

CorAL

The Correlations Analysis Library *

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1 Introduction

CorAL is a code base for analysis of 2-particle correlations at small relative momentum. It includes routines for reading, storing and manipulating three-dimensional correlation functions and source functions. Three-dimensional data can be stored in a Cartesian mesh or in terms of expansion coefficients using either Cartesian Harmonics or spherical harmonics as a basis. Routines are provided to translate between different three-dimensional realizations. CorAL routines provide both the means to calculate correlation functions from source functions, and to perform the inverse, by either fitting to a parameterized form, or to perform imaging.

Classes for wave functions and the associated kernels in CorAL are provided for the following pairs: $\pi^+\pi^-$, $\pi^+\pi^+$, pp , nn , pn , pK^+ , $K^+\pi^+$, $K^+\pi^-$, $p\Lambda$, $\Xi^+\pi^-$ and $\Lambda\Lambda$.

CorAL is designed to exploit the simple equation linking three-dimensional source functions to three-dimensional correlation functions:

$$R(\mathbf{q}) \equiv C(\mathbf{q}) - 1 = \int d^3r \{ |\phi(\mathbf{q}, \mathbf{r})|^2 - 1 \} S(\mathbf{r}), \quad (1)$$

by providing the obtain numerical arrays describing $C(\mathbf{q})$ from numerical representations of $S(\mathbf{r})$ for a variety of relative wave functions. A more detailed description of theory can be found in [1], and is included in the `doc/papers/` directory. Eq. (1) can be re-expressed with Correlation and source functions represented by expansion coefficients in either the spherical-harmonic or Cartesian-harmonic basis. In these bases, an angular function $F(\Omega)$ can be reproduced from coefficients via the expressions:

$$F(\Omega) = \sum_{\ell_x, \ell_y, \ell_z} \frac{(\ell_x + \ell_y + \ell_z)!}{\ell_x! \ell_y! \ell_z!} F_{\ell_x, \ell_y, \ell_z} n_x^{\ell_x} n_y^{\ell_y} n_z^{\ell_z} \quad (2)$$

$$F(\Omega) = (4\pi)^{1/2} \sum_{\ell, m} F_{\ell, m} Y_{\ell, m}(\Omega), \quad (3)$$

where $F_{\vec{\ell}}$ and $F_{\ell, m}$ are defined

$$F_{\vec{\ell}} = \frac{(2\ell + 1)!}{\ell!} \int \frac{d\Omega}{4\pi} F(\Omega) \mathcal{A}_{\vec{\ell}}(\Omega), \quad (4)$$

$$F_{\ell, m} = (4\pi)^{-1/2} \int d\Omega F(\Omega) Y_{\ell, m}(\Omega). \quad (5)$$

Here $\mathcal{A}_{\vec{\ell}=\ell_x, \ell_y, \ell_z}$ and $Y_{\ell, m}(\Omega)$ are angular functions and are referred to as Cartesian and Spherical Harmonics. Forms and properties of Cartesian Harmonics can be found in the `doc` directory. Forms and properties for Spherical Harmonics can be found in standard texts, such as J.D. Jackson, *Classical Electrodynamics*.

The expansion coefficients for the correlation and source functions are related on a one-to-one basis.

$$\mathcal{R}_{\ell, m}(q) = \int 4\pi r^2 dr \mathcal{K}_{\ell}(q, r) \mathcal{S}_{\ell, m}(r), \quad (6)$$

$$\mathcal{R}_{\vec{\ell}}(q) = \int 4\pi r^2 dr \mathcal{K}_{\ell}(q, r) \mathcal{S}_{\vec{\ell}}(r). \quad (7)$$

For equations the kernel K_{ℓ} plays the role of the wave function in Eq. (1) linking the correlation and source functions:

$$\mathcal{K}_{\ell}(q, r) \equiv \frac{1}{2} \int d\cos\theta_{qr} [|\phi(q, r, \cos\theta_{qr})|^2 - 1] P_{\ell}(\cos\theta_{qr}). \quad (8)$$

The functions of CorAL are to :

- Derive source functions from output of theoretical models or parameterizations. Source functions would then be stored numerically in arrays. The arrays could either be three dimensional cartesian arrays or arrays of expansion coefficients for either basis mentioned above.
- Calculate wave functions and kernels for a variety of interaction types. Kernels could then be discretized and stored.

- Provide simple convolutions to generate correlation functions from source functions.
- Perform deconvolutions to generate source functions from experimentally determined source functions. This will be done either through parameter fitting or through *imaging* which involves fitting to source functions whose radial dependences are defined by splines or other very general forms.

2 Directory Structure

The base directory of CorAL has four directories:

`doc/ include/ lib/ src/ examples/ codetests/`

The `doc` directory includes this document, plus some related papers along with a sub-directory `printAY` which has codes used to generate printouts showing the relationship between $Y_{\ell,m}$ s and Cartesian harmonics.

The `include/` directory will store copies of all include files required by CorAL. The `lib/` directory contains the makefile used to compile the project and will store the static library `libcoral.a`.

The `src/` directory stores all source and header files in several sub-directories:

Amongst the header files are two catch-all files, `coral.h` and `Utilities/utilities.h` which allow the programmer to include all CorAL files with a simple command `#include "coral.h"`. The header file `utilities.h` is automatically included by `coral.h`, and is useful if the user wishes to use the code for other purposes than analyzing correlation functions.

The routines for generating `CWaveFunction` objects are in `src/WaveFunctions/`, code related to kernels are in `src/Source2CF/` and `src/SourceCalc/` contains code used for filling arrays with information about source functions. Imaging and fitting routines are stored in `src/Imaging/` and `src/Fitting/`. The directory `src/Utilities` holds a panoply of useful codes, which can be applied to projects other than correlations.

Example codes are kept in `examples/`. These codes are meant to provide examples which can be useful in getting started with CorAL. The directory `codetests` also has some examples which can be used to test whether CorAL is operating correctly.

3 Installing, Compiling and Running CorAL

CorAL will be downloaded as a file `coral.tar.gz`. After decompression, the project will appear in a directory `CorAL/`, which can be moved at the user's discretion.

3.1 Compiling CorAL

CorAL will not function without the installation of the GSL (Gnu Scientific Library). This can be found by first dialing into:

<http://www.gnu.org/software/gsl>

GSL routines are used for random numbers, Bessel functions, Coulomb wave functions, spherical harmonics and Clebsch-Gordan coefficients. The code is only tested for the GNU compiler (g++, version 4.0 or higher). **GSL** and g++ are available for Linux, Windows and Mac OS X. Rather than using makefiles, CorAL uses a replacement for the `make` command, `scons`, which provides a simple means to ensure that object files, libraries, and executables are recompiled on a must-compile basis. The package is available at:

<http://www.scons.org>

CorAL compilation requires a few environmental variables, which allow CorAL to find header files, source files and libraries. These variables are:

VARIABLE	DESCRIPTION	EXAMPLE
CORAL_HOME	base directory, where src/, include/, examples... reside	/Users/Barbara/CorAL/
CORAL_GSLPATH	root of ../lib/gsl and ../include/gsl	/usr/local/
CORAL_X11PATH	root of ../lib/X11 and ../include/X11	/usr/X11R6/
CORAL_CCFLAGS	optimization flags for g++	-O2

If the environment variables are not set, CorAL will do its best to guess the appropriate values given your operating system. ?????? can it even guess CORAL_HOME ??????

To compile the libraries, one need only go into the CorAL home directory enter:

```
~/CorAL% scons
```

If all works, the code will be compiled and three static libraries will appear in lib/: libcoral.a, libcoralutils.a and libxgraph.a. The xgraph library is non-essential and is mainly used for the code tests, so if it fails to compile, or if you simply have no need for it, you can delete the one pertinent line in the SConstruct file in the CorAL home directory.

To incorporate CorAL functionality into your code, you need only add an include statement to your code, then compile. If you wish to use the static CorAL library (libcoral.a in lib/), you need to add the line:

```
#include "coral.h"
```

to the beginning of your source code, then compile with the command:

```
g++ -I ${CORALHOME}/include -L ${CORALHOME}/lib -lcoral -lgsl -lgslcblas myprog.cc -o myprog,
```

where CORALHOME is the root CorAL directory. If you would prefer to recompile the source code along with your main program, you would add the line:

```
#include "coral.cc"
```

to your main routine rather than coral.h, and compile with the command:

```
g++ -I ${CORALHOME}/include -lgsl -lgslcblas myprog.cc -o myprog,
```

The codes in the sample/ directory can be compiled with make (e.g. make wfsample) using the makefile found in that directory. The header file coral.h simply contain include statements to the remaining headers.

The result of the compilation will be a shared library file libcoral.a which will reside in the lib directory. The make command will also copy all header and source files from the src/ tree into the include/ directory (unless new versions already exist). If new source files are added to the src directory, the makefile must be edited. To make a new makefile, simply run the shell script:

```
makemaker.sh > makefile,
```

then change the \${OPT} variable in the makefile if needed. In the makefile in the samples/ directory, one can also recompile the source with command make coral, which changes to the lib/ directory, compiles CorAL and returns to the original directory.

