Problem 5 Peer Assessment

October 22, 2023

[37]:
```
# Needed for lib import, since it is a local module
import sys
sys.path.insert(0, "..")

import numpy as np
from lib.matrix_operations import add_row, swap_row, multiply_row
from numpy.linalg import det
```

[38]:
```
A = lambda z: np.array([[1, 1, z, 0], [1, 1, 1, z], [0, 1, 1, 1], [0, 0, 1, 1]])
detA = lambda z: det(A(z))
```

0.1 b.

$z = 0$ and $z = 5$

[39]:
```
print(detA(5))
print(detA(0))
```

```
8.999999999999998
-1.0
```

0.2 d.

$rref(A)|z = \frac{1}{2}$

[40]:
```
A = A(0.5)
A
```

[40]:
```
array([[1. , 1. , 0.5, 0. ],
       [1. , 1. , 1. , 0.5],
       [0. , 1. , 1. , 1. ],
       [0. , 0. , 1. , 1. ]])
```

0.2.1 Clean C1

1. $A[0,0] = 1 \rightarrow pivot$
2. $R_2 \leftarrow R_2 - R_1$

[41]:
```
A = add_row(A, 1, 0, -1, "float64")
A
```

[41]:
```
array([[1. , 1. , 0.5, 0. ],
       [0. , 0. , 0.5, 0.5],
       [0. , 1. , 1. , 1. ],
       [0. , 0. , 1. , 1. ]])
```

0.2.2 Clean C2

1. Swap $R_2 \leftrightarrow R_3$

[42]:
```
A = swap_row(A, 1, 2, "float64")
A
```

[42]:
```
array([[1. , 1. , 0.5, 0. ],
       [0. , 1. , 1. , 1. ],
       [0. , 0. , 0.5, 0.5],
       [0. , 0. , 1. , 1. ]])
```

2. $A[1,1] = 1 \rightarrow pivot$
3. $R_1 \leftarrow R_1 - R_2$

[43]:
```
A = add_row(A, 0, 1, -1, "float64")
A
```

[43]:
```
array([[ 1. ,  0. , -0.5, -1. ],
       [ 0. ,  1. ,  1. ,  1. ],
       [ 0. ,  0. ,  0.5,  0.5],
       [ 0. ,  0. ,  1. ,  1. ]])
```

0.2.3 Clean C3

1. Swap $R_3 \leftrightarrow R_4$

[44]:
```
A = swap_row(A, 2, 3, "float64")
A
```

[44]:
```
array([[ 1. ,  0. , -0.5, -1. ],
       [ 0. ,  1. ,  1. ,  1. ],
       [ 0. ,  0. ,  1. ,  1. ],
       [ 0. ,  0. ,  0.5,  0.5]])
```

2. $A[2,2] = 1 \rightarrow pivot$
3. $R_4 \leftarrow R_4 - \frac{1}{2}R_3$
4. $R_2 \leftarrow R_2 - R_3$
5. $R_1 \leftarrow R_1 + \frac{1}{2}R_3$

[45]:
```
A = add_row(A, 3, 2, -0.5, "float64")
A = add_row(A, 1, 2, -1, "float64")
A = add_row(A, 0, 2, 0.5, "float64")
A
```

[45]:
```
array([[ 1. ,  0. ,  0. , -0.5],
       [ 0. ,  1. ,  0. ,  0. ],
       [ 0. ,  0. ,  1. ,  1. ],
       [ 0. ,  0. ,  0. ,  0. ]])
```

$$null(A) = \left\{ \begin{pmatrix} 0.5 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}$$