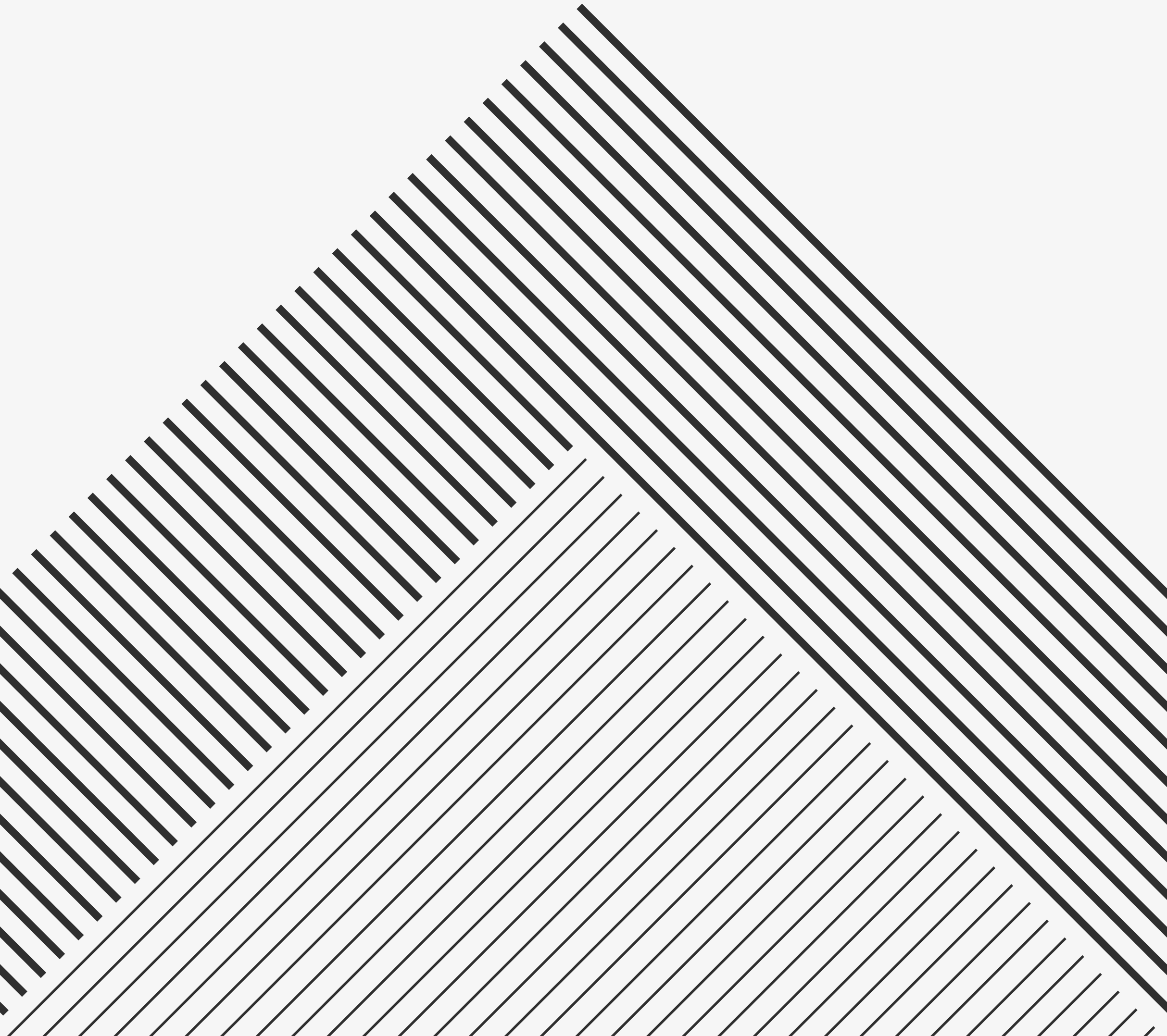
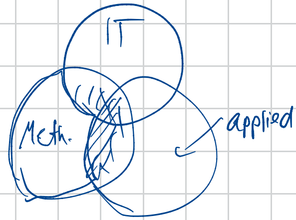


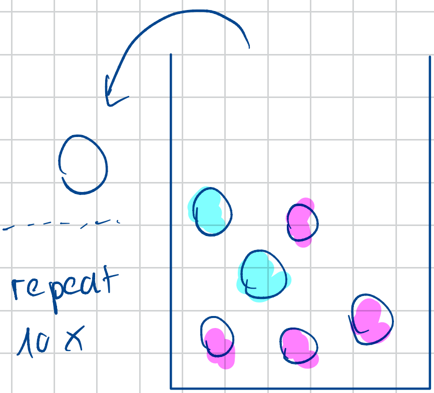
pwd: Stats24

What is statistics?

