# Applied Signal Processing Laboratory

## Assignment 4 - Array signal processing

Daniel Gaetano Riviello

A.A. 2024-2025

28/05/2025

# Exercise 1 - Linear array processing and beamforming

1. Set the initial parameters: number of antennas $M = 8$, $f_c = 900$ MHz, $c = 3 \cdot 10^8$ m/s, calculate $\lambda$ and set $d = \lambda/2$. Set the DoA $\theta_1 = 0°$.
2. Compute $\Psi_1$ as $2\pi \frac{d}{\lambda} \sin\theta_1$ and the conventional beamforming vector for the corresponding $\theta_1$. Normalize the vector by $\sqrt{M}$.
3. Create a vector of angles $\boldsymbol{\theta}$ of $S = 3601$ elements with `linespace` from $-\pi/2$ to $\pi/2$. Use only one for loop to compute the array pattern: for each $\theta_i$, compute the vector $\mathbf{a}(\theta_i)$ and stack them into a $M \times S$ matrix $\mathbf{V}$, finally compute the pattern as a vector by matrix product $\mathbf{w}^H \mathbf{V}$.
4. Plot the array factor (magnitude of the pattern) with the standard `plot` function and compare it with the plot of the `diric` function with $\Psi = 2\pi \frac{d}{\lambda} (\sin\theta - \sin\theta_1)$ and $M$ as arguments, after proper normalization. Finally use the MATLAB code at slide 17 of `Lec12` for the polar plot.

# Exercise 1

5 Test the script for different configurations and make a $2 \times 2$ subplot of polar plots for each of the 2 following scenarios:
- Use initial $d$ and $\theta_1$ and choose 4 different values of $M$ from 2 to 50. Check that the maximum is $\sqrt{M}$ for each case.
- With fixed $M$ and initial $\theta_1$, test different values of $d/\lambda = 0.25, 0.5, 1, 2$. How many and where are the maxima for each case? The maxima can be computed as

$$\theta_m^{\text{MAX}} = \arcsin\left(\sin\theta_1 \pm m\frac{\lambda}{d}\right), \quad m = 0, 1, 2, \ldots$$

6 Considering the locations of nulls:

$$\theta_m^{\text{NULL}} = \arcsin\left(\sin\theta_1 \pm \frac{m}{M}\frac{\lambda}{d}\right), \quad m = 1, 2, 3, \ldots \quad m \neq M, 2M, 3M, \ldots$$

For what value of $d/\lambda$ we have maximum directivity and no grating lobes?
- Hint: set the last null to be at $90°$ and solve for $d/\lambda$.

7 Set initial $M = 8$, $d/\lambda = 1/2$ and $\theta_1 = 30°$. Compute the FNBW $\left|\theta_1^{\text{NULL}} - \theta_{-1}^{\text{NULL}}\right|$. Now set $\theta_1 = 60°$, what is the minimum $M$ that allows $\theta_1^{\text{NULL}} < 90°$?

# Exercise 1

8. Set the same initial parameters of step 1, $\theta_1 = 0°$ is the DoA of the user of interest (UE).
9. Create a vector with the DoAs of 5 interferers, $\boldsymbol{\theta} = [20°, -40°, 60°, -75°, 80°]$
10. Create a matrix $\mathbf{A}(\theta) \in M \times 6$ with all 6 steering vectors (UE plus interferers), compute the spatial covariance matrix of signal plus noise and interference $\mathbf{R}_y = \mathbf{A}(\theta)\mathbf{A}(\theta)^H + \sigma_n \mathbf{I}$, with $\sigma_n = 10^{-5}$.
11. Compute the beamformer $\mathbf{w}_{\text{MVDR}}$ as in slide 18 of `Lec12`.
12. Compute the pattern as in step 3 and plot only with `polarplot`, add straight dashed lines for the interferers' DoAs with $\rho$ from 0 to the maximum of the pattern.
13. Repeat the plots twice with the first interferer with DoA $\theta_{i=1} = 10°$, and then with $\theta_{i=1} = 5°$ as presented in the slides.
14. Now set $\boldsymbol{\theta}$ with 31 interferers from $-90°$ to $90°$ (`linespace`) and modify the MVDR formula to $\mathbf{w}_{\text{MVDR}} = \mathbf{R}_y^{-1}\mathbf{a}(\theta_1) / \left\|\mathbf{R}_y^{-1}\mathbf{a}(\theta_1)\right\|_2$. What happens to the new pattern (remove the dashed lines for interferers)? Comment the result by comparing it with a conventional beamformer with the same parameters.

