Applied Signal Processing Laboratory

Assignment 2 - Digital filter design

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Outilne

Outline

Exercise 1 - Filter design

1. Consider the following signal:
$$X(t) = \cos(2\pi f_1 t) + 3\left[D_3(2\pi B_D t)\right]^2 \cos(2\pi f_2 t) + W(t)$$
where: $f_1 = 5$ Hz, $f_2 = 250$ Hz, $B_D = 10$ Hz, sampling frequency $f_s = 1$ kHz, D_3 is the Dirichlet function or periodic sinc function of order 3 (use `diric`), $W(t)$ is a zero-mean white Gaussian noise with variance $\sigma^2 = 10$.
2. Generate 20 seconds of the signal. Use `randn` for $W(t)$.
3. Estimate the power spectral density with the following command:
`[Sx, f] = pwelch(x, window, n_overlap, NFFT, fs, 'centered');`
Use $D = N/M$ samples per segment, with N the total length of the signal and $M = 25$, 50% overlap and a Hamming window. Plot the power spectral density (PSD) $S_x(f)$ in logarithmic scale, i.e., `f`, the vector of frequencies versus $10\log_{10}|S_x(f)|$.
4. Filter $X(t)$ with a Butterworth band-pass filter $H_1(z)$ centered at $f_2 = 250$ Hz with the following specifications:
   - Pass-band: 80 Hz centered at 250 Hz.
   - Pass-band ripple: 1 dB.
   - Transition bands: 20 Hz.
   - Stop-band attenuation: 60 dB.

Exercise 1

5. Find pass-band and stop-band edge frequencies `Wp` and `Ws` and estimate the filter order with `buttord` and then design the filter with `butter`.
6. Plot the frequency response of the filter with `freqz` (use 1024 points) and limit the magnitude from -100 dB to 10 dB.
7. Generate $Y_1(t)$ with `filter` and estimate the new PSD of $Y_1(t)$ with `pwelch` and plot it.
8. Repeat twice from step 4 to 7 by using an Elliptic filter $H_2(z)$ and an equiripple FIR filter $H_3(z)$ with the same specifications. Use `ellipord` and `ellip` to generate $Y_2(t)$ and `firpmord` and `firpm` to generate $Y_3(t)$.
9. Answer the following questions in the report:
    - Which filter is the most selective (with steepest slope from pass-band to stop-band)?
    - Which one has more band-pass ripple?
    - Which one has the most stop-band attenuation?
    - Which one needs the lowest order to meet the requirements?

Exercise 1

10 Now design a Chebyshev type 1 low-pass filter $H_4(z)$ with `cheb1ord` and `cheby1` and use the following specifications:
- Pass-band edge frequency: 6 Hz.
- Pass-band ripple: 1 dB.
- Transition band: 8 Hz.
- Stop-band attenuation: 60 dB

11 Repeat step 6 with $H_4(z)$.

12 Create a custom function `my_filter` that filters an input signal vector `x` with a digital filter given in the form of the z-transform of a LCCDE and returns the filtered signal vector `y` of the same length of `x`.

13 The function has 3 input parameters: the filter coefficients b_k and a_k given as vectors `b` and `a` and the signal vector `x` (same structure of the in-built `filter` function).

Exercise 1

14 You have to implement the following structure in a single `for` loop:

$$
\begin{aligned}
y(1) &= b(1)x(1) \\
y(2) &= b(1)x(2) + b(2)x(1) - a(2)y(1) \\
y(3) &= b(1)x(3) + b(2)x(2) + b(3)x(1) - a(2)y(2) - a(3)y(1)
\end{aligned}
$$

with $a(1) = 1$. The general structure is:

$$
\begin{aligned}
y(n) = {} & b(1)x(n) + b(2)x(n-1) + \ldots + b(N_b)x(n - N_b + 1) \\
& - a(2)y(n-1) - \ldots - a(N_a)y(n - N_a + 1)
\end{aligned}
$$

where N_a and N_b are the lengths of `a` and `b` respectively.

15 First check if the input filter is unstable with roots. If that is the case, display an error dialog box with `errordlg` and `return` the function before the filtering computation.

16 The transient can be computed with the same general structure by zero-padding on the left both vectors `x` and `y` with $M = [\max(N_a, N_b) - 1]$ samples.