Not all source code found in the src/ tree is compiled into CorAL, although any *.cc or *.h file will be used as a dependency in the makefile. If you add source files, they should be listed with an include statement in one of the following source and header files which are included in the compilation: coral.cc coral.h misc.cc, misc.h, arrays.cc, arrays.h, sf.cc, sf.h, parametermap.cc, parametermap.h, wavefunction.cc, wavefunction.h, source2CF.cc, source2CF.h, kernel.cc, kernel.h. For instance, if you add a new source file, wf_lambdaxi.cc, which is used to calculate wave functions for Λ s and Ξ s, you would add a line to the file wavefunction.cc so that it would become:

```
#include "wavefunction.h"
#include "wfcommon.cc"
#include "planewave.cc"
#include "partwave.cc"
#include "pipi_phaseshifts.cc"
#include "wf_pp.cc"
#include "wf_pkplus.cc"
#include "wf_pipluspiplus.cc"
#include "wf_pipluspiminus.cc"
#include "wf_nn.cc"
#include "wf_lambdaxi.cc"
```

In fact, there do exist several wf... files in the src/ tree which are not included as they are either in development or are untested.

3.2 Code Tests and Examples

For the purpose of testing CorAL, several source files are located in the `codetests/` directory. All these tests build with the main `SConstruct` file in the top-level directory of the project:

```
> scons tests=True
```

They install to the `codetests/` directory, along with their source and all needed data files.

3.2.1 Unit tests & the like

These are simple tests of specific functions/routines in various codes

- Integrator tests:
 - `dcuhre/dtest1` – FAILS TO BUILD::
 - `dcuhre/dtest2` – FAILS TO BUILD::
- Linear algebra tests:
 - `complexmatrixtest` – linear algebra tests or complex N-dimensional matrices
 - `realmatrixtest` – linear algebra tests or real N-dimensional matrices
 - `tnt_inversions` – matrix math and GLSQR inverter tests
- Random numbers:
 - `gslrantest` – test GSL's random number generator
 - `rantest` – test CorAL's random number generator
- Special function tests:
 - `chttester` – Cartesian harmonics
 - `gslbesstester` – test of GSL's Bessel functions
 - `gslbug` – test of a GSL bug in its Coulomb WF package
 - `ylmchttester` – Spherical to Cartesian harmonic conversions
 - `ylmttester` – a Spherical harmonic (Y_{lm}) tester
- Tests of miscellaneous utilities:
 - `dycast_tester` – test various smart-pointer tools used in CorAL
 - `testBoost` – Lorentz boosts
- Wavefunction test codes:
 - `coulwave/cgttester` –
 - `coulwave/cwttester` – FAILS TO BUILD:: function `CWincoming_smallr` not declared in `wavefunction.h`
 - `iwtest` –
 - `phaseshift_tester` – ?? which is which?
 - `sortTheseOut/phaseshift_tester` – ?? which is which?
 - `sortTheseOut/wftestX` – ??
 - `planewavetest` –
 - `wfsample` –

- stester – simple CorAL regression tests: type “stester -help” to find out details

```
Valid tests are: -pmap      : Test read/write/assignment of parameterMap
                -objio     : Read/write various objects
                -harm      : FAILS:: Initialize a 3d correlation represented in a Spherical
                           Harmonic basis
                -image     : FAILS:: Simple basis spline imaging test in 1d
                -legimage  : FAILS:: Simple Legendre polynomial imaging test in 1d
                -legimage3d : FAILS:: Legendre polynomial imaging test in 3d
                -conv      : FAILS:: Convolute a Gaussian with a kernel
```

This code uses all tests in the imageLegendreTest.h, objectIOTest.h, parameterMapTest.h headers.

- Obsolete tests:
 - besstester – test of Bessel functions
 - coulwave/ptester –
 - fixUncert –
 - ylm_tester – a Spherical harmonic (Y_{lm}) tester

3.2.2 Integration tests

These are more complex tests that check whether components of CorAL work together.

- Kernels:
 - kernelsample –
 - sortTheseOut/kerneltestX – FAILS TO BUILD::
 - kzeros – zeros of the $K_l = 0(q, r)$ and $K_l = 2(q, r)$ pi0-pi0 kernel
- Miscellaneous:
 - boltztest –
 - cgctester –
 - testConvolve –
- Source functions:
 - sourcesample_blast –
 - sourcesample_gauss –
 - sourcesample_OSCAR – FAILS TO BUILD:: wrong function prototype for CSourceCalc_OSCAR::CalcS ??

3.3 Known Issues

GSL has a bug in the Coulomb partial wave routines for versions 1.7 and earlier. The sign of the wave function for larger (>3) values of $x = qr/\hbar$ oscillates between the correct and incorrect sign. The authors have provided a fix to the source code (see below). If you do not have this fix to **GSL**, you can edit the file `src/SpecialFunctions/CoulWave/coulwave.cc` and comment away the line `#define NO_GSLCOULWAVE_BUG`. If you want to change the **GSL** source file, then recompile, a bug fix was provided by the author and is described in the text file “gsibugfix.txt” which is included in `doc/`.

4 Parameter files

CorAL uses its own input and parameter file format for controlling both the executables distributed with CorAL and many of the subroutines in the CorAL library itself. The main use of files in the CorAL format is to populate instances of the associative array class called `parameterMap` (defined in the `parameter_map.h` file). The data types supported by CorAL are given in Table 1. CorAL's general format is straightforward and consists of a few variations:

- **simple:** used for simple scalar data or small strings and has the format:

```
type name value
```

For example:

```
int lmax 2
string filename output.dat
```

Note, strings do not need to be quoted and whitespace separates each token.

- **multi-line:** used for vectors and lists and has the format:

```
type name {
    value0
    value1
    ...
}
```

Again, strings do not need to be quoted and whitespace separates each token. In the array, each value is given on its own line. The dimension of the vector is computed from the length of the vector.

- **extended multi-line:** used for matrices exclusively and have the form

```
type name {
    value00 value01 value02 ...
    value10 value11 value12 ...
    ...
}
```

We assume that the matrices are rectangular and get the dimensions for the matrix from the number of columns in the first line and the number of rows in the entire data structure. We do not check that each line has the proper number of elements in it.

- **nested multi-line:** used for storing `parameterMaps` and may contain any simple, multi-line or extended multi-line inside. The format is as follows:

```
type name {
    subtype0 subname0 subvalue0
    subtype1 subname1 {
        value11
        ...
    }
    ...
}
```

The order of items in a `parameterMap` is not preserved so is not important for the input files.

- **comments:** If the character “#” is encountered, the remainder of the line is considered a comment. For example:

```
#int lmax 0 # the old value
int lmax 2 # the new value
string filename output.dat # this is the output correlation
```

The follow set of tables, Tables ??-??, detail the various input parameters in each class.

CorAL type	C++ equivalent	Syntax
bool	bool	simple
int	int	simple
double	double	simple
string	std::string	simple
vector_bool	std::vector< bool >	multi-line
vector_int	std::vector< int >	multi-line
vector_double	std::vector< double >	multi-line
vector_string	std::vector< std::string >	multi-line
matrix_double	std::vector< std::vector< double > >	extended multi-line
parameter_map	parameterMap	nested multi-line

Table 1: Data types supported by CorAL and their mappings onto C++ data structures inside CorAL

Table 2: supported particles and their keys

"pi+"
"pi-"
"pi0"
"K+"
"K-"
"p"
"n"
"Xi"
"lambda"
"antilambda"

5 Sample codes

The main part of the project are the static libraries in the `lib/` directory. However, using them is not currently well documented, so we provide examples and executables that use the libraries. In the `bin/` directory, there are several executables.

The CorAL package includes a `examples/` directory. Within that directory are several example codes. With these codes you can additionally

All have inline help (invoke using e.g. `> crab -h`).

5.1 bfplot

The code `bfplot` is a simple tool that produces linear interpolable tables of the basis functions of each basis. Additionally, the `-convolve` option allows us to fold the basis function with a kernel specified by the "particles" option in the input file.

5.2 scplot

Similar to `bfplot`, the `scplot` is a simple tool to make linear interpolable tables of sources and correlations suitable for plotting. Because each representation of the sources and correlations is a little different and

Table 3: supported pairs

pi+ pi+ *	wf_pipluspiplus_nostrong.cc
pi+ pi+	wf_pipluspiplus_phaseshift.cc
pi+ pi+	wf_pipluspiplus_sqwell.cc
pi+ pi-	wf_pipluspiminius_phaseshift.cc
pi+ pi-	wf_pipluspiminius_sqwell.cc
pi+ pi-, K+ K-	wf_pipluspiminius_nostrong.cc
K+ pi+, K- pi-	wf_kpluspiplus_sqwell.cc
K+ pi-, K- pi+	wf_kpluspiminius_phaseshift.cc
K+ pi-, K- pi+	wf_kpluspiminius_sqwell.cc
p pi+	wf_ppiplus_phaseshift.cc
p pi+	wf_ppiplus_sqwell.cc
p pi-	wf_ppiminius_sqwell.cc
p K+	wf_pkplus_sqwell.cc
p K+	wf_pkplus_phaseshift.cc
p p	wf_pp_phaseshift.cc
p p	wf_pp_schrod.cc
n n	wf_nn_phaseshift.cc
p n	wf_pn_phaseshift.cc
Lambda Lambda	wf_lambdalambda_phaseshift.cc
Lambda Lambda	wf_lambdalambdaantiparspin_phaseshift.cc
Lambda Lambda	wf_lambdalambdaparspin_phaseshift.cc
p Lambda	wf_plambda_phaseshift.cc
Xi pi	wf_Xipi_phaseshift.cc

Table 4: Command line options for `bfplot`, the widget to plot the basis functions using CorAL. Usage: `bfplot <option> [inputFile.dat]`

-h, -help, -help	All print this message, then quit
-legendre	Print out Legendre polynomial basis
-bspline	Print out Basis Spline basis
-laguerre	Print out Laguerre function basis
-chebyshev	Print out Chebyshev polynomial basis
-histogram	Print out Histogram basis
-hermite	Print out Hermite function basis
-convolve	Convolve basis funcs with kernel too

Type	Map	Name	Default Value	Description
string	inMap	particle1	"pi+"	
string	inMap	particle2	"pi+"	
string	inMap	basis_func_plot_file	"output_r_basis_funcs.dat"	
string	inMap	convoluted_basis_func_plot_file	"output_q_basis_funcs.dat"	
double	inMap	qmin	0.0	
double	inMap	qmax	100.0	
int	inMap	l	0	

Table 5: Input file controls for `bfplot` (src/bfplot/bfplot.cc).

