16_P4_Assessment_GionRubitschung

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[1]:
```python
# Needed for lib import, since it is a local module
import sys

sys.path.insert(0, "..")

from lib.matrix_operations import add_row, multiply_row
import numpy as np
```

Set

$$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{pmatrix}$$

[2]:
```python
F = np.array(
    [
        [1, 2, 3, 4, 5],
        [2, 3, 4, 5, 6],
        [3, 4, 5, 6, 7],
        [4, 5, 6, 7, 8],
        [5, 6, 7, 8, 9],
    ]
)
print(F)
```

```
[[1 2 3 4 5]
 [2 3 4 5 6]
 [3 4 5 6 7]
 [4 5 6 7 8]
 [5 6 7 8 9]]
```

1 a.

Compute the $rref(F)$, with all-zero rows dropped.

1.1 Clean C1

1. $F[0,0] = 1 \rightarrow pivot$

2. $R_1 \leftarrow R_1 - 2R_0$
3. $R_2 \leftarrow R_2 - 3R_0$
4. $R_3 \leftarrow R_3 - 4R_0$
5. $R_4 \leftarrow R_4 - 5R_0$

[3]:
```
F = add_row(F, 1, 0, -2)
F = add_row(F, 2, 0, -3)
F = add_row(F, 3, 0, -4)
F = add_row(F, 4, 0, -5)
print(F)
```

```
[[  1   2   3   4   5]
 [  0  -1  -2  -3  -4]
 [  0  -2  -4  -6  -8]
 [  0  -3  -6  -9 -12]
 [  0  -4  -8 -12 -16]]
```

1.2 Clean C2

1. $R_1 \ldots R_4 \leftarrow (-1)(R_1 \ldots R_4)$

[4]:
```
F = multiply_row(F, 1, -1)
F = multiply_row(F, 2, -1)
F = multiply_row(F, 3, -1)
F = multiply_row(F, 4, -1)
print(F)
```

```
[[ 1  2  3  4  5]
 [ 0  1  2  3  4]
 [ 0  2  4  6  8]
 [ 0  3  6  9 12]
 [ 0  4  8 12 16]]
```

2. $F[1,1] = 1 \rightarrow pivot$
3. $R_0 \leftarrow R_0 - 2R_1$
4. $R_2 \leftarrow R_2 - 2R_1$
5. $R_3 \leftarrow R_3 - 3R_1$
6. $R_4 \leftarrow R_4 - 4R_1$

[5]:
```
F = add_row(F, 0, 1, -2)
F = add_row(F, 2, 1, -2)
F = add_row(F, 3, 1, -3)
F = add_row(F, 4, 1, -4)
print(F)
```

```
[[ 1  0 -1 -2 -3]
 [ 0  1  2  3  4]
 [ 0  0  0  0  0]
 [ 0  0  0  0  0]
 [ 0  0  0  0  0]]
```

$rref(F)$, with all-zero rows dropped ->

$$\underline{\underline{rref(F) = \begin{pmatrix} 1 & 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 3 & 4 \end{pmatrix}}}$$

2 b.

Compute the $rank(F)$

$$rref(F) \rightarrow 2pivots \Rightarrow \underline{\underline{rank(F) = 2}}$$

3 c.

Compute a basis of the nullspace of F

$$\underline{\underline{null(F) = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -4 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}}}$$

4 d.

Compute a basis of the column space of F.

$$F \rightarrow \mathbb{R}^5$$

$$null(F) \rightarrow \mathbb{R}^3$$

$$dim(colsp(F)) = dim(F) - dim(null(F)) = 5 - 3 = 2 \Rightarrow colsp(F) \rightarrow \mathbb{R}^2$$

4.1 Transpose of F

$$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{pmatrix} \Rightarrow F^T = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{pmatrix} = F$$

4.2 rref of F^T

Since

$$F = F^T \Rightarrow rref(F) = rref(F^T) = \begin{pmatrix} 1 & 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 3 & 4 \end{pmatrix}$$

4.3 Basis of columspace F

$$\underline{\underline{colsp(F) = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ -2 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \right\}}}$$

4.3.1