THE STARTING POINT FOR THE PARAMETERIZATION OF THE SAXOPHONE MOUTHPIECE GEOMETRY AND VERIFICATION USING FEM

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ABSTRACT

This paper proposes to find the significant geometrical length of the saxophone mouthpiece, to compare the acoustical properties of shape variations. For this purpose I created a parametric model of a mouthpiece, and then I used the finite element method (FEM) to calculate its acoustical behavior. I investigated several models to validate the geometric behavior. Then, I compared the simulation results with the literature [1], The comparison of the acoustic characteristics between different mouthpiece shapes shows that the selected parameters are suitable. In addition, I explained my settings in the software COMSOL v5.6 to achieve high-performance computing and reduce simulation time.

***Index Terms*—** Saxophone, mouthpiece, geometry parametrization, FEM.

1. INTRODUCTION

The sound of the saxophone depends on the mouth of the player, mouthpiece, reed and saxophone body [2]. I focused on the mouthpiece because it can be studied separately, the mouth of the player doesn't affect the mouthpiece behavior [3], and because I assumed that the saxophone body is an ideal truncated cone.

The saxophone is an auto-oscillating musical instrument that works as follows: a source (player) injects energy into a non-linear element (mouthpiece-reed valve [2]) which generates a pressure wave. The resonator (saxophone body) amplifies the wave, which interacts with the reed, and the cycle starts again. So reed transforms the constant pressure in the player mouth, into pressure wave. Mouthpiece drives this reed process.

There are many models of mouthpieces available. Variations in mouthpiece's geometry can have an important role in the sound and playability of the instrument. For this reason I developed a method to calculate the acoustic response of the mouthpiece. For this purpose, I created a parametric model that allows us to change the shape easily. Then, I calculated the acoustics response using two differential models. Finally, I compared the results to the measurement [1].

1.1. State of the art

In the most recent study [4], various mouthpiece studying methods are compared: lumped, equivalent cylinder and transfer matrix. Those studies propose a transfer matrix model that derives from FEM analysis. In [4] the authors start from a geometry, which is drawn with external CAD software, and then, they validate the transfer matrix method with empirical measurements. The purpose of this research is to synthesize the sound of a saxophone, so it has nothing to do with mouthpiece production. For this reason, to study different geometries, it is necessary to restart the process described in [4]. Therefore, the advantage of this research is that you can choose whether to start with a known geometry and simulate its acoustics, or start with acoustics and simulate its geometry.

From a reverse engineering point of view, there are two principal ways to study the acoustics behavior of mouthpiece: analytical (using FEM) and empirical. In [5] the authors propose a way to implement mouthpiece analysis with FEM. On the empirical side, the work from Stearns Wyman [1] proposes how to compare and analyse different mouthpieces experimentally.

In this paper I propose a parametric model for the mouthpiece that has some advantages:

- it does not need external CAD software,
- it does not need a laser scanner, but the geometry measurements can be done with a calliper,
- it allows us to quickly study various phenomena, like a vortex inside the chamber, or the interaction between the tube and the mouthpiece,
- it allows us to print a 3D model,
- it allows us to study the acoustics behavior of different geometries without restarting the the implementation,
- it allows us to define the geometry with only 8 variables, that can be measured or chosen in a range, 5 semi-variables (lengths that are usually constant) and 6 fixed parameters.

2. BACKGROUND

2.1. Mouthpiece

The saxophone mouthpiece is made starting from a cylinder of ebonite or brass. The most important thing is that the side rails must be smooth and must coincide with the reed print. Some mouthpiece's sizes are imposed by the interaction between the reed and the kiever. The Reed shape is standard and does not change with brands or quality. The diameter of the bore must be equal or greater than the saxophone joint on the kiever, so I assumed these lengths fixed. The remaining lengths, see Fig. 1, can be derived or defined in a range that makes the mouthpiece buildable.