Table 6: Command line options for `scplot`, a widget to plot sources and correlation using CorAL. Usage: `scplot <option> [inputFile.dat]`

-h, -help, -help	all print this message, then quit
-legendre	Print out using Legendre polynomial basis
-bspline	Print out using Basis Spline basis
-laguerre	Print out using Laguerre function basis
-chebyshev	Print out using Chebyshev polynomial basis
-histogram	Print out using Histogram basis
-hermite	Print out using Hermite function basis
-gaussian	Print out using using Gaussian class
-correlation	inputFile.dat is a correlation, only histogram basis available
-3dsphr	inputFile.dat is a 3d object, in spherical harmonics
-3dcarhisto	inputFile.dat is a 3d Cartesian histogram object

Type	Map	Name	Default Value	Description
string	inMap	plot_file	"output_plot.dat"	
string	inMap	plot_file_x	"output_plot.x.dat"	
string	inMap	plot_file_y	"output_plot.y.dat"	
string	inMap	plot_file_z	"output_plot.z.dat"	

Table 7: Input file controls for `scplot` (src/scplot/scplot.cc).

there is no way to denote these in the CorAL input files, you must specify the representation on the command line using the options in the command line options table.

5.3 chum

Make OSCAR formatted events from simple core-halo model CHUM: the Core-Halo Ur-Model

Usage: `chum <options> [inputFile.dat]`

5.4 shark

Simple HARmonic Kernels (SHARK) is a tool to pregenerate the full 3D kernel, expanded in Legendre polynomials, for use in Harmonic expansions of the sources and correlations.

5.5 crab

reimagining of S. Pratt's CoRrelation AfterBurner

Usage: `crab <mode> [inputFile.dat]`

5.6 converts2c

fold a source w/ kernel to make a correlation

Type	Map	Name	Default Value	Description
double	main	T	175.0	Core temperature (in MeV)
double	main	Rx	4.0	Core radius in x direction (in fm)
double	main	Ry	4.0	Core radius in y direction (in fm)
double	main	Rz	4.0	Core radius in z direction (in fm)
double	main	tau_fo	10.0	Core freeze-out duration (in fm/c)
double	main	f	1.0	Core fraction
int	main	id	id	??
string	main	flowprofile	flowprof	??
double	main	etamin	etamin	??
double	main	etamax	etamax	??
double	main	ymin	ymin	??
double	main	ymax	ymax	??
double	main	pTmin	pTmin	??
double	main	pTmax	pTmax	??
string	main	filename		output file name
int	main	initial_seed	197	an integer to serve as a random seed
int	main	num_particles	100000	number of particles to sample

Table 8: src/chum/chum.cc

Type	Map	Name	Default Value	Description
string	main	kernel_particle1	""	
string	main	kernel_particle2	""	

Table 9: Input file options for Simple HARMonic Kernels (SHARK) using CorAL (src/shark/shark.cc). Usage: shark <option> [inputFile.dat]

Table 10: default

-1doldway	Generate 1d source and correlation, using old code for source
-3dcart	Generate 3d source and correlation directly in Cartesian
-3dcart_ex	Generate 3d source and correlation directly in Cartesian, then expand
-3dspbr	Generate 3d source and correlation directly in Ylm's
-crab	Generate 1d & 3d correlations and output them in the style of Scott Pratt's CRAB code
-sourcemaker	Generate 1d source & 3d source slices and output them in the style of Dave Brown's SourceMaker code
-skip_correlation	Skip generation of the correlations if accompanying modes would generate them
-make_plots	Make plots for all the sources and correlations generated

Type	Map	Name	Default Value	Description
mode -1d: Generate 1d source and correlation				
string	main	source_file	"output_source_1d.dat"	
string	main	correlation_file	"output_correlation_1d.dat"	
parameterMap	main	source_settings	None	
string	main	source_file	"output_source_1d.dat"	
string	main	correlation_file	"output_correlation_1d.dat"	
string	main	source_file	"output_source_3d.dat"	
string	main	correlation_file	"output_correlation_3d.dat"	
string	main	source_file.cart	"output_source_3d_cart.dat"	
string	main	source_file	"output_source_3d_sphr.dat"	
string	main	correlation_file	"output_correlation_3d_cart.dat"	
string	main	crab_correlation_1d_filename	"correlation_qinv.dat"	
string	main	crab_correlation_3d_filename	"correlation_qinv3d.dat"	

Table 11: src/crab/crab.cc

Table 12: Widget to Convert Sources to Correlations (ConvertS2C) using CorAL. Usage: converts2c <option> [inputFile.dat]

-h, -help, --help	all print this message, then quit
-legendre	inputFile.dat uses Legendre polynomial basis
-bspline	inputFile.dat uses Basis Spline basis
-laguerre	inputFile.dat uses Laguerre function basis
-chebyshev	inputFile.dat uses Chebyshev polynomial basis
-histogram	inputFile.dat uses Histogram basis
-hermite	inputFile.dat uses Hermite function basis
-gaussian	inputFile.dat is a 1d Gaussian
-3d	inputFile.dat is a 3d source
-convolve	Use convoluter, not imager for this (uncertainty not included)

Table 13: Demonstration InVERter: a CorAL imaging code. Usage: diver [mode] <input file>

-help	help message
-bspline	Use Basis Spline basis for source
-chebyshev	Use Chebyshev polynomial basis for source
-hermite	Use Hermite polynomial basis for source
-histogram	Use Histogram basis for source
-laguerre	Use Laguerre polynomial basis for source
-legendre	Use Legendre polynomial basis for source
-norestore	Don't restore the correlation
-restore	Restore the correlation
-1d	Problem is 1d
-3d	Problem is 3d

Type	Map	Name	Default Value	Description
string	main	correlation_file	None	
string	main	imaged_source_file	None	
string	main	restored_correlation_file	None	
parameterMap	main	source_settings	None	
parameterMap	main	source_settings	None	

Table 14: src/diver/diver.cc

5.7 diver

source imaging code

Type	Map	Name	Default Value	Description
int	s	l	l	
int	s	m	m	
bool	s	realpart	realpart	
bool	s	imagpart	!realpart	
int	s	lx	lx	
int	s	ly	ly	
int	s	lz	lz	
bool	s	spherical_harmonic	sphrHarm	
bool	s	cartesian_harmonic	!sphrHarm	

Table 15: (src/coral/ObjectInterfaces/objects1d.cc)

Type	Map	Name	Default Value	Description
int	com	ndata	data.dim	
matrix_double	com	covmtx_datablock	empty_mat	
vector_double	com	y_datablock	empty_vec	
matrix_double	com	ydy_datablock	empty_mat	

Table 16: (src/coral/ObjectInterfaces/dataset.cc)

Type	Map	Name	Default Value	Description
All Basis Function classes inherit these (src/coral/ObjectInterfaces/func_expansion1d.cc)				
double	m	xmin	xmin	
double	m	xmax	xmax	
(WHAT HAPPENED TO ORTHOGFUN_EXPANSION?)				
(src/coral/ObjectInterfaces/generic_spline1d.cc)				
int	m	spline_degree	3	
vector_double	m	knots_datablock	empty_vec	
(WHAT HAPPENED TO BSPLINE?)				
(src/coral/ObjectInterfaces/histogram1d.cc)				
bool	m	fixed_width_bins	fixed_width_bins	
double	m	dx	None	
double	m	xmax	None	
vector_double	m	knots	empty_vec	
matrix_double	m	xy_datablock	empty_mat	
matrix_double	m	xydy_datablock	empty_mat	
(WHAT HAPPENED TO CHEBYSHEV?)				
(WHAT HAPPENED TO LEGENDRE?)				
(src/coral/ObjectInterfaces/hermitefunction_expansion1d.cc)				
double	m	xscale	xscale	
(src/coral/ObjectInterfaces/laguerrepoly_expansion1d.cc)				
double	m	xscale	xscale	

