NUMERICAL MODELING AND SIMULATION FOR ACOUSTICS
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LECTURER: Prof. I. Mazzieri

HOMEWORK 2

EXERCISE

Let us consider the following wave propagation problem in $\Omega = (-\delta L, L + \delta L) \times (0, T]$:

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2}(x,t) - c^2 \dfrac{\partial^2 u}{\partial x^2}(x,t) + \sigma(x)\left(\widetilde{c}\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial t}\right) = g(x,t) & \text{in } \Omega \times (0,T], \\ u(x,0) = u_0(x) & \text{in } \Omega, \\ \dfrac{\partial u}{\partial t}(x,0) = v_0(x) & \text{in } \Omega, \\ u(-\delta L, t) = 0 & t \in (0,T], \\ u(L + \delta L, t) = 0 & t \in (0,T], \end{cases} \tag{1}$$

where g, u_0, v_0 are given regular functions, c is a positive constant, σ is given by

$$\sigma(x) = \begin{cases} 0 & x \in (0,L), \\ \alpha > 0 & \text{otherwise}, \end{cases}$$

and $\widetilde{c}$ is definded as

$$\widetilde{c} = \begin{cases} c & x \in (L, L+\delta L), \\ -c & x \in (-\delta L, 0). \end{cases}$$

- Write the weak formulation for problem (1). State precisely the functional spaces where the problem is formulated.
- Write the Galerkin formulation of the problem at the previous point. Consider a spectral element space discretization.
- Show that the proposed discrtezation leads to the following system of ordinary differential equations
$$M\ddot{\mathbf{u}}(t) + D_\sigma \dot{\mathbf{u}}(t) + (A + C_\sigma)\mathbf{u}(t) = \mathbf{F}(t), \tag{2}$$
for $t > 0$. Define precisely the entries of the matrices M, A C_σ and D_σ and of the right hand side $\mathbf{F}$.
- Propose a time integration scheme for (2) and implement it in Matlab. Consider the following data: $L = 2$, $c = 1$, $f = 0, v_0 = 0$, $u_0(x) = e^{-5(x-1)^2}$ and $T = 3$. Check the behaviour of the solution for different choices of δL and α by computing the norm $\|u_h\|_{L^2(0,L)}$ at the final observation time.
- Compute the solution of problem (1) by using spectral elements coupled with the leap-frog scheme. Consider $\Omega = (0, L)$, $\sigma = 0$ and first order absorbing boundary conditions for $x = 0$ and $x = L$. Comment on the results and compare them with those obtained at the previous point.