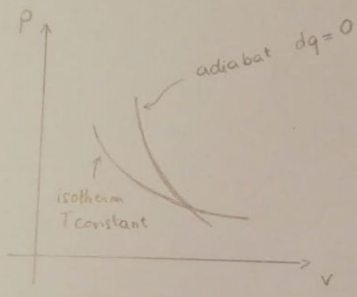
Homework 1 Giorgio Checola

Exercise 1

① Giorgio Checola



In a (p, v) diagram, the adiabatic and the isotherm curve are drawn differently because they represent two different processes: in the first case heat remains constant ($dq = 0$); pressure and volume are in the relation $pv^{\gamma} = \text{const}$ (with $\gamma > 1$). In an isotherm transformation the temperature stays constant, and for the ideal gas law we write $pv = \text{const}$. As we can see in the adiabatic process pressure has a stronger inversely dependence on volume, and so it will fall fastly as volume increases.

From a physical point of view, I would say that the heat that goes out during an isotherm compression helps bring down the pressure. Moreover in an adiabatic compression temperature increases and we know that it's easier to compress cooler gases than hotter ones.

Exercise 2

② I decided to start from the solution:

$$N^2 = \frac{g}{\theta_0}\frac{d\theta}{dz} = \frac{g}{\theta_0}\frac{d}{dz}\left(T\left(\frac{p_0}{p}\right)^{R/c_p}\right) = \frac{g}{\theta_0}\left(\frac{dT}{dz}\left(\frac{p_0}{p}\right)^{R/c_p} + \frac{d}{dz}\left(\frac{p_0}{p}\right)^{R/c_p} T\right) =$$

$$= \frac{g}{\theta_0}\left(\frac{dT}{dz}\left(\frac{p_0}{p}\right)^{R/c_p} - \frac{R}{c_p}\cdot\frac{1}{p_0}\left(\frac{p}{p_0}\right)^{-\frac{R}{c_p}-1}\cdot\frac{dp}{dz}\cdot T\right) =$$

$\frac{dp}{dz} = -\rho g$ hydrostatic balance

$p = \rho R T$ ideal gas law

$$= \frac{g}{\theta_0}\left(\frac{dT}{dz}\left(\frac{p_0}{p}\right)^{R/c_p} + \frac{g}{c_p}\frac{1}{p_0}\left(\frac{p}{p_0}\right)^{-\frac{R}{c_p}-1}\rho R T\right) =$$

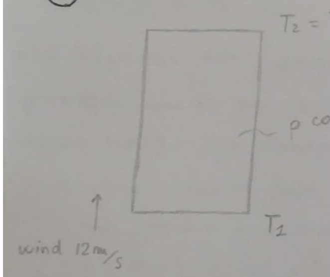
$$= \frac{g}{\theta_0}\left(\frac{dT}{dz}\left(\frac{p_0}{p}\right)^{R/c_p} + \frac{g}{c_p}\frac{p}{p_0}\left(\frac{p_0}{p}\right)^{R/c_p}\left(\frac{p}{p_0}\right)^{-1}\right) =$$

$$= \frac{g}{\theta_0}\left(\frac{dT}{dz} + \frac{g}{c_p}\right)\left(\frac{p_0}{p}\right)^{R/c_p} \qquad \theta_0 = T_0\left(\frac{p_0}{p}\right)^{R/c_p}$$

$$= \frac{g}{T_0\cancel{\left(\frac{p_0}{p}\right)^{R/c_p}}}\underbrace{\left(\frac{dT}{dz} + \frac{g}{c_p}\right)}_{\Gamma_d - \Gamma_e}\cancel{\left(\frac{p_0}{p}\right)^{R/c_p}} =$$

$$= \frac{g}{T_0}\left(\Gamma_d - \Gamma_e\right)$$

Exercise 3



(3)

$\dot q = 0.3\ m^2/s^3$ $\quad y = 30$ Km

$T_2 - T_1$

$$\dot q = c_p \frac{DT}{Dt} - \frac{1}{\cancel{\rho}}\cancel{\frac{Dp}{Dt}} = c_p\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right)$$

$$= 1005\,\frac{J}{K\cdot Kg}\left(\frac{\partial T}{\partial t} + 12\,\frac{m}{s}\left(\frac{-4}{30000}\right)\right)$$

$$\frac{\partial T}{\partial t} = \frac{\dot q}{1005} + 12\cdot\frac{4}{30000} = \frac{0.3}{1005} + 12\cdot\frac{4}{30000} = 1.9\cdot 10^{-3}\,\frac{K}{s}$$

