SAXOPHONE MOUTHPIECE GEOMETRY PARAMETRIZATION

Gioele Greco

Dipartimento di Elettronica, Informazione e Bioingegneria (DEIB), Politecnico di Milano
Piazza Leonardo Da Vinci 32, 20122 Milano, Italy
gioele.greco@mail.polimi.it

ABSTRACT

This paper proposes to find the significant geometrical length of the saxophone mouthpiece, in order to compare the acoustical properties of shape variations. For this purpose we have created a parametric model of a mouthpiece, and then we have used the finite element method (FEM) to calculate their acoustical behaviour. We have investigated several models to find the mouthpiece acoustic behaviour. Then, the simulation results were compared with literature [1] to validate the model. The comparison of acoustic properties among different mouthpiece shapes shows that the chosen parameters are significant. Furthermore, we have explained how to find the best settings, in COMSOL, to have high performance computing and reduce the simulation time.

.

1. INTRODUCTION

The saxophone sound depends on: player mouth, mouthpiece, reed and saxophone body [2]. The player mouth is variable and so can be neglected [3], and we assume that saxophone body is a truncated cone.

Saxophone is an auto-oscillating musical instrument that works as follows: a source (player) injects energy into a non-linear element (mouthpiece-reed valve [2]) which generate a pressure wave. The resonator (saxophone body) amplifies the wave, which interact with the reed and cycle starts again. So reed transforms the constant pressure in the player mouth, into pressure wave. Mouthpiece drives this reed process.

There are many models of mouthpieces available. Variations in mouthpiece's geometry can have an important role on the sound and playability of the instrument. The goal is to develop a method to compute the acoustics response of mouthpieces. For this purpose we have created a parametric model that allow us to change the shape easily. Then we have calculated the acoustics response using two differential models. Finally we have compared the results to the measured ones [1].

1.0.1. State of art

In the most recent studies [4], various mouthpiece studying methods are compared: lumped, equivalent cylinder and transfer matrix. They propose a transfer matrix model that derives from FEM analysis. In [4] they start from a geometry, that is draft with external CAD software, and then they validate the transfer matrix method with empirical measurements. This research is oriented to synth the saxophone sound, so it is not correlated to the mouthpiece crafting. For this reason, to study different geometries, it is necessary to restart all the process described in [4].

From the reverse engineering point of view, there are two principal ways to study the acoustics behaviour of mouthpiece: analytical (using FEM) and empirical. In this [5] study they propose how to implement mouthpiece analysis with FEM. On the empirical side, [1] they propose how to compare and analyse different mouthpieces experimentally.

In this paper we propose a parametric model of mouthpiece that has some advantages:

- it does not needs external CAD software,
- it does not needs laser scanner, but the geometry measurements can be done with calliper,
- it allows us to study quickly various phenomena, like vortex inside chamber, or interaction between tube and mouthpiece,
- it allows us to print a 3D model,
- it allows us to study the acoustics behaviour of different geometries without restart the implementation,
- it allows us to define the geometry with only 8 variables, that can be measured or chosen in a range, 5 semi-variables (lengths that are usually constant) and 6 fixed parameters.
- it was developed before the [4] publication, but the final results are similar.

2. BACKGROUND

2.1. Mouthpiece

Saxophone mouthpiece is crafted starting from a cylinder of ebonite or brass. The most important thing is that the side rails must be smooth and must coincide with the reed print. Reed shape is standard and does not change with brands or quality, so it can be assumed fixed. As the same, the diameter of the bore must be equal or greater than the saxophone joint on the kiever, so it can be assumed fixed too. The remaining lengths, see 1, can be derived or defined in a range that make it buildable.

2.2. Reed

Why we don't study reed behaviour? From [2] we know that reed acts as a valve that generates the pressure wave. We assumed the mouthpiece as passive, so the reed is assimilated in the mouthpiece's boundaries. In this way, the excitation and the nonlinearity of the system can be modeled separately, and the mouthpiece can be treated as a linear and passive acoustic system. We have studied three way to simulate reed behaviour, that are discussed in section 6.

