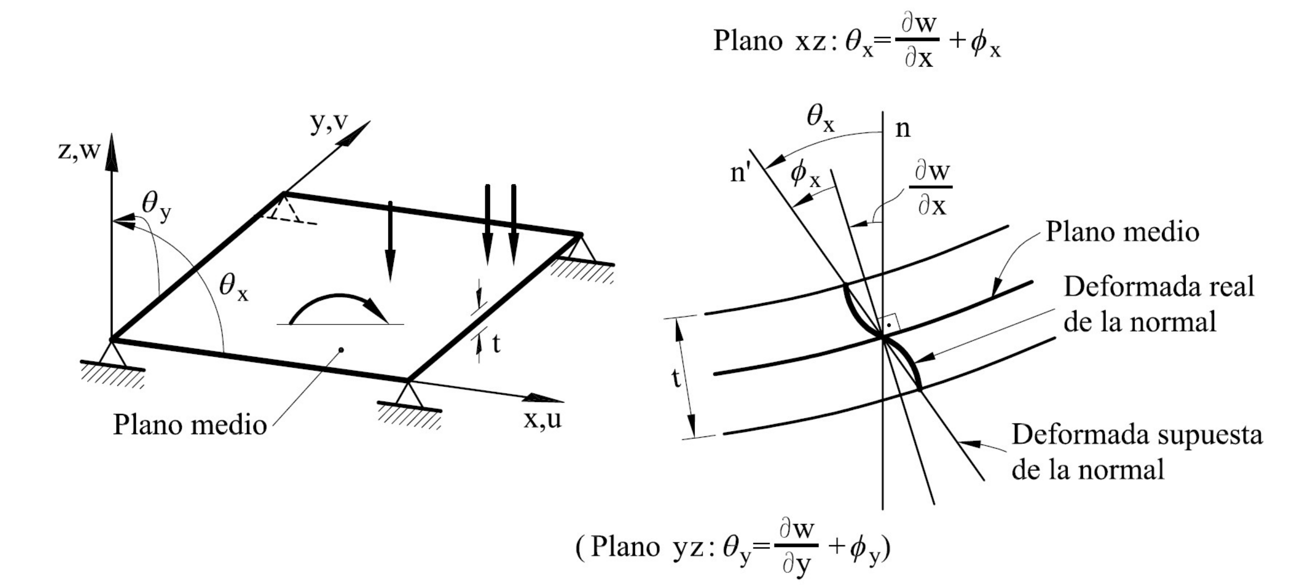


Figura 8.19 Teoría de placas de Reissner-Mindlin. Convenio de signos para los movimientos y giro de la normal.

Fuente: *Calculo de estructuras por el método de los elementos finitos (Oñate)*

- *Modulo de Elasticidad:* $E := 2.535$

- *Coeficiente de Poisson:* $\nu := 0.20$

- *Modulo de Elasticidad a corte:* $G := \dfrac{E}{2 \cdot (1+\nu)} = 1.056$

- *Espesor de la placa:* $d := 250$

- *Coeficiente de corrección por corte:* $\kappa := \dfrac{5}{6}$

- *Matriz constitutiva a flexión:*

$$D_f := \frac{E \cdot d^3}{12 \cdot (1-\nu^2)} \cdot \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix}$$

- *Matriz constitutiva a corte:*

$$D_s := \kappa \cdot G \cdot d \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- *Funciones de interpolación:*

$$N_1(\xi,\eta) := \frac{1}{4} \cdot (1-\xi) \cdot (1-\eta) \qquad N_3(\xi,\eta) := \frac{1}{4} \cdot (1+\xi) \cdot (1+\eta)$$

$$N_2(\xi,\eta) := \frac{1}{4} \cdot (1+\xi) \cdot (1-\eta) \qquad N_4(\xi,\eta) := \frac{1}{4} \cdot (1-\xi) \cdot (1+\eta)$$

$$N(\xi,\eta) := \begin{bmatrix} N_1(\xi,\eta) & N_2(\xi,\eta) & N_3(\xi,\eta) & N_4(\xi,\eta) \end{bmatrix}$$