2.2. Reed

I don't study reed behavior because from [2] I know that reed acts as a valve that generates a pressure wave. If I assumed the mouthpiece as passive, the reed is assimilated in the mouthpiece's boundaries. In this way, the excitation and the nonlinearity of the system can be

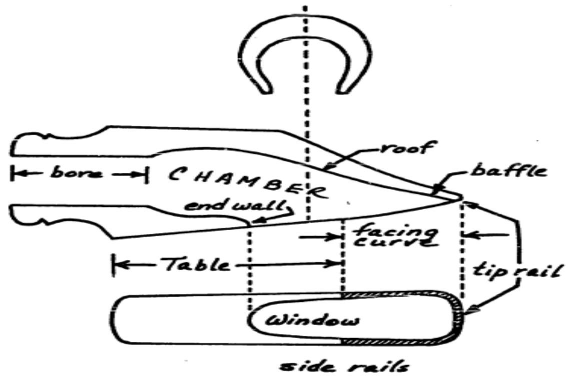

Figure 1: Terminology of the saxophone mouthpiece [1]

modeled separately, and the mouthpiece can be treated as a linear and passive acoustic system. I studied three way to simulate reed behavior discussed in Section 5.

2.3. Finite Element Method (FEM)

The FEM is a general numerical method for solving partial differential equations [6]. To solve a problem, the FEM subdivides a large system into smaller parts called finite elements. This is achieved by a space discretization (mesh of the object) of the numerical domain of the solution, which has a finite number of points. The simple equations that model these finite elements are assembled into a larger system of equations that models the entire problem. To have a great mesh I need to have an high number of finite elements in the wavelength.

$$h = \frac{c}{fn} \tag{1}$$

with element size h, wave speed c, frequency f and number of elements per wavelength n. It is important to mention that the larger number of elements requires more the computational power for converging the solution.

3. THE MODEL, MATHEMATICAL BEHAVIOR AND GEOMETRY PARAMETRIZATION

As suggested in [5], I implemented the mouthpiece analysis using the Helmholtz equation, and I also chose the Navier-Stokes equation to compare the results. Both equations are computed in the frequency domain.

3.1. Helmholtz equation (H)

The Helmholtz equation is used to study vibratory phenomena [2]. It is implemented in COMSOL v5.6 within the "Acoustic pressure" module as follows

$$\begin{cases} \nabla(-\frac{1}{\rho_c}\nabla p_t) - \frac{k_{eq}^2 p_t}{\rho_c} = 0 \\ k_{eq}^2 = (\frac{\omega}{c_c})^2 \end{cases} \tag{2}$$

I implement the air as an ideal gas using the COMSOL v5.6 material library and imposing $T = 298.15K$ and $c = 346.112[m/s]$

$$\begin{array}{l|l} \omega = 2\pi frequency[\frac{rad}{s}] & \rho_c = air\ density[\frac{kg}{m^3}] \\ p_t = total\ pressure[Pa] & c_c = sound\ speed[\frac{m}{s}] \end{array}$$

3.1.1. Boundary conditions

The main differences with [5] are in the boundary conditions, they propose 2D conditions that don't work in the 3D domain. Therefore, the entire surface of the air volume contacting the solid material is considered a rigid wall, so the normal flux and force are zero.

$$-\mathbf{n}(-\frac{1}{\rho_c}\nabla p_t) = 0$$

On the exit side I considered an additional sphere, $r_{sphere} = 2r_ex$ and I imposed the spherical wave propagation condition.

$$-\mathbf{n}(-\frac{1}{\rho_c}\nabla p_t) + (ik_{eq} + \frac{1}{r_{rf}})\frac{p}{\rho_c} - \frac{r_{rf}\Delta_{||}p}{2\rho_c(1 + ik_{eq}r_{rf})} = 0 \tag{3}$$

with $r_{rf} = |\mathbf{x} - L_{tube}|$, $p = p_t - p_{atm}$. Finally, I assumed the reed behavior as a pressure generator on the tip rail in three different ways, see Fig. 3.