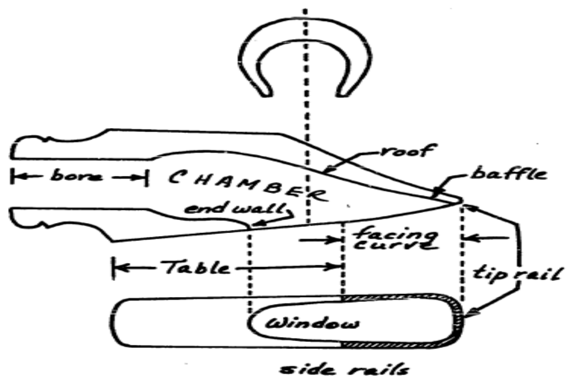

Figure 1: Terminology of the saxophone mouthpiece [1]

2.3. Finite Element Method (FEM)

The FEM is a general numerical method for solving partial differential equations in two or three space variables (i.e., some boundary value problems). To solve a problem, the FEM subdivides a large system into smaller, simpler parts that are called finite elements. This is achieved by a particular space discretization (mesh of the object) of the numerical domain of the solution, which has a finite number of points. The simple equations that model these finite elements are then assembled into a larger system of equations that models the entire problem. The whole of finite elements is called mesh. To have a great mesh we need to have an high number of finite elements for wavelength.

$$h = \frac{c}{fn} \tag{1}$$

with h =element size, $c =$ wave speed, f =frequency, n =number of elements per wavelength. Be careful, because the more the number of elements, the more the computational power needed for converging the solution.

3. MATHEMATICAL MODEL

We have implemented the mouthpiece analysis with two models and computed in the frequency domain. In this section we have discussed the physical equation and the imposed boundary conditions.

3.1. Helmholtz equation (H)

The Helmholtz equation is used to study the wave propagation in space and frequency. It is implemented in COMSOL within the "Acoustic pressure" module as follows

$$\nabla(-\frac{1}{\rho_c}\nabla p_t) - \frac{k_{eq}^2 p_t}{\rho_c} = 0 \tag{2}$$

$$k_{eq}^2 = (\frac{\omega}{c_c})^2 - k_m^2$$

with air material $\omega = 2\pi frequency[\frac{rad}{s}]$,
$\rho_c = air \quad density[\frac{kg}{m^3}]$,
$p_t = total \quad pressure[Pa]$,
$c_c = sound \quad speed[\frac{m}{s}]$,
$k_m = imaginary \quad attenuation.$

3.1.1. boundary conditions

All the surface of the air volume that touch the solid material are considered rigid walls, so the normal flux and force are nulls.

$$-\mathbf{n}(-\frac{1}{\rho_c}\nabla p_t) = 0$$

On the exit side we consider an additional sphere, $r_{sphere} = 2r_e x$ and we impose the spherical wave propagation condition.

$$-\mathbf{n}(-\frac{1}{\rho_c}\nabla p_t) + (ik_{eq} + \frac{1}{r_{rf}})\frac{p}{\rho_c} - \frac{r_{rf}\Delta_{||}p}{2\rho_c(1+ik_{eq}r_{rf})} = 0 \tag{3}$$

with $r_{rf} = |\mathbf{x} - L_{tube}|$, $p = p_t - p_{atm}$. Finally we simulate the reed like a pressure generator on the tip rail in three different ways, see 3.

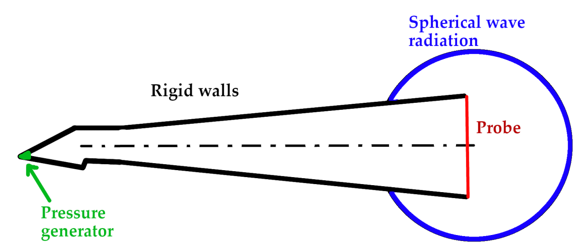

Figure 2: Sketch of the boundary condition in the Helmholtz case, in green the pressure generator 3, in black the rigid walls, in red the probe, in blue the spherical wave radiation condition.

$$p(t) = Pe^{i\omega t}$$

3.2. Linearized Navier-Stokes equation (LNS)

In order to analyze the air flux inside the mouthpiece, we have considered the Linearized Navier-Stokes equation. We have assumed the system as adiabatic without external perturbations.

