KALMAN FILTERING FOR UNCERTAIN NOISE COVARIANCES

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To my family, anna and ammu

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ABSTRACT

Aircraft health monitoring has been a challenging task for over decades. In turbofan jet engines all the parameters which describe the health of the engine cannot be measured explicitly. One possible solution to this problem is Kalman filter. The traditional Kalman filter is *optimal* as long as the modeling of the plant is accurate. The turbofan jet engine being highly non-linear makes the task difficult. This thesis shows a way of linearizing the jet engine model so that theoretically proven estimation techniques can be applied to this problem. This thesis presents the application of Kalman filter to health parameter monitoring of the gas turbine engine. It is shown that the standard Kalman filter will not be robust enough if there are uncertainties in the modeling of the plant. A new filter is developed in this thesis which addresses the uncertainties in the process noise and measurement noise covariances without assuming any bounds on them. A hybrid gradient descent algorithm is proposed to tune the new filter gain. This filter is then implemented for the health parameter estimation. The results show significant decrease in the estimation error covariance. It is shown in the conclusions that advanced search algorithms like Genetic Algorithms proves to be superior to hybrid gradient descent algorithm in searching for better minima.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER I

INTRODUCTION

In many approaches to the design of control systems, it is assumed that all the state variables are available for feedback [5]. In practice, however, none of the state variables may be directly available for feedback. We rather have measurements that consist of noisy linear combinations of the state variables. In that case we need to estimate the state variables. If the state estimator estimates all the states of the system, regardless of whether some state variables are available for measurement, it is called full order state estimator.

Estimation is the process of extracting information from data. In systems engineering this leads to extracting information about state variables from noisy measurements. Modern estimation methods use known relationships to compute the desired information from the measurements, taking account of measurement errors, the effects of disturbances and control actions of the systems and prior knowledge of the information [1]. If the process does not contain any stochastic elements then the process of estimation leads to the straightforward technique of the well known least squares estimation.

If the process has non-stationary stochastic variables then the solution to the optimal state estimation is the celebrated Kalman filter. The derivation of the discrete time Kalman filter is presented in detail in Chapter 2. The extensions of the Kalman filter for non-linear state estimation are the linearized Kalman filter and extended Kalman filter. These topics are also covered in Chapter 2. For the rest of the work in this thesis the linearized Kalman filter is used.

The problem under consideration in this thesis is the health parameter estimation of a gas turbine engine which is the principal engine in most modern day jets. The performance of the engines degrades over time. This deterioration effects the fuel economy, impact emissions, component life consumption and thrust response of the engine. Airlines periodically collect data to determine the maintenance schedules. The data that is collected does not directly correspond to the health of the engine. But this can be incorporated into the model required by the linearized Kalman filter and for further work that is carried on in this thesis. This information is used to obtain the estimates of the health parameters which determine the maintenance schedules of the engines. The operation of jet engines and their modeling are discussed in detail in Chapter 3.

In a standard Kalman filter, all the system characteristics (i.e., the system model, initial conditions, and noise characteristics) have to be specified a priori. However, if there is uncertainty in any of these characteristics, the filter may not be robust enough. In this thesis, an alternate filter is proposed which performs better than the standard Kalman filter for uncertainties in both process and measurement noise covariances. The mathematical derivation of the new filter and its application are discussed in Chapter 4.

The results of the proposed method for the health parameter estimation for the gas turbine engine are discussed in Chapter 5. Conclusions and possible future work are discussed in Chapter 6.

CHAPTER II

OPTIMAL ESTIMATION

This thesis is aimed at obtaining optimal state estimates in the presence of uncertainties in the noise characteristics of the system model for a gas turbine engine. But before getting into the details of modeling gas turbine engines and Kalman filtering it is necessary to look into the development of optimal estimation methods. This chapter looks into the derivation of optimal estimation techniques. Section 2.1 gives a brief overview and history of state estimation. Section 2.2 and Section 2.3 derives the least squares estimation and recursive least squares estimation algorithms. It is to be noted that there is no element of uncertainty in these two methods. Section 2.4 looks into the propagation of uncertainty. Section 2.5 deals with the derivation of discrete time Kalman filter. This forms the backbone of this study. Section 2.6 looks into state estimation for non-linear systems. The two methods discussed in this section are the linearized Kalman filter approach and the extended Kalman filter approach.

2.1 Introduction

The development of estimation techniques from available noisy measurements can be dated back to Gauss in the 18th century [1]. Gauss was the first person to use least squares method for estimation. Later on Fisher, Wiener and Kalman worked on advanced optimal recursive filter techniques [1]. The issue of the number of measurements required for the determination of the unknown quantities, which in today's world is called "observability," was addressed by Gauss in his work. Based on the work done in estimation methods, an optimal estimator can be defined as follows: "An optimal estimator is a computational algorithm that processes observations to deduce a minimum error estimate of the state of a system by utilizing: knowledge of system and measurement dynamics, assumed statistics of system noises and measurement errors, and initial condition performance" [1]. The major advantages of this kind of technique is that it minimizes the estimation error in a well defined statistical sense with mathematical proofs. Moreover it utilizes all the observation data and the a priori knowledge of the system. The presumed disadvantages are its sensitivity to modeling errors of the system and its statistics and the computational intensity involved with the system estimator.

The three types of estimation techniques can be described as follows. If the time at which the estimated state is desired is exactly equal to the time at which the measurement is available, the problem is a filtering problem; if the time of estimation is after the measurement is available then it is a prediction problem; if the time of interest is before the measurement is available then it is a smoothing problem.

A popular filtering technique that is used for estimating the state of a linear system is the Kalman filter. Given a linear system model and its measurement model, statistical models which characterize the system and measurement errors and the initial conditions of the system, the Kalman filter describes how to process the measurements for obtaining the optimal estimates of the state of the system. But there are many limitations to the Kalman filter: it does not deal with the presence of uncertainties in the system model or the measurement model; it does give optimal measurement schedules; etc. This thesis is to robustify the filter in the presence of uncertainties in the process noise covariance and measurement noise covariance. But in order to do so, it is necessary to understand the performance of the Kalman filter. This chapter describes the design and implementation of the Kalman filter.

2.2 Least squares estimation

Let us assume that the *r* measurements, *y*, can be expressed as the linear combination of *n* element vector *x* and the measurement error vector *v*. The measurement error is a random, additive element to the measurement model. Thus the measurement process is modeled as

$$y = Cx + v$$

Here y is an $r \times 1$ vector, x is an $n \times 1$ vector, v is an $r \times 1$ vector and C is an $r \times n$ matrix. If $\hat{x}$ is defined as the estimate of x, we would want to minimize the sum of the squares of the error. This is termed as the *least squares estimator*. The cost function for this estimator can be given as follows

$$J = (y - C\hat{x})^T (y - C\hat{x})$$

The estimate $\hat{x}$ which minimizes this cost function is obtained by setting $\partial J / \partial \hat{x} = 0$. This results in

$$\hat{x} = (C^T C)^{-1} C^T y \tag{A}$$

If one needs to minimize the weighted sum of squares of deviations, the cost function would be,

$$J = (y - C\hat{x})^T W^{-1} (y - C\hat{x})$$

Here, W^{-1} is an $r \times r$ symmetric, positive definite matrix. Then the weighted least square estimates are given as

$$\hat{x} = (C^T W^{-1} C)^{-1} C^T W^{-1} y \tag{B}$$

The above mentioned *least squares estimation* is a deterministic process. There is no probabilistic interpretation involved with it. If the statistics of v are known, then the *maximum likelihood* method can be adopted to estimate x. This means maximizing the likelihood of the occurrence of the measurements y that actually occurred, given the chosen probability distribution model of v. This gives the conditional probability density function for y, conditioned on the given values of x, is the density of v centered around Cx. If v is assumed to be zero mean, Gaussian distributed noise with covariance matrix *R*, the conditional probability density function is given as

$$p(y \mid x) = \frac{1}{(2\pi)^{r/2} \mid R \mid^{1/2}} \exp\left[-\frac{1}{2} (y - Cx)^T R^{-1} (y - Cx) \right]$$

To maximize $p(y \mid x)$, the exponent in the braces has to be minimized. This is similar to *weighted least squares estimation*.

The other approach to obtain better estimates is *Bayesian estimation*.

In this case the statistical data of x is also available along with the statistics of v. Here the conditional probability distribution of x given all y is calculated. Bayes' theorem is given as follows

$$p(x \mid y) = \frac{p(y / x) p(x)}{p(y)}$$

where $p(x)$ is the probability density function of x and $p(y)$ is the probability density function of the measurements. The objective here is to find the minimum variance of the Bayes' estimate. If the objective was to maximize the probability that $\hat{x} = x$ and the probability density function $p(x)$ is uniform, then the solution is the maximum likelihood estimate. But as the objective now is to minimize the variance of Bayes' estimate, the cost function to be minimized is given as

$$J = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} . \quad . \quad . \int_{-\infty}^{\infty} (\hat{x} - x)^T S (\hat{x} - x) p(x \mid z) dx_1 dx_2 . \quad . \quad . dx_n$$

where S is a positive definite matrix. To minimize this cost function, we simply set $\partial J / \partial \hat{x} = 0$. This results in

$$\hat{x} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} . \quad . \quad . \int_{-\infty}^{\infty} x p(x \mid z) dx_1 dx_2 . \quad . \quad . dx_n = E[x \mid z]$$

Assuming Gaussian probability distribution functions for x and v, the estimate in the above equation can be simplified as follows

$$\hat{x} = (P_0^{-1} + C^T R^{-1} C)^{-1} C^T R^{-1} z$$

where P_0 is the a priori estimation error covariance of x

In the Bayes' estimation case if the estimation error covariance is large, i.e. P_0^{-1} is small, then the solution becomes equation (B). Moreover if the measurement errors are all uncorrelated in (B), i.e. W is a diagonal matrix and all of them have equal variance, i.e. $W = \sigma^2 I$, then equation (B) reduces to equation (A). This proves that identical results are obtained by all these methods as long as the assumptions are the same in each case [1].

It is clearly seen that the $\hat{x}$ is a linear operator on the measurement. It has also been proved in [2] that, for a Gaussian time varying signal, the minimum mean square error predictor is a linear predictor. Let us now consider the recursive form of the linear estimator.

2.3 Recursive least square estimation

A recursive filter is one in which we need not store past measurements for evaluating the present estimates. We need measurements at the present time to get the estimates at the present time. The concept of the recursive filter is best demonstrated from the following example.

Let us consider the problem of estimating a scalar constant, x, based on k noise corrupted measurements which are modeled as follows

$$z_i = x + v_i$$
$$i = 1, 2, ..., k$$

Here, v_k is the measurement noise which is assumed to be a white noise.

An unbiased, minimum variance estimate $\hat{x}_k$ can be obtained by averaging all the measurements which is given as:

$$\hat{x}_k = \frac{1}{k} \sum_{j=1}^{k} z_i$$

When an additional measurement is available, the new estimate is updated as

$$\hat{x}_{k+1} = \frac{1}{k+1}\sum_{j=1}^{k+1} z_i$$

In order to use this expression all the measurements before obtaining new measurement have to be stored. But manipulating this equation by including previous estimate, the need to store all the measurements can be avoided. This is shown as follows

$$\hat{x}_{k+1} = \frac{k}{k+1}\left(\frac{1}{k}\sum_{i=1}^{k} z_i\right) + \frac{1}{k+1} z_{k+1}$$
$$\hat{x}_{k+1} = \frac{k}{k+1}\hat{x}_k + \frac{1}{k+1} z_{k+1}$$
$$\hat{x}_{k+1} = \hat{x}_k + \frac{1}{k+1}\left(z_{k+1} - \hat{x}_k\right)$$

In the above equation to compute $\hat{x}_{k+1}$, the need to store all the previous measurements has been eliminated. Thus, we have a *recursive*, linear estimator. The term $(z_{k+1} - \hat{x}_k)$ is known as the *measurement residual* in the above equation. It can be easily understood from the last part of the above equation that the new estimate is given by the previous estimate plus a weighted difference between the new measurement and its expected value.