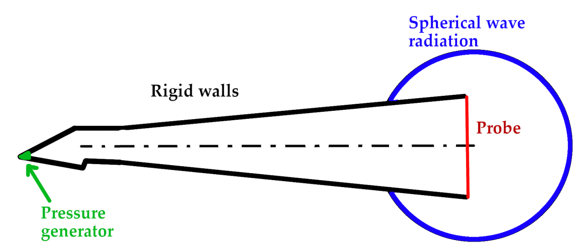

Figure 2: Sketch of the boundary condition in the Helmholtz case, in green the pressure generator Fig. 3, in black the rigid walls, in red the probe, in blue the spherical wave radiation condition.

$$p(t) = Pe^{i\omega t}$$

3.2. Linearized Navier-Stokes equation (LNS)

In order to analyse the air flux inside the mouthpiece, I considered the Linearized Navier-Stokes equation [7]. I assumed the system as adiabatic without external perturbations. It is implemented in COMSOL v5.6 within the "Linearized Navier-Stokes" module as follows

$$\begin{cases} i\omega\rho_t + \nabla\cdot(\rho_t\mathbf{u}_0 + \rho_0\mathbf{u}_t) = M, \\ \boldsymbol{\sigma} = -p_t\mathbf{I} + \mu(\nabla\mathbf{u}_t + (\nabla\mathbf{u}_t)^T) + (\mu_b - \frac{2}{3}\mu)(\nabla\cdot\mathbf{u}_t)\mathbf{I}, \\ \rho_0(i\omega\mathbf{u}_t + (\mathbf{u}_t\cdot\nabla)\mathbf{u}_t) + \rho_t(\mathbf{u}_0\cdot\nabla)\mathbf{u}_0 = \nabla\cdot\boldsymbol{\sigma} + \mathbf{F} - \mathbf{u}_0 M. \end{cases}$$

With spatial vector displacement $\mathbf{u}$, dynamic viscosity μ, Bulk viscosity μ_b, identity tensor $\mathbf{I}$, reaction forces $\mathbf{F}$.

3.2.1. Boundary conditions

As before I imposed a null normal flux on the solid surface, while pressure is applied to the corners of the tip. On the exit side surface I added a little sphere, like in 2, and I imposed the flowing condition $\mathbf{n}\cdot\mathbf{u}_t = \mathbf{0}$ on its surface.

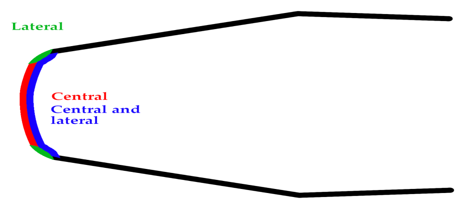

Figure 3: Three locations in which I put the pressure generator on the tip. Red central, green lateral, blue central and lateral together

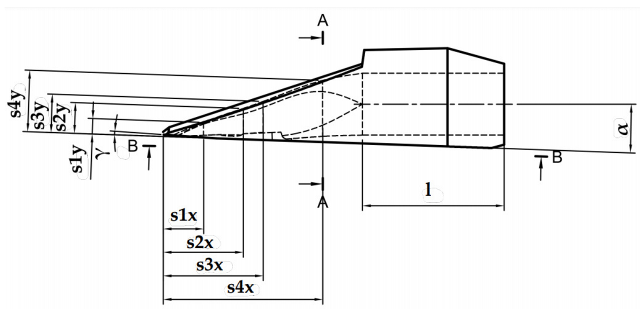

Figure 4: Mechanical design project of the parametric mouthpiece geometry

3.3. Geometry model parameters

I developed the geometry parameters versatile and cheap to be implemented. It must reproduce lots of mouthpiece types in [1] by changing few parameters. Additionally, it has to be significant and to neglect unnecessary acoustic details, like sharp edges. In the end, it must be cheap and fast, because I need to change the geometry without going through a laser scanner.
From [1] I found 19 parameters that can be grouped into three types: fixed, semi-variable, and variable.
Fixed parameters are lengths imposed from other interactive objects (reed, kiever, ligature), like bore diameter, table, facing curve, external diameter and total length of the mouthpiece.
Semi-variable parameters are lengths that depend on the reed interaction, and are usually fixed, like tip rail lengths, window and facing curve.
Variable parameters are the lengths that describe better the mouthpiece type, like roof profile function (in four points Fig. 4), side rails inclination, percentage of joint with kiever and other parameters that describe chamber behavior.
All parameters can be measured, on a real mouthpiece, using a caliper. In this paper I analyse the three types of mouthpieces, A, B and C2 explained in [1]. In the end, I put a cone with semi-angle $= 1.6\deg$ and I chose the $L_{tube} = 0.735m$ to have the resonance frequency around $f_0 = 174Hz$, corresponding to the note F3, without the mouthpiece, or to the note Ab3 (low F for the saxophone) by adding the mouthpiece.

