

Plotting and Integrating Signals in MATLAB

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1. For the first part of the project, we were asked to plot the following piecewise continuous signal in MATLAB:

$$h(t) = \begin{cases} \cos(\pi t / 2) & -2 \leq t < -1 \\ 1+t^3 & -1 \leq t < 0 \\ 1 & 0 \leq t < 1 \\ 1+\sin(2\pi t) & 1 \leq t < 2 \\ 0 & \text{elsewhere} \end{cases}$$

The signal has non-zero values within the interval [-2;2]. Therefore, the range of time vector in MATLAB is 4. Since MATLAB cannot store an infinite number of data, I divided the time interval into 10000 segments. I implemented that logic with the following MATLAB code:

```
Range = 2-(-2);
step = Range/(10^4);
t = -2:step:2;
```

The signal $h(t)$ is consist of four signals. We can represent $h(t)$ with unit step functions. I need to define the following pulses:

$u(t+2) - u(t+1)$ – pulse has magnitude 1 over range $-2 \leq t < -1$. Implemented with code:

```
pulse_neg2_neg1 = (t>=(-2))&((t)<(-1)); % pulse from -2 to -1
```

$u(t+1) - u(t)$ – pulse has magnitude 1 over range $-1 \leq t < 0$ Implemented with code:

```
pulse_neg1_0 = (t>=(-1))&((t)<(0)); % pulse from -1 to 0
```

$u(t) - u(t-1)$ – pulse has magnitude 1 over range $0 \leq t < 1$ Implemented with code:

```
pulse_0_1 = (t>=(0))&((t)<(1)); % pulse from 0 to 1
```

$u(t-1) - u(t-2)$ – pulse has magnitude 1 over range $1 \leq t < 2$ Implemented with code:

```
pulse_1_2 = (t>=(1))&((t)<(2)); % pulse from 1 to 2
```

After defining those pulses, I can represent the signal as follows:

$$h(t) = \cos\left(\frac{\pi t}{2}\right) \cdot [u(t+2) - u(t+1)] + (1+t^3)[u(t+1) - u(t)] \\ + [u(t) - u(t-1)] + (1 + \sin(2\pi t))[u(t-1) - u(t-2)]$$

I implemented that signal with the following code:

```
h = pulse_neg2_neg1.*cos(pi*t/2) + pulse_neg1_0.*(1+t.^3) + pulse_0_1.*1 +  
pulse_1_2.*(1+sin(2*pi*t));
```

MATLAB generated the following plot:

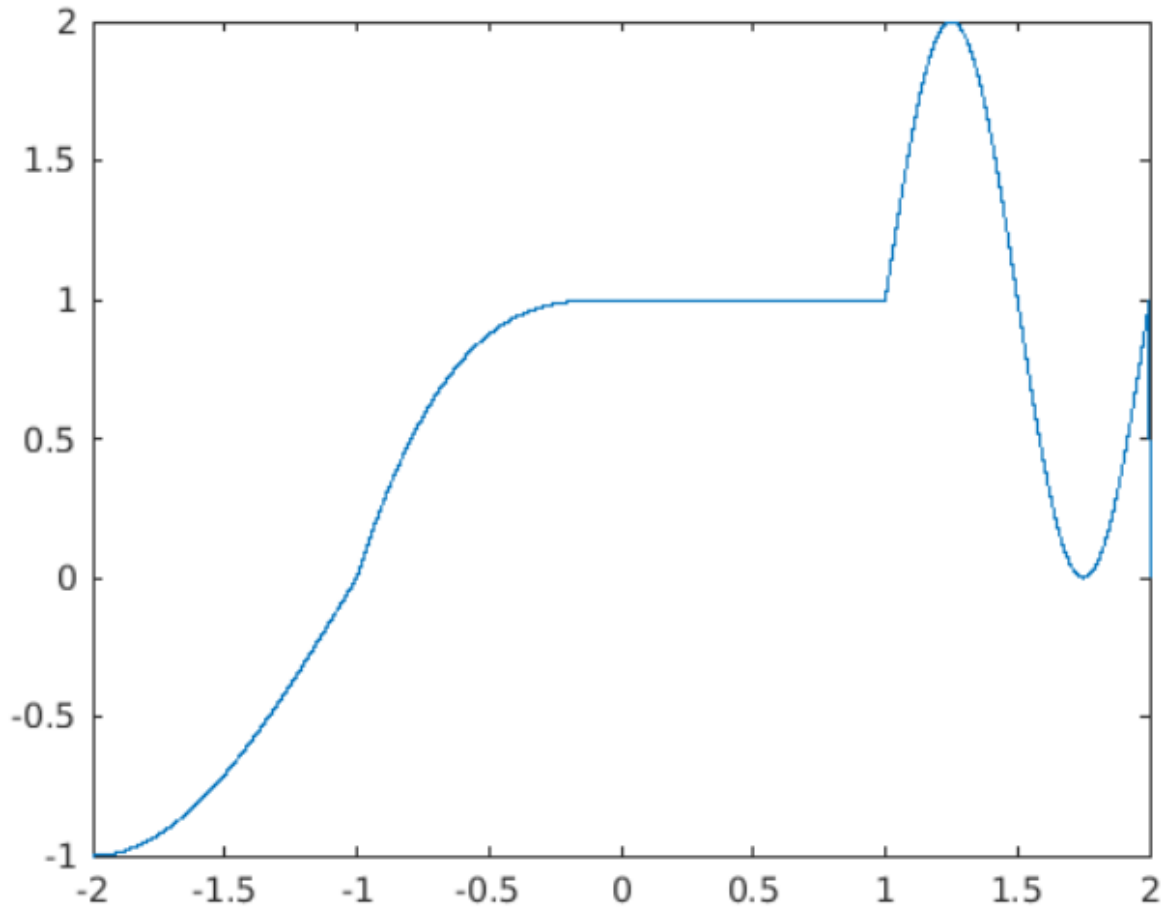


Figure 1

2. For the second part, we were asked to integrate the above-described function by hand and in MATLAB. I used trapz() command to integrate the signal in MATLAB.

$$\text{a) } I1 = \int_{-2}^1 h(t) dt = \int_{-2}^{-1} \cos\left(\frac{\pi t}{2}\right) dt + \int_{-1}^0 (1+t^3) dt + \int_0^1 dt = \left. \frac{2}{\pi} \sin\left(\frac{\pi t}{2}\right) \right|_{-2}^{-1} + \\ \left. t \right|_{-1}^0 + \left. \frac{t^4}{4} \right|_{-1}^0 + 1 = \frac{2}{\pi} \left(\sin\left(\frac{-\pi}{2}\right) - \sin(-\pi) \right) + 1 - \frac{1}{4} + 1 = 1.11338$$

To evaluate that integral in MATLAB, I used the following code:

```
t1 = t(t>=(-2)&t<=(1)); % time interval [-2,1]
h1 = h(t>=(-2)&t<=(1)); % values of the function at time interval [-2,1]
I1 = trapz(t1,h1); % integral of h over time interval [-2,1]
```

The result was 1.1134 (rounded up to 4th decimal). The discrepancy between hand and MATLAB calculation is 1.8734e-05 %

$$\begin{aligned} \text{b) } I_2 &= \int_{-2}^2 h(t) dt = \int_{-2}^{-1} \cos\left(\frac{\pi t}{2}\right) dt + \int_{-1}^0 (1+t^3) dt + \int_0^1 dt + \int_1^2 (1 + \sin(2\pi t)) dt \\ &= \left. \frac{2}{\pi} \sin\left(\frac{\pi t}{2}\right) \right|_{-2}^{-1} + \left. t + \frac{t^4}{4} \right|_{-1}^0 + 1 + 1 - \left. \frac{1}{2\pi} \cos(2\pi t) \right|_1^2 = \\ &= \frac{2}{\pi} \left(\sin\left(\frac{-\pi}{2}\right) - \sin(-\pi) \right) + 1 - \frac{1}{4} + 1 + 1 - \frac{1}{2\pi} [\cos(4\pi) - \cos(2\pi)] = 2.11338 \end{aligned}$$

To evaluate that integral in MATLAB, I used the following code:

```
t2 = t(t>=(-2)&t<=(2)); % time interval [-2,2]
h2 = h(t>=(-2)&t<=(2)); % values of the function at time interval [-2,2]
I2 = trapz(t2,h2); % integral of h over time interval [-2,2]
```

The result was 2.1132 (rounded up to 4th decimal). The discrepancy between hand and MATLAB calculation is -0.0094536%.