In the example we have dealt with only scalars. The process can be directly applied to vector quantities. Let us now consider the case with vector quantity. The vector notation for the measurements available is given as follows

$$z_k = H_k x + v_k$$

Here, x = constant unknown vector

z_k = kth measurement

H_k = Measurement matrix

v_k = measurement noise vector

The estimate at the present time is given as

$$\hat{x}_k = \hat{x}_{k-1} + K_k(z_k - H_k\hat{x}_{k-1})$$

where K_k is the estimator gain matrix that has to be determined and the term $(z_k - H_k\hat{x}_{k-1})$ is the *correction* term which is equivalent to the *measurement residual* in the scalar case.

Let the estimation error be defined as the difference between the unknown vector and the estimate at the present time given by

$$\varepsilon_{x_k} = x - \hat{x}_k$$

Taking the expected value of the estimation error we get,

$$\begin{aligned}
E\left(\varepsilon_{x_k}\right) &= E\left(x - \hat{x}_k\right) \\
&= E\left(x - \hat{x}_{k-1} - K_k(z_k - H_k\hat{x}_{k-1}\right) \\
&= E(x - \hat{x}_{k-1}) - K_k E(z_k - H_k\hat{x}_{k-1}) \\
&= E\left(\varepsilon_{x_{k-1}}\right) - K_k E(H_k x + v_k - H_k\hat{x}_{k-1}) \\
&= E\left(\varepsilon_{x_{k-1}}\right) - K_k E(v_k) - K_k H_k E(x - \hat{x}_{k-1})
\end{aligned}$$

$$E(\varepsilon_{x_k}) = (I - K_k H_k)E(\varepsilon_{x_{k-1}}) - K_k E(e_k)$$

This estimation can be termed as *unbiased* because the expected value of the estimate can be proved to be equal to the expected value of the actual unknown quantity.

This can proved with couple of assumptions.

$E(\varepsilon_{x_{k-1}}) = E(x - \hat{x}_{k-1}) = 0$ and $E(v_k) = 0$ which is a reasonable assumption that the average estimation error would be zero and the noise is also of zero mean.

Then it is straight forward that

$E(\hat{x}_k) = E(x)$ which proves the estimator to be unbiased.

Now, the cost function is defined as the estimation error covariance given by

$$
\begin{aligned}
J_k &= \frac{1}{2} E(\varepsilon_{x_k}^T \varepsilon_{x_k}) \\
&= \frac{1}{2} E\left[(x - \hat{x}_k)^T (x - \hat{x}_k) \right]
\end{aligned}
$$

But by definition the estimation error covariance matrix P_k is given as

$$P_k = E[(x - \hat{x})(x - \hat{x})^T]$$

Therefore,

$$J_k = \frac{1}{2} Tr(P_k)$$

where Tr denotes the trace of the matrix.

Now, P_k can be expressed as follows

$$
\begin{aligned}
P_k &= E(\varepsilon_{x_k} \varepsilon_{x_k}^T) \\
&= E\left[\left\{ (I - K_k H_k)\varepsilon_{x_{k-1}} - K_k e_k \right\} \left\{ (I - K_k H_k)\varepsilon_{x_{k-1}} - K_k e_k \right\}^T \right] \\
&= (I - K_k H_k) P_{k-1} (I - K_k H_k)^T + K_k R K_k^T
\end{aligned}
$$

where R is defined as the covariance of the measurement noise vector and is assumed to be zero mean.

The objective now would be to minimize the cost function with respect to the estimator gain, i.e., to evaluate the partial of the cost function with respect to the gain and set it equal to zero and then solve for the estimator gain.

$$\frac{\partial J_k}{\partial K_k} = \frac{1}{2}\left[\frac{\partial}{\partial K_k} Tr(I - K_k H_k)P_{k-1}(I - K_k H_k)^T + \frac{\partial}{\partial K_k} Tr(K_k R_k K_k^T)\right]$$
$$= \frac{1}{2}\left[2(I - K_k H_k)P_{k-1}(-H_k)^T + 2K_k R_k\right]$$

Now, this partial derivative has to be set equal to zero and solved which results in

$$K_k = P_{k-1} H_k^T (H_k P_{k-1} H_k^T + R_k)^{-1}$$

which is the recursive least square estimator gain which results in the minimum variance. This is the basic formula for the recursive least square estimator gain but many equivalent formulae can be found in various text books which can be obtained by performing simple linear algebra on the equation mentioned above.

The same results could be extended to non-linear systems. The non-linear systems have to be linearized and the change in the states from their nominal states will have to be estimated rather than the true states. It is obvious here that the estimation error will largely depend on the accuracy of linearization; i.e., the larger the error in linearization the greater will be the estimation error. In this study we discuss linear systems but the logic can be easily extended to non-linear systems which can be linearized satisfactorily.

2.4 Propagation of uncertainty

Till now we have considered only static systems. Let us now consider the case of dynamic systems. Let the linear, time variant system be represented as

$$x_k = A_{k-1} x_{k-1} + B_{k-1} u_{k-1} + \Lambda_{k-1} w_{k-1}$$

where w_k is a Gaussian random variable with zero mean and covariance Q_k.

However the initial condition x_0 is a Gaussian random variable prescribed by its mean value and covariance matrix as follows

$$E(x_0) = m_0$$
$$E[(x_0 - m_0)(x_0 - m_0)^T] = P_0$$

The expected value of the state at the present time is given as

$$\begin{aligned} E(x_k) = m_k &= E(A_{k-1}x_{k-1} + B_{k-1}u_{k-1} + \Lambda_{k-1}w_{k-1}) \\ &= A_{k-1}E(x_{k-1}) + B_{k-1}u_{k-1} \\ &\quad \text{since } E(w_{k-1}) = 0 \end{aligned}$$

The covariance of state perturbations can be propagated in a similar way. The outer product of the state perturbations at the present time is given as

$$\begin{aligned} (x_k - m_k)(x_k - m_k)^T &= [A_{k-1}(x_{k-1} - m_{k-1}) + B_{k-1}(0) + \Lambda_{k-1}w_{k-1}] \\ &\quad \cdot [A_{k-1}(x_{k-1} - m_{k-1}) + B_{k-1}(0) + \Lambda_{k-1}w_{k-1}]^T \end{aligned}$$

Taking the expected values on both the sides of the above equation,

$$\begin{aligned} E[(x_k - m_k)(x_k - m_k)^T] &= P_k \\ &= E[A_{k-1}(x_{k-1} - m_{k-1})(x_{k-1} - m_{k-1})^T A_{k-1}^T \\ &\quad + A_{k-1}w_{k-1}w_{k-1}^T A_{k-1}^T + A_{k-1}(x_{k-1} - m_{k-1})w_{k-1}^T \Lambda_{k-1}^T \\ &\quad + A_{k-1}w_{k-1}(x_{k-1} - m_{k-1})^T A_{k-1}^T] \\ &= A_{k-1}P_{k-1}A_{k-1}^T + \Lambda_{k-1}Q_{k-1}\Lambda_{k-1}^T \end{aligned}$$

This is an important equation as it relates as how the estimation error covariance propagates with time.

2.5 Discrete time Kalman filter

Keeping track of the previous sections in this chapter, the estimate of the state is specified by its conditional probability density function. The purpose of a filter is to compute the state estimate, while an optimal filter minimizes the spread of the estimation error probability density. A recursive optimal filter propagates the conditional probability density function from one sampling instant to the next, keeping in view the system dynamics and inputs, and it incorporates measurements and measurement error statistics in the estimate. Therefore, the recursive generation of the mean and covariance in finite time can be expressed as the following five steps.

1. State estimate extrapolation (Propagation)
2. Covariance estimation extrapolation (Propagation)
3. Filter gain computation
4. State estimate update
5. Covariance estimate update

The first two processes were discussed in the previous section and the last three procedures follow directly from the recursive weighted least squares estimation. Let us now look at how to put together all the pieces together and derive the Kalman filter equations.

Lets us consider the linear, time varying stochastic system represented by the following equations

$$x_k = A_{k-1}x_{k-1} + B_{k-1}u_{k-1} + \Lambda_{k-1}w_{k-1}$$
$$z_k = C_k x_k + n_k$$

The first equation of the set is the state equation where x is the state vector , u is the control vector. w is the process noise which is white, zero-mean Gaussian random sequence. The process noise statistics are given by

$$E(w_k) = 0$$
$$E(w_k w_k^T) = Q_k$$

The second equation of the set is the measurement model. z is the noisy measurement vector and n is the measurement noise which is also white, zero-mean Gaussian random sequence that is uncorrelated with the disturbance input. The measurement noise statistics are given by

$$E(n_k) = 0$$
$$E(n_k n_k^T) = R_k$$

In deriving the Kalman filter equations, results from the previous sections are used extensively. In the following equations, one must distinguish between the estimates made before the measurements are processed and after the measurements are processed. This also applies to the estimation error covariance matrix P. The distinction is illustrated with plus (+) and minus (−) superscripts to the terms respectively.

From the previous section the state estimate extrapolation is given by

$$\hat{x}_k^{(-)} = A_{k-1}\hat{x}_{k-1}^{(+)} + B_{k-1}u_{k-1}$$

while the covariance estimate extrapolation is given by

$$P_k^{(-)} = A_{k-1}P_{k-1}^{(+)}A_{k-1}^T + \Lambda_{k-1}Q_{k-1}\Lambda_{k-1}^T$$

The recursive mean-value estimator propagation provides the filter gain computation given by

$$K_k = P_k^{(-)}C_k^{(-)}[C_k P_k^{(-)}C_k^T + R_k]^{-1}$$

From the recursive least square estimation in the previous section the update equations after the measurements are processed are given as follows

$$\hat{x}_k^{(+)} = \hat{x}_k^{(-)} + K_k(z_k - C_k\hat{x}_k^{(-)})$$
$$P_k^{(+)} = (I - K_kC_k)P_k^{-}(I - K_kC_k)^T + K_kR_kK_k^T$$

These are the basic Kalman filter equations. Various other forms are seen in text books which can be derived applying linear algebra to the equations provided. Thus a Kalman filter is a minimum variance filter assuming the filter to be affine.

So we have now derived the basic equations of the highly celebrated Kalman filter. There are other forms of this filter like

1. Sequential KF
2. Information KF
3. Square root KF
4. UD KF

All these forms are slight variations to the basic KF based on the problem specification [5].

2.6 Nonlinear state estimation

Optimal state estimation is considered more difficult when the system contains nonlinear elements because the probability density functions of signals and noise are altered as they are transmitted through these elements [5]. In non-linear case the Gaussian inputs cause non-Gaussian response, and the criteria for optimal state estimation is not straight forward as in the linear case.

The Kalman filter can be used for non-linear state estimation but the condition for optimality is not retained in the exact same form as it was in the linear systems case. In this section two methods of dealing with the nonlinearities are discussed briefly. The first one is the linearized KF and the second is the extended KF.

Let a general non-linear system be represented as follows

$$\dot{x} = f(x, u, w, t)$$
$$z = h(x, t) + n(t)$$

where x is the state vector, u is the input vector, w and n are the process and measurement noises with covarainces Q and R respectively.

