Goods transportation with family-split penalties

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Table of contents

Introduction and example

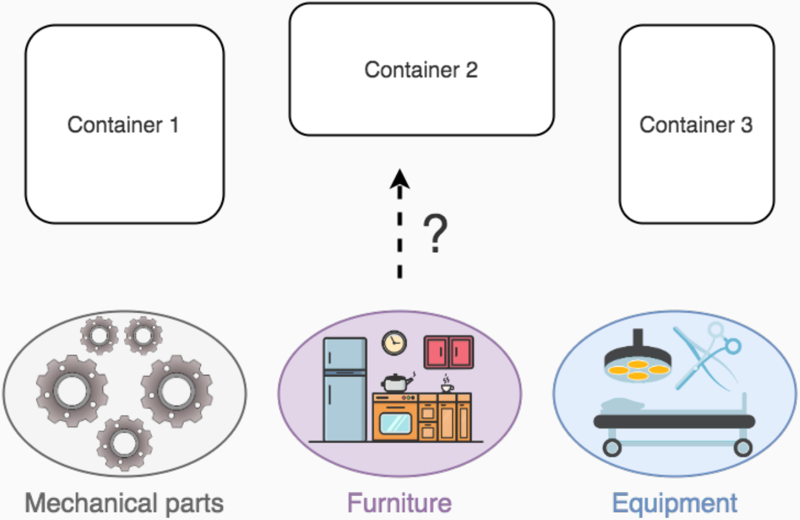
Motivation

- Shipping goods is a complex task
- Shipping **large goods** is an even more complex task as they must be **split into smaller units** (single parcels) shipped separately

The same problem appears at different scales, e.g.

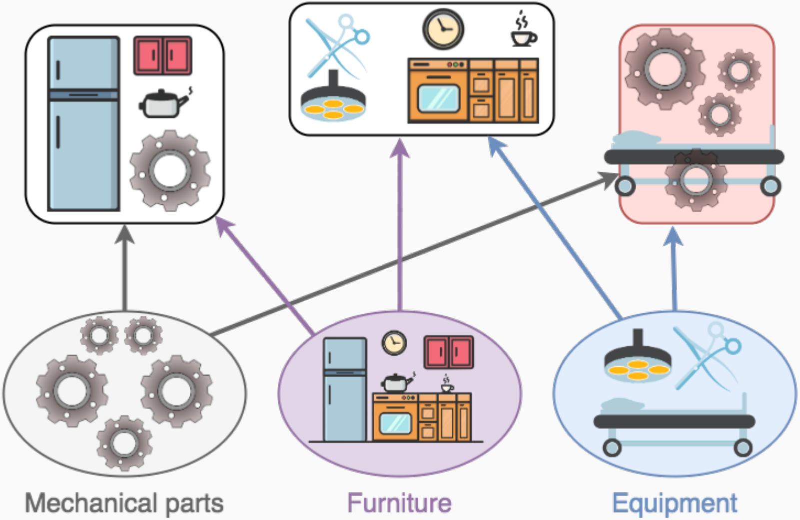
- Assembling pallets in a warehouse for delivery
- Loading containers for transoceanic shipment