- *Derivadas de las funciones de interpolación:*

$$dN(\xi,\eta) := \begin{bmatrix} \frac{\partial}{\partial \xi} N_1(\xi,\eta) & \frac{\partial}{\partial \xi} N_2(\xi,\eta) & \frac{\partial}{\partial \xi} N_3(\xi,\eta) & \frac{\partial}{\partial \xi} N_4(\xi,\eta) \\ \frac{\partial}{\partial \eta} N_1(\xi,\eta) & \frac{\partial}{\partial \eta} N_2(\xi,\eta) & \frac{\partial}{\partial \eta} N_3(\xi,\eta) & \frac{\partial}{\partial \eta} N_4(\xi,\eta) \end{bmatrix} \rightarrow \begin{bmatrix} \frac{\eta-1}{4} & \frac{-\eta+1}{4} & \frac{\eta+1}{4} & \frac{-\eta-1}{4} \\ \frac{\xi-1}{4} & \frac{-\xi-1}{4} & \frac{\xi+1}{4} & \frac{-\xi+1}{4} \end{bmatrix}$$

- *Coordenadas del elemento:*

$L1 := 3000$ $\qquad$ $L2 := 1500$

X	Y
0	0
$L1$	0
$L1$	$L2$
0	$L2$

$$XY := \operatorname{augment}(X, Y) = \begin{bmatrix} 0 & 0 \\ 3 \cdot 10^3 & 0 \\ 3 \cdot 10^3 & 1.5 \cdot 10^3 \\ 0 & 1.5 \cdot 10^3 \end{bmatrix}$$

- *Jacobiano:*

$$J(\xi,\eta) := dN(\xi,\eta) \cdot XY \rightarrow \begin{bmatrix} 1500 & 0 \\ 0 & 750 \end{bmatrix}$$

- *Derivadas de las funciones de interpolación con respecto al sistema original:*

$$d'N(\xi,\eta) := \left(J(\xi,\eta)^{-1}\right) \cdot dN(\xi,\eta) \rightarrow \begin{bmatrix} \frac{\eta-1}{6000} & \frac{-\eta+1}{6000} & \frac{\eta+1}{6000} & \frac{-\eta-1}{6000} \\ \frac{\xi-1}{3000} & \frac{-\xi-1}{3000} & \frac{\xi+1}{3000} & \frac{-\xi+1}{3000} \end{bmatrix}$$

- *Construcción de la matriz de deformación para flexión:*

$f(i,j) := 0$

```
B_b(ξ,η) := ‖ t ← matrix(3, 12, f)
            ‖ for i ∈ 1 .. 4
            ‖   ‖ t[1, 3·i−1] ← (d'N(ξ,η))[1, i]
            ‖   ‖ t[2, 3·i]   ← (d'N(ξ,η))[2, i]
            ‖   ‖ t[3, 3·i−1] ← (d'N(ξ,η))[2, i]
            ‖   ‖ t[3, 3·i]   ← (d'N(ξ,η))[1, i]
            ‖ t
```

$$\rightarrow \begin{bmatrix} 0 & \frac{\eta-1}{6000} & 0 & 0 & \frac{-\eta+1}{6000} & 0 & 0 & \frac{\eta+1}{6000} & 0 & 0 & \frac{-\eta-1}{6000} & 0 \\ 0 & 0 & \frac{\xi-1}{3000} & 0 & 0 & \frac{-\xi-1}{3000} & 0 & 0 & \frac{\xi+1}{3000} & 0 & 0 & \frac{-\xi}{30} \\ 0 & \frac{\xi-1}{3000} & \frac{\eta-1}{6000} & 0 & \frac{-\xi-1}{3000} & \frac{-\eta+1}{6000} & 0 & \frac{\xi+1}{3000} & \frac{\eta+1}{6000} & 0 & \frac{-\xi+1}{3000} & \frac{-\eta}{60} \end{bmatrix}$$

- *Construcción de la matriz de deformación por corte:*

$$B_s(\xi,\eta) := \left\| \begin{array}{l} t \leftarrow \text{matrix}(2,12,f) \\ \text{for } i \in 1..4 \\ \quad \left\| \begin{array}{l} t_{1,3\cdot i-2} \leftarrow (d'N(\xi,\eta))_{1,i} \\ t_{1,3\cdot i-1} \leftarrow -(N(\xi,\eta))_{1,i} \\ t_{2,3\cdot i-2} \leftarrow (d'N(\xi,\eta))_{2,i} \\ t_{2,3\cdot i} \leftarrow -(N(\xi,\eta))_{1,i} \end{array} \right. \\ t \end{array} \right. \rightarrow \begin{bmatrix} \frac{\eta-1}{6000} & \frac{-((1-\xi)\cdot(1-\eta))}{4} & 0 & \frac{-\eta+1}{6000} & \frac{-((1-\eta)}{4} \\ \frac{\xi-1}{3000} & 0 & \frac{-((1-\xi)\cdot(1-\eta))}{4} & \frac{-\xi-1}{3000} & 0 \end{bmatrix}$$