$$i\omega\rho_t + \nabla\cdot(\rho_t\mathbf{u}_0 + \rho_0\mathbf{u}_t) = M,$$

$$\boldsymbol{\sigma} = -p_t\mathbf{I} + \mu(\nabla\mathbf{u}_t + (\nabla\mathbf{u}_t)^T) + (\mu_b - \frac{2}{3}\mu)(\nabla\cdot\mathbf{u}_t)\mathbf{I}, \tag{4}$$

$$\rho_0(i\omega\mathbf{u}_t + (\mathbf{u}_t\cdot\nabla)\mathbf{u}_t) + \rho_t(\mathbf{u}_0\cdot\nabla)\mathbf{u}_0 = \nabla\cdot\boldsymbol{\sigma} + \mathbf{F} - \mathbf{u}_0 M,$$

3.2.1. boundary conditions

As before we have imposed a null normal flux on the solid surface, while pressure is applied to the corners of the tip. On the exit side surface we add a little sphere, like in 2, and we have imposed the flowing condition $\mathbf{n}\cdot\mathbf{u}_t = \mathbf{0}$ on its surface.

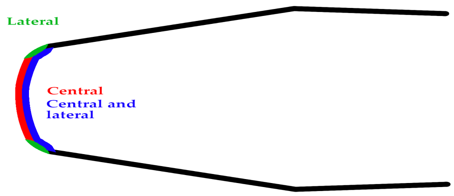

Figure 3: Three locations in which we put the pressure generator on the tip. Red central, green lateral, blue central and lateral together

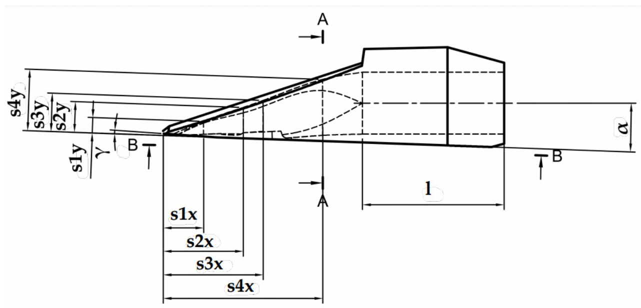

Figure 4: Mechanical design project of the parametric mouthpiece geometry

4. GEOMETRY

We have developed the geometry parameters in order to be versatile and cheap to be implemented. It must reproduce lots of mouthpiece types in [1] by changing few parameters. It has to be significant and it has to neglect unnecessary acoustics details, like sharp edges. It has to be cheap and quickly, cause we need to change the geometry without pass through a laser scanner.
From [1] and personal experience, we have found 19 parameters that can be grouped into three types: fixed, semi-variable, and variable.
Fixed parameters are lengths that are imposed from other interaction objects (reed, kiever, ligature), like bore diameter, table, facing curve, external diameter and total length of the mouthpiece. Semi-variable parameters are lengths that depends on the reed interaction, and are usually fixed, like tip rail lengths, window and facing curve. Variable parameters are the lengths that describe better the mouthpiece type, like roof profile function (in four points 4), side rails inclination, percentage of joint with kiever and other parameters that describe chamber behaviour.
All parameters can be measured, on real mouthpiece , using only a calliper. In this paper we have chosen to analyse only three type of mouthpieces, A, B and C2 in [1]. We have implemented also the A1, A2, B1, B2, C1 and C2 geometry. In the end we have put a cone with semi-angle $= 1.6\,\text{deg}$ and we have chosen the $L_{tube} = 0.735mm$ to have the resonance frequency around $f_0 = 174Hz$, the F3.

5. FINITE ELEMENT IMPLEMENTATION

We have used COMSOL to implement the FE model. As seen in 2.4, we have created a free tetrahedral mesh with some boundary layers, that are necessary to describe air/wall interaction. We want to analyse the behaviour below the following condition $f_{max} = 3000Hz$, so with $h_{max} = 8.25[mm]$ and $c = 346.12[m/s]$, so from 1 $n = 14$ is enough, see 5.

h_{max}	8.25 mm
h_{min}	2.54 mm
grow factor	1.25
n° boundary layers	4

We have put one probe on the generator surfaces and another on the exit side of the cone, see 2. We have implemented H with the "Acoustics pressure, in frequency domain" module, with the boundary conditions in 3 and we have qualitative compared the exit side probe pressure with [1] spectrum measurements. To validate this configuration, we have started from the simple 3D cone shape, as [5], with semi-angle $= 1.6\,\text{deg}$, $L_{tube} = 0.735mm$ and $r_{in} = 6.5mm$.
As the same, in LNS case we need to watch out the h_{max} value that may be increased. The finer the mesh, the more precise the solution, the longer the time, see 5. For both cases, we have studied from 100Hz to 1kHz with 10Hz steps, with an inlet pressure $p = 1000Pa$. In the end we have normalized the results and compared it with the theoretical impedance of the cone [2].

