The Multidimensional Knapsacks problem with Family-Split Penalties (MKFSP)

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Important: for this project you are required to collaborate using `git`[1] on the GitHub platform. In order to get access to the source code you must register a free account on `github.com`. Once you own a GitHub account, you must send an email to `lorenzo.moreschini@unibs.it` specifying your username on GitHub and asking to join the Java and/or the Python repository of the project. The urls of the repositories are the following (you will not be able to see them until you have been granted the permission):

- `https://github.com/lmores/optalg-mkfsp-java`
- `https://github.com/lmores/optalg-mkfsp-python`

The teaching assistant is always available if you need help with your project. To schedule a (group) meeting send an email to `lorenzo.moreschini@unibs.it` asking for an appointment (Monday to Thurday, from 9:00 to 17:00).
You are actually encouraged to schedule at least two meetings with the teaching assistant: one at the beginning of the project to discuss your strategy and one at the end to assess your results.

1 Problem statement

1.1 Definition

Given $n, m \in \mathbb{N}^+$, let $I = \{1, \dots, n\}$ be a set of items, let $F = \{1, \dots m\}$ and let $\{F_j\}_{j \in F}$ be a partition of I, i.e.

$$\begin{aligned} F_j &\neq \emptyset, && \forall j \in F, \\ F_j \cap F_l &= \emptyset, && \forall j, l \in F : j \neq l, \\ \bigcup_{j \in F} F_j &= I. \end{aligned}$$

[1] `Git` is the most widely adopted version control system and a *de facto* standard in the software industry. If you do not know how to use it, ask your friends, read the tutorial available at `https://www.atlassian.com/git/tutorials` or check out the official manual `https://git-scm.com/docs/gittutorial`.

Given $k_{\max}, r_{\max} \in \mathbb{N}^+$, let $K = \{1, \ldots, k_{\max}\}$ be the set of available knapsacks, each having a maximum capacity $C_{kr} \in \mathbb{N}$ for each resource type $r \in R = \{1, \ldots, r_{\max}\}$. Each item $i \in I$ requires an amount $w_{ir} \in \mathbb{N}$ of resource $r \in R$ to be allocated into knapsack $k \in K$. The Multidimensional Knapsacks problem with Family-Split Penalties (MKFSP) requires to find the best assignment(s) of the items among the available knapsacks in order to maximize the total value under the following conditions.

1. If an item $i \in F_j$ ($j \in F$) is selected, then all the items of the family F_j must be selected,
2. the items of any family may be assigned to different knapsacks,
3. each family of items may or may not be selected,
4. knapsacks capacities for each resource $r \in R$ can not be exceeded,
5. selecting family $j \in F$ provides a given profit $p_j \in \mathbb{N}$,
6. each time a family $j \in F$ is split among more than one knapsack, a penalty δ_j has to be paid.

1.2 ILP model

The proposed ILP model makes use of the following variables.

- $x_j = \begin{cases} 1 & \text{if family } j \in F \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$
- $y_{ik} = \begin{cases} 1 & \text{if item } i \in I \text{ is allocated to knapsack } k \in K, \\ 0 & \text{otherwise.} \end{cases}$
- $z_{jk} = \begin{cases} 1 & \text{if at least one item of family } j \in F \\ & \text{is allocated to knapsack } k \in K, \\ 0 & \text{otherwise.} \end{cases}$
- $s_j \in \mathbb{N}$: how many times the family $j \in F$ has been split among more than one knapsack.

The proposed ILP formulation is as follows.

$$\max \quad \sum_{j \in F} (p_j x_j - \delta_j s_j), \tag{1}$$

$$\sum_{k \in K} y_{ik} = x_j, \qquad \forall j \in F,\ \forall i \in F_j, \tag{2}$$

$$\sum_{i \in I} w_{ir} y_{ik} \leq C_{kr}, \qquad \forall k \in K,\ \forall r \in R, \tag{3}$$

$$\sum_{i \in F_j} y_{ik} \leq |F_j| z_{jk}, \qquad \forall j \in F,\ \forall k \in K, \tag{4}$$

$$\sum_{k \in K} z_{jk} - 1 \leq s_j, \qquad \forall j \in F, \tag{5}$$

$$x_j,\ y_{ik},\ z_{jk} \in \{0, 1\}, \qquad \forall i \in I,\ \forall j \in F,\ \forall k \in K, \tag{6}$$

$$s_j \in \mathbb{N}, \qquad \forall j \in F. \tag{7}$$

The model maximizes the sum of the profits of the selected families minus the sum of the penalties due to the splitting of a family among more than one knapsack. Constraints (2) ensure that each item is loaded in exactly one knapsack if and only if the family it belongs to is selected (loaded among one or more knapsacks). Constraints (3) ensure that for each knapsack its maximum capacity for each resource type is not exceeded. Constraints (4) of type "BigM" set the value of variables z_{jk} ($j \in F$, $k \in K$) to 1 if at least one item $i \in F_j$ is assigned to knapsack k. Constraints (5) set the value of variables s_j ($j \in F$) equal to the numer of times each family is split among more than one knapsack Both constraints (4) and (5) relay on the objective function maximization sense together with δ_j being positive values. Finally, (6) and (7) define the domain of each variable.