Example

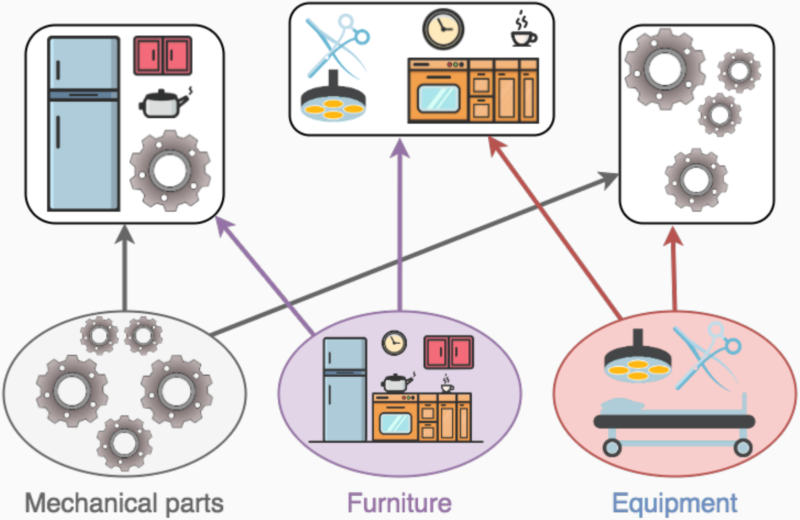


Container 1
Container 2
Container 3
?
Mechanical parts
Furniture
Equipment

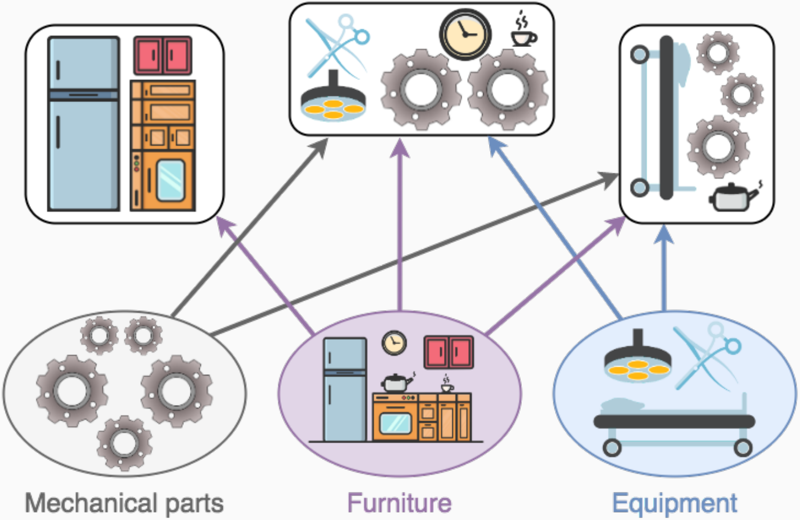
Example: infeasible solution



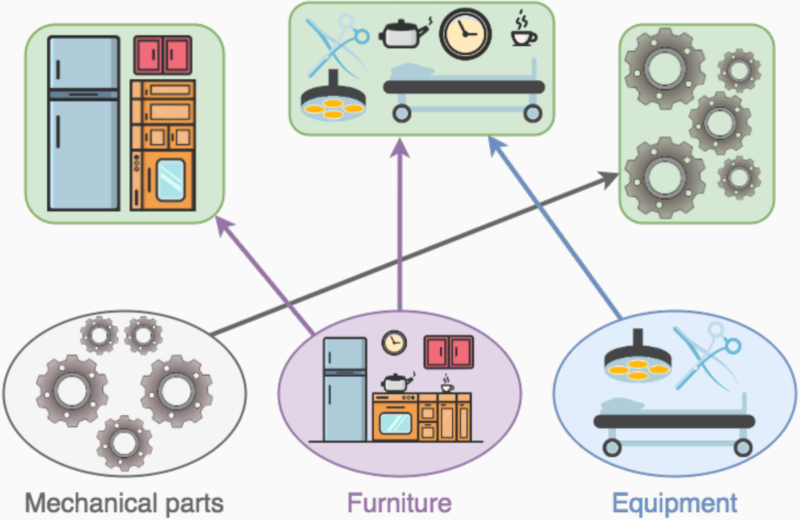
Example: infeasible solution



Example: feasible solution



Example: optimal solution



Problem statement

The Multidimensional Knapsacks Problem with Family-Split Penalties

Data

- The set $I = \{1, \dots, n\}$ of items,
- The set $F = \{1, \dots, m\}$ of families of items,
- The set F_j of the items that belong to the family $j \in F$,
- The set $K = \{1, \dots, k_{\max}\}$ of available knapsacks,
- The set $R = \{1, \dots, r_{\max}\}$ of resources to take into account,
- The capacity $C_{kr} \in \mathbb{N}$ of knapsack $k \in K$ for resource $r \in R$,
- The amount $w_{ir} \in \mathbb{N}$ of resource $r \in R$ required for item $i \in I$,
- The profit $p_j \in \mathbb{N}$ of each family $j \in F$,
- The penalty $\delta_j \in \mathbb{N}$ paid each time the family $j \in F$ is split.

Constraints

- Each family of items may or may not be shipped
- If an item of a family is shipped, then all the items of the same family must be shipped
- The items of a family may be assigned to different knapsacks
- Knapsacks capacities can not be exceeded
- Shipping a family provides a given amount of profit
- Each time a family is split among more than one knapsack, a penalty has to be paid

Decision variables

- $x_j = \begin{cases} 1 & \text{if family } j \in F \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$

- $y_{ik} = \begin{cases} 1 & \text{if item } i \in I \text{ is allocated to knapsack } k \in K, \\ 0 & \text{otherwise.} \end{cases}$

- $z_{jk} = \begin{cases} 1 & \text{if at least one item of family } j \in F \\ & \text{is allocated to knapsack } k \in K, \\ 0 & \text{otherwise.} \end{cases}$

- $s_j \in \mathbb{N}$: how many times the family $j \in F$ has been split among different knapsack.

ILP model

$$\max \quad \sum_{j \in F} (p_j x_j - \delta_j s_j), \tag{1}$$

$$\sum_{k \in K} y_{ik} = x_j, \qquad \forall j \in F, \ \forall i \in F_j, \tag{2}$$

$$\sum_{i \in I} w_{ir} y_{ik} \leq C_{kr}, \qquad \forall k \in K, \ \forall r \in R, \tag{3}$$

$$\sum_{i \in F_j} y_{ik} \leq |F_j| z_{jk}, \qquad \forall j \in F, \ \forall k \in K, \tag{4}$$

$$\sum_{k \in K} z_{jk} - 1 \leq s_j, \qquad \forall j \in F, \tag{5}$$

$$x_j, \ y_{ik}, \ z_{jk} \in \{0, 1\}, \qquad \forall i \in I, \ \forall j \in F, \ \forall k \in K, \tag{6}$$

$$s_j \in \mathbb{N}, \qquad \forall j \in F. \tag{7}$$

Assignment

Your task

- Analize the ILP model on test instances
- Improve the ILP model
- **Develop an heuristic algorithm**
- Implement your algorithm
- Gather data about its performance
- Compare its performance to Gurobi's one

You are provided with

- A formal definition of the problem
- A set of test instances
- A simple codebase able to read the test instances, build the ILP model with Gurobi and solve it
- A function to check if your solutions are feasible and their objective function value is correct
- Python or Java


Now it’s your turn!