- *Puntos de integración:*

ξ'	η'	W
$\frac{-1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$	1
$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	1
$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	1
$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	1

- *Matrices aumentadas para integración reducida:*

$$D_f := \text{stack}(\text{augment}(D_f, \text{matrix}(3,2,f)), \text{matrix}(2,5,f))$$

$$D_f = \begin{bmatrix} 3.438\cdot10^6 & 6.877\cdot10^5 & 0 & 0 & 0 \\ 6.877\cdot10^5 & 3.438\cdot10^6 & 0 & 0 & 0 \\ 0 & 0 & 1.375\cdot10^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_s := \text{stack}(\text{matrix}(3,5,f), \text{augment}(\text{matrix}(2,3,f), D_s))$$

$$D_s = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 220.052 & 0 \\ 0 & 0 & 0 & 0 & 220.052 \end{bmatrix}$$

$$B_b(\xi,\eta) := \text{stack}(B_b(\xi,\eta), \text{matrix}(2,12,f))$$

$$B_s(\xi,\eta) := \text{stack}(\text{matrix}(3,12,f), B_s(\xi,\eta))$$

- *Matriz de rigidez:*

$j := 1 \,..\, 4$

$$\Phi_j := \left(B_b\left(\xi'_j, \eta'_j\right)\right)^{\mathrm{T}} \cdot D_f \cdot \left(B_b\left(\xi'_j, \eta'_j\right)\right) \cdot \det\left(J\left(\xi'_j, \eta'_j\right)\right) \cdot W_j \qquad K_b := \sum_{j=1}^{4} \Phi_j$$

$$K_s := 2 \cdot 2 \cdot \left(B_s(0,0)\right)^{\mathrm{T}} \cdot D_s \cdot \left(B_s(0,0)\right) \cdot \det\left(J(0,0)\right)$$

$$K_T := K_b + K_s$$

$$K_b = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1489935.981 & 515747.07 & 0 & -114610.46 & -171915.69 & 0 & -744967.99 & -515747.07 & 0 & -630357.53 & 171915.69 \\
0 & 515747.07 & 2521430.122 & 0 & 171915.69 & 916883.681 & 0 & -515747.07 & -1260715.061 & 0 & -171915.69 & -2177598.741 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -114610.46 & 171915.69 & 0 & 1489935.981 & -515747.07 & 0 & -630357.53 & -171915.69 & 0 & -744967.99 & 515747.07 \\
0 & -171915.69 & 916883.681 & 0 & -515747.07 & 2521430.122 & 0 & 171915.69 & -2177598.741 & 0 & 515747.07 & -1260715.061 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -744967.99 & -515747.07 & 0 & -630357.53 & 171915.69 & 0 & 1489935.981 & 515747.07 & 0 & -114610.46 & -171915.69 \\
0 & -515747.07 & -1260715.061 & 0 & -171915.69 & -2177598.741 & 0 & 515747.07 & 2521430.122 & 0 & 171915.69 & 916883.681 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -630357.53 & -171915.69 & 0 & -744967.99 & 515747.07 & 0 & -114610.46 & 171915.69 & 0 & 1489935.981 & -515747.07 \\
0 & 171915.69 & -2177598.741 & 0 & 515747.07 & -1260715.061 & 0 & -171915.69 & 916883.681 & 0 & -515747.07 & 2521430.122
\end{bmatrix}$$