4. FINITE ELEMENT IMPLEMENTATION

I used COMSOL v5.6 to implement the FE model. I created a free tetrahedral mesh with some boundary layers, that are necessary to describe air/wall interaction. I analysed the behavior below the following condition $f_{max} = 3000Hz$, so with $h_{max} = 8.25[mm]$, $h_{min} = 2.54[mm]$, four boundary layers and $c = 346.12[m/s]$ from eq. (1) $n = 14$, that is enough.
I put one probe on the generator surfaces and another on the exit side of the cone, see Fig. 2. I implemented H with the ”Acoustics pressure, in frequency domain” module, with the boundary conditions in Section 3 and I qualitative compared the exit side probe pressure with [1] spectrum measurements. To validate this configuration, I started from the simple 3D cone shape, as [5], with semi-angle $= 1.6\deg$, $L_{tube} = 0.735mm$ and $r_{in} = 6.5mm$.
Likewise, in LNS I needed to watch out the h_{max} value because, the finer the mesh, the more precise the solution, the longer the time, see Fig. 5. For both cases, I started from 100Hz to 1kHz with 10Hz steps, with an inlet pressure $p = 1kPa$. In the end, I normalized the results and compared it with the theoretical impedance of the cone [2].

$$Z_{in} = \frac{i \sin(kL)\sin(k\theta)}{\sin(k(L+\theta))} \tag{4}$$

with $k\theta = kx_1$ and x_1 =distance of the input from the virtual apex L needs to be corrected with
$L \simeq L + 0.6r_{ex}$.

model	h_{max}	$t_{calculation}$
LNS	80mm	180s
LNS	8mm	427s
H	80mm	29s
H	8mm	160s

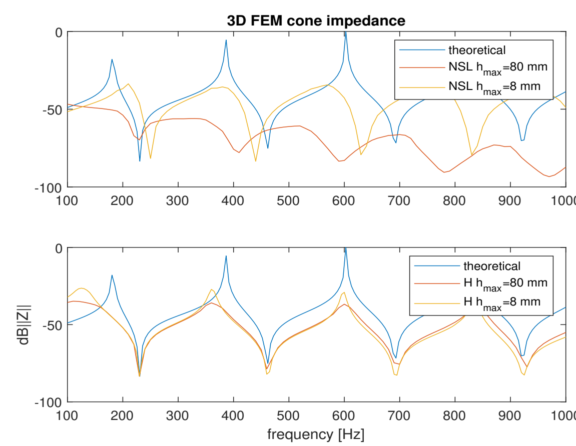

Figure 5: Difference between the LNS method and the H method.

The LNS can describe it better than H in the low frequency range, but requires a lot of elements and time. H can also describe it, but without giving us information on the local air speed. See tab. 4 for time details. In first analysis, the best model is H because it needs less time than LNS. I computed also LNS to have more details about the air flux, see Fig. 6
The computer behavior are processor Intel(R) Core(TM) i7-6500U CPU 2.50GHz and RAM 8,00 GB.

5. RESULTS

In [1] there are measurements of acoustic pressure, performed on a playing saxophone with different mouthpieces. So I have to compare these results by putting a pressure probe on the exit side cross

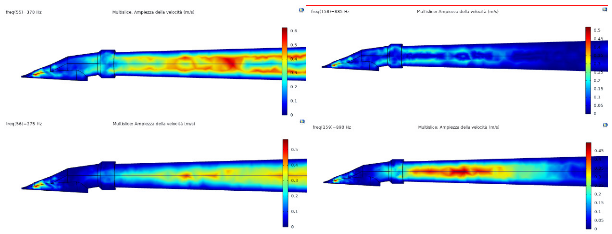

Figure 6: Speed inside mouthpiece B from LNS, longitudinal cross-section. Different velocity profiles, two toroidal up, two Poiseulle down

section (see Fig. 2).
For first I improved is the location of the generator, see Fig. 7. Using H, if I change the inlet pressure position I have different main modes, so it needs to be investigated.

