NUMERICAL MODELING AND SIMULATION FOR ACOUSTICS
A.A. 2019/2020

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HOMEWORK 3

EXERCISE

Let us consider the following wave propagation problem in $(0, L) \times (0, T]$:

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,t) + \dfrac{\partial F(u)}{\partial x}(x,t) = 0 & \text{in} \quad (0, L) \times (0, T], \\ u(x, 0) = u_0(x) & \text{in} \quad (0, L), \end{cases} \tag{1}$$

where $F(u) = \dfrac{u^2}{2}$ and u_0 is a given function, regular enough. Consider the following boundary conditions

$$u(0, t) = \phi(t) \quad t \in (0, T]. \tag{2}$$

for a regular enoguh function ϕ.

1. Write the discontinuous Galerkin formulation for problem (1)–(2) by considering $\mathbb{P}^0$ finite element. Write the semidiscretized problem under the form:

$$\begin{cases} M\dot{\mathbf{u}}_h(t) = \mathcal{L}(\mathbf{u}_h(t), t), \quad t \in (0, T) \\ \mathbf{u}_h(0) = \mathbf{u}_h^0, \end{cases}$$

   $\mathbf{u}_h(t)$ being the vector of degrees of freedom, $\mathcal{L}(\mathbf{u}_h(t), t)$ the vector resulting from the discretization of the flux term $-\frac{\partial F}{\partial x}$ and M the mass matrix.

2. Starting from the provided code `DGSEM_1D_BURGER` implement the 3^{rd} order Runge-Kutta scheme

$$\begin{cases} M(\mathbf{u}_h^* - \mathbf{u}_h^n) = \Delta t \mathcal{L}(\mathbf{u}_h^n, t^n), \\ M(\mathbf{u}_h^{**} - (\frac{3}{4}\mathbf{u}_h^n + \frac{1}{4}\mathbf{u}_h^*)) = \frac{1}{4}\Delta t \mathcal{L}(\mathbf{u}_h^*, t^{n+1}), \\ M(\mathbf{u}_h^{n+1} - (\frac{1}{3}\mathbf{u}_h^n + \frac{2}{3}\mathbf{u}_h^{**})) = \frac{2}{3}\Delta t \mathcal{L}(\mathbf{u}_h^{**}, t^{n+1/2}) \end{cases}$$

   for the solution of problem (1)–(2). Consider the following data:

   (a) $L = 10$, $h = 0.02$,

   (b) $T = 3$, $\Delta t = 0.01$,

   (c) $\phi(t) = 0$, $u_0(x) = e^{-(x-3)^2}$.

   Report your results and comment on them

3. Compute analitically the critical time t_c being the first instant where characteristics lines intersect. Compute numerically σ being the speed of propagation of the discontinuity after t_c. Report the results and comment on them.