Exercise 1

17 Perform a sliding window on the vectors `x` and `y` and use the command `fliplr` on the filter coefficient vectors `a` and `b`.

18 No more than one `for` loop is allowed. Hint: use twice the command `sum` on the element-wise multiplication between `x` and time reversed `b`, and between `y` and time reversed `a`.

19 In the main script, compute the impulse response $h_4(n)$ of the Chebyshev type 1 filter with coefficients `b` and `a` generated at step 10 by filtering a kronecker delta (zero-padded with $M-1$ samples) with your custom function `my_filter`. Compare it with the command `impz(b,a,M)` with $M = 100$ in the same figure (`impz` returns a `stem` of $h_4(n)$, therefore use `hold on` and `plot` for the result of your function).

Exercise 1

20 With the same Chebyshev filter coefficients, generate $Y_{4a}(t)$ by filtering $X(t)$ with your custom function, then generate $Y_{4b}(t)$ by using the `filter` command on $X(t)$.

21 Plot in the same figure the first 4 seconds of $X(t)$, $Y_{4a}(t)$ and $Y_{4b}(t)$ (use `hold on`).

22 Repeat the previous plot twice with noise variance $\sigma^2 = 1$ and $\sigma^2 = 0.1$. Check if you can clearly see a 5 Hz sinusoid.

23 Finally test your custom function with the following unstable filter coefficients:

$$\texttt{b} = [1 \quad 1.3 \quad 0.49 \quad -0.013 \quad -0.029]$$
$$\texttt{a} = [1 \quad -0.4326 \quad -1.6656 \quad 0.1253 \quad 0.2877].$$

Check that an error dialog box is correctly displayed.

Outline

Exercise 2 - Window method for FIR filter design

1. Design the two following FIR filters with window method. The sampling frequency is $f_s = 1$ kHz for both specifications:
   - Filter type: band-pass
     Passband region: 70 Hz centered at 200 Hz.
     Transition bands: 30 Hz.
     Minimum stop-band attenuation $A_s = 50$ dB.
   - Filter type: band-stop
     Passband region: from 0 Hz to 255 Hz and from 395 Hz to 500 Hz
     Transition bands: 40 Hz.
     Minimum stop-band attenuation $A_s = 70$ dB.
2. Choose the window from the table at slide 42 of Lec07 that satisfies the given requirements with the lowest number of coefficients.
3. Find the corresponding cut-off frequencies $f_1, f_2 \in \left[0, \frac{1}{2}\right]$ and number of required coefficients N.

Exercise 2

4. Generate the corresponding window function $w(n)$ without using any MATLAB in-built function.
5. Compute the corresponding delay in samples M to make the filter causal.
6. Use the spectral transformation formulas to generate the proper ideal filter $h_{id}(n)$ (each Sinc must be properly delayed) and multiply it by $w(n)$.
7. Plot the impulse response of the filter with `impz` and save the frequency response with `[H,f]=freqz(h,1,1024,fs)`.
8. Plot the magnitude of the frequency response and add a vertical black dotted line for each frequency edge and a vertical red dashed line for the cut-off frequencies (use `xline`).

Exercise 2

9. Write now a custom function called `my_Kaiser_filter` that outputs the FIR filter coefficients b_k, the number of coefficients N and the parameter β, and has as input arguments the required stop-band attenuation A_s, transition band B_T, the sampling frequency f_s, cut-off frequency f_c and the following char array for the type of filter:
   - `'-lp'` for low-pass
   - `'-hp'` for high-pass
   - `'-bp'` for band-pass
   - `'-bs'` for band-stop
10. Use `strcmp` to check the input string with each one of the 4 possibilities in an `if-elseif...-else` statement. Use the final `else` for any other possible string, output an error with `errordlg` and use `return` to terminate the function.
11. In case of band-pass or band-stop filter, $f_c = [f_1, f_2]$ is a 2-element vector.

