup B} = \bar{A} \cap \bar{B}$ (De Morgan)

$$\bar{A} \rightarrow \mu_{\bar{A}} = 1 - \mu_A(x), \quad \bar{B} \rightarrow \mu_{\bar{B}} = 1 - \mu_B(x)$$

$$A \cup B \rightarrow \mu_{A \cup B} = \max[\mu_A(x), \mu_B(x)]$$

$$\bar{A} \cap \bar{B} \rightarrow \mu_{\bar{A} \cap \bar{B}} = \min[1 - \mu_A(x), 1 - \mu_B(x)]$$

$$\overline{A \cup B} \rightarrow \mu_{\overline{A \cup B}} = 1 - \mu_{A \cup B} = 1 - \max[\mu_A, \mu_B]$$

$$\mu_A(x) = 0, \mu_B(x) = 0$$

$$= \min[1, 1] = 1$$

$$= 1 - \max[0, 0] = 1$$

$$\mu_A(x) = 0, \mu_B(x) = 1$$

$$= \min[1, 0] = 0$$

$$= 1 - \max[0, 1] = 0$$

$$\mu_A(x) = 1, \mu_B(x) = 0$$

$$= \min[0, 1] = 0$$

$$= 1 - \max[1, 0] = 0$$

$$\mu_A(x) = 1, \mu_B(x) = 1$$

$$= \min[0, 0] = 0$$

$$= 1 - \max[1, 1] = 0$$

② As propriedades são consideradas válidas.

③. (n1): $N(0) = 1, N(1) = 0$

(n2): $N(a) \geq N(b) \text{ se } a \leq b$

$$N(0) = \frac{1-0}{1+5 \cdot 0} = 1$$

$$\frac{1-a}{1+5a} \geq \frac{1-b}{1+5b}$$

$$N(1) = \frac{1-1}{1+5 \cdot 1} = 0$$

$$(1-a)(1+5b) \geq (1-b)(1+5a)$$

$$1+5b-a-5ab \geq 1+5a-b-5ab$$

$$5b+b \geq 5a+a$$

(n4): $N(N(a)) = a$

$$b(5+1) \geq a(5+1)$$

$$a \leq b$$

$$1 - \left(\frac{1-a}{1+5a} \right) = \frac{1+5a-1+a}{1+5a} = \frac{a(5+1)}{5+1} = a$$

$$1 + 5 \left(\frac{1-a}{1+5a} \right) = \frac{1+5a+5-a}{1+5a} = \frac{5+4a}{5+4a} = 1$$

④. $S(a,b) = a+b-ab$

(S1): $S(0,0) = 0+0-0 \cdot 0 = 0$

$S(A,0) = A+0-A \cdot 0 = A$

$S(0,A) = 0+A-0 \cdot A = A$

(S2): $S(a,b) \leq S(c,d) \text{ se } a \leq c \text{ e } b \leq d$

$$a+b-ab \leq c+d-cd$$

$$\hookrightarrow ab \leq cd$$

(S3): $S(a,b) = S(b,a) \rightarrow a+b-ab = b+a-ba$

(S4): $S(a, S(b,c)) = S(S(a,b), c)$

$S(b,c) = b+c-bc$

$S(a, S(b,c)) = a + (b+c-bc) - a(b+c-bc) = a+b+c-bc-ab-ac+abc$

$S(a,b) = a+b-ab$

$S(S(a,b), c) = (a+b-ab) + c - (a+b-ab)c = a+b-ab+c-ac-bc+abc$

⑤. $S(a,b) = \min(1, a+b)$

(S1): $S(0,0) = 0, S(a,0) = S(0,a) = a$

$S(0,0) = \min(1, 0+0) = 0, S(a,0) = \min(1, A+0) = A$

$S(0,a) = \min(1, 0+A) = A$

$$(S2): S(a,b) \leq S(c,d) \text{ se } a \leq c \text{ e } b \leq d$$

$$S(a,b) = \min(1, a+b) = a+b$$

$$S(c,d) = \min(1, c+d) = c+d, \text{ como } a+b \leq c+d, S(a,b) \leq S(c,d)$$

$$(S3): S(a,b) = S(b,a) \rightarrow \min(1, a+b) = \min(1, b+a)$$

$$(S4): S(a, S(b,c)) = S(S(a,b), c)$$

$$S(b,c) = b+c$$

$$S(a, S(b,c)) = \min(1, a+b+c)$$

$$S(S(a,b), c) = \min(1, a+b+c)$$

$$(6) \cdot T(a,b) = ab$$

$$(T1): T(0,0) = 0, T(a,1) = T(1,a) = a$$

$$T(0,0) = 0$$

$$T(A,1) = A \cdot 1 = A$$

$$T(1,A) = 1 \cdot A = A$$

$$(T2): T(a,b) \leq T(c,d) \text{ se } a \leq c \text{ e } b \leq d$$

$$T(a,b) \leq T(c,d)$$

$$ab \leq cd$$

$$(T3): T(a,b) = T(b,a)$$

$$ab = ba$$

$$(T4): T(a, T(b,c)) = T(T(a,b), c)$$

$$T(a,bc) = T(ab, c)$$

$$abc = abc$$

$$(7) \cdot (T1): T(0,0) = 0, T(a,1) = T(1,a) = a$$

$$\max(0, 0+0-1)$$

$$\max(0, -1) = 0$$

$$\max(0, a+1-1) = a$$

$$\max(0, 1+a-1) = a$$

$$(T2): T(a,b) \leq T(c,d) \text{ se } a \leq c \text{ e } b \leq d$$

$$\max(0, a+b-1) \leq \max(0, c+d-1)$$

$$\text{se } (a+b) \text{ e } (c+d) > 1$$

$$a+b-1 \leq c+d-1$$

$$a+b \leq c+d$$

$$(T3): T(a,b) = T(b,a)$$

$$\max(a, a+b-1) = \max(0, b+a-1)$$

$$(4): T(a, T(b, c)) = T(T(a, b), c)$$

$$T(b, c) = \max(0, b+c-1) = b+c-1$$

$$T(T(a, b), c) = T(a+b-1, c)$$

$$T(a, b+c-1) = \max(0, a+b+c-1-1)$$

$$= \max(0, a+b+c-1-1)$$

$$⑧. N(a) = 1-a$$

$$N(b) = 1-b$$

$$T(a, b) = N(S(N(a), N(b)))$$

$$= N(S(1-a, 1-b))$$

$$= N(1-a+1-b - (1-a)(1-b))$$

$$= N(2-a-b-1+b+a-ab)$$

$$= N(1-ab)$$

$$= 1-1+ab$$

$$T(a, b) = ab$$

$$\hookrightarrow ab = ab$$