# Exercise 2: Beamforming techniques with planar arrays

1. Complete the missing sections of the script on slides 8 and 9 from line 1 to 69 to generate the array factor of a UPA in conventional beamforming:
   - Set $N_z = N_y = 8$.
   - Define the following directivity function by means of anonymous function:

     $$d(\theta, \phi) = 0.25 \cdot [1 - \cos(2\theta)] \cdot [1 + \cos(\phi)].$$

   - Compute the beamforming filter (you can use the `kron` function).
   - Use `meshgrid` to obtain the 2 matrices containing the grids of angles (to be used with antenna element pattern generation and conversion from spherical to Cartesian cordinates).
   - Generate the UPA steering vectors for each angle pair in a double `for` loop (use `kron`). You will need an array of 3 dimensions to store all steering vectors.
   - Compute the UPA pattern or equivalently the AF by multiplying your beamforming vector with each steering vector, only use one `for` loop to apply vector by matrix multiplication (`squeeze` on your 3-D array).
   - Apply pattern multiplication with the directivity function.

# Exercise 2

2. Plot the 3-D UPA pattern (use `mesh`) with isotropic antenna elements (`directive=0`).
3. Now set `directive=1`, generate again the pattern and comment on the differences.
4. Test your conventional beamformer with both isotropic and directive antenna elements with the following angle pairs:
   - $(\theta, \phi) = (105^\circ, 30^\circ)$
   - $(\theta, \phi) = (70^\circ, -45^\circ)$
5. Finally set $N_z = 4$, $N_y = 32$, plot and compare $(100^\circ, 60^\circ)$ with both isotropic and directive elements.

# Exercise 2

6. Now complete the missing parts of the script on slide 10 from line 70 to 116 to generate the pattern of a MVDR beamformer with a planar array. User's and interferers' DoAs are specified in the parameters section ($P_k = 1$, $\forall k$ in the equation of slide 13 in Lec13). Use $N_z = N_y = 16$.
   - Compute the BF vector as $\mathbf{w} = \mathbf{R}_y^{-1}\,\mathbf{a}(\theta_1, \phi_1) / \left\|\mathbf{R}_y^{-1}\,\mathbf{a}(\theta_1, \phi_1)\right\|_2$.
   - Recall to multiply the steering vector of each interferer by its element directivity function.
   - For the pattern generation, perform the elementwise multiplication between the already computed 3-D array with all steering vectors for each $(\theta, \phi)$ pair, and the 3D-array with the directivity function repeated $N_{tot}$ times (use `repmat` and `permute`).
7. First plot the pattern with `directive=0`, then set `directive=1` and re-plot the pattern. Discuss the differences.
8. Now change the DoA of the first interferer to $(\theta, \phi) = (88°, 2°)$ and check how the pattern changes.
9. Finally set $\sigma_n^2 = 10^3$. What happens to the pattern and why? Compare it with the Array Factor in conventional beamforming.

# MATLAB script to reuse and complete for Exercise 2

```matlab
clc; clear, close all;
%% Parameters
c = 3e8;            % speed of light
fc = 6e9;           % frequency
lambda = c/fc;      % wave length
Nz = 8;            % number of sensors along z
Ny = 8;            % number of sensors along y
N_tot = Nz*Ny;
d_z = lambda/2;    % sensor spacing along z [m]
d_y = lambda/2;    % sensor spacing along y [m]
theta_1deg = 90;   % UE el. angle from 0 to 180
theta_1 = deg2rad(theta_1deg);
phi_1deg = 0;   % UE az. from -90 to 90
phi_1 = deg2rad(phi_1deg);
directive = 1;
%ARRAY LIES IN THE Z-Y PLANE
Tz=(2*[-Nz/2+1:Nz/2]-1)/3;
Ty=2*[-Ny/2+1:Ny/2]-1;
T = Tz'*ones(1,Ny)./Nz+1i*ones(Nz,1)*Ty/Ny;
T = T(:);
%% Antenna element directivity function
%%% 1) Define as anonymous function the directivity
    function of the antenna element if directive
    ==1, otherwise define it equal to 1
if(directive==1)
    %TO COMPLETE
else
    %TO COMPLETE
end
%% Compute the beamforming filter for Conventional BF
%%% 2) Compute the BF vector as in slide 11 of Lec13
%TO COMPLETE
%% Compute UPA pattern for Conventional BF
angles_1 = 361;
theta = linspace(0,pi,angles_1);
angles_2 = 721;
phi = linspace(-pi, pi, angles_2);
%%% 1) Use "meshgrid" to get matrices of all phi and
    theta
%[PHI, THETA] = %TO COMPLETE
%%% 2) Generate the UPA pattern or AF (3D array) with
    isotropic antennas
%TO COMPLETE
```

```matlab
%% Perform pattern multiplication
%%% 3) Call you directivity function with input parameters the matrices PHI and THETA and perform elementwise multiplication with AF
%TO COMPLETE
%% Plot the pattern
%%% 4) Convert the pattern into cartesian coordinates (use matrices PHI and THETA), save into the variable r the square root of the array gain towards the user of interest (UE).
%Z = %TO COMPLETE
%Y = %TO COMPLETE
%X = %TO COMPLETE
%r = %TO COMPLETE
r_C = sqrt(X.^2+Y.^2+Z.^2);
%%% 5) Plot the pattern with "mesh" with color scaling r_C and options 'FaceAlpha' and 'EdgeAlpha' at 0.5, set the axes equal and the "view" at (140, 15).
figure(1)
%TO COMPLETE
hold on;
hidden on;
plot3(zeros(Nz*Ny,1), imag(T), real(T), 'ks', 'MarkerFaceColor', 'r'); %%draws the array
%%% 6) Draw a red dashed line at broadside direction (90, 0) with length equal to r with "plot3" and place the writing 'Broadside' at position (r,0,0) with "text" and 'Fontsize' 12.
%plot3( %TO COMPLETE
%txt1 = %TO COMPLETE
%text( % TO COMPLETE
%%% 7) Add one more red dashed line with same length only if the array is NOT steered towards broadside, add a text with UE el. angle and azimuth at UE position (use char(176) for the degree symbol).
%if( %TO COMPLETE
    %x_1 = %TO COMPLETE
    %y_1 = %TO COMPLETE
    %z_1 = %TO COMPLETE
    %plot3(%TO COMPLETE
    %txt2 = %TO COMPLETE
    %text(%TO COMPLETE
%end
%%% 8) Add labels and the title with BF technique, array type and size, and elevation angle and azimuth of UE, set 'FontSize' to 12.
```