2.6.1 Kalman Filter for linearized systems

In this method the approach is to linearize the system about a nominal operating condition. Expanding the non-linear system using Taylor series and ignoring higher order terms, we get the following

$$\dot{x} \approx f(x_0, u_0, w_0, t) + \left.\frac{\partial f}{\partial x}\right|_{x_0, u_0, w_0} (x - x_0) + \left.\frac{\partial f}{\partial u}\right|(u - u_0) + \left.\frac{\partial f}{\partial w}\right|(w - w_0)$$
$$z \approx h(x_0, t) + \left.\frac{\partial h}{\partial x}\right|_{x_0} (x - x_0) + n(t)$$

where x_0, u_0, w_0 are known nominal values. The partial fractions in the above equations can be expressed as follows $F = \frac{\partial f}{\partial x}, G = \frac{\partial f}{\partial u}, L = \frac{\partial f}{\partial w}, H = \frac{\partial f}{\partial x}$. All these partials are evaluated at the nominal operating points. So the linearized equations can be given as

$$\dot{x} = f(x_0, u_0, w_0, t) + F(x - x_0) + G(u - u_0) + L(w - w_0)$$
$$z = h(x_0, t) + H(x - x_0) + n(t)$$

Let

$$
\begin{aligned}
f(x_0, u_0, w_0, t) &= \dot{x}_0 \\
x - x_0 &= \Delta x \\
u - u_0 &= \Delta u = 0 \\
w - w_0 &= \Delta w = W \\
\dot{x} - \dot{x}_0 &= \Delta \dot{x} \\
z - z_0 &= \Delta z
\end{aligned}
$$

Thus, the final linearized equations can be given as follows

$$
\begin{aligned}
\Delta \dot{x} &= F \Delta x + LW \\
\Delta z &= H \Delta x + n
\end{aligned}
$$

Now, a standard Kalman filter can be applied to the linearized system to estimate the change in the states. The update equations can be given as follows

$$
\begin{aligned}
\Delta \dot{\hat{x}} &= F \Delta x + K(\Delta z - H \Delta \hat{x}) \\
K &= PH^T R^{-1} \\
\dot{P} &= FP + PF^T + LQL^T - PH^T R^{-1} HP
\end{aligned}
$$

Once the perturbations in the states are estimated, the true state estimates can be obtained by summing the perturbation estimate and the nominal solution as shown

$$
\hat{x} = x_0 + \Delta \hat{x}
$$

This kind of state estimate is clearly not accurate but is an approximation based on the known nominal state trajectory. However, if the control perturbation and the measurement residual are kept small, the state estimate will be close to the true value. In the following chapters where health parameters of a gas turbine are to be estimated, the linearized KF is used.

2.6.2 Extended Kalman filter (EKF)

An improved state estimate can be obtained with no prior knowledge of a nominal trajectory. The EKF retains the linear calculation of the covariance and filter gain matrices, and it updates the state estimate using a linear function of the filter residual. However it uses the original non-linear equations for state propagation and output vector. The filter equations are given as follows

$$\dot{\hat{x}} = f(\hat{x}, u, t) + K\left[z - h(\hat{x}, t)\right]$$
$$K = PH^T R^{-1}$$
$$\dot{P} = FP + PF^T + LQL^T - PH^T R^{-1} HP$$

The important aspect to be noted in EKF is that the partial derivatives are evaluated at the current values of the state estimates and control inputs rather than their nominal values. These time varying matrices cannot be precomputed as they are functions of the state estimates. This makes the computational effort required for the update equations more tedious.

For both the *linearized* KF and EKF, improved state estimates could be obtained from a second order or higher order filter that retains more terms in Taylor series expansion [5]. But the computational effort also increases considerably. In general in most applications, *linearized* KF or EKF yields satisfactory results.

CHAPTER III

THE AIRCRAFT GAS TURBINE ENGINE AND ITS OPERATION

The objective of this thesis is to obtain optimal health parameter estimates for a gas turbine engine. Before getting into the estimation techniques, it is necessary to understand the physics of the gas turbine engine. This chapter discusses the operation and modeling of the gas turbine engine. Section 3.1 gives an overview of the gas turbine engine fundamentals. Section 3.1.1 looks into the historical perspective of gas engines. Section 3.1.2 outlines the jet propulsion theory. Section 3.1.3 looks into the operation of jet turbo engines and section 3.1.4 analyzes the types of jet engines. Section 3.2 looks into the software called Digital Computer Program for generating Dynamic Turbofan Engine Models (DIGTEM) which was developed by NASA. This is the main engine simulator that is used in this study. Section 3.3 looks into obtaining the small signal linearized model that is used in simulating the system to obtain the health parameter estimates.

3.1 Gas turbine engine fundamentals

The study of jet engines is a separate discipline on its own. But this section explains the fundamentals that are necessary to understand the rudiments of a jet engine. The following sections explain the operation and performance of the jet engine [3], [4], [16] .

3.1.1 Background

The progress of powered flight is quite an interesting field in the past century. But during this time period it became evident that without a lightweight and yet adequately powerful engine, controlled flight of sufficient distance to serve useful purpose was not possible. It was Germany's Dr. N.A.Otto who created the first four stroke internal combustion engine in 1876. Twenty years later Daimler perfected the eight horsepower engine which enabled the making of first gasoline powered dirigible flight. The famous duo of the Wright brothers had to develop their own flight to achieve the first recorded successful flight in 1903. So it has gone down in aviation industry that larger and more efficient aircrafts lead to faster and safer aircrafts.

The first aircrafts were powered by piston engines that turned a propeller. Yet the use of jet propulsion to carry an aircraft aloft precedes the piston engine by hundreds of years. Many historians agree that the Chinese used 'propulsion theory' to use rockets as early as the 13th century. These early Chinese experimenters were the first people to discover the principle of jet thrust, that is, that a stream of hot gases exiting from one end of a tube could generate a 'push.' The same principle is the basis for today's jet engines.

3.1.2 Jet Propulsion Theory

Briefly, jet propulsion is the propelling force generated in the direction opposite to the flow of the mass of gas or liquid under pressure which is escaping through a hole or opening called a jet nozzle. The thrust that sends a rocket upwards is a classic example of jet propulsion. Whatever the form of apparatus utilizing jet propulsion, the device is essentially a reaction engine that operates through the practical application of the laws of motion that were first stated by the English physicist Sir Isaac Newton.

The first-of-its-kind reaction engine was believed to be built in Alexandria by Heron around 250 B.C. The reaction engine consisted of a sphere into which steam under pressure was introduced. When the steam escaped from two bent tubes which essentially formed the jet nozzles, the sphere rotated about its own axis. A pulley and ropes were attached to the sphere to pull open temple doors. The same principle was later used by Newton to move a wagon propelled by a steam jet.

Although the history of gas turbine propulsion goes back much further into the 18th century, jet propelled aircrafts became practical in the mid 1940's. Sir Frank Whittle of Great Britain and Han von Ohain of Germany were the first people to design applications of the jet propulsion principle that could be used to power an airplane.

In a basic jet engine both the piston engine and the gas turbine develop thrust by burning a combustive mixture of fuel and air. Both convert the energy of the expanding gases into propulsive force. The piston engine does this by changing the energy of combustion into mechanical energy which is used to turn a propeller. Aircraft propulsion is obtained as the propeller imparts a relatively small acceleration to a large mass of air. The gas turbine, in its basic turbo jet configuration, imparts a relatively large acceleration to a small mass of air and thus produces thrust or propulsive force directly.

The simplest gas turbine engine for aircraft is a turbojet. A can-like horizontal container which is open at both ends and is called an engine case encloses the internal parts. Large quantities of air enter the front of the case. After the air is largely heated and accelerated by being burned with fuel, the air that remains and the gases produced by the combustion are exhausted through the rear opening. A rotating compressor is located in the forward section of the engine case. The compressor is followed by the combustion chamber, burner section or combustor. Next are the driving turbines which develop the power needed to operate the compressor. Last, there is the engine tail pipe or exhaust duct. The opening to the outside air at the rear of the pipe is called the jet nozzle.

The compressor raises the pressure and temperature of the incoming air before passing it on to the combustion chamber. Fuel is sprayed through nozzles into the front of the chamber. The resulting mixture of fuel and air is burned to produce hot, expanding gases that rush into the turbine section, causing the turbine rotors to rotate. On leaving the turbine section, the gases are expelled to the outside air through the exhaust duct and the jet nozzle. The power that the turbine rotors extract from the gases is used to drive the compressor. Since the turbine rotors and the compressor are both mounted on the same shaft, they operate as a unit. Thus, it might be said that a single compressor turbojet engine has only one major moving part. The other members of the gas turbine engine family such as dual compressor (twin spool) engines, turbo fans, turbo props and turbo shaft engines are different versions of the basic single compressor turbojet.

3.1.3 Operation of turbojet engine

A turbojet engine is essentially a machine designed for the sole purpose of producing high velocity gases at the jet nozzle. The engine is started by rotating the compressor with a starter, and then igniting a mixture of fuel and air in the combustion chamber with one or more igniters. When the engine has started and its compressor is rotating at sufficient speed, the starter and the igniter are turned off. The engine will then run without further assistance as long as fuel and air in proper proportions continue to enter the combustion chamber and burn.

The main reason for the operation of the turbojet engine is its compressor. The gases created by a fuel and air mixture burning under normal atmospheric pressure do not expand enough to do commendable work. Air under pressure must be mixed with fuel before gases produced by combustion can be used to make a turbojet engine operate. The more air that an engine can compress and use the greater is the power it can deliver. Finding a way to accomplish the compressing of air was the biggest challenge for designers during the early years of turbojet engine development. High power is necessary to drive the compressor in a turbojet engine. Workable gas turbine engines would have been developed sooner if anyone had known how to build a turbine that would produce sufficient power to turn the compressor and yet leave enough energy in the exhaust gases to push an airplane. Superior compressor and turbine combinations eventually led to successful engines. To indicate how much power is absorbed by a compressor of a moderately large turbojet, let us assume that we have an engine with about a 12:1 compression ratio that produces 10,000 pounds of thrust for takeoff. In this engine, the turbine has to produce approximately 35,000 shaft horsepower just to drive the compressor when the engine is operating at full thrust.

3.1.4 Types of jet engines for aircraft

The types of jet engines can be classified according to the design and internal arrangement of their component parts. This section briefly discusses turbojets, turbo props and turbofans. The engine type used in this thesis study is the turbofan.

Turbojet

If an aircraft engine gas turbine engine uses only the thrust developed within the engine to produce its propulsive force, it is known as a turbojet. As they have no added features such as a fan, propeller or free turbine, turbojets are sometimes referred to as straight jets. There are two kinds of turbojets: the centrifugal flow compressor type and the axial flow compressor type. Either may have one or more compressors and some engines use both a centrifugal compressor and an axial flow compressor.

Turboprop

When the exhaust gases from the basic part of a turbojet are used to rotate an additional turbine that drives a propeller through a speed-reducing gear system, the engine becomes a turboprop or propjet. In some turboprops, an extra turbine stage is incorporated in the engine assembly that rotates the compressor. The additional power that is produced drives the propeller reduction gearing directly from the compressor drive shaft. Engines of this type are known as direct-drive turboprops. A free turbine is incorporated in most modern turboprops. This turbine is independent of the compressor drive turbines and is free to rotate by itself in the engine exhaust gas stream. The shaft on which the free turbine is mounted drives the propeller through the propeller reduction gear system.

Although the turboprop aircraft is more complicated and heavier than a turbojet engine of equivalent size and power, it will deliver more thrust at low subsonic speeds. However the advantage decreases as the flight speed increases. In normal cruising speed ranges the propulsive efficiency of a turboprop decreases as speed increases whereas it is the other way around for a turbojet engine. The spectacular performance of a turboprop during takeoff and climb is the result of the ability of the propeller to accelerate a large mass of air while the aircraft is moving at relatively low ground and flight speed.

Turbofan

In principle a turbofan is much like a turboprop except that the ratio of secondary air flow (the airflow through the fan or propeller) to the primary airflow through the basic engine is less. This is called the bypass ratio. Also, in the turbofan, the gear driven propeller is replaced by an axial flow fan with rotating blades and stationary vanes which are considerably larger, similar to the blades and vanes of an axial flow compressor. The fan is enclosed in a duct. In a turbofan, the fan makes a substantial contribution to the total thrust. Over and above the thrust develop by the basic engine, the fan accelerates the air passing through it, similar to the function of the propeller of a turboprop. The fans of turbofan engines produce between 30 and 75 percent of the total thrust, the actual amount depending principally upon the bypass ratio.