Exercise 2

12. Use the formulas at slide 44 of Lec07 to compute β and N according to the required A_s and B_T.
13. Compute the corresponding delay in samples M to make the filter causal, then use the spectral transformation formulas to generate the proper ideal filter $h_{id}(n)$ and multiply it by $w(n)$.
14. Set the sampling frequency $f_s = 4$ kHz and test the function with these 2 sets of parameters:
    - Filter type: high-pass.
      Stop-band attenuation $A_s = 40$ dB.
      Transition band $B_T = 200$ Hz.
      Cut-off frequency $f_c = 1.6$ kHz.
    - Filter type: band-pass
      Stop-band attenuation $A_s = 60$ dB.
      Transition band $B_T = 100$ Hz.
      Cut-off frequencies $f_1 = 800$ Hz and $f_2 = 1.2$ kHz.

Exercise 2

15 Use the same parameters β and N as the number of filter coefficients and check with `freqz(h,1,1024,fs)` that you obtain the same frequency response with the filters generated by the following commands:

- a `b1=fir1(N-1,Wn,kaiser(N,beta),'high');`
- b `b2=fir1(N-1,Wn,kaiser(N,beta),'bandpass');`
  where `Wn` is the cut-off frequency (or a 2-element vector) normalized between 0 and 1.

16 Save the frequency response of both filters with `[H, f] = freqz(h,1,1024,fs)` and plot their magnitude in dB in the same figure (use `hold on`).

Outline

Digital signal processing application to pulse oximetry

A **pulse oximeter** is an instrument that uses two frequencies of light, **red and infrared**, to compute:

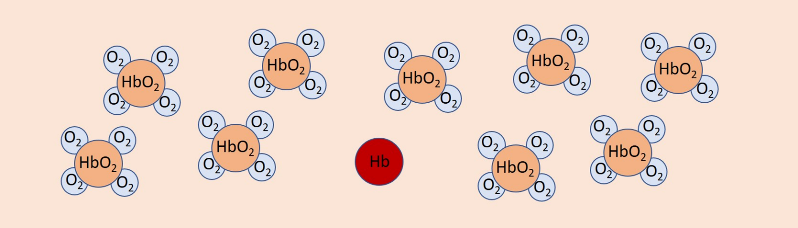
- the **blood oxygen saturation**, i.e., the percentage (%) of hemoglobin in the blood that carries oxygen;
- the **pulse rate**, i.e., the number of heartbeats per minute.

Introduction

- **Air** contains approximately 21% **oxygen**.
- When we inhale, **oxygen enters the body** and travels to the lungs.
- **Lungs** contain millions of alveoli which are surrounded by blood capillaries.
- Alveoli inflate and, since their walls and capillary walls are very thin, **oxygen is transferred into the blood** capillaries.
- **Most of the oxygen** entering the blood **binds to hemoglobin** in the red blood cells, while a small part of the oxygen dissolves in the blood plasma.
- **Blood enriched with oxygen** is then sent to the all the organs and their cells.

Saturation

Inside arterial blood, **most of hemoglobin molecules are bound to oxygen and a few are not**.

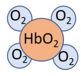


$$SaO_2 = \frac{[HbO_2]}{([HbO_2] + [Hb])}$$

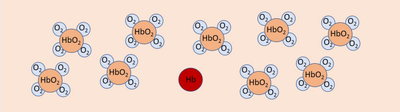
Hemoglobine

- Hemoglobin with no oxygen bound to it is called **deoxygenated hemoglobin** Hb and has a **dark red color**.

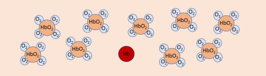
- One molecule of hemoglobin can bind to 4 molecules of oxygen. Hemoglobin bound to oxygen is called **oxygenated hemoglobin** HbO_2 and has a **bright red color**.



- Inside arterial blood, **most of hemoglobin molecules are bound to oxygen and few are not**.

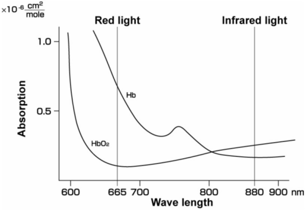


Absortion



$$SaO_2 = \frac{[HbO_2]}{([HbO_2] + [Hb])}$$

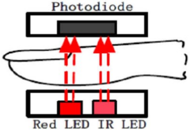
- **Dark red** deoxygenated hemoglobin Hb and **bright red** oxygenated hemoglobin HbO_2 have **different color absorptive** properties.
- Deoxygenated hemoglobin Hb absorbs more **red light**, while HbO_2 absorbs more **infrared light**.
- This property can be exploited to compute their percentage.