Type	Map	Name	Default Value	Description
(src/coral/CF2Source/minuitstuff/sfit_minuit_3dgaussian.cc)				
double	sourcecalc- >spars	lambda	1	
double	sourcecalc- >spars	Rx	5	
double	sourcecalc- >spars	Ry	5	
double	sourcecalc- >spars	Rz	5	
double	sourcecalc- >spars	Xoff	0	
(src/coral/CF2Source/minuitstuff/sfit_minuit_blast.cc)				
double	sourcecalc- >spars	lambda	1	
double	sourcecalc- >spars	R	13.0	
double	sourcecalc- >spars	Tau	12.0	
double	sourcecalc- >spars	DelTau	5	
(src/coral/CF2Source/sfit_3dgaussian.cc)				
double	sourcecalc- >spars	lambda	0.5	
double	sourcecalc- >spars	Rx	5	
double	sourcecalc- >spars	Ry	5	
double	sourcecalc- >spars	Rz	5	
double	sourcecalc- >spars	Xoff	0.0	
(src/coral/CF2Source/sfit_blast.cc)				
double	sourcecalc- >spars	lambda	1	
double	sourcecalc- >spars	R	13	
double	sourcecalc- >spars	Tau	12	
double	sourcecalc- >spars	DelTau	5	
src/coral/CF2Source/minuitstuff/sfit_minuit_GX1d.cc)				
double	sourcecalc- >spars	lambdaG	0.6	
double	sourcecalc- >spars	Xfrac	0.5	
double	sourcecalc- >spars	R	5	
double	sourcecalc- >spars	X	10	
double	sourcecalc- >spars	a	5	
src/coral/CF2Source/sfit_GX1d.cc)				
double	sourcecalc- >spars	lambdaG	0.3	
double	sourcecalc- >spars	R	5	
double	sourcecalc- >spars	lambdaX	0.3	
double	sourcecalc- >spars	X	10.0	
double	sourcecalc- >spars	a	5.0	

Table 17: Source fitting input parameters.

6 parameterMap Reference

6.1 1d Function Parameterizations/Basis Functions

6.2 Fitting classes (using MINUIT)

6.3 Kernels and Wavefunctions

6.4 1d source functions

7 Data Array Reference

Data are stored in several classes of arrays, including three-dimensional Cartesian meshes, Spherical Harmonic expansions, Cartesian Harmonic expansions and in the future, basis splines. In the next section we will discuss a namespace for operations involving multiple arrays, such as those used to translate into different array types, or multiply and divide arrays.

7.1 Cartesian Meshes

Cartesian meshes are described by a three dimensional array of size $NXMAX$, $NYMAX$, $NZMAX$ and granularities $DELX$, $DELY$, $DELZ$. The mesh is further defined by three parameters $XSYM$, $YSYM$, $ZSYM$ which are booleans set to `true` for the corresponding reflection symmetries. When set to `false` the program assigns additional memory to describe data for negative values of x , y , z . The ℓ_x, ℓ_y, ℓ_z components refer to a range $\ell_x \Delta x < x < (\ell_x + 1) \Delta x, \ell_y \Delta y < y < (\ell_y + 1) \Delta y \dots$.

The capabilities of the class are best explained by viewing the public members of the class:

```
class C3DArray{
public:
    C3DArray(char *arrayparsfilename);
    C3DArray(int NXYZMAX,double DELXYZ,bool XSYM,bool YSYM,bool ZSYM);
    C3DArray(int NXMAX,double DELX,int NYMAX,double DELY,int NZMAX,double DELZ,
        bool XSYM,bool YSYM,bool ZSYM);
    ~C3DArray();

    int GetNXMAX();
    int GetNYMAX();
    int GetNZMAX();
    double GetDELX();
    double GetDELY();
    double GetDELZ();
    bool GetXSYM();
    bool GetYSYM();
    bool GetZSYM();
    void PrintPars();

    double GetElement(double x,double y,double z);
    double GetElement_NoInterpolation(double x,double y,double z);
    double GetElement(int isx,int ix,int isy,int iy,int isz,int iz);
    void SetElement(int isx,int ix,int isy,int iy,int isz,int iz,double value);
    void IncrementElement(int isx,int ix,int isy,int iy,int isz,int iz,
```

```

double value);
void SetElement(double x,double y,double z,double value);
void IncrementElement(double x,double y,double z,double increment);

void ZeroArray();
void MakeConstant(double c);
void ScaleArray(double scalefactor);

void Randomize(double c);
void RandomizeGaussian(double c);

void PrintArray();
double GetBiggest();

void ReadArray(char *dirname);
void WriteArray(char *dirname);
};

```

Three constructors are used to initialize the arrays and set the parameters: XSYM, YSYM, ZSYM, NXMAX, NYMAX, NZMAX, DELX, DELY, DELZ, which can also be set by reading a parameters file. The parameters can be printed by calling PrintPars() and can be accessed via the function GetXSYM(),... If tC3DArray(char *arrayparsfilename) is used, the parameters are read from a file formatted in the form:

```

bool YSYM 1
int NXMAX 36
double DELZ 2.5
:

```

If any of the parameters are missing, default values will be used. The boolean parameter IDENTICAL can also be set, which is identical to setting the three symmetry parameters to true. For the reflection-symmetry parameters, XSYM, YSYM and ZSYM are set to false by default.

The functions that set, get and increment elements should probably not need much explanation. They are over-loaded so that one can access the values either by the indices directly, or through the coordinates. If values are accessed outside the range, e.g., $x > \Delta x \cdot NXMAX$, zero is returned. The arguments isx, isy, isz which appear in several functions refer to the sign of the argument. For $x > 0$ isx=0, while for $x < 0$, isx=1. When XSYM='true' memory is not allocated for isx=1.

ZeroArray() will set all the elements to zero and MakeConstant(c) sets all elements to c. The entire array can be scaled by a constant with ScaleArray. Every element can be randomized according to a Gaussian distribution, $\exp -x^2/2c^2$, with RandomizeGaussian(c) or to a uniform distribution between $-c$ and c with Randomize(c).

The element with the largest absolute value is returned with GetBiggest().

The functions WriteArray(char *dirname) and ReadArray(char *dirname) will write and read arrays into the named directory. The same format is used for both reading and writing, so once arrays are written they can be read in easily. The directory will include a file, 3Darraypars.dat, describing the dimensions and symmetry of the array, so only one array can be stored in a given directory. If the array being read has different dimensions or symmetry, the arrays will be deleted and re-created with the correct amount of allotted memory.

7.2 Cartesian Harmonics Data Arrays

Angular information can also be stored in terms of expansion coefficients describing expansions in terms of powers of unit vectors \hat{e} . For a given radial bin ir, coefficients $F_{\ell_x, \ell_y, \ell_z}$ can describe the function,

$$F(\Omega) = \sum_{\ell_x, \ell_y, \ell_z} \frac{(\ell_x + \ell_y + \ell_z)!}{\ell_x! \ell_y! \ell_z!} F_{\ell_x, \ell_y, \ell_z} e_x^{\ell_x} e_y^{\ell_y} e_z^{\ell_z}, \quad (9)$$

where the arrays satisfy the tracelessness constraint $F_{\ell_x, \ell_y, \ell_z+2} + F_{\ell_x+2, \ell_y, \ell_z+2} + F_{\ell_x, \ell_y+2, \ell_z} = 0$. With this constraint the elements for a given rank $\ell = \ell_x + \ell_y + \ell_z$ are determined by the $\ell_x = 0, 1$ elements. Thus, there are $(2\ell + 1)$ independent coefficients for each ℓ .

The public members of the class are:

```
class CCHArray{
  CCHArray(char *arrayparsfilename);
  CCHArray(int LMAXset,int NRADIALset,double RADSTEPset);
  CCHArray(int LMAXset,int NRADIALset,double RADSTEPset,
    bool XSYMset,bool YSYMset,bool ZSYMset);
  ~CCHArray();

  int GetLMAX();
  int GetNRADIAL();
  double GetRADSTEP();
  void SetLMAX(int LMAXset);
  void SetRADSTEP(double RADSTEPset);
  bool GetXSYM();
  bool GetYSYM();
  bool GetZSYM();
  void PrintPars();

  void ZeroArray();
  void ZeroArray(int lx,int ly,int lz);
  void ZeroArray(int ir);
  void ZeroArray_Partial(int LMAX_Partial);
  void ZeroArray_Partial(int LMAX_Partial,int ir);
  void ScaleArray(double scalefactor);
  void ScaleArray(double scalefactor,int ir);

  double GetElement(int lx,int ly,int lz,int ir);
  double GetElement(int lx,int ly,int lz,double r);
  void SetElement(int lx,int ly,int lz,int ir,double Element);
  void SetElement(int lx,int ly,int lz,double r,double Element);
  void IncrementElement(int lx,int ly,int lz,int ir,double increment);
  void IncrementElement(int lx,int ly,int lz,double r,double increment);

  void PrintArrayFixedIR(int ir);
  void PrintArrayFixedIR(int LMAXPrint,int ir);
  //FillRemainder functions use identity
  // A_{lx+2,ly,lz}+A_{lx,ly+2,lz}+A_{lx,ly,lz+2}=0
  // to find entire array if (2L+1) values for lx=0,1 are known

  double GetBiggest(int ir);

  // Read and Write to a given directory, "AX" suffix means only lx=0,1 elements
  // will be read/written. To write only up to WLMAX, use WriteShort()
  void ReadAX(char *dirname);
  void WriteAX(char *dirname);
  void ReadAllA(char *dirname);
  void WriteAllA(char *dirname);
  void WriteShort(char *filename,int WLMAX);

  void IncrementAExpArray(double x,double y,double z,double weight);
  void IncrementAExpArrayFromE(double ex,double ey,double ez,
    double weight,int ir);
  void AltIncrementAExpArrayFromE(double ex,double ey,double ez,
```

```

    double weight,int ir);
void AltAltIncrementAExpArrayFromE(double ex,double ey,double ez,
    double weight,int ir);
void IncrementMArrayFromE(double ex,double ey,double ez,double weight,int ir);
void IncrementAExpArrayFromThetaPhi(double theta,double phi,
    double weight,int ir);
void IncrementMArrayFromThetaPhi(double theta,double phi,
    double weight,int ir);

// Returns < ex^lx ey^ly ez^lz > where (ex,ey,ez) is unit vector
double GetMElementFromAExpArray(int lx,int ly,int lz,int ir);
// Given array, < ex^lx ey^ly ez^lz >, this returns the CH expansion
// element
double GetAExpElementFromMArray(int lx,int ly,int lz,int ir);

void FillRemainderX(int ir);
void FillRemainderY(int ir);
void FillRemainderZ(int ir);
void FillRemainderX();
void FillRemainderY();
void FillRemainderZ();

// For CH Expansion array, this returns
// F(Omega)=\sum_{lx,ly,lz} (L!/(lx!ly!lz!))
// F_{lx,ly,lz} e_x^lx e_y^ly e_z^lz
double AExpand(double ex,double ey,double ez,int ir);
double AExpand(double x,double y,double z);
double AExpand(double theta,double phi,int ir);

void Detrace(int ir);
void Detrace();

void Randomize(double mag,int ir);
void RandomizeA(double mag,int ir);
void Randomize(double mag);
void RandomizeA(double mag);
void RandomizeA_Gaussian(double mag,int ir);
void RandomizeA_Gaussian(double mag);
};