$$Z_{in} = \frac{i\sin(kL)\sin(k\theta)}{\sin(k(L+\theta))} \tag{5}$$

with $k\theta = kx_1$ and x_1 =distance of the input from the virtual apex L needs to be corrected with
$L \simeq L + 0.6r_{ex}$.

model	h_{max}	$t_{calculation}$
LNS	80mm	180s
LNS	8mm	427s
H	80mm	29s
H	8mm	160s

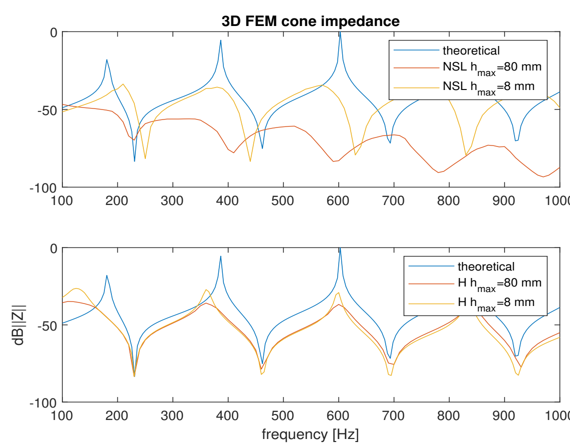

Figure 5: Difference between the LNS method and the H method. LNS can describe better in the low frequency range, but it needs lots of elements and time. H can describe as well, but it has not information about local air speed. See 5 for time details.

In first analysis the best model is H because it needs less time than LNS. We have computed also LNS to have more details about

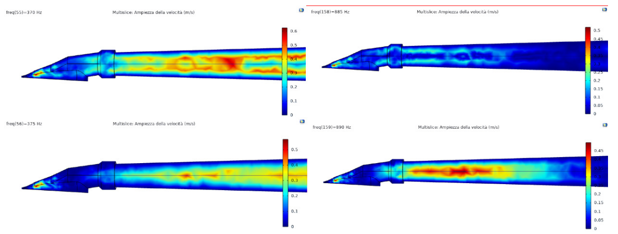

Figure 6: Speed inside mouthpiece B from LNS, longitudinal cross section. Different velocity profiles, two toroidal up, two Poiseulle down

the air flux. For example see 6
Computer behaviour are: processor Intel(R) Core(TM) i7-6500U CPU @ 2.50GHz and RAM 8,00 GB.

6. RESULTS

In [1] there are measurements of acoustic pressure, performed on a playing sax with different mouthpieces. So we have to compare this results by putting a pressure probe on the exit side cross section (see 2).
The first thing that we need to improve is the generator's location, see 7. Using H, if we change the inlet pressure position we have different main modes, so it needs to be investigated.

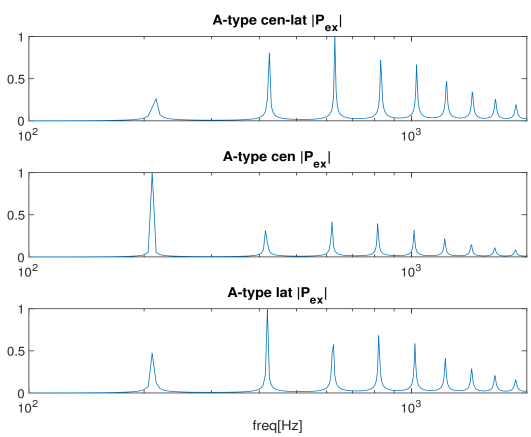

Figure 7: Mouthpiece type A. Difference between central and lateral, only central, only lateral, position of the pressure generator. See 3

The study starts from 100Hz to 1200Hz with 5Hz steps. In the H study the computing time is always around 410s an the LNS around 29000s, about 8h.