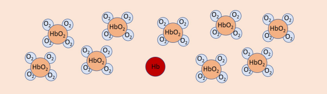


Pulse oximeter

The pulse oximeter

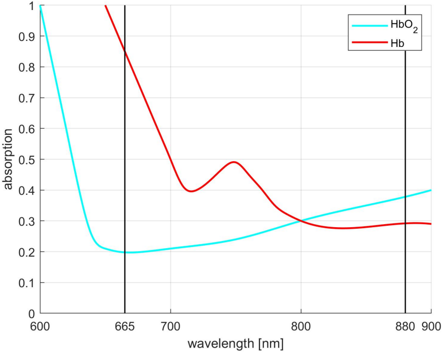
- emits red (R) and infrared (IR) LED light that passes through the finger,
- receives data from a photodetector,
- calculates the oxygen saturation (percentage of oxygenated and deoxygenated hemoglobin) by processing the ratio of the two received signals.



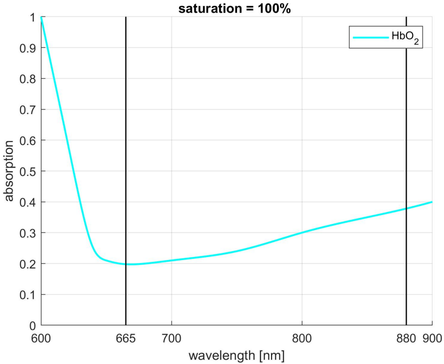


$$SaO_2 = \frac{[HbO_2]}{([HbO_2] + [Hb])}$$

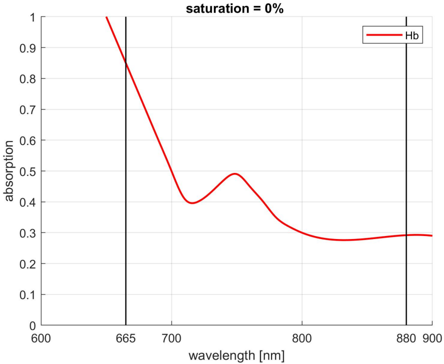
Saturation and Absorption



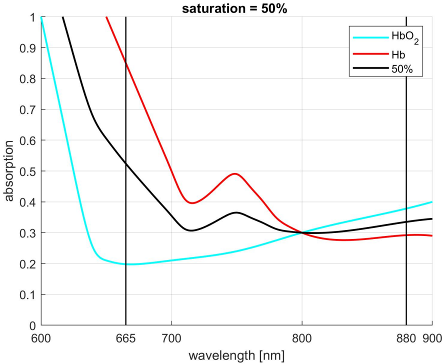
Saturation and Absorption



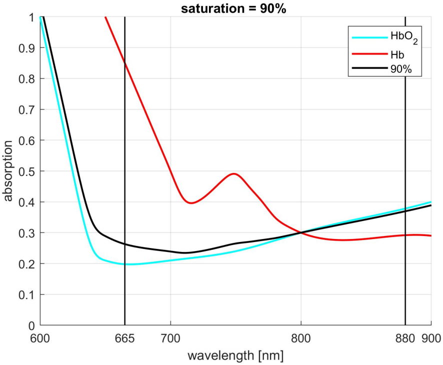
Saturation and Absorption



Saturation and Absorption



Saturation and Absorption



Absorbed light

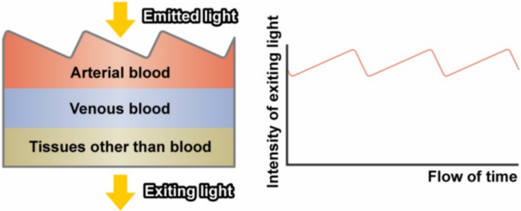
The amount of absorbed light is proportional to:

- The concentration of the absorbing substance (Beer-Lambert law).
- The traveled distance.

Traveled distance

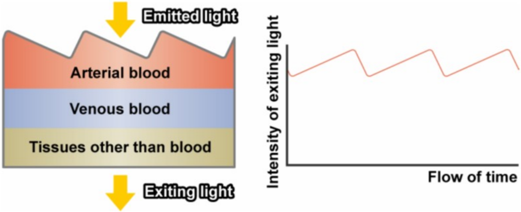
Problem: in the finger, the light must pass through:

- arterial blood.
- venous blood.
- tissue, bones...