```

The parameters describing the size of the array are LMAX, NRADIAL, RADSTEP and XSYM, YSYM, ZSYM and can be set either with the constructors or in the parameters file as described for the cartesian arrays in the previous subsection. These parameters are printed by calling PrintPars().

Some of the functions are overloaded with and without dependences on the radial coordinate r or the radial index ir . The variations without radial dependencies operate on all the radial indices.

Individual elements can be accessed, set or incremented with the functions GetElement(...), SetElement(...) and IncrementElement(...). The array is set to zero with ZeroArray(...) and Randomize(...) sets the elements to random values between -1 and 1.

The reading and writing routines ReadAllA and WriteAllA will read and write all elements of the array, whereas ReadAX and Write AX read and write only those components with $\ell_x = 0, 1$. This is useful for traceless arrays since the remaining components are easily generated from those with $\ell_x = 0, 1$. The function FillRemainderX() will accomplish this feat using the tracelessness constraint above, which can be re-expressed as: $F_{\ell_x+2,\ell_y,\ell_z} = -F_{\ell_x,\ell_y+2,\ell_z} - F_{\ell_x,\ell_y,\ell_z+2}$. FillRemainderY() and FillRemainderZ() fill the remainders of the arrays starting with the $\ell_y = 0, 1$ and $\ell_z = 0, 1$ components respectively. The function WriteShort(char *filename,int WLMAX) writes the $\ell_x = 0, 1$ components (skipping those that are zero due to symmetry) up to $L \leq WLMAX$ to a single file in format that is convenient for graphing, but not

sufficiently accurate for re-reading.

The value of the function expanded with Eq. (9) can be found with the functions `AExpand`.

The detracing operation is used by the array operations which multiply, divide and invert arrays. If one has an arbitrary non-detraced array M which describes a function

$$M(\Omega) = \sum_{\ell_x, \ell_y, \ell_z} \frac{\ell!}{\ell_x! \ell_y! \ell_z!} M_{\ell_x, \ell_y, \ell_z} e_x^{\ell_x} e_y^{\ell_y} e_z^{\ell_z}, \quad (10)$$

The `Detrace` function returns a detraced array that yields the identical angular function $M(\Omega)$.

Moments of unit-vector components $\langle e_x^{\ell_x} e_y^{\ell_y} e_z^{\ell_z} \rangle$ can be calculated from arrays of Cartesian-Harmonic coefficients with the functions `GetMElementFromAExpArray`. Similarly, if an array stores the moments, the equivalent element for an array of Cartesian-Harmonic expansion coefficients that gives those moments can be found with `GetAExpElementFromMArray(...)`.

The `IncrementAExpArray(...)` and `IncrementMexpArray()` can be used to generate arrays which will, when expanded, reproduce functions with the same angular distribution as the sampling of coordinates used to increment the function. For instance, the following set of calls will produce a numerator for a correlation function given a set of relative momenta qx , qy and qz ,

```
CHArray *numerator;
numerator=new CHArray(parfilename);
for(ipair=0;ipair<NPAIRS;ipair++){
    numerator->IncrementAExpArray(qx[ipair],qy[ipair],qz[ipair],1.0);
}
```

Similarly an array of moments $\langle e_x^{\ell_x} e_y^{\ell_y} e_z^{\ell_z} \rangle$ can also be calculated with by using `IncrementMArray(...)` then scaling the array to make into an average.

If the object stores moments, `GetAExpElementFromMArray(...)` will return the Cartesian-Harmonic expansion coefficient, and if the object stores expansion coefficients, `GetMElementFromAExpArray(...)` will return specific moments.

7.3 Arrays for Expansions in Spherical Harmonics

A class is also provided to accommodate expansions in spherical harmonics. The $Y_{\ell,m}$ expansion coefficients are used to define angular functions,

$$F(\Omega) = (4\pi)^{1/2} \sum_{\ell,m} F_{\ell,m} Y_{\ell,m}(\Omega). \quad (11)$$

This class is rather incomplete. However, one can do all calculations using the Cartesian Harmonics of the previous subsection, then use one of the `CopyArray` functions described in the next section to translate expansion coefficients stored in a `CCHArray` arrays to those for a `CYlmArray`.

```
class CYlmArray{
public:
    CYlmArray(int LMAXset,int NRADIALset);
    ~CYlmArray();
    int GetLMAX();
    complex<double> GetElement(int L,int M,int ir);
    void SetElement(int L,int M,int ir,complex<double>);
    void IncrementElement(int L,int M,int ir,complex<double> increment);
    void ScaleArray(double scalefactor);
    void ScaleArray(double scalefactor,int ir);
    void ZeroArray();
    void ZeroArray(int ir);
    void PrintArrayFixedIR(int ir);
```

```

    void PrintArrayFixedIR(int LMAXPrint,int ir);
};

```

7.4 Array Operations Involving Multiple Arrays

In addition to the functionality described in the various member functions for the array objects, CorAL also provides functions involving more than one array through the namespace ArrayCalc:

```

    namespace ArrayCalc{
void CopyArray(CCHArray *A,CCHArray *B);
void CopyArray(CCHArray *A,int ira,CCHArray *B,int irb);
void CopyArray(C3DArray *A,C3DArray *B);

    void CalcMArrayFromAExpArray(CCHArray *A,CCHArray *M);
void CalcMArrayFromAExpArray(CCHArray *A,int ira,CCHArray *M,int irm);
void CalcAExpArrayFromMArray(CCHArray *M,CCHArray *A);
void CalcAExpArrayFromMArray(CCHArray *M,int irm,CCHArray *A,int ira);
void CalcAExpArrayFromXExpArray(CCHArray *X,CCHArray *A);
void CalcAExpArrayFromXExpArray(CCHArray *X,int irx,CCHArray *A,int ira);
void CalcXExpArrayFromAExpArray(CCHArray *A,CCHArray *X);
void CalcXExpArrayFromAExpArray(CCHArray *A,int ira,CCHArray *X,int irx);
void CalcYlmExpArrayFromAExpArray(CCHArray *A,int ir,
    CYlmArray *YlmArray,int irlm);
void CalcAExpArrayFromYlmExpArray(CYlmArray *YlmArray,int irlm,
    CCHArray *A,int ira);
void CalcAExpArrayFrom3DArray(C3DArray *threedarray,CCHArray *A);
void Calc3DArrayFromAExpArray(CCHArray *A,C3DArray *threed);

    void AddArrays(CCHArray *A,CCHArray *B,CCHArray *C);
void AddArrays(CCHArray *A,int ira,CCHArray *B,int irb,CCHArray *C,int irc);
void AddArrays(C3DArray *A,C3DArray *B,C3DArray *C);

    // If C(Omega)=A(Omega)*B(Omega), this finds A in terms of A and B
void MultiplyArrays(CCHArray *A,CCHArray *B,CCHArray *C);
void MultiplyArrays(CCHArray *A,int ira,CCHArray *B,
    int irb,CCHArray *C,int irc);
void MultiplyArrays(C3DArray *A,C3DArray *B,C3DArray *C);
// If you know array is zero up to given Ls, or don't care to detrace,
// this can save time
void MultiplyArrays_Partial(int LMAXA,CCHArray *A,int ira,
    int LMAXB,CCHArray *B,int irb,
    int LMAXC,CCHArray *C,int irc);

    // If A(Omega)=B(Omega)*C(Omega), this finds C in terms of A and B
void DivideArrays(CCHArray *A,CCHArray *B,CCHArray *C);
void DivideArrays(CCHArray *A,int ira,CCHArray *B,int irb,
    CCHArray *C,int irc);
void DivideArrays(C3DArray *A,C3DArray *B,C3DArray *C);

    void Detrace(CCHArray *M,CCHArray *A);
void Detrace(CCHArray *M,int irm,CCHArray *A,int ira);

```

```

    bool CompareArrayParameters(C3DArray *threed, CCHArray *A);
    bool CompareArrayParameters(CCHArray *A, C3DArray *threed);
    bool CompareArrayParameters(CCHArray *A, CCHArray *B);
    bool CompareArrayParameters(C3DArray *threeda, C3DArray *threedb);
};

```

The CopyArray(...) functions check to make sure that the array parameters are equal before proceeding.

