Termination and Partial Correctness of eliminate

Theorem (Termination of eliminate)

Given G=(V,E), after one elimination step, we obtain G'=(V',E'), such that $V'\subsetneq V$.

Lemma (Soundness of $is_simplicialb$)

Given G = (V, E), and node $u \in V$, if is_simplicial is true, then $G \vdash u$: Simplicial.

Lemma (Completeness of is_simplicialb)

Given G = (V, E), and node $u \in V$, if $G \vdash u$: Simplicial, then is_simplicials is true.

Theorem (Partial Correctness of eliminate)

Given G = (V, E) such that G has a PEO, if eliminate_step returns node u, then $G \vdash u$: Simplicial, otherwise $G = (\emptyset, \emptyset)$.