



UNIVERSITÀ DEGLI STUDI DI TRENTO

Department of Industrial Engineering

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Course of Systems and techniques for digital signal processing

Exercise 12 – Digital PWM modulator REPORT

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1 Introduction

Pulse width modulation (PWM) controller is commonly used for direct current (DC) motor control in robotics and other applications. Its switching frequency is a key parameter to reduce torque ripples and acoustic noise, and to meet the severe requirements to the power converters [4].

The problem is that the control resolution is inversely proportional to the switching frequency. Consequently, a higher switching frequency results in lower PWM resolution which may cause undesirable lower order harmonic components in the output current.

Quantization noise shaping based on $\Sigma\Delta$ modulation has been applied extensively to mitigate this problem and to improve the output waveform. $\Sigma\Delta$ operation consists in quantizing the input command (a duty cycle or a sinewave) to the low resolution of the DPWM and shaping the quantization noise. In this way it's possible to minimize requirements on its resolution.

As the loop integrates the error between the sampled signal and the input signal, it low-pass filters the signal and high-pass filters the noise. The differentiator factor $NTF(z)$ pushes the noise at high frequencies and converts the lower order harmonics caused by poor resolution into higher order harmonics. Then, noise and disturbance are attenuated by the output LC filter of the converter [1], [2]. This report summaries the work on implementation, simulation and performance evaluation of $\Sigma\Delta$ DPWM modulators.

Figure 1 shows the complete electronic system considered by Norris et al [3]. Figure 2 the error-feedback configuration and a standard linear model of the system.

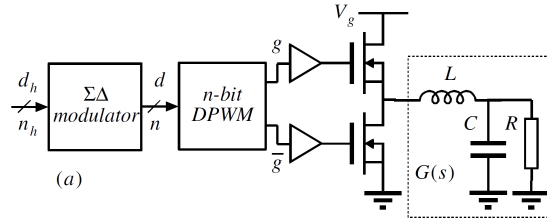


Figure 1: Digital controlled synchronous buck converter

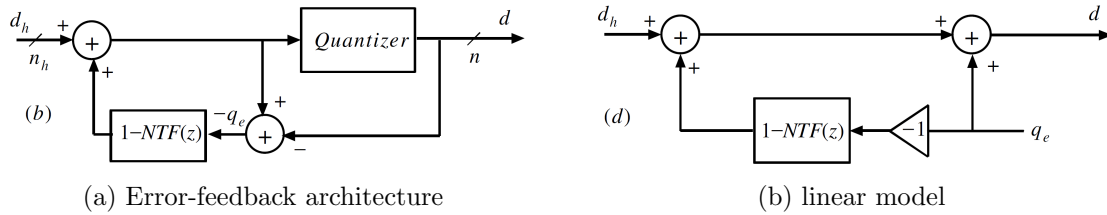


Figure 2: $\Sigma\Delta$ model

The choice of the noise transfer function $NTF(z)$ is a key design consideration and determines how effectively the modulator meets the objectives of noise shaping [3]. The table in figure 3 shows 4 classes of modulator.

TABLE I: NOISE TRANSFER FUNCTIONS (NTF) OF $\Sigma\Delta$ MODULATORS

Modulator	1 st Ord	2 nd Ord	3 rd Ord	3 rd Ord Modified
$NTF(z)$	$1-z^{-1}$	$(1-z^{-1})^2$	$(1-z^{-1})^3$	$(1-z^{-1})(1-Kz^{-1}+z^{-2})$

$$K = 2\cos(2\pi f_o/f_s)$$

Figure 3: Noise transfer functions

2 Implementation and frequency response

I started plotting the frequency response of $\Sigma\Delta$ noise shapers. In the figure below we can see the first, second and third order, plus the third order modified and the second order buck output filter, as described in [3]. For the digital filters I used the function `freqz` using the coefficients of figure 3, while for the buck filter `freqs` with transfer function $H(s) = \frac{1}{LCs^2 + Cs + 1}$.