The Calc*ArrayFrom*Array(...) functions translate one type of format into another. The AExp qualifier is for arrays of Cartesian-Harmonic expansion coefficients, whereas YlmExp refers to expansion coefficients using spherical harmonics. The 3D is for Cartesian meshes. These three forms of arrays were described previously.

The AddArrays, MultiplyArrays and DivideArrays functions perform as advertised. For functions $F(\mathbf{r})$ described by the expansion coefficients in the arrays, the results provide arrays describing the arrays in \mathbf{r} space. For instance, DivideArrays(A,B,C) operates on Cartesian-Harmonic coefficients, the resulting array C would satisfy $C(\mathbf{r}) = A(\mathbf{r})/B(\mathbf{r})$ when expanded. These functions do some checking to make sure that the symmetries of the resulting arrays are appropriate, but this checking is not yet complete.

The Detrace(A,B) operation provides an array of Cartesian-Harmonic coefficients B that satisfies the tracelessness condition while expanding identically as A, i.e., $A(\mathbf{r}) = B(\mathbf{r})$.

The boolean functions CompareArrayParameters check whether arrays have identical dimensions and symmetries (XSYM, YSMY and ZSYM). For comparisons of CCHArray and C3DArray objects, only symmetries are tested.

8 Wave Function Reference

One of the basic elements of the code base are objects which derive from the CWaveFunction class. The main functionality of these classes is to calculate the squared wave function. The public members of the base class are:

```

class CWaveFunction{
public:
    int GetNQMAX();
    double GetQ(int iq);
    void PrintCdelta(double Rx,double Ry,double Rz);
    double GetPsiSquared(double *pa,double *xa,double *pb,double *xb);
    double GetPsiSquared(double q,double r,double ctheta);
    virtual double CalcPsiSquared(int iq,double r,double ctheta);
    CWaveFunction();
    ~CWaveFunction();
}

```

Calculations for wave functions are based on stored parameters for specific magnitudes of the relative momentum. Examples of such information might be phase shifts. To view the values of q used for calculations, one can use the functions GetNQMAX() and Get Q(int iq). The relative momenta are the canonical momenta as measured in the two-particle frame, i.e., in that frame the momenta are \mathbf{q} and $-\mathbf{q}$, and are measured in MeV/c.

The function printCdelta is not meant to be used often, as it provides a plot of the correlation function for a Gaussian source in terms of phase shifts, ignoring Coulomb, and assuming that $qR \gg \hbar$. It is really only used for checking certain types of calculations.

The functions GetPsiSquared(...) and CalcPsiSquared(iq,r,ctheta) provide the squared wave function in terms of the relative momentum, or in terms of the two momenta of the particles. The two GetPsiSquared(...) functions call CalcPsiSquared by interpolating for different values of iq.

New CWaveFunction objects must be created for each class of interaction, e.g., pp , $\pi^+\pi^-$ or pK^+ . The function CalcPsiSquared is virtual as it is different for each class which derives from it, e.g., CWaveFunction_pp,

CWaveFunction_pipluspiplus, etc.. The constructors are not listed in this class, though they all have the same form, e.g., CWaveFunction_pp(char *parsfilename). The parameters file must have the form:

```
int NQMAX 20
double DELQ 4.0
double EPSILON 1.0
```

The momentum mesh is defined by NQMAX and DELQ, and EPSILON refers to a distance within which ϕ^2 is a constant. As long as the characteristic source size is larger than EPSILON resulting correlation functions should be independent of EPSILON. To better understand this, one can read the file “corrtail.pdf” and “longpaper.pdf” in “doc/papers/”. This method requires good knowledge of experimental phase shifts. Unfortunately, such phase shifts are often provided as if the Coulomb interaction does not exist, so one must then modify the phase shifts to account for Coulomb which is done with the `CoulWave::phaseshifts_CoulombCorrect(...)` utilities described in Sec. 12. It should be emphasized that this approximation can significantly affect the answer, and that stable results require that the phaseshifts and their derivatives are consistent to a high level. To reduce the sensitivity to this approximation, *pp* phase shifts for the *s*-wave are calculated by solving the Schrödinger equation with the Reid soft-core potential.

Classes were defined in such a way to easily accommodate adding additional classes of interactions. Currently, the classes that inherit from CWaveFunction are:

CWaveFunction_pp	proton-proton [†]
CWaveFunction_pn	proton-neutron [†]
CWaveFunction_nn	neutron-neutron [†]
CWaveFunction_Xipi	Ξ^0, π^+
CWaveFunction_kpluspiminus	$K^+ \pi^-$ (Coulomb and K^*)
CWaveFunction_lambdalambda	$\Lambda\Lambda$ (will prompt for scattering length)
CWaveFunction_plambda	$p\Lambda$
CWaveFunction_piplusplus	$\pi^+ \pi^{+\dagger}$
CWaveFunction_pipluspiminus	$\pi^+ \pi^{-\dagger}$
CWaveFunction_pkplus	proton- $K^{+\dagger}$
CWaveFunction_ppiplus	proton- $\pi^{+\dagger}$

[†]Uses fairly complete set of experimental phase shifts

9 Kernels and Convolution Reference

Kernels are used to provide a connection between both correlations functions and source functions when both are expressed in terms of expansion coefficients for either the Cartesian-Harmonic or the $Y_{\ell,m}$ basis. Since squared wave functions $|\phi(q, r, \cos \theta_{qr})|^2$ is rotationally invariant (\mathbf{q} and \mathbf{r} rotate together) there is a one-to-one correspondence between angular moments of the correlation and source functions with the same indices:

$$\mathcal{R}_{\ell,m}(\mathbf{q}) = \int 4\pi r^2 dr \mathcal{K}_{\ell}(q, r) \mathcal{S}_{\ell,m}(\mathbf{r}) \quad (12)$$

$$\mathcal{R}_{\ell_x, \ell_y, \ell_z}(\mathbf{q}) = \int 4\pi r^2 dr \mathcal{K}_{\ell}(q, r) \mathcal{S}_{\ell_x, \ell_y, \ell_z}(\mathbf{r}) \quad (13)$$

$$\mathcal{K}_{\ell}(q, r) = \frac{1}{2} \int d \cos \theta_{qr} |\phi(q, r, \cos \theta_{qr})|^2 \quad (14)$$

Thus, the kernel $\mathcal{K}_{\ell}(q, r)$ plays the basic role when linking source properties to correlations when analyzing with angular correlations and is determined by the wave function. All types of interactions provide resolving power for kernels, even for values of ℓ greater than those that matter for the interactions. See “doc/papers/longpaper.pdf”.

The members of CKernel are:

```
class CKernel{
public:
```



```

CWaveFunction *wf;
double GetValue(int ell,double q, double r);
double GetValue(int ell,int iq,int ir);
void Read(char *datadirname);
void Write(char *datadirname);
void Print();
int GetLMAX();
double GetDELR();
double GetDELQ();
int GetNQMAX();
int GetNRMAX();
void Calc();
void Calc.ClassCoul(double ma,double mb,int zazb);
void Calc.PureHBT();
CKernel(CWaveFunction *wf,char *kparsfilename);
~CKernel();
double GetPsiSquared(int iq,int ir,double ctheta);
double GetPsiSquared(int iq,double r,double ctheta);
double GetPsiSquared(double q,double r,double ctheta);
}

```

The constructor sets parameters but does not calculate the kernels. To calculate the kernels, one uses one of three calls:

Calc	Will use the wave function object to generate kernel
Calc_ClassCoul	Will calculate kernel for classical Coulomb interaction
Calc_PureHBT	Identical bosons, no Coulomb or strong interaction

Kernels are stored in meshes denoted by the parameters `ell`, `iq` and `ir` with values bounded by `LMAX`, `NRMAX` and `NQMAX`, and the granularities set by `DELR` and `DELQ`. They can be written to files with the `Write()` function which needs a character string as an argument to name the directory. The `Read()` function will read from the same format. In the named directory files will be written for each (`ell`,`q`) combination with filenames set by the values of `q`, e.g., `ell2_q52.dat` would store kernel information for `ell=2`, `q=52`. The first line of the file give `NRMAX` and `DELR`, with the subsequent lines giving the kernel values for `ir=0,1,...`. Thus, one should store information for different kernels in different directories.

Once wave functions or kernels are calculated and a source function is stored in an array, one can calculate the correlation function with functions in the namespace `Source2CF`:

```

namespace CS2CF{
    void s2c(C3DArray *s,CWaveFunction *wf,C3DArray *c);
    void s2c(CCHArray *s,CKernel *kernel,CCHArray *c);
};

```

One uses kernels to perform convolutions with angular decompositions, and wave-functions for convolutions connecting source and correlation functions stored in three-dimensional Cartesian meshes. However, since the kernels are a fairly efficient way of storing the calculated squared wave functions, you can also reproduce the squared wave functions with the functions `GetPsiSquared()`.