$$K_s = \begin{bmatrix}
0.138 & 41.26 & 82.52 & 0.083 & 41.26 & 82.52 & -0.138 & 41.26 & 82.52 & -0.083 & 41.26 & 82.52 \\
41.26 & 61889.648 & 0 & -41.26 & 61889.648 & 0 & -41.26 & 61889.648 & 0 & 41.26 & 61889.648 & 0 \\
82.52 & 0 & 61889.648 & 82.52 & 0 & 61889.648 & -82.52 & 0 & 61889.648 & -82.52 & 0 & 61889.648 \\
0.083 & -41.26 & 82.52 & 0.138 & -41.26 & 82.52 & -0.083 & -41.26 & 82.52 & -0.138 & -41.26 & 82.52 \\
41.26 & 61889.648 & 0 & -41.26 & 61889.648 & 0 & -41.26 & 61889.648 & 0 & 41.26 & 61889.648 & 0 \\
82.52 & 0 & 61889.648 & 82.52 & 0 & 61889.648 & -82.52 & 0 & 61889.648 & -82.52 & 0 & 61889.648 \\
-0.138 & -41.26 & -82.52 & -0.083 & -41.26 & -82.52 & 0.138 & -41.26 & -82.52 & 0.083 & -41.26 & -82.52 \\
41.26 & 61889.648 & 0 & -41.26 & 61889.648 & 0 & -41.26 & 61889.648 & 0 & 41.26 & 61889.648 & 0 \\
82.52 & 0 & 61889.648 & 82.52 & 0 & 61889.648 & -82.52 & 0 & 61889.648 & -82.52 & 0 & 61889.648 \\
-0.083 & 41.26 & -82.52 & -0.138 & 41.26 & -82.52 & 0.083 & 41.26 & -82.52 & 0.138 & 41.26 & -82.52 \\
41.26 & 61889.648 & 0 & -41.26 & 61889.648 & 0 & -41.26 & 61889.648 & 0 & 41.26 & 61889.648 & 0 \\
82.52 & 0 & 61889.648 & 82.52 & 0 & 61889.648 & -82.52 & 0 & 61889.648 & -82.52 & 0 & 61889.648
\end{bmatrix} 10^3$$

$$K_T = \begin{bmatrix}
0.138 & 41.26 & 82.52 & 0.083 & 41.26 & 82.52 & -0.138 & 41.26 & 82.52 & -0.083 & 41.26 & 82.52 \\
41.26 & 63379.584 & 515.747 & -41.26 & 61775.038 & -171.916 & -41.26 & 61144.68 & -515.747 & 41.26 & 61259.291 & 171.916 \\
82.52 & 515.747 & 64411.079 & 82.52 & 171.916 & 62806.532 & -82.52 & -515.747 & 60628.933 & -82.52 & -171.916 & 59712.05 \\
0.083 & -41.26 & 82.52 & 0.138 & -41.26 & 82.52 & -0.083 & -41.26 & 82.52 & -0.138 & -41.26 & 82.52 \\
41.26 & 61775.038 & 171.916 & -41.26 & 63379.584 & -515.747 & -41.26 & 61259.291 & -171.916 & 41.26 & 61144.68 & 515.747 \\
82.52 & -171.916 & 62806.532 & 82.52 & -515.747 & 64411.079 & -82.52 & 171.916 & 59712.05 & -82.52 & 515.747 & 60628.933 \\
-0.138 & -41.26 & -82.52 & -0.083 & -41.26 & -82.52 & 0.138 & -41.26 & -82.52 & 0.083 & -41.26 & -82.52 \\
41.26 & 61144.68 & -515.747 & -41.26 & 61259.291 & 171.916 & -41.26 & 63379.584 & 515.747 & 41.26 & 61775.038 & -171.916 \\
82.52 & -515.747 & 60628.933 & 82.52 & -171.916 & 59712.05 & -82.52 & 515.747 & 64411.079 & -82.52 & 171.916 & 62806.532 \\
-0.083 & 41.26 & -82.52 & -0.138 & 41.26 & -82.52 & 0.083 & 41.26 & -82.52 & 0.138 & 41.26 & -82.52 \\
41.26 & 61259.291 & -171.916 & -41.26 & 61144.68 & 515.747 & -41.26 & 61775.038 & 171.916 & 41.26 & 63379.584 & -515.747 \\
82.52 & 171.916 & 59712.05 & 82.52 & 515.747 & 60628.933 & -82.52 & -171.916 & 62806.532 & -82.52 & -515.747 & 64411.079
\end{bmatrix} 10^3$$