In a turbofan engine, after the secondary air leaves the fan, it does not pass through the basic engine for burning with fuel. The fan discharge air may be exhausted into the outside air through a fan jet-nozzle soon after it leaves the fan or it may be carried rearward by an annular fan discharge duct which surrounds the basic engine for its full length.

The turbofan combines the good operating efficiency and high thrust capability of a turboprop and the high speed, high altitude capability of a turbojet. The complexity and weight of the propeller reduction gearing and the intricate propeller governing system in a turboprop is completely eliminated in a turbofan. A turbofan is therefore not only lighter than a turboprop but is less complex. One fundamental difference between a turbofan and a turboprop is that the airflow through the fan is controlled by the design of the engine air inlet duct in such a manner that the velocity of the air through the fan blades is not greatly affected by the speed of the aircraft. In the modern jet engine scenario turbofan engines with afterburners are used for supersonic planes that attain speeds in excess of Mach 2. When compared with a turbojet of equal thrust, the turbofan has the advantage of a lower noise level for the engine exhaust, which is an important feature at all commercial airports. The lower level of noise occurs because a turbofan engine has at least one additional turbine stage to drive the fan. Extraction of more power from the engine exhaust gases as they pass through the additional turbine or turbines reduces the velocity of the engine exhaust. Less velocity through the jet nozzle results in less noise.

For the reasons mentions above and due to its lower fuel consumption, the modern turbofan engine has taken over completely as the most widely used engine for all conventional large aircraft, both military and commercial.

3.2 Digital Computer Program for generating Dynamic Turbofan Engine Models (DIGTEM)

This section describes DIGTEM, a digital computer program that simulates two-spool, two-stream turbofan engines [16]. The turbofan engine model in DIGTEM contains steady state performance maps for all of the components and has control volumes where continuity and energy balances are maintained. Rotor dynamics and duct momentum dynamics are also included. Altogether there are 16 state variables and state equations in DIGTEM. DIGTEM features a backward difference integration scheme for integrating stiff systems. It trims the model equations to match a prescribed design point by calculating correction coefficients that balance out the dynamic equations. It uses the same coefficients at off design points and iterates to balanced engine conditions.

DIGTEM is generalized in the aero-thermodynamic treatment of components. This feature along with its trimming calculations at a design point makes it a very useful tool for developing models of specific engines having the same two spool, two stream configuration. Also subsets of the turbofan engine configuration such as a turbojet or a turbo shaft can be simulated with minor modifications to the Fortran coding. With extensive modifications to the coding, arbitrary configurations can be modeled.

The development of aircraft propulsion systems depends, to a great extent, on being able to predict the performance of the propulsion system and its associated controls. Computer simulations provide the means for analyzing the behavior and interactions of these complex systems prior to the building and testing of expensive hardware. Simulations can also serve as aids in understanding and solving problems that arise after the propulsion system is developed. Computer simulations can be either generalized or specific to a particular propulsion system. Generalized simulations are desirable in that they allow for paper studies of many different engine configurations. Many generalized digital engine simulations exist today. Most of them are limited to steady state performance calculations for a fixed number of engine configurations. Most of them are reviewed in detailed in the literature [3-4]. DIGTEM is the most generalized turbofan engine simulator developed. This model is used in this study and is supplied by the NASA Glenn Research Center in Cleveland [16].

The engine model supplied with DIGTEM represents a two spool, two stream augmented turbofan engine. Figure 3.1 shows a schematic representation of this type of engine.

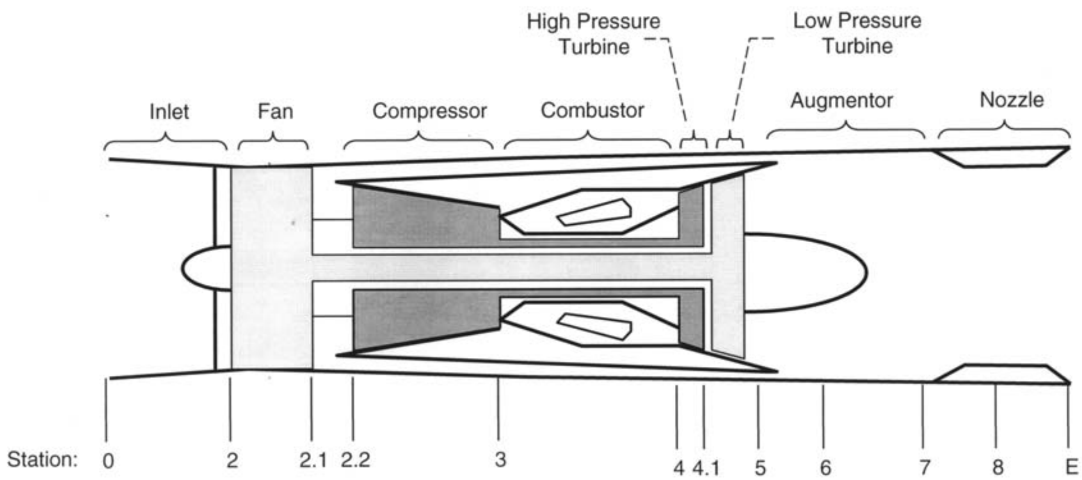


Figure 3.1 Two spool, two stream turbofan engine

A single inlet is used to supply airflow to the fan. Air leaving the fan is separated into two streams - one passing through the engine core and another passing through an annular bypass duct. The fan is driven by a low pressure turbine. The core airflow passes through a compressor that is driven by a high pressure turbine. Both the fan and compressor are assumed to have variable geometry for better stability at low speeds. Engine airflow bleeds are extracted at the compressor exit and used for turbine cooling. Fuel flow is injected in the main combustor and burned to produce hot gas for driving the turbines. The engine core and bypass streams combine in an augmentor duct, where the flows are assumed to be thoroughly mixed. Additional fuel is added to further increase the gas temperature and thrust. The augmentor flow is discharged through a variable convergent-divergent nozzle area. The nozzle throat area and exhaust nozzle area are varied to maintain engine airflow and to minimize drag during augmentor operation. The analytical model includes multivariate maps to model the steady-state performance of the engines rotating components. Fluid momentum in the bypass duct and the augmentor, mass and energy storage within control volumes, and rotor inertias are included in the model to provide transient capability.

The integration technique used in DIGTEM is a backward-difference integration scheme that is well suited for integrating "stiff" systems. A typical engine model will have time constants that differ by three or four orders of magnitude. This requires the use of very small time steps when using forward difference integration scheme to insure stability. The backward difference scheme uses a multi variable Newton-Raphson iteration method for convergence at each time point. In DIGTEM the iteration variables correspond to the state variables. The sixteen state variables that are used to describe the engine model in DIGTEM are given as follows.

1. Low Pressure Turbine (LPT) rotor speed
2. High Pressure Turbine (HPT) rotor speed
3. Compressor volume stored mass
4. Combustor inlet temperature
5. Combustor volume stored mass

6. HPT inlet temperature
7. HPT volume stored mass
8. LPT inlet temperature
9. LPT volume stored mass
10. Augmentor inlet temperature
11. Augmentor volume stored mass
12. Nozzle inlet temperature
13. Duct airflow
14. Augmentor airflow
15. Duct volume stored mass
16. Duct temperature

The turbofan measurements are given as follows.

1. LPT rotor speed
2. HPT rotor speed
3. Duct pressure
4. Duct temperature
5. Compressor inlet pressure
6. Compressor inlet temperature
7. Combustor inlet pressure
8. Combustor inlet temperature
9. LPT inlet pressure
10. LPT inlet temperature
11. Augmentor inlet pressure

12. Augmentor inlet temperature

The turbofan controls are given as follows.

1. Combustor fuel flow
2. Augmentor fuel flow
3. Nozzle throat area
4. Nozzle exit area
5. Fan vane angle
6. Compressor vane angle

3.3 Turbofan engine health monitoring

The performance of gas turbine engines deteriorate over time. This deterioration can affect the fuel economy, and impact emissions, component life consumption and thrust response of the engine. Airlines periodically collect engine data in order to evaluate the health of the engine and its components. The health evaluation is then used to determine the maintenance schedules. Reliable health evaluations are used to anticipate future maintenance needs. This offers the benefits of improved safety margins and reduced safety margins. The money saving potential of such evaluations is substantial, but only if the evaluations are reliable. The data used to perform health evaluations is collected during flight and later transferred to ground based computers for post flight analysis. Data is collected each flight at approximately the same engine operating conditions and corrected to account for variability in ambient conditions and power setting levels. Typically data is collected for about 3 seconds at about 10 or 20 Hz for each flight.

The main aim of this work is to estimate engine component efficiencies and flow capacities which are referred to as health parameters.

The health parameters that are to be estimated are given as follows.

1. Fan airflow
2. Fan efficiency
3. Compressor airflow
4. Compressor efficiency
5. HPT airflow
6. HPT enthalpy change
7. LPT airflow
8. LPT enthalpy change

We can use our knowledge of the physics of the turbofan engine in order to obtain the dynamic model using DIGTEM. The health parameters that we are trying to estimate can be modeled as slowly varying biases. The state vector of the dynamic model is augmented to include the health parameters, which are then estimated using a Kalman filter.

The nonlinear, time-invariant equations used in DIGTEM can be summarized as follows

$$\dot{x} = f(x,u,p) + w(t)$$
$$y = g(x,u,p) + e(t)$$

x is the 16 element state vector, u is the 6 element control vector, p is the 8 element vector of health parameters and y is the 12 element vector of measurements. The noise term *w(t)* represents the inaccuracies in the model and *e(t)* represents the measurement noise. This equation can be linearized about the nominal operating point by using the first order approximation of the Taylor series expansion. This gives a linear small signal system model defined for small excursions from the nominal operating point.

$$\delta \dot{x} = A_1 \delta x + B \delta u + A_2 \delta p + w(t)$$
$$\delta y = C_1 \delta x + D \delta u + C_2 \delta p + e(t)$$

We note that

$$A_1 = \frac{\partial f}{\partial x}$$
$$A_1(i,j) \approx \frac{\Delta \dot{x}(i)}{\Delta x(j)}$$

Similar equations hold for A_2, C_1 and C_2 matrices. These matrices were obtained using numerical approximations by varying x and p from their nominal values (one element at a time) and recording the new $\dot{x}$ and y vector in DIGTEM.

The goal of this thesis is to obtain an accurate estimate of δp, which varies slowly with time. It is therefore assumed that δp remains essentially a constant during a single flight. We also assume that the control input is known, so $\delta u = 0$ .This will give us the following discrete time system.

$$\delta x_{k+1} = A_{1d} \delta x_k + A_{2d} \delta p_k + w_k$$
$$\delta y_k = C_1 \delta x_k + C_2 \delta p_k + e_k$$

where $A_{1d} = \exp(A_1 T)$ and $A_{2d} = A_1^{-1}(A_{1d} - I)A_2$

We next augment the state vector with the health parameters vector to obtain the system equation as

$$\begin{bmatrix} \delta x_{k+1} \\ \delta p_{k+1} \end{bmatrix} = \begin{bmatrix} A_{1d} & A_{2d} \\ 0 & I \end{bmatrix} \begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + \begin{bmatrix} w_{1k} \\ w_{2k} \end{bmatrix}$$

$$\delta y_k = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + e_k$$

where w_{2k} is a small noise term uncorrelated with w_{1k} that represents model uncertainty and allows the Kalman filter to estimate time-varying health parameter variations. The discrete time small signal model can be written as follows

$$\begin{bmatrix} \delta x_{k+1} \\ \delta y_{k+1} \end{bmatrix} = A \begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + w_k$$

$$\delta y_k = C \begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + e_k$$

where the definitions of A and C are apparent from previous discussion. Now we can use Kalman filter to estimate δx_k and δp_k. Actually, we are only interested in estimating δp_k (the health parameter deviations), but the Kalman filter gives us the bonus of also estimating δx_k (the excursions of the original turbofan state variables).