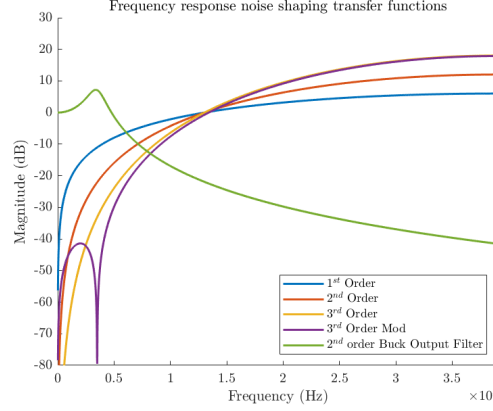


Figure 4: Frequency response of the filters

All noise shapers have a zero at DC ($z = e^{j\omega} = 1 \rightarrow \omega = 0$), which eliminates DC output errors.

In order to implement a $\Sigma\Delta$ DPWM modulator, I performed a time-domain analysis based on the error-feedback architecture in figure 2a. It's a very simple system loop obtainable from the inverse Z-transform. The quantized output d is compared to the carrier of the DPWM.

I will show the case for the first order and leave the rest in the MATLAB code.

```

1  for i = 1:length(t)
2      if i == 1
3          x1(i) = d_sine_sim(i);
4          d1(i) = quant(x1(i), Q);
5          q_e_sim1(i) = d1(i) - x1(i);
6      else
7          x1(i) = d_sine_sim(i) - q_e_sim1(i-1);
8          d1(i) = quant(x1(i), Q);
9          q_e_sim1(i) = d1(i) - x1(i);
10     end
11 end
12
13 for i = 1:1:length(t)
14     if car_q(i) ≤ d1(i)
15         pwm1(i)=1;
16     else
17         pwm1(i)=0;
18     end
19 end

```

As input, `d_sine_sim`, I used a sinusoidal reference signal with fundamental frequency of 300 Hz and amplitude $A = 0.8$, while for the DPWM carrier a triangular wave with switching frequency of 50 kHz, quantized with quantization step $Q = 2/2^4$. Sampling frequency $f = 1$ MHz and simulation time $T_{sim} = 5$ ms. Fig.5 shows the signal changes through the three noise shapers. This additional noise, which increase with the order of the system, allows to reconstruct perfectly the quantized output after passing through the converter output filter ($f_0 = 35$ kHz), unlike in the case without shaping.