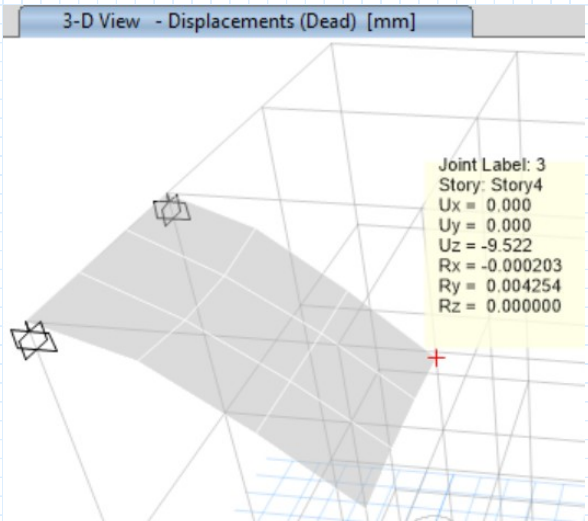
$$K_{red} := \text{submatrix}\left(K_T, 7, 12, 7, 12\right) = \begin{bmatrix} 137.533 & -41259.766 & -82519.531 & 82.52 & -41259.766 & -82519.531 \\ -41259.766 & 63379584.418 & 515747.07 & 41259.766 & 61775037.977 & -171915.69 \\ -82519.531 & 515747.07 & 64411078.559 & -82519.531 & 171915.69 & 62806532.118 \\ 82.52 & 41259.766 & -82519.531 & 137.533 & 41259.766 & -82519.531 \\ -41259.766 & 61775037.977 & 171915.69 & 41259.766 & 63379584.418 & -515747.07 \\ -82519.531 & -171915.69 & 62806532.118 & -82519.531 & -515747.07 & 64411078.559 \end{bmatrix}$$

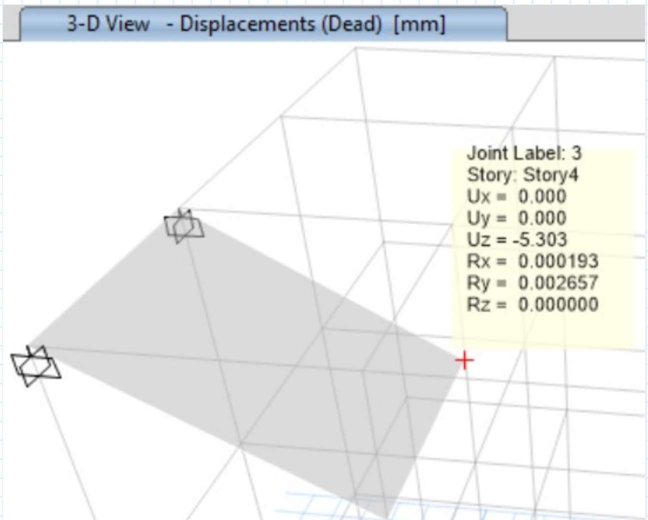
$$P := 3$$

$$Q := \begin{bmatrix} P \\ 0 \\ 0 \\ P \\ 0 \\ 0 \end{bmatrix} \qquad D := K_{red}^{-1} \cdot Q = \begin{bmatrix} 0.515 \\ 0 \\ 0.001 \\ 0.515 \\ 0 \\ 0.001 \end{bmatrix}$$

$$I := L1 \cdot \frac{d^3}{12}$$

$$D_{exac} := \left(\frac{L1^3}{3 \cdot E \cdot I}\right) \cdot 2\ P = 5.453$$





ALGORITMO PARA ENSAMBLAR LA MATRIZ DE RIGIDEZ GLOBAL:

$Lx := 3000$ $\qquad$ $Ly := 3000$

- *Coordenadas:*

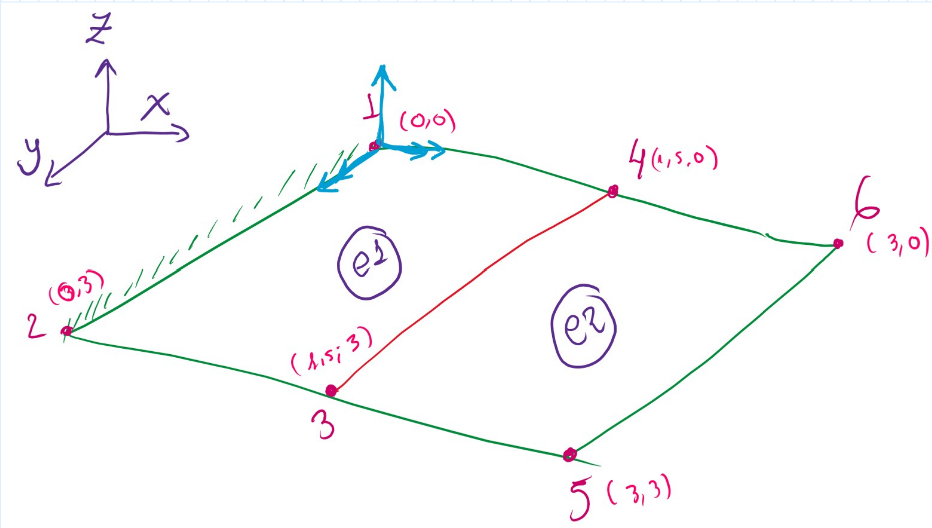
Nudo	X	Y
1	0	0
2	0	Ly
3	$0.5 \cdot Lx$	Ly
4	$0.5 \cdot Lx$	0
5	Lx	Ly
6	Lx	0