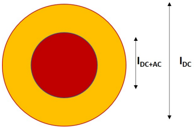


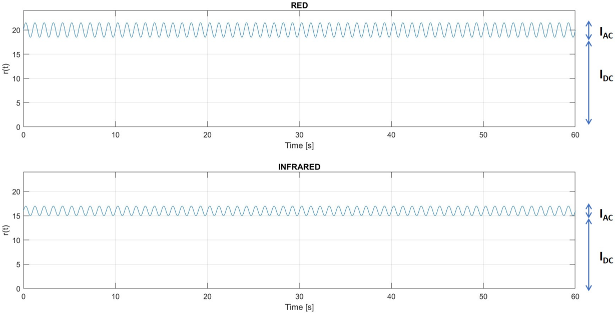
Arterial size

- **Arterial size changes**: blood pumped from the **heart** moves through arterial vessels in the form of waves.
- Instead, venous size does not change: blood inside does not move in pulse waves.
- Then, since **venous, tissue, bone sizes do not change**, the amount of **absorbed light changes with arterial size and is bigger when the artery is bigger.**



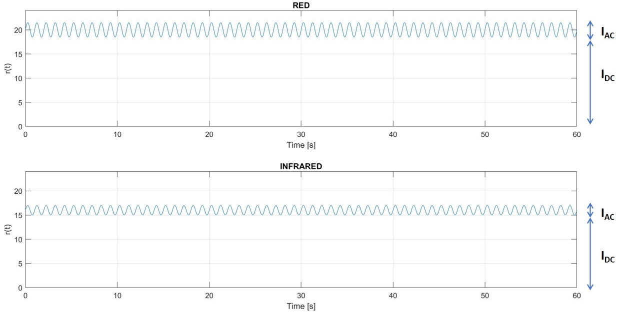
Min and max





- When the arterial size is maximum, the absorption is bigger and we receive less signal (we call it I_{DC}).
- When the arterial size is minimum, the absorption is smaller and we receive more signal (we call it I_{DC+AC}).
- I_{AC} represents the received signal variation due to arterial size change.
- We always have $I_{AC} << I_{DC}$.

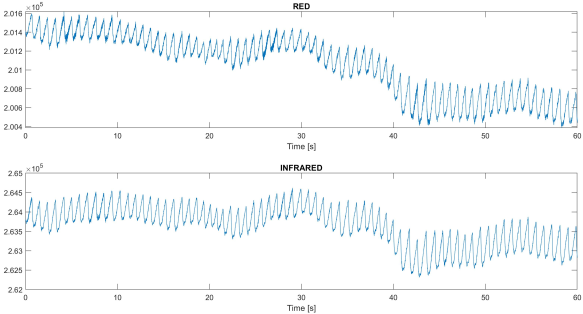
Ideal behavior



It is possible to show (see Appendix) that the saturation can be computed starting from the measured AC and DC signals at RED and INFRARED frequencies by applying these formulas:

$$R = \frac{\frac{I_{AC}(RED)}{I_{DC}(RED)}}{\frac{I_{AC}(INFRARED)}{I_{DC}(INFRARED)}} \qquad \Rightarrow \qquad SaO_2 = 110 - 25R$$

Actual behavior

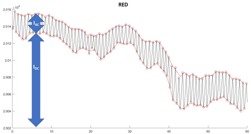


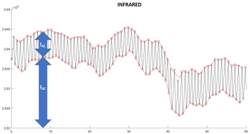
Main differences:

- noise,
- high frequency muscular movements (above 3 Hz),
- low frequency breathe movements (below 0.5 Hz),
- other impairments.