The spectrum has different frequency peaks, this is the reason why mouthpiece can change its longitudinal position to adjust intonation. We have not computed all the mouthpiece positions, so intonation can change.

We can say that for A and B types the second harmonic is higher than the fundamental, both in the FEM model and in the literature [1]. On the contrary in the C-type the second harmonic is equal or

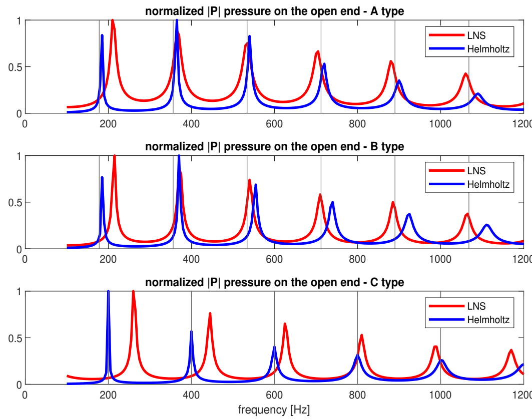

Figure 8: COMSOL FEM results of computed absolute normalized pressure results for mouthpieces A,B,C2. with pressure generator in lateral position.

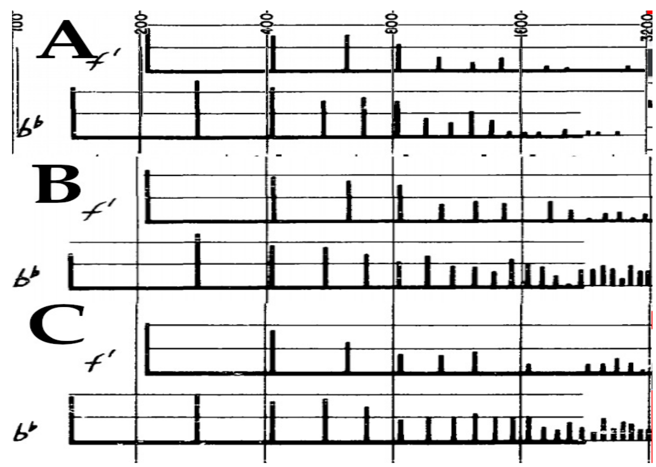

Figure 9: Qualitative mean spectrum measured [1] for type A,B,C2 mouthpiece.

smaller than the fundamental. From the third harmonic onwards it goes down, this occurs in the literature [1] and FEM results. This analysis can be only qualitative, because we don't have numerical behaviour in [1].
For more accurate analysis we need to test each geometry and take measurements.

7. CONCLUSION

This document is a first analysis to find a relation between mouthpiece shape and sound, but there are many elements that needs to be investigated yet. For example print models and perform measurements on them. Or study in deep the reed behaviour, in order to define accurate pressure boundary conditions and to avoid complex systems and long computation time. Print more interesting models and perform measurements on them.
In the end a simple parametric model, with less than ten variables, may simulate the real shape changes. This model may be tested

with different input pressure values (from 1kPa to 5kPa [2]) and other mouthpiece position to evaluate intonation changes.

8. ACKNOWLEDGMENT

I thank prof. Mazzieri for his help, his availability and the time spent following me in the development of this project. I thank prof. Augusto Sarti and prof. Alberto Bernardini for their trust and for giving me the opportunity to develop this project within the Milan Polytechnic.

9. REFERENCES

[1] F. S. Wyman, "An acoustical study of alto saxophone mouthpiece chamber design," *Eastman School of Music of the University of Rochester*, 1972.

[2] N. H. F. T. D. Rossing, "Wood-wind instruments," in *The physics of the musical instruments*, vol. II, 1998, pp. 803–806.

[3] https://www.academia.edu/43524735/SAXOPHONE_FROM_WOOD_TO_BRASS_BETWEEN_PHYSICS.

[4] E. M. S. Wang and G. Scavone, "Acoustical modeling of the saxophone mouthpiece as a transfer matrix," *The Journal of the Acoustical Society of America 1*, vol. 149, p. 1901–1912, Mar. 2021.

[5] M. Harazia, "A numerical and experimental analysis of the acoustical properties of saxophone mouthpieces using the finite element method and impedance measurements," *Ećole Normale Supeŕieure, Paris, France and Computational Acoustic Modeling Laboratory, McGill University, Montreal, Canada.*