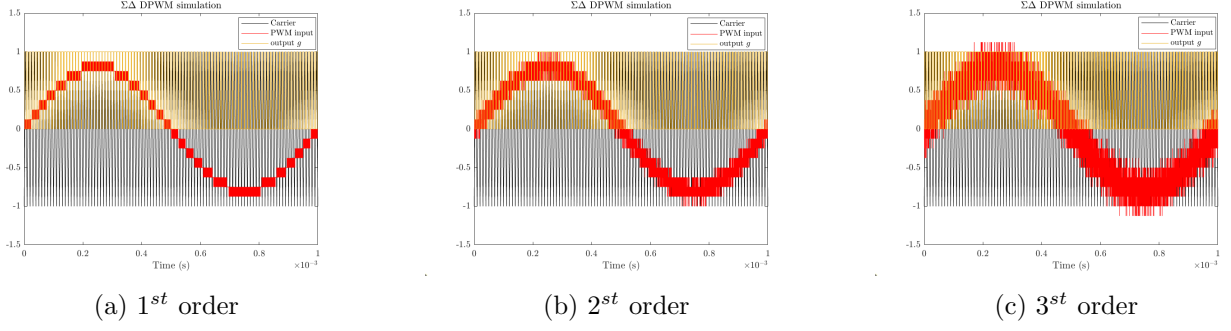


Figure 5: $\Sigma\Delta$ DPWM signals

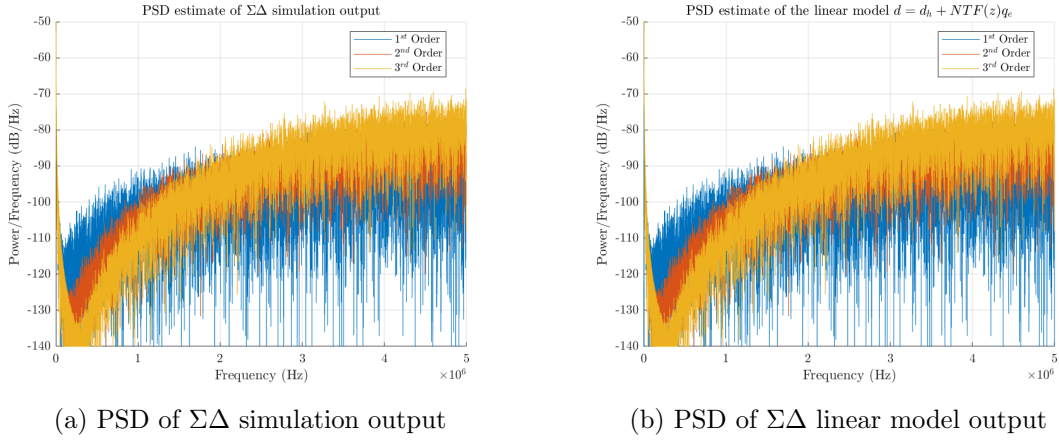


Figure 6: Comparison of $\Sigma\Delta$ output PSD

The figure above shows a comparison between the PSD of the output d , obtained from the simulation in time-domain, and by using the linear model of Fig.2b. It's visible the reduction of noise power at low frequencies, and the increase at high frequencies for higher order systems. The correctness of the implementation is confirmed by the identical spectral components of $\Sigma\Delta$ output and linear model.

3 Performance evaluation

We can test the various $\Sigma\Delta$ architectures in terms of signal-to-noise and distortion ratio (SINAD) and total harmonic distortion (THD). The reason is that DPWM sampling results in signal distortion affecting the noise-shaping performance. Moreover its non-linearity adds distortion which ends up moving back some noise from high to low frequencies [3], [4].

SINAD is a measure of the quality of a signal. It's the ratio between the total received power and the noise-plus-distortion power. Total harmonic distortion (THD) is a measurement of the distortion that an electronic device introduces in a signal which passes through, and depends on the amplitude of the harmonics. Normally, we could compute this value manually, but in this case, since it cannot be measured directly due to quantization and PWM non linear function, we need to do an estimation using periodogram analysis. This is done with the function `sinad` and `thd`.

I started by plotting the power spectrum of the quantized output d and the pulsating DPWM outputs g (Fig.7).

In the left figure we can see the characteristic behavior due to the noise transfer function $NTF(z)$ and the case without shaping, in which noise is high at low frequencies and lower at higher ones. In the figure on the right, instead, the spectra after passing through the staircase PWM function. The output

g is a square wave made up of multiple harmonics summed together. The visible peaks are situated at the fundamental frequency (50 kHz) and at its higher harmonics. By going up with the orders of $\Sigma\Delta$, noise power increases at high frequencies, while peaks are slightly lowered at small frequencies.

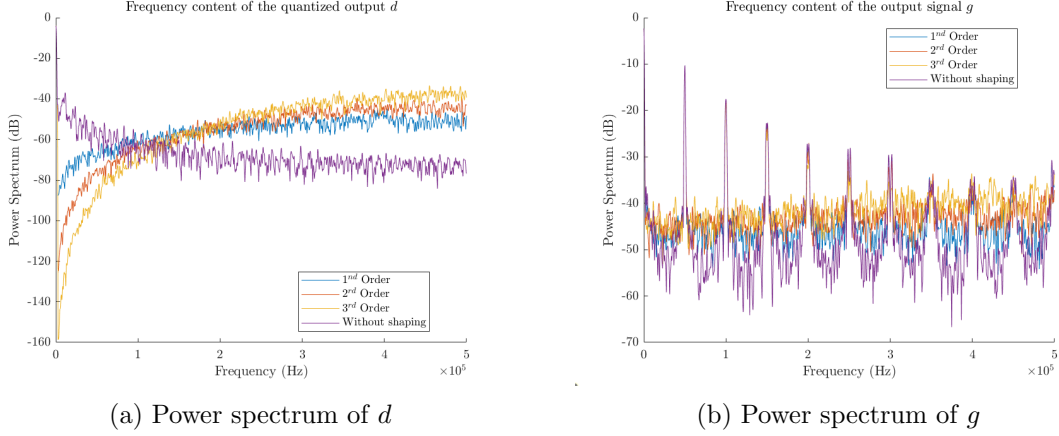


Figure 7: Power spectrum of the main signals of the system

At this point, I analyzed the final outputs.