- Probability
- Make sense from data
- Linear regression
- Larger group vs. single case
- Plots
- Machine Learning



Probability vs. Statistics



8 pink balls

2 blue balls

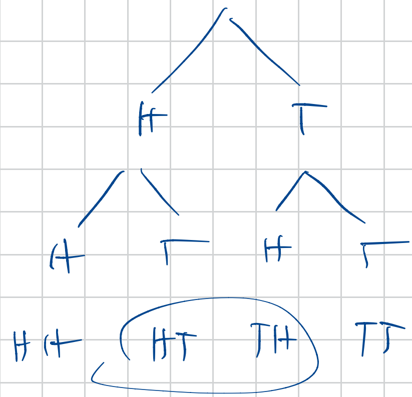
$P(\text{ball is pink?}) = 0.8$

$$P(8 \text{ pink balls } \& \text{ } 2 \text{ blue balls}) = \binom{10}{2} \, 0.8^8 \cdot 0.2^2$$

?

$$\binom{10}{2} = 45$$

Example: Coin



$$P(1 \times H \,\&\, 1 \times T) = \tfrac{1}{2} = \binom{2}{1} \tfrac{1}{2} \cdot \tfrac{1}{2}$$

Height $\in \,]0, \infty[\, \subset \mathbb{R}$

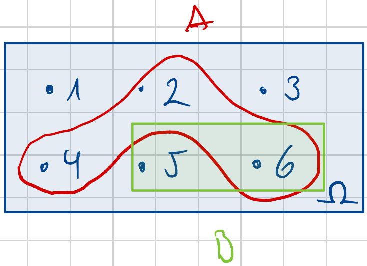
\# students $\in \mathbb{N}_0$

Notation

$\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$ $\quad \omega_i$ - all possible outcomes

Example: Die thrown once

$\Omega = \{1, 2, 3, 4, 5, 6\}$



E.g. $A = \{$"die shows an even number"$\}$ $= \{2, 4, 6\} \subset \Omega$

- ω - elementary events

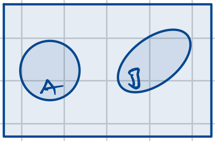
$B = \{$"spots ≥ 5"$\} = \{5, 6\}$

$\mathcal{F} = \{$"even number", "odd number", "≤ 2", $\{1, 3, 6\}$, $\{2\}, \dots\}$

- "and" link: $A \cap B$ ↳ A and B occur
- "or" link: $A \cup B$ ↳ A or B occur
- complementary: A^c ↳ not A
- "without": $A \setminus B$ ↳ A without B occur

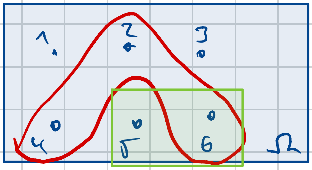
Comment:

Two events are called disjoint, if $A \cap B = \emptyset$



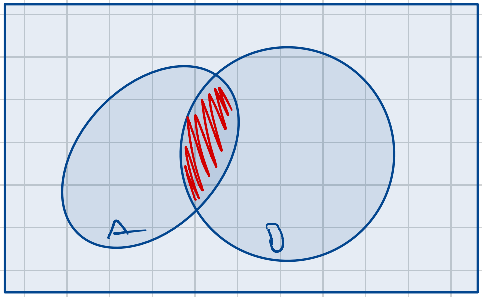
E.g. Die $\Omega = \{1, 2, 3, 4, 5, 6\}$

$A = \{\text{"even"}\}$, $B = \{\text{"}\geq 5\text{"}\}$



De Morgan's laws

$$(\boxed{A \cap B})^c \overset{?}{=} A^c \cup B^c$$



Probability

$$\Omega = \{1, 2, \ldots, 6\}$$

$$\rightsquigarrow P(\{6\}) = 1/6 = P(\{1\}) = \ldots$$

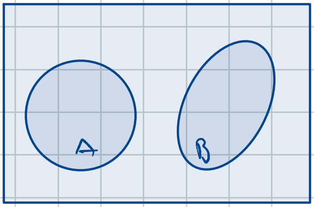
Die is unfair, e.g. $P(\{6\}) = 1/2$, $P(\{1\}) = P(\{2\}) = \ldots = 1/10$

Notation

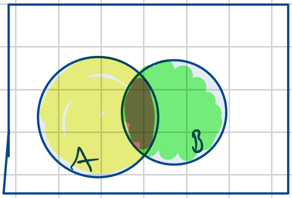
$$P: \mathcal{F} \to [0,1]$$

$$A \mapsto P(A)$$

iii)