- *Reacciones:*

Apoyo	Rw	Rθx	Rθy
1	1	1	1
2	1	1	1

- *Conectividad:*

Ele	Ini	Int	Int'	Fin
1	1	2	3	4
2	4	3	5	6



- *Numero de nudos:* $N_{nud} := \text{rows}(Nudo) = 6$
- *Numero de grados por nudo:* $n_{nudo} := 3$
- *Numero de grados de libertad:* $N_{gl} := N_{nud} \cdot n_{nudo} = 18$
- *Numero de apoyos:* $N_{apo} := \text{rows}(Apoyo) = 2$
- *Numero de reacciones:* $N_r := \sum_{i=1}^{N_{apo}} Rw_i + \sum_{i=1}^{N_{apo}} R\theta x_i + \sum_{i=1}^{N_{apo}} R\theta y_i = 6$
- *Numero de elementos:* $N_{ele} := \text{rows}(Ele) = 2$ $\qquad$ $i := 1 .. N_{ele}$
- *Función para crear matrices nulas:* $f(i, j) := 0$

- *Matriz que identifica las reacciones en los nudos:*

$$Mre := \left\| \begin{array}{l} Mre \leftarrow \text{matrix}\left(N_{nud}, n_{nudo}, f\right) \\ \text{for } i \in 1 .. N_{apo} \\ \quad \left\| \begin{array}{l} Mre_{Apoyo_i, 1} \leftarrow Rw_i \\ Mre_{Apoyo_i, 2} \leftarrow R\theta x_i \\ Mre_{Apoyo_i, 3} \leftarrow R\theta y_i \end{array} \right. \\ Mre \end{array} \right.$$

$$Mre = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

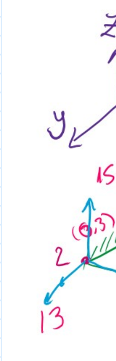
- *Matriz que asigna los grados de libertad a cada nudo:*

$$Mgl := \left\| \begin{array}{l} i \leftarrow N_{gl} + 1 \\ j \leftarrow 0 \\ \text{for } k \in 1 .. N_{nud} \\ \quad \left\| \begin{array}{l} \text{for } l \in 1 .. n_{nudo} \\ \quad \left\| \begin{array}{l} \text{if } Mre_{k,l} = 1 \\ \quad \left\| \begin{array}{l} i \leftarrow i - 1 \\ Mgl_{k,l} \leftarrow i \end{array} \right. \\ \text{else} \\ \quad \left\| \begin{array}{l} j \leftarrow j + 1 \\ Mgl_{k,l} \leftarrow j \end{array} \right. \end{array} \right. \end{array} \right. \\ Mgl \end{array} \right.$$

$$Mgl = \begin{bmatrix} 18 & 17 & 16 \\ 15 & 14 & 13 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

- *Matriz que asigna los grados de libertad a cada elemento:*

$$a := \left\| \begin{array}{l} \text{for } i \in 1 .. N_{ele} \\ \quad \left\| \begin{array}{l} \text{for } j \in 1 .. n_{nudo} \\ \quad \left\| \begin{array}{l} a_{j,i} \leftarrow \left(Mgl^{T}\right)_{j, (Ini)_i} \\ a_{j + n_{nudo}, i} \leftarrow \left(Mgl^{T}\right)_{j, (Int)_i} \\ a_{j + n_{nudo} \cdot 2, i} \leftarrow \left(Mgl^{T}\right)_{j, (Int')_i} \\ a_{j + n_{nudo} \cdot 3, i} \leftarrow \left(Mgl^{T}\right)_{j, (Fin)_i} \end{array} \right. \end{array} \right. \\ a \end{array} \right. = \begin{bmatrix} 18 & 4 \\ 17 & 5 \\ 16 & 6 \\ 15 & 1 \\ 14 & 2 \\ 13 & 3 \\ 1 & 7 \\ 2 & 8 \\ 3 & 9 \\ 4 & 10 \\ 5 & 11 \\ 6 & 12 \end{bmatrix}$$