Computation

- Filter out at least high frequency components.
- Compute the value of R at each second.
- Compute its mean value.





$$\overline{R} = \left\langle \frac{\frac{I_{AC}(RED)}{I_{DC}(RED)}}{\frac{I_{AC}(INFRARED)}{I_{DC}(INFRARED)}} \right\rangle \Rightarrow SaO_2 = 110 - 25\overline{R}$$

Step 1 - Load the data

The starting point is the “pulse.txt” file, containing the pulse oximeter data.
The sampling frequency is $f_s = 100$ Hz.
The file format is:

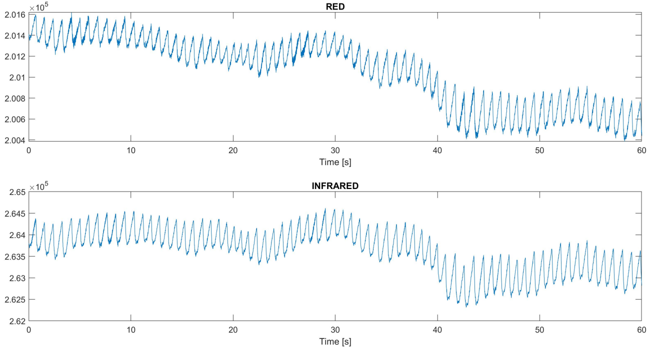
RED	IR
107825	107873
156950	156989
...	...

As an example, you can load it with:

```
filename = 'pulse.txt';
delimiterIn = ' ';
headerlinesIn = 1;
Data_struct = importdata(filename,delimiterIn,headerlinesIn);
Led_R = Data_struct.data(:,1); % RED (R)
Led_IR = Data_struct.data(:,2); % INFRARED (IR)
```

Step 2: Plot R and IR signals

- Skip the first 10 seconds (remember that $f_s = 100$ Hz).
- Consider 60 seconds.
- Plot R and IR signals on the time axis.



Step 3 - Low-pass filtering

- The maximum pulse rate is 180 bpm (3 Hz). Filter the signals (muscular movement) above this frequency.
- Design a low-pass filter to attenuate all frequencies above 3 Hz. Use the formulas of Lec08 to design the analog prototype of a Butterworth low-pass filter with the following specification:
  - Pass-band frequency edge Ω_p: 3 Hz.
  - Stop-band frequency edge Ω_s: 6 Hz.
  - Pass-band ripple R_p: 1 dB.
  - Stop-band attenuation A_s: 60 dB.
  - No `buttord`, `butter` or `buttap` allowed.
- Estimate the filter order N and the two versions of the cut-off frequency $\Omega_{c,1}$ and $\Omega_{c,2}$ of slide 16 (recall to convert first Ω_p and Ω_s into angular frequencies in rad/s).
- Choose Ω_c as the middle point between Ω_p and Ω_s and make sure that Ω_c in Hz is a rational number with three decimal places (use `round`)

Step 3 - Low-pass filtering

- Compute the squared magnitude of the analog filter $|H_a(j\Omega)|^2$ (slide 12) and plot it with the frequency axis in Hz from 0 to 40 Hz.
- Find the poles of the transfer function (slide 20).
- Compute the transfer function $H_a(s)$ with `zp2tf`. Use an empty vector for the zeros and the numerator in the last formula of slide 21 as scalar gain.
- Use `zplane` to check that all poles lie on the left half-plane s.
- Set the sampling frequency $f_s = 100$ Hz. Convert your analog filter to the discrete-time domain with the impulse invariance method by using `impinvar` to generate $H(z)$.
- Plot the filter frequency response with `freqz`.
- since $I_{DC} >> I_{AC}$, the use of `filter` can be problematic, use instead `filtfilt` (zero-phase forward and reverse digital IIR filtering) to filter the signals.

Step 4 - High-pass filtering

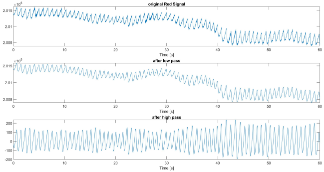
- Before computing the pulse rate and the AC components, apply a high pass filter for attenuating frequencies below 0.5 Hz.
- Use a high-pass FIR digital filter with window method, you can use parts of the code of Exercise 2. Use the following specifications:

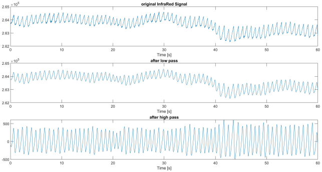
Ω_s	Ω_p	Window type	f_s
0.05 Hz	0.75 Hz	Hamming	100 Hz

- Find the cut-off frequency f_c and compute the number of required coefficients N according to the input transition band and window type.
- Compute the corresponding delay in samples M to make the filter causal and use the spectral transformation formula for high-pass (Lec07) to generate the proper ideal filter and multiply it by the window.
- Plot the filter frequency response with `freqz`.
- Use again `filtfilt` on the signals after low-pass filtering.