CHAPTER IV

ROBUST FILTERING

In this chapter the robustness of Kalman filtering against uncertainties in process and measurement noise covariances is discussed. It is shown that a standard Kalman filter may not be robust enough if the process and measurement noise covariances are changed. A new filter is proposed which addresses the uncertainties in process and measurement noise covariances and gives better results than the standard Kalman filter. This new filter is used in simulation to estimate the health parameters of an aircraft gas turbine engine.

The next section gives an overview of the important approaches that have been proposed in the field of robust Kalman filtering. The remaining sections focus on the general state estimation problem, the presence of uncertainties in the process and measurement noise covariances of the system, and a new way to design a robust Kalman filter.

4.1 Approaches to robust Kalman filtering

For the sake of discussion in this section the Kalman filter equations are briefly reviewed here.

The equations for a stochastic discrete time system are given as follows.

$$
\begin{aligned}
x_{k+1} &= Ax_k + B_u u_k + B_w w_k \\
y_k &= Cx_k + v_k
\end{aligned}
$$

where x is the system state, y is the measurement, w and v are the process and measurement noises with zero mean and covariances of Q and R respectively. The Kalman filter estimator equations are given as

$$
\begin{aligned}
K_k &= P_k C^T (CP_k C^T + R)^{-1} \\
\hat{x}_{k+1} &= A\hat{x}_k + B_u u_k + K_k (y_k - C\hat{x}_k) \\
P_{k+1} &= (I - K_k C) AP_k A^T + Q
\end{aligned}
$$

where K is the Kalman gain, P is the estimation error covariance and $\hat{x}$ is the a priori state estimate.

4.1.1 Four basic approaches to adaptive filtering

Mehra proposes four different methods of adaptive filtering where uncertainties in noise statistics are considered [6]. He discusses ways of estimating the noise characteristics online based on the available data. He also proposes estimating the optimal Kalman gain without estimating the covariances of the process and measurement noise. He categorizes adaptive filtering into four categories: Bayesian, maximum likelihood, correlation, and covariance matching.

Bayesian Estimation

This method is for obtaining better state estimates in the presence of uncertainties but not obtaining a better Kalman gain. Mehra considers uncertainties in the system model as well as Q and R but emphasizes uncertainties in Q and R. If α is a vector of unknown parameters in the system and A is the set of all α's, in this approach to obtain better state estimates it is required to perform integrations over the high dimensional space A. This process can be very time consuming and may lead to an exhaustive search based on the problem.

Maximum Likelihood Estimation

In this method, Mehra comes up with an algorithm to estimate the optimal value of the steady state Kalman gain. He makes certain assumptions. For instance, the system has to be time invariant, the system has to be completely controllable and observable, and the estimator gain has reached a steady state value. In this method Mehra does not use the estimate of K to come up with the state estimates directly. Instead he uses the estimate of K to estimate Q and R. With this method there is a chance of uncertainty being added after every iteration. That is, the error in K may lead to a larger error in Q or R, which in turn may lead to larger errors in the state estimates.

Correlation Methods

Correlation methods have been in use for a long time for estimation in time series. The main idea of correlation methods is to correlate the output of the system with a known autocorrelation either directly or by performing certain linear operations on it [2].

These methods are mainly applicable to systems with constant coefficients or LTI systems and the system has to be completely controllable and observable. In this method an analytical solution for K can be calculated. The main drawback of this method is that either the autocorrelation of the measurements or the autocorrelation of the innovations have to be known a priori.

Covariance Matching

The main idea in this method is to make the true values of the residuals match their theoretical covariances. This is a way of estimating the Q and R matrices rather than realizing a better Kalman gain. This method is more suitable for cases in which Q is known but R is unknown.

4.1.2 Bounded covariance estimation

Lihua Xie and Yeng Chai Soh wrote a paper in which the state estimation problem for linear systems with parameter uncertainty in both the state and output equations is considered [7]. In this method a state estimator is designed such that the estimation error covariance will have a bound for all admissible uncertainties. In this method the state estimation error is augmented to the state vector and the model is so posed that it has zero mean process noise with identity covariance matrix. The drawback of this method is the stability of the estimator after the error is augmented to the state vector. The stability of the estimator has not been discussed. In this work only uncertainties in system model were considered.

4.1.3 Optimal Guaranteed Cost Control and Filtering for Uncertain Linear Systems

Ian R. Petersen and Duncan C. McFarlane wrote a paper about designing robust state feedback controllers and steady state robust state estimators for uncertain linear systems with norm bounded uncertainties [8]. The main approach adopted in this paper is to develop an optimal state feedback quadratic guaranteed cost control. Then an optimal guaranteed cost state estimator is proposed which is based on duality with the quadratic guaranteed cost control. Using this technique the authors come up with an upper bound on the estimation error covariance using fixed quadratic Lyapunov function. The uncertainties considered in the derivation are assumed to be additive uncertainties in the state space model of the system. It is assumed that the uncertainties are less than a known bound. The Riccati equation method proposed for guaranteed cost assumes that there exists a constant greater than zero which makes the closed loop control stable.

4.1.4 Fuzzy Neural Network Aided Adaptive Extended Kalman Filtering for GPS navigation

Dah-Jing Jwo and Hung-Chih Huang wrote a paper in which a fuzzy method combined with neural networks is proposed to identify the noise covariance matrices of a system [9]. In this method the innovations produced by the Kalman filter are used as the inputs to a fuzzy neural network (FNN) and the outputs of the FNN are the noise spectral strengths. The FNN is trained offline using a steepest descent technique. A back propagation algorithm is used to acquire the fuzzy inference rules and tune the membership functions but this requires data of the system from which the rules can be inferred.

4.1.5 Robust, Reduced-Order, Nonstrictly Proper State Estimation via the Optimal Projection Equations with Guaranteed Cost Bounds

Wassim M. Haddad and Dennis S. Bernstein devised a method in which a robust state estimator over a range of parameter uncertainties is considered. The main idea is to bound the effect of uncertain parameters on the estimation error over the uncertainty range and then choose estimator gains to minimize the estimation bound [10]. It should however be noted that a bound on the estimation error is minimized rather than the estimation error itself. Additive uncertainty is considered in this paper.

4.1.6 Guaranteed Error Estimation in Uncertain Systems

Bijendra N. Jain wrote a short paper in which an estimator for uncertain systems with a bound on the uncertainty is modeled using fuzzy dynamic programming. The uncertainties are considered to be present in the initial state, additive plant disturbances, and measurement errors [11]. It is assumed that the uncertainties are unknown but bounded. The problem is first formulated for nonlinear systems and then the results are applied to a linear system. It is shown that such an estimator reduces to an optimal filter for linear systems with only additive disturbances but no uncertainties.

4.1.7 Robust Nonfragile Kalman Filtering for Uncertain Systems with Estimator Gain Uncertainty

Guang-Hong Yang and Jian Liang Wang wrote a paper that is concerned with the problem of robust nonfragile Kalman filter design for uncertain linear systems with norm-bounded uncertainties [12]. The uncertainties considered here are in the plant model and the estimator gain. In this paper robustness is a concern due to uncertainties in the plant and the fragility relates to the inaccuracies in the implementation of the designed estimator. In this paper multiplicative uncertainties in the gain matrix are considered. A Riccati equation method is proposed to guarantee an upper bound on the steady-state error covariance matrix. This method assumes that there exits two positive constants which make the Riccati equation solutions exist for the nonfragile robust Kalman filter. In an example given it is stated that they realized those constants to come up with an optimal guaranteed cost but it is not mentioned how they achieved them.

4.2 State estimation problem

This analysis is based on [13], which applies to continuous time systems, and is extended in this thesis to discrete time systems and applied to aircraft gas turbine engine health estimation.

Consider a linear stochastic system represented by

$$\begin{aligned} x_{k+1} &= Ax_k + B_u u_k + B_w w_k \\ y_k &= Cx_k + v_k \end{aligned} \tag{1}$$

Here x is the system state vector, y is the measurement vector, u is the input vector, w is the process noise vector and v is the measurement noise vector. A, B_u, B_w and C are matrices of appropriate dimensions. w and v in this case are assumed to be mutually independent and zero mean white noise. The covariances of w and v are given as

$$\begin{aligned} E[w_k w_k^T] &= Q \\ E[v_k v_k^T] &= R \end{aligned} \tag{2}$$

The state estimate equations before and after the measurements are processed are given as

$$\begin{aligned}\hat{x}_{k+1}^{-} &= A\hat{x}_{k}^{+} + B_u u_k \\ \hat{x}_{k+1}^{+} &= \hat{x}_{k+1}^{-} + K_k(y_{k+1} - C\hat{x}_{k+1}^{-})\end{aligned} \tag{3}$$

Where K_k is the Kalman filter gain.

The estimation error is defined as follows:

$$e_{k+1} \equiv x_{k+1} - \hat{x}_{k+1}^{-}$$

From Equations (1) and (3) the estimation error satisfies the equation

$$e_{k+1} = (A - AK_k C)e_k + B_w w_k - AK_k v_k \tag{4}$$

Using the noise characteristics in Equation (2) the steady state error covariance P becomes the solution to the following equation:

$$P = (A - AKC)P(A - AKC)^T + B_w Q B_w{}^T + (AK)R(AK)^T \tag{5}$$

Where P is defined as

$$P = E[ee^T] \tag{6}$$

When $R = 0$ (no measurement noise), Equation (5) becomes

$$X_1 = (A - AKC)X_1(A - AKC)^T + B_w Q_k B_w{}^T \tag{7}$$

Where X_1 is the estimation error covariance due to process noise only.

When $Q = 0$ (no process noise), Equation (5) becomes

$$X_2 = (A - AKC)X_2(A - AKC)^T + (AK)R(AK)^T \tag{8}$$

Where X_2 is the estimation error covariance due to observation noise only.

Adding Equations (7) and (8) gives the following:

$$(X_1 + X_2) = (A - AKC)(X_1 + X_2)(A - AKC)^T + B_w Q B_w{}^T + (AK)R(AK)^T \tag{9}$$

This shows that when Q and R are not zero at the same time, the solution P of Equation (5) becomes:

$$P = X_1 + X_2 \tag{10}$$

This is the estimation error covariance in the presence of both the process and measurement noise. Thus, it is shown to be a linear combination of the estimation error covariances when only one of the noises is present.

This linear combination helps in realizing the performance index of the Kalman Filter, which would be a linear combination of functions of X_1 and X_2.

Therefore, in the standard Kalman filter, the filter gain K minimizes the following performance index:

$$J = tr[E(e_k e_k^T)] = tr(P) = tr(X_1) + tr(X_2) \tag{11}$$

where tr( ) denotes the trace of a matrix. If there are no uncertainties in the process and measurement noise covariances the performance index J attains a global minimum using the standard Kalman filter. But if there were uncertainties in Q and R, J would not attain a minimum. Let us now consider the case where there are uncertainties in Q and R.