$$K' := K_T$$

$$K_i := K'$$

- *Numero de vértices del elemento:* $nv := 4$ $\qquad n_{nudo} \cdot nv = 12$

- *Ensamblaje de la matriz de rigidez de toda la estructura:*

$$K_{g_i} := \left\| \begin{array}{l} f(i, j) \leftarrow 0 \\ K_{g_i} \leftarrow \text{matrix}\left(N_{gl}, N_{gl}, f\right) \end{array} \right.$$

$$
\begin{array}{l}
\text{for } j \in 1,2..n_{nudo} \cdot nv \\
\quad EG \leftarrow \left(a^{\langle i \rangle}\right)_j \\
\quad \text{for } k \in 1,2..n_{nudo} \cdot nv \\
\quad\quad CG \leftarrow \left(a^{\langle i \rangle}\right)_k \\
\quad\quad \left(K_{g_i}\right)_{EG,CG} \leftarrow \left(K_i\right)_{j,k} \\
K_{g_i}
\end{array}
$$

$$
K_{g_1} = \begin{bmatrix}
137.533 & -4.126\cdot10^4 & -8.252\cdot10^4 & 82.52 & -4.126\cdot10^4 & -8.252\cdot10^4 & 0 & 0 & 0 & 0 & 0 & 0 \\
-4.126\cdot10^4 & 6.338\cdot10^7 & 5.157\cdot10^5 & 4.126\cdot10^4 & 6.178\cdot10^7 & -1.719\cdot10^5 & 0 & 0 & 0 & 0 & 0 & 0 \\
-8.252\cdot10^4 & 5.157\cdot10^5 & 6.441\cdot10^7 & -8.252\cdot10^4 & 1.719\cdot10^5 & 6.281\cdot10^7 & 0 & 0 & 0 & 0 & 0 & 0 \\
82.52 & 4.126\cdot10^4 & -8.252\cdot10^4 & 137.533 & 4.126\cdot10^4 & -8.252\cdot10^4 & 0 & 0 & 0 & 0 & 0 & 0 \\
-4.126\cdot10^4 & 6.178\cdot10^7 & 1.719\cdot10^5 & 4.126\cdot10^4 & 6.338\cdot10^7 & -5.157\cdot10^5 & 0 & 0 & 0 & 0 & 0 & 0 \\
-8.252\cdot10^4 & -1.719\cdot10^5 & 6.281\cdot10^7 & -8.252\cdot10^4 & -5.157\cdot10^5 & 6.441\cdot10^7 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-8.252\cdot10^4 & 1.719\cdot10^5 & 5.971\cdot10^7 & -8.252\cdot10^4 & 5.157\cdot10^5 & 6.063\cdot10^7 & 0 & 0 & 0 & 0 & 0 & 0 \\
-4.126\cdot10^4 & 6.126\cdot10^7 & -1.719\cdot10^5 & 4.126\cdot10^4 & 6.114\cdot10^7 & 5.157\cdot10^5 & 0 & 0 & 0 & 0 & 0 & 0 \\
-82.52 & -4.126\cdot10^4 & 8.252\cdot10^4 & -137.533 & -4.126\cdot10^4 & 8.252\cdot10^4 & 0 & 0 & 0 & 0 & 0 & 0 \\
-8.252\cdot10^4 & -5.157\cdot10^5 & 6.063\cdot10^7 & -8.252\cdot10^4 & -1.719\cdot10^5 & 5.971\cdot10^7 & 0 & 0 & 0 & 0 & 0 & 0 \\
-4.126\cdot10^4 & 6.114\cdot10^7 & -5.157\cdot10^5 & 4.126\cdot10^4 & 6.126\cdot10^7 & 1.719\cdot10^5 & 0 & 0 & 0 & 0 & 0 & 0 \\
-137.533 & 4.126\cdot10^4 & 8.252\cdot10^4 & -82.52 & 4.126\cdot10^4 & 8.252\cdot10^4 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
K_{g_2} = \begin{bmatrix}
137.533 & -4.126\cdot10^4 & 8.252\cdot10^4 & 82.52 & -4.126\cdot10^4 & 8.252\cdot10^4 & -82.52 \\
-4.126\cdot10^4 & 6.338\cdot10^7 & -5.157\cdot10^5 & 4.126\cdot10^4 & 6.178\cdot10^7 & 1.719\cdot10^5 & -4.126\cdot10^4 \\
8.252\cdot10^4 & -5.157\cdot10^5 & 6.441\cdot10^7 & 8.252\cdot10^4 & -1.719\cdot10^5 & 6.281\cdot10^7 & -8.252\cdot10^4 \\
82.52 & 4.126\cdot10^4 & 8.252\cdot10^4 & 137.533 & 4.126\cdot10^4 & 8.252\cdot10^4 & -137.533 \\
-4.126\cdot10^4 & 6.178\cdot10^7 & -1.719\cdot10^5 & 4.126\cdot10^4 & 6.338\cdot10^7 & 5.157\cdot10^5 & -4.126\cdot10^4 \\
8.252\cdot10^4 & 1.719\cdot10^5 & 6.281\cdot10^7 & 8.252\cdot10^4 & 5.157\cdot10^5 & 6.441\cdot10^7 & -8.252\cdot10^4 \\
-82.52 & -4.126\cdot10^4 & -8.252\cdot10^4 & -137.533 & -4.126\cdot10^4 & -8.252\cdot10^4 & 137.533 \\
-4.126\cdot10^4 & 6.126\cdot10^7 & 1.719\cdot10^5 & 4.126\cdot10^4 & 6.114\cdot10^7 & -5.157\cdot10^5 & -4.126\cdot10^4 \\
8.252\cdot10^4 & -1.719\cdot10^5 & 5.971\cdot10^7 & 8.252\cdot10^4 & -5.157\cdot10^5 & 6.063\cdot10^7 & -8.252\cdot10^4 \\
-137.533 & 4.126\cdot10^4 & -8.252\cdot10^4 & -82.52 & 4.126\cdot10^4 & -8.252\cdot10^4 & 82.52 \\
-4.126\cdot10^4 & 6.114\cdot10^7 & 5.157\cdot10^5 & 4.126\cdot10^4 & 6.126\cdot10^7 & -1.719\cdot10^5 & -4.126\cdot10^4 \\
8.252\cdot10^4 & 5.157\cdot10^5 & 6.063\cdot10^7 & 8.252\cdot10^4 & 1.719\cdot10^5 & 5.971\cdot10^7 & -8.252\cdot10^4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