My best results in term of SINAD and THD were achieved by considering the periodogram power spectrum estimate, by reducing the number of DFT points to 1000 and 500 respectively, and using a hanning window. They were chosen after a process of trial and error varying several settings. Moreover I had to neglect sampling frequency to compute SINAD. All values are listed in Table 1.

$\Sigma\Delta$ DPWM	SINAD	THD
Carrier	18 dB	-18 dB
Without shaping	30.64 dB	-11.86 dB
1 st order	30.47 dB	-12.50 dB
2 nd order	30.46 dB	-13.94 dB
3 rd order	30.05 dB	-14.77 dB

Table 1: SINAD and THD values after DPWM

Results shows that signal-to-noise and distortion ratio does not change with the order of the system, while total harmonic distortion improves. This confirms the capability of $\Sigma\Delta$ modulation to limit the distortion caused by DPWM. Instead it's natural that SINAD remains constant: quantization noise is not cancelled, but it's only moved to higher frequencies. The figures below displays the frequency content of 3rd-order $\Sigma\Delta$ output with the corresponding parameter values.

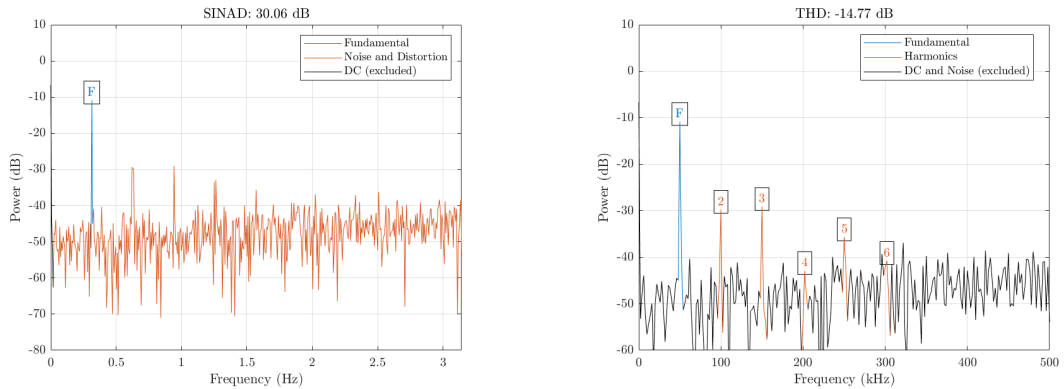


Figure 8: SINAD and THD of 3rd order $\Sigma\Delta$ DPWM output

4 Conclusions

In this report $\Sigma\Delta$ modulators followed by discrete PWM are examined. After a small introduction about its functioning, I described the implementation of the three systems and compared them to each other. Lastly, I computed SINAD and THD to validate their performances. For this purpose spectral estimation was necessary.

By trying many design choices, I noted that signal settings affect a lot the spectral content which directly changes the final value. For example: the type of input signal and its fundamental frequency, switching frequency of the carrier, the type of window functions or the numbers of DFT points over which PSD is estimated.

In the end SINAD remains almost constant for all the system, while THD gets better (it becomes more negative) with higher orders confirming the functionality of this architecture.

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References

- [1] O Deblecker. High-resolution dpwm using sigma-delta modulator implemented on a low-cost buck development board. *International Journal of Electrical Engineering Education*, 48(4):391–404, 2011.
- [2] Zdravko Lukic, Nabeel Rahman, and Aleksandar Prodie. Multibit Σ – Δ PWM digital controller IC for dc–dc converters operating at switching frequencies beyond 10 mhz. *IEEE Transactions on Power Electronics*, 22(5):1693–1707, 2007.
- [3] Mark Norris, Lazaro Marco Platon, Eduard Alarcon, and Dragan Maksimovic. Quantization noise shaping in digital pwm converters. In *2008 IEEE Power Electronics Specialists Conference*, pages 127–133. IEEE, 2008.
- [4] Y Sato and M Jiang. Output waveform improvement of high switching frequency pwm inverters introducing digital signal processing. In *2007 Power Conversion Conference-Nagoya*, pages 800–803. IEEE, 2007.