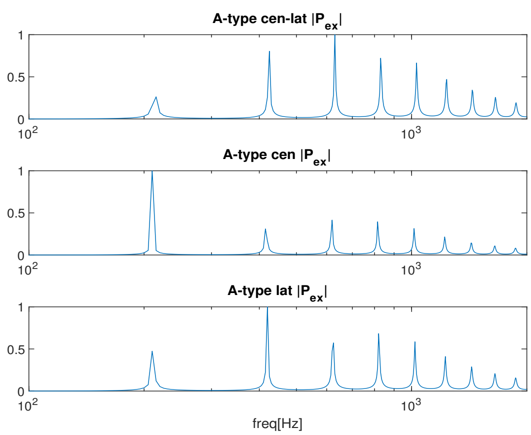

Figure 7: Mouthpiece type A. Difference between central and lateral, only central, only lateral, position of the pressure generator. See Fig. 3

The study starts from 100Hz to 1200Hz with 5Hz steps. In the H study the computing time is always around 410s and the LNS around 29000s, about 8h.

The spectrum has different frequency peaks, this is the reason why mouthpiece can change its longitudinal position to adjust intonation. I have not computed all the mouthpiece positions, so the intonation can change.

I observed that for A and B types the second harmonic is higher than the fundamental, both in the FEM model and in the literature [1]. On the contrary in the C-type the second harmonic is equal or smaller than the fundamental. From the third harmonic onwards it goes down, this occurs in the literature [1] and FEM results. This analysis can be only qualitative, because I don't have numerical behavior in [1].

For more accurate analysis I need to test each geometry and take measurements.

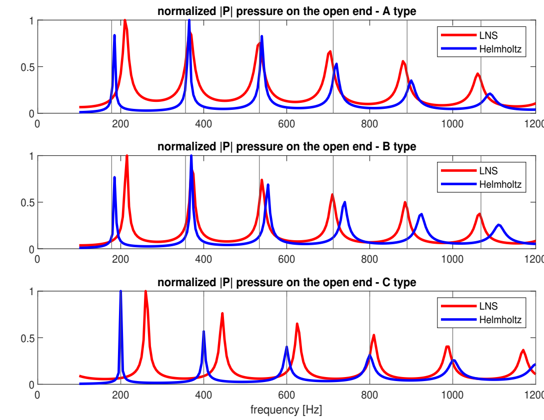

Figure 8: COMSOL v5.6 FEM results of computed absolute normalized pressure results for mouthpieces A,B,C2. with pressure generator in lateral position.

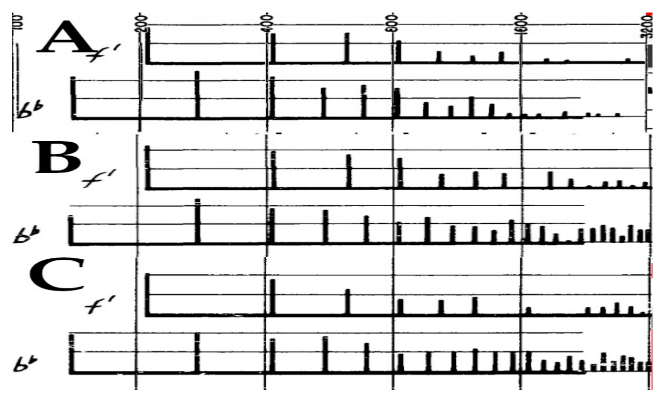

Figure 9: Qualitative mean spectrum measured [1] for type A,B,C2 mouthpiece.

6. CONCLUSION

This document is the first to analyze the relationship between mouthpiece shape and sound, but there are many elements that need to be studied. For example, printing a model and measuring it, or studying thoroughly the reed behavior, to define accurate pressure boundary conditions, to avoid system complexity and long calculation time. Nevertheless, the model proposed was developed before the [4] publication and the simulation results are comparable.
Concluding, in first approximation, a simple parametric model can simulate the real shape changes, with less than ten variables. This model may be tested with different input pressure values (from 1kPa to 5kPa [2]) and other mouthpiece positions to evaluate intonation changes.

7. ACKNOWLEDGMENT

I thank prof. Mazzieri for his help, his availability and the time spent following me in the development of this project. I thank prof. Augusto Sarti and prof. Alberto Bernardini, for the opportunity to develop this project at the Politecnico di Milano.

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