- *Matriz de rigidez total:*

$$K_G := \sum_{i=1}^{N_{ele}} K_{g_i}$$

$$K_G = \begin{bmatrix} 275.065 & -82519.531 & 0 & 165.039 & -82519.531 & 0 & -82.52 & -41259.766 & 8 \\ -82519.531 & 126759168.837 & 0 & 82519.531 & 123550075.955 & 0 & -41259.766 & 61259290.907 & -17 \\ 0 & 0 & 128822157.118 & 0 & 0 & 125613064.236 & -82519.531 & 171915.69 & 5971 \\ 165.039 & 82519.531 & 0 & 275.065 & 82519.531 & 0 & -137.533 & 41259.766 & 8 \\ -82519.531 & 123550075.955 & 0 & 82519.531 & 126759168.837 & 0 & -41259.766 & 61144680.447 & -51 \\ 0 & 0 & 125613064.236 & 0 & 0 & 128822157.118 & -82519.531 & -515747.07 & 6062 \\ -82.52 & -41259.766 & -82519.531 & -137.533 & -41259.766 & -82519.531 & 137.533 & -41259.766 & -8 \\ -41259.766 & 61259290.907 & 171915.69 & 41259.766 & 61144680.447 & -515747.07 & -41259.766 & 63379584.418 & 51 \\ 82519.531 & -171915.69 & 59712049.696 & 82519.531 & -515747.07 & 60628933.377 & -82519.531 & 515747.07 & 6441 \\ -137.533 & 41259.766 & -82519.531 & -82.52 & 41259.766 & -82519.531 & 82.52 & 41259.766 & -8 \\ -41259.766 & 61144680.447 & 515747.07 & 41259.766 & 61259290.907 & -171915.69 & -41259.766 & 61775037.977 & 17 \\ 82519.531 & 515747.07 & 60628933.377 & 82519.531 & 171915.69 & 59712049.696 & -82519.531 & -171915.69 & 6280 \end{bmatrix}$$

- *Matriz reducida y resolución del sistema Q=KD*

$$k_{red} := \mathrm{submatrix}\left(K_G, 1, 12, 1, 12\right) \qquad Q := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 3 \\ 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} \qquad D := {k_{red}}^{-1} \cdot Q = \begin{bmatrix} 1.50726 \\ -0.000283 \\ 0.001992 \\ 1.50726 \\ 0.000283 \\ 0.001992 \\ 5.029848 \\ -0.000129 \\ 0.002687 \\ 5.029848 \\ 0.000129 \\ 0.002687 \end{bmatrix}$$