$P(A \cup B) = P(A) + P(B)$



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Laplace

① Fair die:

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$A = \{6\}$

$\Rightarrow P(A) = \frac{1}{6}$

② Throw 2 dice: $B = \{$ " sum of spots is 6 " $\}$

$\Omega = \{2, 3, 4, \ldots, 12\} \Rightarrow |\Omega| = 11$

$\cancel{\Rightarrow P(B) = \frac{1}{11}}$

with this Ω it is not Laplace

Trick: Change Ω such that it is Laplace

$\Omega = \{(1,1), (1,2), \ldots, (1,6), (2,1), (2,2), \ldots, (6,6)\}$

$\Rightarrow P(B) = \frac{\#\text{ favourable}}{\#\text{ possible}} = \frac{|B|}{|\Omega|} = \frac{5}{36}$

2.1 – 2.8

Problem 2.10

a) $A = \{ 4 \text{ Bauern} \}$

$\Omega = \{(6,6,6,6,7,7,7,7,8), \ldots$
$\ldots, (B,B,B,B,6,6,6,6,7),$
$(B,B,B,B,6,6,7,7,7), \ldots \}$

$P(A) = ?$

4 Bauern

$$P(A) = \frac{\#\text{ favourable}}{\#\text{ possible}} = \frac{\binom{32}{5}}{\binom{36}{9}}$$

b) $B = \{ 3 \text{ Bauern} \}$

$$\Rightarrow P(B) = \frac{\binom{32}{6} \cdot \binom{4}{3}}{\binom{36}{9}}$$

Problem 2.11

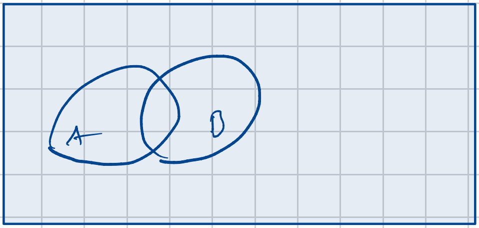
b) $$P(\text{exactly } 5) = \frac{\binom{8}{5}}{2^8}$$

$$\binom{8}{5} = \frac{8!}{5!(8-5)!}$$

c) $P(\geq 5 \text{ burning}) = P(5 \text{ or } 6 \text{ or } 7 \text{ or } 8 \text{ are burning})$

$= P(5) + P(6) + P(8) + P(7)$

disjoint



$P(A \cup B) = P(A) + P(B) - \cancelto{0}{P(A \cap B)}$

Independence

Throw 2 dice independently

$A = \{\text{"die 1 shows 6"}\}$, $B = \{\text{"die 2 shows 6"}\}$

$\Rightarrow P(A \cap B) = P(\{\text{"both dice show 6"}\})$

$= \underbrace{1/6}_{P(A)} \cdot \underbrace{1/6}_{P(B)} = \frac{1}{36}$

Example

Throw die once

$A = \{$" spots are even "$\}$, $B = \{$" spots ≤ 2 "$\}$

$\Rightarrow P(A \cap B) = P(\{2\}) = 1/6$

Q: Are A & B independent?

$P(A) \cdot P(B) = 1/2 \cdot 2/6 = 1/6$

$$P(\text{Clyde} \mid \text{Bonnie}) = \frac{P(\text{both})}{P(\text{Bonnie})} = \frac{0.4}{0.6} = \underline{\underline{2/3}}$$

PCR - Test

D - Person has Covid

T - Test is positive

$$P(T \mid D) = 0.95$$

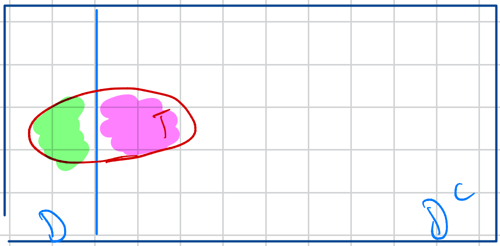
$$P(T^c \mid D^c) = 0.9$$

$$P(D) = 0.01$$

$$P(T) = \boxed{P(T \cap D)} + \boxed{P(T \cap D^c)}$$

$$= P(T \mid D) \cdot P(D) + P(T \mid D^c) \cdot P(D^c)$$

$$P(T \cap D) = P(T \mid D) \cdot P(D)$$



Bayes

$$\frac{P(A \cap B)}{P(B)}$$

$P(A|B)$ - easy to measure

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A|B) \cdot P(B)}{P(A)}$$

PCR - example

$$P(D|T) = \frac{P(T|D) \cdot P(D)}{P(T)} = \frac{0.95 \cdot 0.01}{0.11}$$

$P(T|D) = 0.95$