Step 5 - Plot the filtered signals

Plot RED (R) and INFRARED (IR) signals after low-pass and high-pass filtering on the time axis (compared with the original signals).

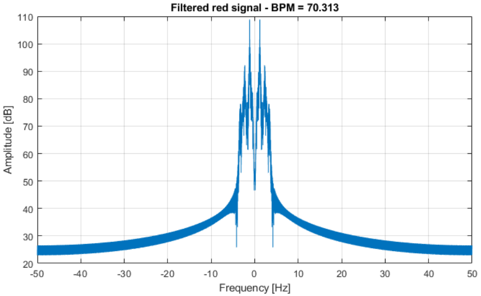




Compute pulse rate and AC components by using these signals (for the DC component you must still use the low-pass filtered signals).

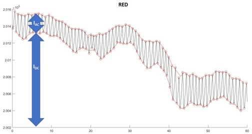
Step 6 - Pulse rate computation

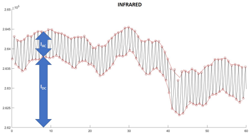
- Work with the filtered RED signal.
- Compute the spectrum with `fft`, round up the number of samples for the FFT to the next power of 2.
- Plot the spectrum in dB scale for the amplitude.
- Compute the pulse rate as the peak frequency of the FFT (use `max`).



Step 7 - Saturation computation

- Compute the value of R at each second.
- Compute its mean value and the saturation value.
- Work on the high-pass filtered signals for the AC and on the low-pass filtered signals for the DC.





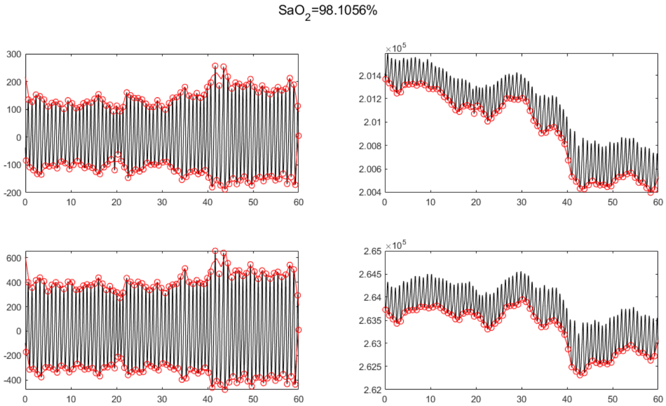
$$\overline{R} = \left\langle \frac{\frac{I_{AC}(RED)}{I_{DC}(RED)}}{\frac{I_{AC}(INFRARED)}{I_{DC}(INFRARED)}} \right\rangle \Rightarrow SaO_2 = 110 - 25\overline{R}$$

Interpolation

- Use `findpeaks` to find the minimum and maximum values for the AC, only minimum for the DC.
- Since the minimum and maximum values are at different time, compute (and plot) the three interpolating curves passing through them.
- Suggestion: identify max (min) and their position, then use `interp1` with `'spline'` option.
- Resample these three curves every second to compute R every second.

Final plots

Plot the interpolating curves and write the saturation value in the title.



Outline

How to compute the saturation value (R signal)

At wavelength λ_1 we measure the received intensity

$$I_1 = I_{in,1} 10^{-(\alpha_1 + (a_{o1} c_o L_{art}) + (a_{d1} c_d L_{art}))}$$

where

- $I_{in,1}$ is the **input** light intensity at λ_1
- α_1 is the **constant** absorption at λ_1 due to bones, tissues, veins,...
- a_{o1} is the **absorption** of HbO_2 at λ_1
- c_o is the **concentration** of HbO_2
- a_{d1} is the **absorption** of Hb at λ_1
- c_d is the **concentration** of Hb
- L_{art} is the arterial width

How to compute the saturation value (IR signal)