2 Benchmark instances

Important: although in the formal statement of the problem indexes start from 1, all benchmark instances use 0-based indexes (as all array-like data structures in Java and Python use 0-based indexes). For example, given a list of n items, the first item has index 0, the second item has index 1 and so on... the last item has index $n - 1$.

2.1 Format description

In each instance file items are sorted by family: if the first family has size f_1, its items are indexed from 0 to $f_1 - 1$, f_1 is the index of the first item of the second family and so on. Benchmark instances are stored in text files with `JSON` format. Each file describes an instance and contains a JSON object with the following properties.

- `id`: a string, a name that uniquely identifies the instance,
- `n_items`: an integer, the number of available items, i.e. n,
- `n_families`: an integer, the number of available families, i.e. m.
- `n_knapsacks`: an integer, the number of available items, i.e. $k_{\max}$,
- `n_resources`: an integer, the number of available resources, i.e. $r_{\max}$,
- `profits`: an array of integers of length m, the value in position $0 \leq j \leq m-1$ is the profit obtained when family j is selected,
- `penalties`: an array of integers of length m, the value in position $0 \leq j \leq m-1$ is the penalty paid each time family j is split among more than on knapsack,
- `first_items`: an array of integers of length m, the value in position $0 \leq j \leq m-1$ is the index of the first item that belongs to family j,
- `items`: an array of length n of arrays of length $r_{\max}$ of integers, for $0 \leq i \leq n-1$ and $0 \leq r \leq r_{\max}-1$ the value of `items[i][r]` equals w_{ir}, i.e. the amount of resource of type r required to assign item i in a knapsack,
- `kanpsacks`: an array of length $k_{\max}$ of arrays of length $r_{\max}$ of integers, for $0 \leq k \leq k_{\max}-1$ and $0 \leq r \leq r_{\max}-1$ the value of `knapsacks[i][r]` equals C_{kr}, i.e. the maximum capacity of knapsack k with respect to resource type r.

2.1.1 Example

Listing 1 shows the data format of a simple instance with 12 items, 4 families, 3 knapsacks and 2 types of resources.
The first family has a profit of 10 and a penalty of 4 for each split, the second family has a profit of 20 and a penalty of 3 for each split, the third family has a profit of 30 and a penalty of 2 for each split and the fourth family has a profit of 40 and a penalty of 1 for each split.
Items 0, 1, 2 and 3 belong to the first family, items 4 and 5 belong to the second family, items 6, 7, 8, 9 and 10 belong to the third family and the fourth family contains only item number 11.
Item number 0 requires 1 unit of the first resource type and 2 units of the second resource type, item number 1 requires 3 units of the first resource type and 4 units of the second resource type, item number 2 requires 8 units of the first resource type and 6 units of the second resource type, and so on.
Finally, knapsack number 0 has a maximum capacity of 10 units for the first resource type and 10 units for the second resource type, knapsack number 1 has a maximum capacity of 10 units for the first resource type and 5 units for the second resource type, knapsack number 2 has a maximum capacity of 15 units for the first resource type and 15 units for the second resource type.

```
{
    "id": "example",
    "n_items": 12,
    "n_families": 4,
    "n_knapsacks": 3,
    "n_resources": 2,
    "profits": [10, 20, 30, 40],
    "penalties": [4, 3, 2, 1],
    "first_items": [0, 4, 6, 11],
    "items": [
        [1, 2], [3, 4], [8, 6], [5, 5],
        [3, 3], [7, 1],
        [2, 0], [0, 7], [3, 2], [3, 1], [1, 8],
        [4, 5]
    ],
    "knapsacks": [
        [10, 10],
        [10,  5],
        [15, 15]
    ]
}
```

Listing 1: Content of instance file `example.json`.

2.2 Instances description

You must test your algorithm against 12 benchmark instances, whose main features are shown in Table 1.

id	n	m	$k_{\max}$	$r_{\max}$
instance01	500	67	5	10
instance02	1000	131	10	10
instance03	1500	202	15	10
instance04	500	67	5	30
instance05	1000	132	10	30
instance06	1500	202	15	30
instance07	1250	167	5	30
instance08	2500	330	10	30
instance09	3750	502	15	30
instance10	2500	334	5	30
instance11	5000	670	10	30
instance12	7500	1000	15	30

Table 1: Benchmark instances