# MATLAB script to reuse and complete for Exercise 2

```matlab
%% Additional parameters for MVDR
n_intf = 5;         % number of interferers
sigman2 = 1e-5;       % noise variance
theta_intfdeg = [86 85 80 100 105]; % el. angles of interferers
theta_intf =  deg2rad(theta_intfdeg);
phi_intfdeg = [4 20 5 -15 15]; % az. of interferers
phi_intf =  deg2rad(phi_intfdeg); %
%% Compute MVDR beamforming filter
%%% 9) Compute the BF vector as in slide 13 of Lec13 with different normalization
% TO COMPLETE
%% Compute UPA pattern for MVDR
%%% 10) Perform elementwise multiplication between two 3-D arrays: the 3-D array containing all steering vector and the one with the directivity function
% TO COMPLETE
%%% 11) Multiply the BF vector with each steering vector (only use one for loop)
%TO COMPLETE
%% Plot the pattern for MVDR
%%% 12) Convert the pattern into cartesian coordinates (use matrices PHI and THETA), save into the variable r the square root of the array gain towards the user of interest (UE).
%Z = %TO COMPLETE
%Y = %TO COMPLETE
%X = %TO COMPLETE
%r = %TO COMPLETE
r_C = sqrt(X.^2+Y.^2+Z.^2);
%%% 13) Plot the pattern with "mesh" with color scaling r_C and options 'FaceAlpha' and 'EdgeAlpha' at 0.5, set the axes equal and the "view" at (96, 3).
figure(2)
% TO COMPLETE
hold on;
hidden on;
plot3(zeros(N_tot,1), imag(T), real(T), 'ks', 'MarkerFaceColor', 'r'); %%draws the array
%%% 14) Draw a red dashed line at UE DoA (90, 0) with length equal to r with "plot3", add a red dot with 'MarkerSize' 8 at the end of the red line and add a text with writing 'User' and UE azimuth and el. angle (use char(176) for the degree symbol).
%x_1 = %TO COMPLETE
%y_1 = %TO COMPLETE
%z_1 = %TO COMPLETE
%plot3( %TO COMPLETE
%plot3( %TO COMPLETE
%txt1 = %TO COMPLETE
%text( %TO COMPLETE
%%% 15) For each interferer intf_i: add a black dashed line with length r, add a text with el. angle and azimuth of intf_i at the end of each line.
%for %TO COMPLETE
    %x_i = %TO COMPLETE
    %y_i = %TO COMPLETE
    %z_i = %TO COMPLETE
    %plot3( %TO COMPLETE
    %txti = %TO COMPLETE
    %text( %TO COMPLETE
    hold on;
%end
%%% 16) Add labels and the title with BF technique, array type and size, and elevation angle and azimuth of UE, set 'FontSize' to 12.
```

# Report and Matlab scripts

- For each exercise include all requested plots and answers.
- Plots must contain labels for all axes, a title and a legend in case of multiple plots in one figure.
- The scripts must run correctly. If the script of an exercise doesn't work, the exercise will be considered failed.
- Deliver a separate Matlab file for each exercise (not a single Matlab file for the entire assignment).
- Both the report and the Matlab files must be uploaded on the portal in a zip file.
- Naming rule: `Group8_Assignment4_lastname#1_lastname#2.zip`.
- Send an email to daniel.riviello@polito.it when you upload it.

# Report delivery and deadlines

- Deliver a single pdf report for Assignment 4 "Array signal processing".
- The report must include all requested plots, comments and answers for all exercises (1 to 2).

### Deadlines to get extra points for Assignment 4

- **Tue. 03/06/2025** at 23:59 for 1 point.
- **Mon. 10/06/2025** at 23:59 for 0.5 points.