4.3 Robustness analysis of the Kalman filter

This section considers variations in the process and measurement noise covariances. The covariance matrices of the process noise and measurement noise are assumed to change from nominal covariances Q, R to $\tilde{Q}, \tilde{R}$ using scalars α, β as follows:

$$\tilde{Q} = (1+\alpha)Q \tag{12}$$

$$\tilde{R} = (1+\beta)R \tag{13}$$

where α, β are random variables. α, β are assumed to be zero mean and uncorrelated. The estimation error P changes to $\widetilde{P}$ when the noise covariances change from Q to $\widetilde{Q}$ and R to $\widetilde{R}$. If $\widetilde{P} = P + \Delta P$ is substituted in Equation (5),

$$\begin{aligned}(P + \Delta P) = (A - AKC)(P + \Delta P)(A - AKC)^T + \\ (1+\alpha)B_w Q_k B_w^{\ T} + (1+\beta)(AK)R_k(AK)^T\end{aligned} \tag{14}$$

$\widetilde{P}$ is the sum of the solution of the following two equations corresponding to $R = 0$ and $Q = 0$ respectively.

$$\tilde{X}_1 = (A - AKC)\tilde{X}_1(A - AKC)^T + \alpha B_w Q_k B_w^{\ T} \tag{15}$$

$$\tilde{X}_2 = (A - AKC)\tilde{X}_2(A - AKC)^T + \beta(AK)R(AK)^T \tag{16}$$

Comparing with (7) and (8) we can see that,

$$\tilde{X}_1 = \alpha X_1, \tilde{X}_2 = \beta X_2 \tag{17}$$

where X_1 and X_2 are the solutions of Equations (7) and (8). Then the change in the estimation error covariance ΔP is described using X_1 and X_2 as follows:

$$\Delta P = \alpha X_1 + \beta X_2 \tag{18}$$

Then the variation of the performance index is as follows:

$$\Delta tr(P) = tr(\Delta P) = \alpha\, tr(X_1) + \beta\, tr(X_2) \tag{19}$$

Let us now consider the sensitivity of the performance index to α, β. Here α, β are random variables expressing uncertainties of noise covariances as follows:

$$E\{\alpha\} = E\{\beta\} = 0, E\{\alpha\beta\} = 0 \tag{20}$$

$$E\{\alpha^2\} = \sigma_1^2, E\{\beta^2\} = \sigma_2^2 \tag{21}$$

From the above characteristics, the mean of the change of the performance index is given as

$$E\{\Delta tr(P)\} = E\{\alpha\} tr\{X_1\} + E\{\beta\} tr(X_2) = 0 \tag{22}$$

So the mean of the variation of the performance index is zero. The variance of the change of the performance index becomes

$$\begin{aligned} Var\{\Delta tr(P)\} &= E\{(\alpha\, tr(X_1) + \beta\, tr(X_2))^2\} \\ &= \sigma_1^2 (tr(X_1))^2 + \sigma_2^2 (tr(X_2))^2 \end{aligned} \tag{23}$$

Considering (22) and (23), if we minimize the variance of the change of the performance index, we make the filter robust to changes in Q and R.

For robustness we would like to have the performance index for the filter with uncertain Q and R to be Equation (23). But we want to balance the performance of the estimator under nominal conditions (nominal Q and R) with the performance of the estimator under off-nominal conditions (off-nominal Q and R). In order to achieve this balance, we want to have the performance index be a combination of the standard cost function and the variance of the change in the standard cost function.

From this discussion, the performance index for the robust Kalman filter can be given as:

$$J = \rho_1 \{ tr(X_1) + tr(X_2) \} + \rho_2 \{ \sigma_1^2 (tr(X_1))^2 + \sigma_2^2 (tr(X_2))^2 \} \tag{24}$$

where ρ_1 and ρ_2 provide relative weighting to nominal performance and robustness. This results in a new Kalman gain to minimize the new performance index. The robust Kalman filter is developed with the steady state gain of the standard Kalman filter and using the hybrid gradient descent algorithm a new Kalman gain is realized which minimizes the new performance index. Using this new Kalman gain the estimation error will be found to be less than the standard Kalman filter when there is an uncertainty in the noise covariances.

The hybrid gradient descent algorithm to realize the gain can be summarized as follows

To find the minimum of new $J, \frac{\partial J}{\partial K}$ has to be found. But $\frac{\partial J}{\partial K}$ cannot be found analytically as J is not an explicit function of K. J is an analytical function of X_1 and X_2 i.e. $J= f(X_1, X2)$ and X_1 and X_2 are numerical functions of K.

Therefore $$\frac{\partial J}{\partial K} = \frac{\partial J}{\partial X_1}\frac{\partial X_1}{\partial K} + \frac{\partial J}{\partial X_2}\frac{\partial X_2}{\partial K} \tag{25}$$

$\frac{\partial J}{\partial X_1}$ and $\frac{\partial J}{\partial X_2}$ given analytically which are given as

$$\begin{aligned} \frac{\partial J}{\partial X_1} &= \rho_1 I + 2\rho_2 \sigma_1 tr(X_1) I, \\ \frac{\partial J}{\partial X_2} &= \rho_1 I + 2\rho_2 \sigma_1 tr(X_2) I \end{aligned} \tag{26}$$

where I stands for the identity matrix of appropriate dimension. $\frac{\partial X_1}{\partial K}, \frac{\partial X_2}{\partial K}$ are computed numerically. The calculation of these partial derivatives is complex as the numerator and denominators are both matrices. In order to compute these partials each element of K is perturbed from its nominal value and then the new X_1 and X_2 are calculated.

This calculation of partial of X_1 and X_2 with respect to K is not straightforward as both X_1, $X2$ and K are all matrices. In order to compute these partials each element of K is perturbed and the corresponding change in X_1 and X_2 is calculated numerically. So every time X_1 and X_2 are calculated a discrete time Ricatti equation has to be solved, which is computationally very expensive. Only the calculation of the partial of X_1 with respect to K is shown here because of space limitations. Similar results apply to the calculation of the partial of X_2.

$$\frac{\partial X_1(i,j)}{\partial K(i,j)} = \frac{X_1 - \Delta X_1}{\varepsilon K(i,j)} \tag{27}$$

where ΔX_1 is the change in X_1 caused by a perturbation of the element in the ith row and jth column of K. This perturbation is carried out for all the elements of K. Then, these partials are so multiplied that the change in each element of $X1$ for change in all the elements of K is achieved. After this partial is evaluated the gradient descent steps are given as follows.

Step1. Start with nominal value of K as standard Kalman gain

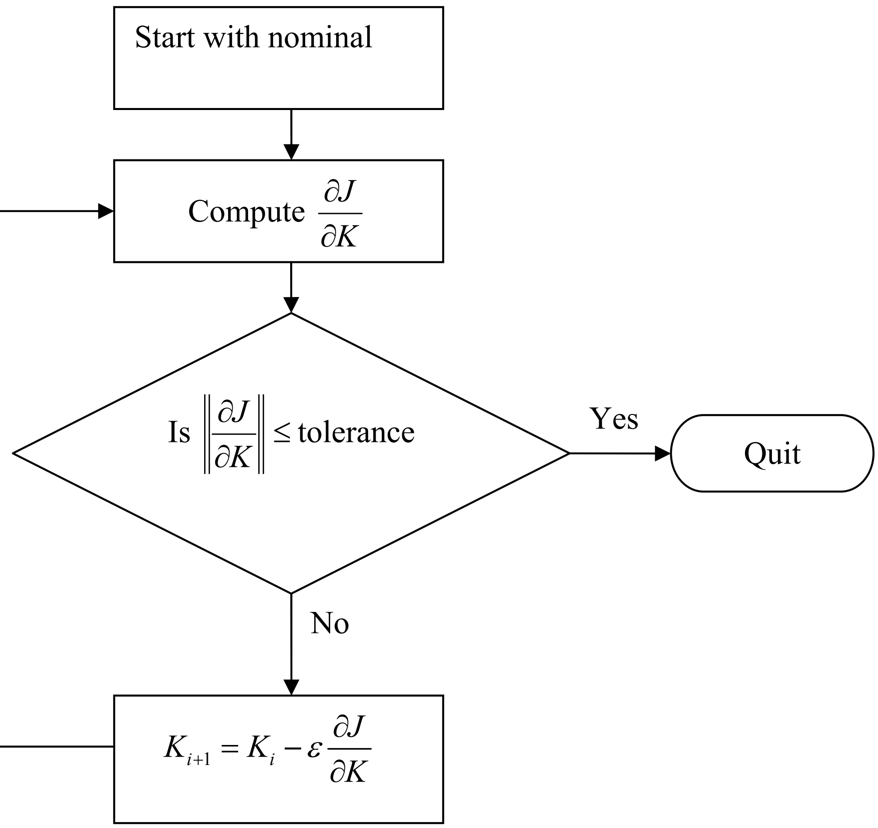
Step2. Compute $\frac{\partial J}{\partial K} \quad at \quad K = K_i$

Step3. If $\left\| \frac{\partial J}{\partial K} \right\| <$ Tolerance, Quit

Step4. $K_{i+1} = K_i - \varepsilon \frac{\partial J}{\partial K}$

Step5. Go to step 2

This process can be illustrated using the following flowchart.



Special care has to be taken to come up with the gradient descent step size and the perturbation size to find the partial derivative.

Computationally this method is time consuming but this is a straightforward method of realizing a new Kalman gain. Computationally it may be better to use more efficient search algorithms rather than gradient descent.

CHAPTER V

RESULTS

This work is about obtaining optimal health parameters for a gas turbine engine. The previous chapters have discussed about operation and modeling of gas turbine engine. They have also looked into the fundamentals of the Kalman filtering. The main contribution of this work is the Robust Kalman filter (RKF). This chapter is about implementation of these methods to the gas turbine problem. The next section gives a brief introduction. Section 5.2 analyzes results obtained for unconstrained and constrained Kalman filter. Section 5.3 evaluates the performance of RKF.

5.1 Introduction

The performance of an aircraft gas turbine engine deteriorates over time. This deterioration reduces the fuel economy of the engine. To determine the health of the engine, data is periodically collected and is used to decide maintenance schedules. The data is then used to come up with the linearized model of the engine using the DIGTEM software, a public domain turbofan software simulation developed by the NASA Glenn

Research Center. Three seconds of data are collected at 10 Hz every flight. In order to apply the theory presented in this thesis, 50 flights are simulated.

5.2 Results of the Kalman filter and the constrained Kalman filter

The Kalman filter used in the course of this thesis is a steady state filter. This is an important aspect to note because the performance of the filter will not be *optimal* when a steady state filter is used instead of a time varying filter. But the computational effort that is reduced by the use of steady state filter is also significant as the Kalman gain and the estimation error covariance does not have to be calculated at every time step. This is more significant in the current scenario of health parameter estimation of gas turbine engines wherein the system matrices dimensions get higher. Thus, it becomes a tradeoff for the engineers to choose between the *optimality* and computational effort. In order to check the feasibility of applying the steady state filter for the gas turbine problem, both steady state and time-varying filters are applied and the results are compared in Figure 5.1. These simulations were obtained by performing a Monte Carlo run of 30 simulations and 500 flights were simulated per Monte Carlo run.

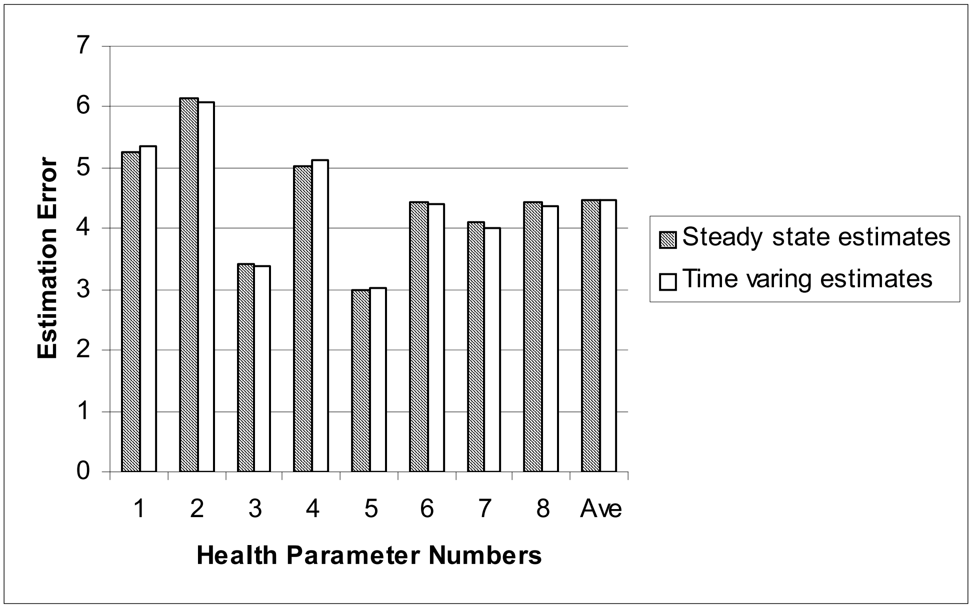


Figure 5.1 Comparison of performance of steady state and time varying filters

It is seen in Figure 5.1 that the performance of the steady state filter is on par with the time varying filter. The individual health parameter estimates do not vary much in the filters and when the average estimation error is observed, there is not much improvement obtained using a time varying filter. The values of the process noise covariance and measurement noise covariance used in these simulations are based on the experience of engineers at NASA. The standard deviation of process noise is typically 10^{-2}. The process noise covariance matrix, Q is then a diagonal matrix of standard deviation of each state and health parameter squared. The measurement noise covariance matrix, R is created based on the signal to noise ration (SNR) of each sensor. The SNRs for the DIGTEM model are given as follows.