10 Source Function Reference

Source and correlation function information are both stored in the same types of arrays. Calculating the source from a model, e.g., blast-wave, requires creating a `CSourceCalc` object, or an object derived from this class. This objects stores parameters describing the source, e.g., Gaussian radii, but the actual source information is stored in arrays of the type described in Sec. 7. The arrays are filled with calls of the type `scalcalc->CalcS(sarray)`, where `scalcalc` would be a pointer to a source object and `sarray` would be a pointer to a the array storing the source function. The public members of the class are:

```

class CSourceCalc{
public:
    parameterMap spars;
    virtual void CalcS(CCHArray *A);
    void ReadSPars(char *sparsfilename);
    void NormCheck(CCHArray *A);
    void CalcEffGaussPars(CCHArray *A);
    CSourceCalc::CSourceCalc();
};

```

All source parameters are stored as a parameterMap objects which are regular c++ maps of strings:
 typedef map<string,string> parameterMap;

The namespace parameter includes a variety of functions which are mostly self explanatory:

```

namespace parameter {
    bool getB(parameterMap ,string ,bool);
    int getI(parameterMap ,string ,int);
    string getS(parameterMap ,string ,string);
    double getD(parameterMap ,string ,double);
    vector< double > getV(parameterMap ,string ,double);
    vector< string > getVS(parameterMap ,string ,string);
    vector< vector< double > > getM(parameterMap ,string ,double);
    void set(parameterMap& ,string ,double);
    void set(parameterMap& ,string ,int);
    void set(parameterMap& ,string ,bool);
    void set(parameterMap& ,string ,string);
    void set(parameterMap& ,string ,char*);
    void set(parameterMap& ,string ,vector< double >);
    void set(parameterMap& ,string ,vector< string >);
    void set(parameterMap& ,string ,vector< vector< double > >);
    void ReadParsFromFile(parameterMap& ,char *filename);
    void PrintPars(parameterMap&);
};

```

The ReadParsFromFile(paramtermap&, char *filename) function will input parameters from a file. For example, the parameters used for reading in parameters for a CSourceCalc.Gaussian object could be specified in a file with the following lines:

```

double Pt 400
double DELPT 30
string OSCARfilename /usr/users/johnson/data/oscarfile.dat
int IDa 211
int IDb -211

```

For specific sources, one must use objects which derive their properties from CSourceCalc. Currently, there are three such objects, CSource_Gaussian, CSource_Blast and CSource_OSCAR, which can be used to generate source functions for Gaussian, blast-waves, and from files where phase space points have been recorded in the OSCAR format. Each type of object has quite different parameters:

Source Parameters

type	parameter	default	class/description
			CSource_Gaussian
double	Rx	4.0	One-particle Gaussian source sizes in c.o.m. frame
double	Ry	4.0	$\rho(\mathbf{r}) \sim e^{-x^2/2R^2}$
double	Rz	4.0	R_{\perp} For non-identicals, $R^2 = (1/2)(R_a^2 + R_b^2)$
double	Xoff	0.0	For non-identicals
double	Yoff	0.0	these are off-sets for separations
double	Zoff	0.0	of centroids of two Gaussians
double	Euler_Phi	0.0	Euler angles for
double	Euler_Theta	0.0	rotations of principle
double	Euler_Psi	0.0	axes
			Csource_Blast*
double	Rx	13	in-plane transverse radius
double	Ry	13	out-of plane (sharp cutoff radii)
double	Tau	12	Bjorken time (All emission at once, $\Delta\tau = 0$)
double	BetaX	0.7	Transverse velocities at surface
double	BetaY	0.7	for in/out-of-plane
double	T	110	Temperature in MeV
double	Pt	600	Total p_t of particles in MeV/c
double	Phi	0.0	Angle of emission relative to x axis
double	EtaG	2.0	Gaussian width of source rapidities ∞ for Bjorken
double	Ma	938.28	mass of first particle
double	Mb	139.58	mass of second particle
int	Nsample	1000	# of points to sample 1-particle distribution source sampling $\sim N_{\text{sample}}^2$
			CSource_OSCAR**
double	Pt	600	Total momentum of pair in MeV/c
double	DELPT	20	Range for accepting momentum of particles, i.e., $ p_{t,a} - m_a P_t / (m_a + m_b) < \Delta P_t$
double	PHIMIN_DEG	0	Range for azimuthal angles, phase space
double	PHIMAX_DEG	360.0	points \mathbf{p}_b and \mathbf{r}_b will be rotated so that \mathbf{p}_a and \mathbf{p}_b are parallel once they pass filter
double	YMIN	-3.0	Range of rapidities, particles
double	YMAX	3.0	boosted to have same rapidity, $y - a = y_b$
int	IDa	211	Particle Data Book
int	IDb	211	IDs
double	Ma	139.58	Masses
double	Mb	139.58	
bool	AEQUALB	0	Set to 1 if particles are non-identical, but one uses same phase space points
int	NMAX	20000	Maximum # of phase space points (array size)
string	OSCARfilename	UNDEFINED	Location of phase space points
int	NEVENTSMAX	10000	Maximum # of events to be read from file

*Blast-wave calculations assume $\mathbf{u}_{\perp} = \gamma \mathbf{v}_{\perp}$ rise linearly from origin.

**All events for which one wishes to mix-and-match phase space points should stored in the same OSCAR file. Data corresponding to different impact parameters should be stored in different files.

11 Imaging Reference

Kernels and wavefunctions provide the means to calculate correlation function objects if given source function objects. The reverse process is more difficult. Correlation functions and source functions are related to one another through the convolution,

$$\mathcal{R}_{\vec{\ell}}(q) = \int dr \mathcal{K}_{\vec{\ell}}(q, r) S_{\vec{\ell}}(r). \quad (15)$$

If one views S as a vector with N_r components and \mathcal{R} as a vector with N_q components, the kernel is a $N_r \times N_q$ matrix which relates the two vectors by

$$\mathcal{R}_i = \sum_j \mathcal{K}_{ij} S_j, \quad (16)$$

where the $\vec{\ell}$ indices are suppressed. The term “imaging” denotes the problem of finding S from \mathcal{R} , i.e., inverting the kernel. Unfortunately, the kernel is usually highly singular, and even if \mathcal{C} is measured to high accuracy, one can often not determine more than a handful of parameters of information describing \mathcal{R} .

Thus all imaging procedures involve fitting routines at their core. Even splines are themselves parameterized functions which are then fit.

```
class CParInfo{
public:
    bool fixed; // 'true' if parameter is not allowed to vary
    // searches are confined to xmin < x < xmax
    double xmin,xmax,error,xbar;
    double bestx,currentx; // bestx is the value that gave smallest chi2
    char *name;
    void Set(string sname,double xset,double error,
    double xminset,double xmaxset);
    CParInfo();
    ~CParInfo();
};

class CCF2SFit{
public:
    /*
    calcflag = 1 if CF is a CCHArray object of specific lx,ly,lz
    and CSourceCalc object makes CCHArray objects
    calcflag = 2 if CF is a C3DArray object
    and CSourceCalc object makes objects
    calcflag =3,4 are used if source functions are calculated through
    intermediate MC lists, but no such implementations yet exist
    */
    void SetCalcFlag(int calcflagset);
    void SetMCSourceFlag(bool MCsourceflagset);

    void SetPar(string parstring,double value);
    void SetPar(string parstring,double value,double error,
    double xmin,double xmax);
    void AddPar(string parstring,double value,double error,
    double xmin,double xmax);
    void PrintPars();
    void PrintErrorMatrix();
    void PrintStepMatrix();
    void FixPar(string parname);
    void FreePar(string parname);
```

```

void UseBestPars();
void SetL(int lxset,int lyset,int lzset);

void Metropolis(int maxcalls);
void SteepestDescent(int maxtries);
void Newton(int maxtries);
void UpdateStepMatrix();
void InitErrorMatrix();

// This calculates source functions
CSourceCalc *sourcecalc;

// ----- The objects below depend on calc_flag -----
// These are used to store information about the source
CCHArray *sourceCH;
C3DArray *source3D;
CMCList *lista;
CMCList *listb;

// Wavefunctions or kernels
CKernel *kernel;
CKernelWF *kernelwf;
CWaveFunction *wf;

// These are arrays for storing correlation functions and errors
C3DArray *cexp3D;
C3DArray *cerror3D;
C3DArray *ctheory3D;
CCHArray *cexpCH;
CCHArray *cerrorCH;
CCHArray *ctheoryCH;

CCF2SFit();
CCF2SFit(CCHArray *sourceCHset,C3DArray *source3Dset,
CMCList *listaset,CMCList *listbset,
CKernel *kernelset,CKernelWF *kernelwfset,
CWaveFunction *wfset,
C3DArray *cexp3Dset,C3DArray *cerror3Dset,
C3DArray *ctheory3Dset,CCHArray *cexpCHset,
CCHArray *cerrorCHset,CCHArray *ctheoryCHset);
    CCF2SFit();

protected:
    int calcflag;
    static bool MCsourceflag; // If the source has a Monte Carlo nature,
    // i.e., it fluctuates for a given parameter set, set this to true

    CParInfo **par;
    static int nmaxpars;
    int nfreepars,npars;
    double **ErrorMatrix;
    double currentchisquared,bestchisquared;

    int lx,ly,lz;
    CRandom *randy;
    void SwitchPars(int ipara,int iparb);
    void SwitchValues(double *a,double *b);