$$\begin{aligned} \text{c) } I_3 &= \int_{-0.5}^{1.5} h(t) dt = \int_{-0.5}^0 (1+t^3) dt + \int_0^1 dt + \int_1^{1.5} (1 + \sin(2\pi t)) dt = \\ &= \left. t + \frac{t^4}{4} \right|_{-0.5}^0 + 1 + 0.5 - \left. \frac{1}{2\pi} \cos(2\pi t) \right|_1^{1.5} = 0.5 - \frac{0.5^4}{4} + 1.5 - \\ &= \frac{1}{2\pi} [\cos(3\pi) - \cos(2\pi)] = 2.302684886 \end{aligned}$$

To evaluate that integral in MATLAB, I used the following code:

```
t3 = t(t>=(-0.5)&t<=(1.5)); % time interval [-0.5,1.5]
h3 = h(t>=(-0.5)&t<=(1.5)); % values of the function at time interval [-0.5,1.5]
I3 = trapz(t3,h3); % integral of h over time interval [-0.5,1.5]
```

The result was 2.3027 (rounded up to 4th decimal). The discrepancy between hand and MATLAB calculation is -7.7027e-06%.

$$d) I4 = \int_{-1}^1 h(t) dt = \int_{-1}^0 (1 + t^3) dt + \int_0^1 dt = t \Big|_{-1}^0 + \frac{t^4}{4} \Big|_{-1}^0 + 1 = 1 - \frac{1}{4} + 1 = 1.75$$

To evaluate that integral in MATLAB, I used the following code:

```
t4 = t(t>=(-1)&t<=(1)); % time interval [-1,1]
h4 = h(t>=(-1)&t<=(1)); % values of the function at time interval [-1,1]
I4 = trapz(t4,h4); % integral of h over time interval [-1,1]
```

The result was 1.75 (rounded up to 4th decimal). The discrepancy between hand and MATLAB calculation is -2.2857e-06%.

MATLAB Source Code:

```
format shortEng;
clc;
clear;

Range = 2-(-2);
step = Range/(10^4);
t = -2:step:2;

pulse_neg2_neg1 = (t>=(-2))&((t)<(-1)); % pulse from -2 to -1
pulse_neg1_0 = (t>=(-1))&((t)<(0)); % pulse from -1 to 0
pulse_0_1 = (t>=(0))&((t)<(1)); % pulse from 0 to 1
pulse_1_2 = (t>=(1))&((t)<(2)); % pulse from 1 to 2

h = pulse_neg2_neg1.*cos(pi*t/2) + pulse_neg1_0.*(1+t.^3) + pulse_0_1.*1 +
pulse_1_2.*(1+sin(2*pi*t));

figure;plot(t,h);

t1 = t(t>=(-2)&t<=(1)); % time interval [-2,1]
h1 = h(t>=(-2)&t<=(1)); % values of the function at time interval [-2,1]
I1 = trapz(t1,h1); % integral of h over time interval [-2,1]

t2 = t(t>=(-2)&t<=(2)); % time interval [-2,2]
h2 = h(t>=(-2)&t<=(2)); % values of the function at time interval [-2,2]
I2 = trapz(t2,h2); % integral of h over time interval [-2,2]
```

```
t3 = t(t>=(-0.5)&t<=(1.5)); % time interval [-0.5,1.5]
h3 = h(t>=(-0.5)&t<=(1.5)); % values of the function at time interval [-0.5,1.5]
I3 = trapz(t3,h3); % integral of h over time interval [-0.5,1.5]
```

```
t4 = t(t>=(-1)&t<=(1)); % time interval [-1,1]
h4 = h(t>=(-1)&t<=(1)); % values of the function at time interval [-1,1]
I4 = trapz(t4,h4); % integral of h over time interval [-1,1]
```

```
I_matlab = [I1 I2 I3 I4];
I_hand = [1.11338 2.11338 2.302684886 1.75];
diff = I_matlab - I_hand;
diff = diff./I_hand;
diff = diff*100;
["Integral from -2 to 1" "Integral from -2 to 2" "Integral from -0.5 to 1.5"
"Integral from -1 to 1"];
I=[ans(:) I_matlab(:) I_hand(:) diff(:)];
I=["time interval" "MATLAB calc" "Hand calc" "% diff"; I] % rounded to 4 decimal
```