1. High pressure turbine rotor speed -- 150
2. Low pressure turbine rotor speed -- 150
3. Duct pressure -- 200

4. Duct temperature -- 200
5. Duct temperature -- 100
6. Compressor inlet pressure -- 200
7. Compressor inlet temperature -- 100
8. Combustor pressure -- 200
9. Combustor inlet temperature -- 100
10. Low pressure turbine inlet pressure -- 100
11. Low pressure inlet temperature -- 100
12. Augmentor inlet pressure -- 100
13. Augmentor inlet temperature -- 70

The R matrix is then created by extracting the noise level from the actual sensor measurement. The standard deviation of initial estimation error covariance is taken to be 10^{-4}. The initial estimation error covariance matrix, P is generated using this information. This information is used in the comparison of steady state and time varying KF. Though the performance of both the filters does not vary significantly, the computational effort required by the steady state filter is reduced by approximately 60%.The results shown in Figure 5.1 are averaged over 30 Monte Carlo runs. So the rest of this work has been based on the steady state filter.

In the application of Kalman filters some known signal information is often either ignored or dealt with heuristically [14]. For instance state variable constraints are often neglected because they do not easily fit into the structure of the Kalman filter. This paper implements a way to generalize the Kalman filter in such a way that known inequality constraints among the state variables are satisfied by the state variable estimates. The method used here for enforcing inequality constraints on the state variable estimates uses hard constraints. Inequality constraints are inherently more complicated than equality constraints, but standard quadratic programming results can be used to solve the Kalman filtering problem with equality constraints. At each time step of the constrained filter, a quadratic programming problem is solved to obtain the constrained state estimate. A family of constrained state estimates is obtained, where the weighting matrix of the quadratic programming problem determines which family member forms the desired solution. The current problem of health parameter estimation is a very good example of constrained state estimation. The constraint on the states is that the health parameters do not improve with time. They degrade or at most remain the same. As the approach indicated in the previous chapters discusses, we augment the health parameters onto the state vector, and this information can be used as constraint on the states. Figures 5.2 and 5.3 show the actual degradations in the case of standard Kalman filter and constrained Kalman filter. The plots shown here are for a one particular run of 500 flights. Table 5.1 shows the performance of the filters averaged over 30 Monte Carlo simulations.

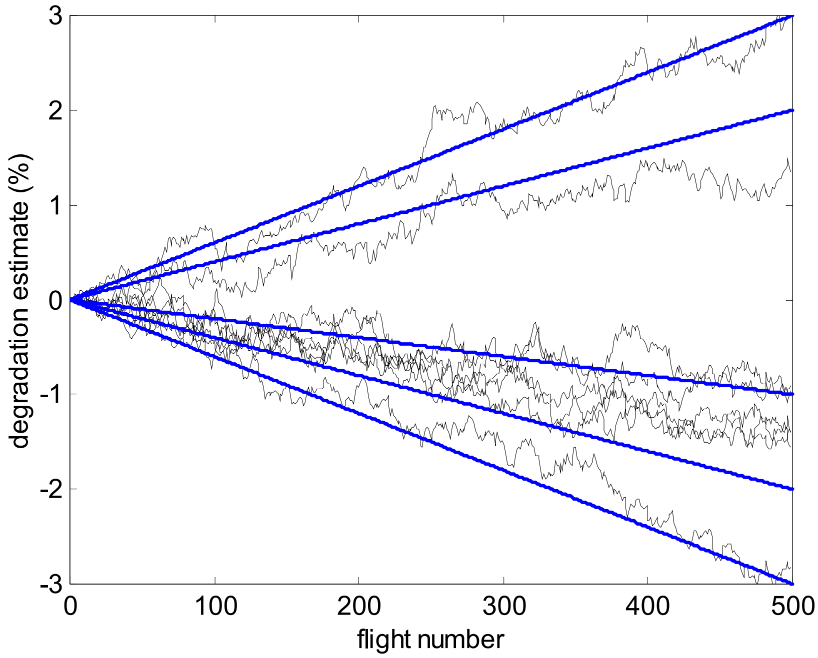


Figure 5.2 Unconstrained Kalman filter estimates of health parameters. True health parameter changes are shown as heavy lines.

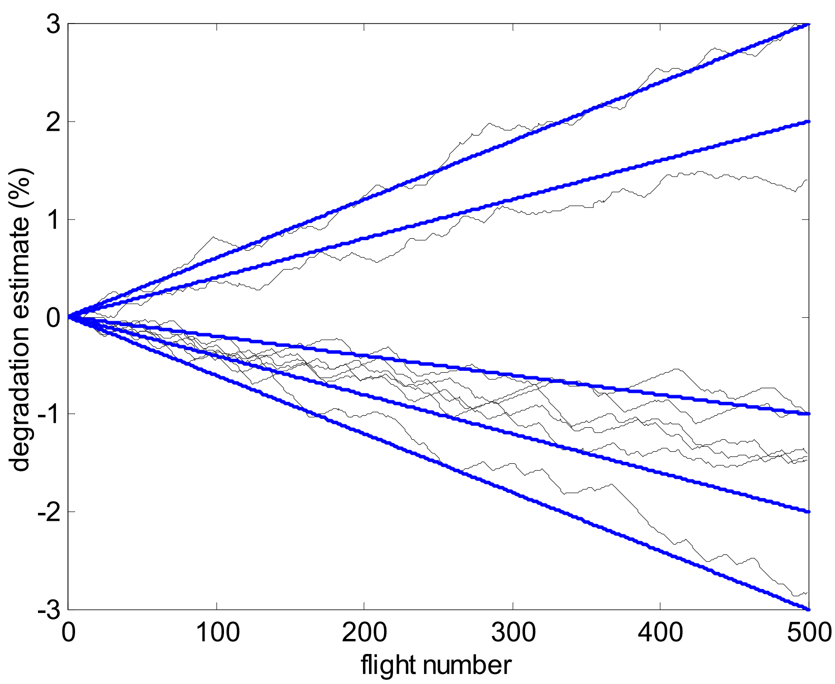


Figure 5.3 Constrained Kalman filter estimates of health parameters. True health parameter changes are shown as heavy lines.

Table 5.1 – Comparision of RMS health parameter estimation errors (percent) for unconstrained and constrained Kalman filter

Health Parameter #	Unconstrained KF estimation error	Constrained KF estimation error
1	4.8	4.4
2	5.8	4.6
3	3.4	2.7
4	4.8	3.8
5	3.0	2.4
6	4.5	3.8
7	4.5	4.2
8	6.3	4.3
Average	4.6	3.7

The improved performance of the constrained filter comes with a price of extra computational effort. The constrained filter requires about four times the computational effort of the unconstrained filter. This is because of the additional quadratic programming problem that is required.

5.3 Results of Hybrid Gradient Descent Algorithm

As discussed in the previous chapter, a hybrid gradient descent algorithm was used to change the gain of the Kalman filter to minimize the cost function.

$$J = \rho_1 \{ tr(X_1) + tr(X_2) \} + \rho_2 \{ \sigma_1^2 (tr(X_1))^2 + \sigma_2^2 (tr(X_2))^2 \}$$

As the system is highly complex, the results presented here are not optimal, as the algorithm had not converged in a given time frame (see Figure 5.4). Nevertheless we can see that the cost function decreased by about 80 percent.

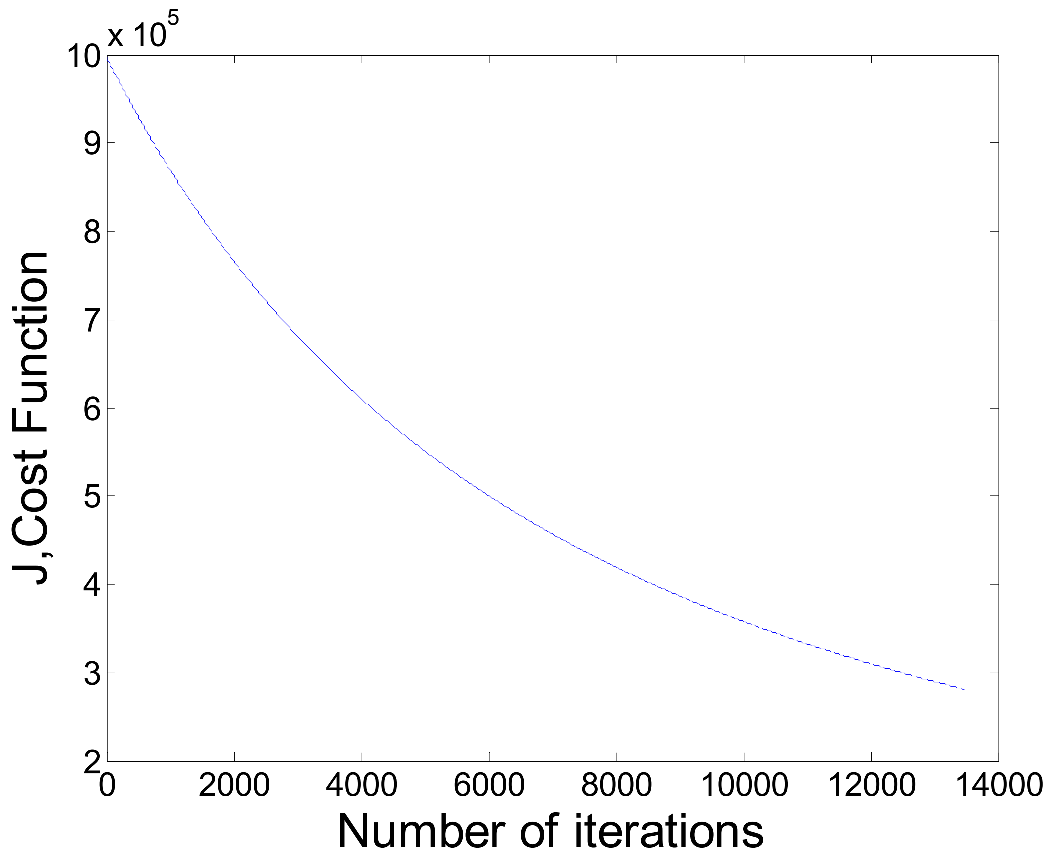


Figure 5.4 Cost function vs. number of iterations

Although the gradient descent algorithm did not converge, the suboptimal results verify that the robust Kalman filter may provide an attractive filtering option when there are uncertainties in the noise covariances. The results here are shown only for the presence of measurement noise uncertainty but no process noise uncertainty as the system model obtained by DIGTEM is assumed to be accurate. In other words, we used $\sigma_1^2 = 0$ and $\sigma_2^2 = 1$ in the following equation

$$E\{\alpha^2\} = \sigma_1^2, E\{\beta^2\} = \sigma_2^2$$

It took 96 hours of computation on a Pentium-IV, 1.8 GHz, 256 MB RAM system for the system to complete the iterations shown in Figure 5.4.This is because for every iteration in gradient descent algorithm there are 1152 (2 x 2 x 12 x 24) Riccati equations to be solved in MATLAB© (the Kalman gain matrix is 12 x 24, there are 2 Riccati equations to solve for [X_1 and X_2], and 2 evaluations of each Riccati equation is required to approximate the partial derivative). Each Riccati takes about 5.5 seconds to be solved.