Same formula holds at wavelength λ_2:

$$I_2 = I_{in,2} 10^{-(\alpha_2 + (a_{o2} c_o L_{art}) + (a_{d2} c_d L_{art}))}$$

where

- $I_{in,2}$ is the **input** light intensity at λ_2
- α_2 is the **constant** absorption at λ_2 due to bones, tissues, veins,. . .
- a_{o2} is the **absorption** of HbO_2 at λ_2
- c_o is the **concentration** of HbO_2
- a_{d2} is the **absorption** of Hb at λ_2
- c_d is the **concentration** of Hb
- L_{art} is the arterial width

Ratio

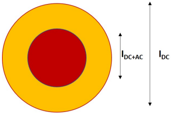
We define the ratio

$$R = \frac{10\log_{10}\frac{I_1}{I_{in,1}}}{10\log_{10}\frac{I_2}{I_{in,2}}}$$

It is easy to show that it is linked to the saturation by:

$$SaO_2 = \frac{c_o}{c_o + c_d} = \frac{a_{d2}R - a_{d1}}{(a_{d2} - a_{o2})\,R - (a_{d1} - a_{o1})}$$

DC and AC components



To remove the dependency on the input light intensity we use the DC and the AC components.
At the smallest width we have:

$$I_{1,DC} = I_{in,1}10^{-(\alpha_1+(a_{o1}c_oL_{art,DC})+(a_{d1}c_dL_{art,DC}))}$$

At the largest width we have:

$$I_{1,DC+AC} = I_{in,1}10^{-(\alpha_1+(a_{o1}c_oL_{art,DC})+(a_{d1}c_dL_{art,DC})+(a_{o1}c_oL_{art,AC})+(a_{d1}c_dL_{art,AC}))}$$

Ratio and saturation

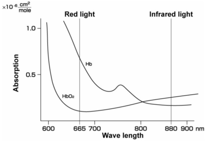
This way, it is easy to show that an equivalent definition of the previously introduced R is given by:

$$R = \frac{10\log_{10}\frac{I_{DC+AC,1}}{I_{DC,1}}}{10\log_{10}\frac{I_{DC+AC,2}}{I_{DC,2}}}$$

And then we apply the formula

$$SaO_2 = \frac{c_o}{c_o + c_d} = \frac{a_{d2}R - a_{d1}}{(a_{d2} - a_{o2})\,R - (a_{d1} - a_{o1})}$$

where the values of the absorption must be derived from the figure below:



Final saturation formula

Finally, we can consider the modified ratio

$$R' = \frac{10\log_{10}\frac{I_{AC,1}}{I_{DC,1}}}{10\log_{10}\frac{I_{AC,2}}{I_{DC,2}}}$$

and compute the saturation value from the formula

$$SaO_2 = \alpha - \beta R'$$

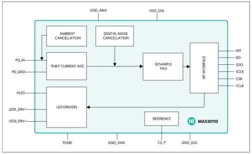
In our case, the values of the parameters from the curve for the two given wavelengths are $\alpha = 110$ and $\beta = 25$.

Maxim Integrated

The pulse oximeter data have been obtained with this device by Maxim Integrated:

MAX30110 - Optimized Pulse-Oximeter and Heart Rate AFE for Wearable Health

maxim integrated™



Outline

Report and Matlab scripts

- For each exercise include all requested plots and answers.
- Plots must contain labels for all axes, a title and a legend in case of multiple plots in one figure.
- The scripts must run correctly. If the script of an exercise doesn't work, the exercise will be considered failed.
- Deliver a separate Matlab file for each exercise (not a single Matlab file for the entire assignment). For exercise 1 and 2, the main script plus the function (2 separate .m files).
- Both the report and the Matlab files must be uploaded on the portal in a zip file.
- Naming rule: `Group8_Assignment2_lastname#1_lastname#2.zip`.
- Send an email to daniel.riviello@polito.it when you upload it.

Report delivery and deadlines

- Deliver a single pdf report for Assignment 2 “Digital filter design”.
- The report must include all requested plots, comments and answers for all exercises (1 to 3).

Deadlines to get extra points for Assignment 2

- **Tue. 29/04/2025** at 23:59 for 1 point.
- **Tue. 06/05/2025** at 23:59 for 0.5 points.