```

```

    int ncalls;
    double **StepMatrix;
    void Init();
    double GetChiSquared(double *x);
    void CalcErrorMatrixFromCurvature(double **C);
};

class CCF2SFit_Blast : public CCF2SFit{
public:
    CCF2SFit_Blast(CSourceCalc *scset,C3DArray *cexpset,
        C3DArray *cerrorset,C3DArray *ctheory3Dset,
        CCHArray *ctheoryset,CCHArray *sourceset,
        CKernel *kernelset);
};

class CCF2SFit_GX1D : public CCF2SFit{
public:
    CCF2SFit_GX1D(CSourceCalc *scset,CCHArray *cexpset,
        CCHArray *cerrorset,CCHArray *ctheoryset,
        CCHArray *sourceset,CKernel *kernelset);
};

class CCF2SFit_3DGaussian : public CCF2SFit{
public:
    CCF2SFit_3DGaussian(CSourceCalc *scset,C3DArray *cexpset,
        C3DArray *cerrorset,C3DArray *ctheory3Dset,
        CCHArray *ctheoryset,CCHArray *sourceset,
        CKernel *kernelset);
};

```

12 Utility Function Reference

The directory `src/Utilities` contains code used for special functions, random-number generation, parameter maps, lorentz boosts, Clebsch Gordan coefficients, Monte-Carlo generation of momenta for boltzmann distributions and the “triangle function”. We devote space below to describing some of the utilities which we think might likely be of some interest to a user:

The random number generation routines are fairly self explanatory:

```

class CRandom{
public:
    double ran(void);
    unsigned long int iran(unsigned long int imax);
    double gauss(void);
    void gauss2(double *g1,double *g2);
    CRandom(int seed);
    void reset(int seed);
    void generate_boltzmann(double mass,double T,double *p);
}

```

Here, `ran` returns a random real number between 0 and 1, `iran` returns an integer $0 \leq i < imax$, and `gauss` returns a gaussian random number consistent with the distribution $e^{-x^2/2}$. The function `gauss2` returns a pair of gaussian numbers which is efficient if one is creating more than one gaussian number since the fundamental algorithm generates pairs of numbers. The function `generate_boltzmann()` returns momenta consistent with a Boltzmann distribution. These routines use the **GSL** library with the “`ranlxd1`”

choice of random number generators. The properties of the various algorithms are documented in the **GSL** documentation. By editing the code `src/Utilities/Random/gslrandom.cc`, one can easily switch to different classes of random number generators.

Including `src/Utilities/Misc/misc.h` and `src/Utilities/Misc/misc.cc` provides a user with several functions. Clebsch Gordan coefficients can be generated with `double cgc(double j1,double m1,double j2,double m2,double j,double m)`. A four vector `p[4]` can be boosted by a four velocity `u[4]` to `pprime[4]` with `void lorentz(double *u,double *p,double *pprime)`. The “triangle function”, `double triangle(double M,double ma,double mb)`, defined by $\sqrt{M^4 + m_a^2 + m_b^2 - 2M^2m_a^2 - 2M^2m_b^2 - 2m_a^2m_b^2/(4M^2)}$, gives the relative momentum of a object of mass M decaying. The utility `bool comparestrings(char *s1,char *s2)` returns true if the strings are identical. Finally, the routine `outsidelong(...)` can be used to find the projections of the relative momentum in the pair frame from two four vectors p_a and p_b . The header file `src/Utilities/Misc/misc.h` has the following prototypes:

```
void lorentz(double *u,double *p1,double *p1prime);
double cgc(double j1,double m1,double j2,double m2,double j,double m);
bool comparestrings(char *s1,char *s2);
double triangle(double m0,double m1,double m2);
void outsidelong(double *pa,double *pb,
    double &qinv,double &qout,double &qside,double &qlong);
```

which are hopefully self-explanatory. Note that `outsidelong` includes the outwards boost which makes $q_{inv}^2 = q_{out}^2 + q_{side}^2 + q_{long}^2$. Thus, the returned value of q_{out} is shorter by a factor $\gamma = (1 - v_{\perp}^2)^{-1/2}$ than what most experimental groups use for their conventions. Also, the canonical relative momentum is used, $q = (1/2)(p'_a - p'_b)$ in the pair frame. This also differs from the usual convention for $\pi\pi$ interferometry, but follows the convention used for most other correlations such as pp analyses.

Special functions include spherical harmonics, Legendre Polynomials, Cartesian Harmonics, Coulomb wave functions and a variety of Bessel functions, which are all defined in `src/Utilities/SpecialFunctions/sf.h`. Many of the special functions are simply **GSL** routines in disguise.

Legendre polynomials and spherical harmonics are functions of the namespace `SpherHarmonics`:

```
namespace SpherHarmonics{
double legendre(int ell,double ctheta);
complex <double> Ylm(int ell,int m,double theta,double phi);
}
```

Bessel functions are incorporated as a namespace, with the function names being self-explanatory:

```
namespace Bessel{
double J0(double x);
double J1(double x);
double Jn(int n, double x);

double K0(double x);
double K1(double x);
double Kn(int n, double x);

double Y0(double x);
double Y1(double x);
double Yn(int n, double x);

double I0(double x);
double I1(double x);
double In(int n, double x);

double j0(double x);
double j1(double x);
double jn(int n, double x);
```

```

    double y0(double x);
    double y1(double x);
    double yn(int n, double x);

    complex<double> h0(double x);
    complex<double> h1(double x);
    complex<double> hn(int n, double x);

    complex<double> hstar0(double x);
    complex<double> hstar1(double x);
    complex<double> hstarn(int n, double x);
};

```

The namespace `CoulWave` provides functions for calculating Coulomb partial waves. Here, `CWincoming` and `CWoutgoing` are the incoming/outgoing waves defined in terms of the regular and irregular solutions $F_L \pm iG_L$ respectively. The gamma function of a complex argument $\Gamma(z)$ is `cgamma()`.

```

namespace CoulWave{
    void GetFG(int L,double x,double eta,double *FL,double *GL);
    void GetFGprime(int L,double x,double eta,double *FL,double *GL,
        double *FLprime,double *GLprime);
    complex<double> CWincoming(int ell,double x,double eta);
    complex<double> CWoutgoing(int ell,double x,double eta);
    complex<double> cgamma(complex<double> cx);
    void phaseshift_CoulombCorrect(int ell,double q,double eta,
        double &delta,double &deltadq);
};

```

Here, the function `phaseshift_CoulombCorrect` provides a **crude** way to scale phase shifts to account for their distortion due to the Coulomb interaction. In many instances, models or parameterizations provide phase shifts assuming there is no Coulomb interaction, e.g., $\delta_s \sim qa$. This correction assumes that $\tan \delta_{\text{withCoul.}} = \tan \delta_{\text{noCoul.}} \cdot \text{Gamow}_\ell(q)$, where the Gamow factor is $F_\ell(q, r=0)/qr|^2$ for a Coulomb wave function.

Cartesian harmonic functions are included as a class rather than as a namespace for the sake of efficiency. Since many of the calculations use binomial and trinomial distributions, and since the overlap function might be used repeatedly, arrays are strategically stored so that subsequent calls can be performed more quickly. The public members of the `CCHCalc` object are:

```

class CCHCalc{
public:
    CCHCalc();
    ~CCHCalc();
    double GetAFromE(int lx,int ly,int lz,double ex,double ey,double ez);
    double GetAFromThetaPhi(int lx,int ly,int lz,double theta,double phi);
    double GetMFromE(int lx,int ly,int lz,double ex,double ey,double ez);
    double GetMFromThetaPhi(int lx,int ly,int lz,double theta,double phi);

    double GetOverlap(int lx,int ly,int lz,int lxprime,int lyprime,int lzprime);
    double GetOverlap0(int lx,int ly,int lz,int lxprime,int lyprime,int lzprime);

    double Factorial(int n);
    double DoubleFactorial(int n);
    double Binomial(int lx,int ly);
    double Trinomial(int lx,int ly,int lz);
};

```

Here, the `GetA...` functions give the Cartesian harmonics \mathcal{A}_ℓ as functions of either $\cos \theta$ and ϕ or in terms of unit vector components e_x, e_y, e_z . The `GetM` functions are simply moments of unit-vector components, $M_\ell =$

$e_x^{\ell_x} e_y^{\ell_y} e_z^{\ell_z}$. Unlike spherical harmonics, Cartesian harmonics are not orthogonal, and `GetOverlap0(...)` and `GetOverlap(...)` return $(1/4\pi) \int d\Omega A_{\vec{\ell}}(\Omega) A_{\vec{\ell}'}(\Omega)$. The function `GetOverlap(...)` stores the calculated values, creating the needed memories as they are calculated. This saves time but can use significant memory if one is calculating for $\ell \geq 50$. Thus, if one only plans to calculate a few overlaps, one should use `GetOverlap0(...)`. Binomial, factorial and trinomial functions also used stored values for increased speed.

References

- [1] M. A. Lisa, S. Pratt, R. Soltz and U. Wiedemann, arXiv:nucl-ex/0505014.