Figure 5.5 illustrates the nonlinearity of the performance index with respect to two different elements of the gain K for a turbofan health parameter estimation problem.

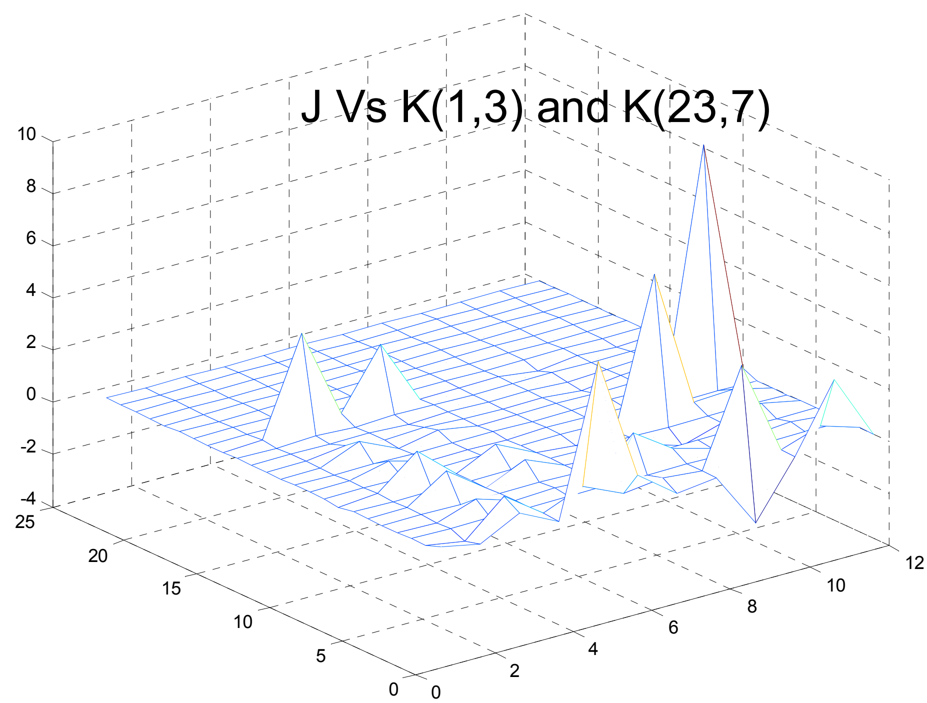


Figure 5.5 Performance index as a function of two elements of K

The new Kalman gain realized after these iterations resulted in a filter that was unstable. So we had to realize a new gain which is a linear combination of the standard Kalman gain (K_s) and the robust Kalman gain (K_r). So the new gain is given as follows

$$K_{new} = \eta K_s + (1-\eta)K_r$$

Where η is determined heuristically to give a stable but robust filter.

In the condition that there are no uncertainties in the noise covariances, the standard Kalman filter is expected to perform better than the robust Kalman filter. Simulation results are in accordance with this theory. These results are shown in Table 5.2 and Table 5.3. Although the robust filter does not perform as well as the standard filter in the nominal case, the drop off in performance is slight, and it may be worth the extra robustness that is obtained as seen in Table 5.2.

Table 5.2 – Health parameter estimation errors (percent) when the variation in the measurement noise covariance is two standard deviations; η=0.7 averaged over 30 Monte Carlo runs

Health Parameter #	Standard KF RMS error	Robust KF RMS error
1	14.0	12.8
2	16.6	17.3
3	8.1	7.3
4	12.5	12.3
5	6.7	6.7
6	12.1	11.0
7	9.3	8.5
8	10.5	9.7
Average	11.3	10.7

Table 5.3 – Health parameter estimation errors (percent) when there is no change in the measurement noise covariance; η=0.7 averaged over 30 Monte Carlo runs

Health Parameter #	Standard KF RMS error	Robust KF RMS error
1	5.5	4.6
2	6.7	7.1
3	2.9	3.0
4	5.6	5.7
5	2.6	5.5
6	4.7	5.0
7	4.4	4.1
8	6.8	6.7
Average	4.9	5.2

Figure 5.6 shows the average RMS estimation error for the two filters averaged for 30 Monte Carlo simulations for the various changes in the measurement noise covariance. As expected, when the covariance change is zero, the standard Kalman filter outperforms the robust Kalman filter. However, as the covariance changes more and more, the robust filter gains more and more performance relative to the standard filter. η which is used for the simulation results shown here is 0.7.

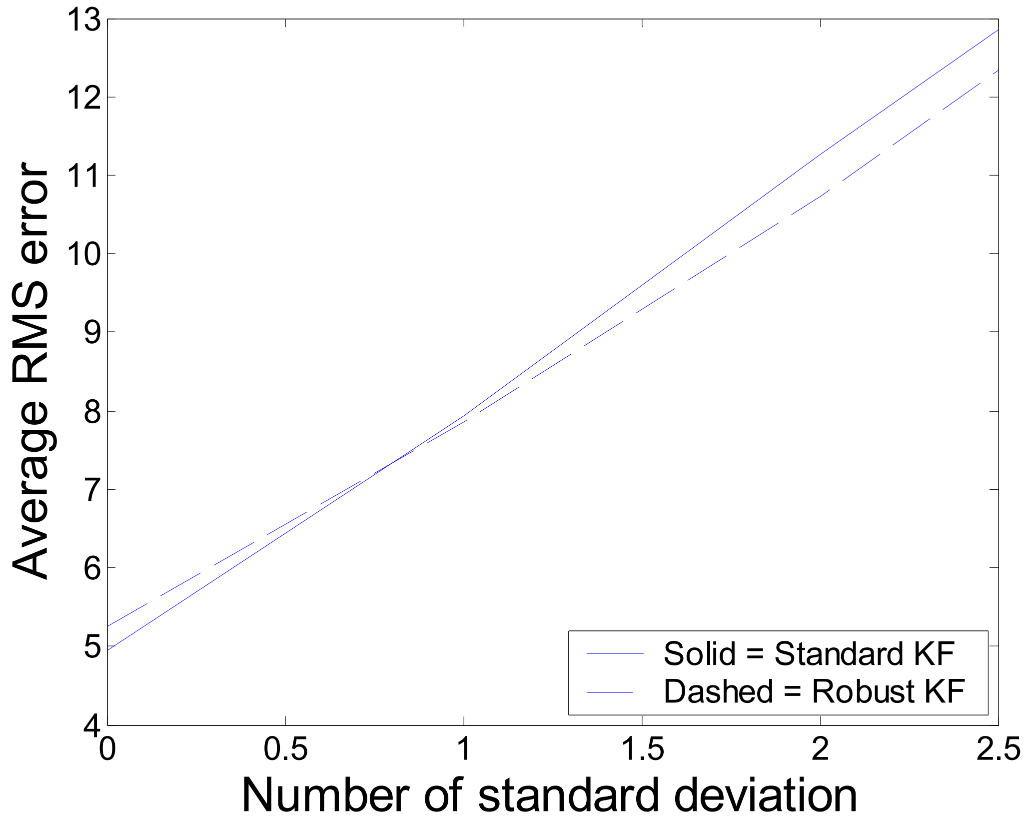


Figure 5.6 Performance of filters for various noise perturbations

CHAPTER VI

CONCLUSIONS AND FUTURE WORK

This thesis is aimed at obtaining optimal state estimates for the gas turbine engine. The area itself is very wide and offers a wide scope of research. The first step in approaching this problem was to model the highly nonlinear dynamics of the gas turbine engine with a linearized model. This involved modifying the DIGTEM code to calculate the Jacobian matrices used to obtain the linearized model of the jet engine. Though intuitively it can be argued that there would be a wide mismatch between the nonlinear model and the linearized model, test results at NASA have indicated that the mismatch is about 1% or 2% only. So it was decided in this study to stick with the linearized model. But proper care was taken in the initial part of the study to check the linearization of the model. But it is still an area of concern whether to use the nonlinear model or linearized model. The other area of research that has to be explored is the uncertainties in the system modeling. This study was limited to the uncertainties in the process noise and measurement noise models.

The analytical approach used in this study was not able to solve the case of the individual uncertainties in different sensors. It was also limited in its approach towards uncertainties in the system modeling. The problem of optimal state estimation in the presence of general uncertainties is still a wide open research area. As seen in the literature search, most of the research done in this field has been to minimize the upper and lower bounds on the estimation error. But the interest of this work has been to minimize the actual average error.

The other possibilities that are currently being explored are the use of interval Kalman filters and the use of evolutionary algorithms as better search algorithms for finding the *optimal global minima*. But these algorithms turn out to be computationally expensive in most of the cases. So the question of *optimality* versus *computational effort* has to be answered in these situations. To check the application of these algorithms a small scale problem was used for simulations rather than the turbofan health parameter estimation problem.

The problem under consideration was a vehicle moving along a straight line, with position and velocity being the states of the system and acceleration being the input to the system. The only measured state is a noisy measurement of the position of the vehicle. A continuous parameter genetic algorithm (GA) is coded for this problem. The details of the GA are not discussed here but some interesting conclusions are drawn here. The cost function that the GA tries to minimize is the actual estimation error rather than the bounds on the estimation error. In this process, the chromosomes which are the actual solutions of the problem were coded as the gain elements of the Kalman filter. The selection process used is a rank based selection and an elitism factor of 50% is used. The mutation process used is a uniform mutation. During the training of the GA for obtaining the *optimal* gain an actual simulation of the process is run to evaluate the fitness of each individual.

The GA trained filter is then compared with the standard Kalman filter and the robust Kalman filter proposed in this thesis. To make this comparison, only uncertainties in process noise and measurement noise covariance matrix were considered. In this scenario the robust filter is expected to outperform the standard KF which is backed by the result that the robust filter performs 25% better than the standard KF. But the interesting result is that the GA trained filter outperformed standard KF by 83% which is also better than the robust filter by 70%. This shows the effectiveness of the GA's in searching for *optimal* solutions. But the effectiveness comes with a price of computational effort. The GA training took about 60% more CPU time than the gradient descent based robust KF. So the engineer has to make a decision based on the resources available and the accuracy needed for the required task. The advantages of the GA trained filter is that the uncertainties on a large map can be considered which is not possible in the algorithm studied in this thesis. Uncertainties in the system modeling for the vehicle moving along straight line are considered. The results compared between standard KF and GA trained filter showed that GA trained filter performs 86% better than the standard filter. So it is shown that that GA training could result in significant improvement in the performance of the state estimators but the computational intensity remains a question of concern. Currently research is being conducted in reducing the complexity of GA's. One way is to incorporate the solution of the gradient descent algorithm in the initial population of GA training. The other solution is the use of hybrid filters with a combination of robust KF and GA trained KF. There is still a lot of scope for exploration in this area.

The other possibility to use the robust KF for the uncertainties in system modeling would be to model the system uncertainties into the uncertainties in the process noise covariance matrix modeling. Mathematically this is feasible as the process noise can also be modeled as disparity in the system model identification. But the biggest drawback of the application of GA based estimation for the health parameter estimation of gas turbine engine would be the computational effort that is required to evaluate the performance index for each individual. The biggest challenge in this area would be to decrease the computational requirement for the training period.

The other area that is of considerable interest in the filed of state estimation in the presence of uncertainties is the interval Kalman filter (IKF). In IKF classical KF techniques are extended to interval linear systems. The basic IKF assumes the systems to be linear but bounded by intervals. It has been proved in [15] that an IKF has the same structure as a classical KF preserving both the statistical optimality and the recursive computational scheme of the standard KF. The drawback of IKF as compared to the goal of this study is that the IKF yields bounds on the estimates rather than the least average estimation error itself. Research is being conducted in this regard to ensure best possible state estimates using IKF.

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