Exercise 4

hydrostatic equilibrium, isoentropic atmosphere $\rightarrow$ $T = T_0 - \Gamma_d z$

$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT}g = -\frac{p}{R(T_0-\Gamma_d z)}g$$

$$\int_{p_0}^{p}\frac{1}{p'}dp' = -\int_0^z \frac{g}{R(T_0-\Gamma_d z')}dz' \qquad p_0 = p(z=0)$$

$$\ln\left(\frac{p}{p_0}\right) = \cancel{-}\frac{g}{R}\Big[\ln(T_0-\Gamma_d z')\Big]_0^z\left(\cancel{-}\frac{1}{\Gamma_d}\right)$$

$$\ln\left(\frac{p}{p_0}\right) = \frac{g}{R\Gamma_d}\ln\left(\frac{T_0-\Gamma_d z}{T_0}\right) \Rightarrow p = p_0\left(\frac{T_0-\Gamma_d z}{T_0}\right)^{\frac{g}{R\Gamma_d}}$$

$$\left(\frac{p}{p_0}\right)^{\frac{R\Gamma_d}{g}} = \left(\frac{T_0-\Gamma_d z}{T_0}\right) = 1 - \frac{\Gamma_d z}{T_0} \Rightarrow z = \left(1-\left(\frac{p}{p_0}\right)^{\frac{R\Gamma_d}{g}}\right)\frac{T_0}{\Gamma_d}$$

where $p = 0 \rightarrow z = \frac{T_0}{\Gamma_d} = \frac{T_0 c_p}{g}\cdot\frac{R}{R} = \frac{RT_0}{\kappa g}$

As reference temperature I chose $T_0 = 15^\circ C = 288.15$ K

$$z_{0\,hPa} = \frac{288.15\cdot 1005}{9.81} = 29.5\ \text{Km}$$

$$z_{500\,hPa} = \left[1-\left(\frac{500}{1000}\right)^{\frac{287}{1005}}\right]\frac{288.15\cdot 1005}{9.81} = 5.3\ \text{Km}$$

$$z_{300\,hPa} = \left[1-\left(\frac{300}{1000}\right)^{\frac{287}{1005}}\right]\frac{288.15\cdot 1005}{9.81} = 8.6\ \text{Km}$$

$$z_{100\,hPa} = \left[1-\left(\frac{100}{1000}\right)^{\frac{287}{1005}}\right]\frac{288.15\cdot 1005}{9.81} = 14.2\ \text{Km}$$

Exercise 5

(5) $T = -60^\circ C = 213.15\ K$

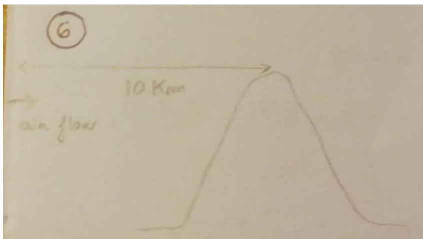
$p = 200\ mb = 200\ hPa$

$$\theta = T\left(\frac{p_0}{p}\right)^{R/c_p} = 213.15\left(\frac{1000}{200}\right)^{\frac{287}{1005}} = 337.5\ K = 64.36^\circ C$$

The value of the potential temperature means that airplanes could not pressurize their cabin adiabatically, even more so to the surface temperature (1 bar), since it would be too hot for the passengers, also by considering low values of reference pressure.

Exercise 6

(6)



$\Gamma_E = 5^\circ C/Km$

$T_0 = 20^\circ C = 293.15\ K$

$\Gamma_d = 9.81^\circ C/Km$

$g = 9.81\ m/s = 9.81 \cdot 10^{-3}\ Km/s$

$$N^2 = \frac{g}{T_0}\left(\Gamma_p - \Gamma_E\right) = \frac{9.81 \cdot 10^{-3}}{293.15}\left(9.8 - 5\right) = 1.61 \cdot 10^{-4}\ s^{-2}$$

$$N = \sqrt{1.61 \cdot 10^{-4}} = 1.27 \cdot 10^{-2}\ s^{-1}$$

$$\tau = \frac{2\pi}{1.27 \cdot 10^{-2}} = 495.5\ s$$

distance between ridges = 10 Km

$\Rightarrow \lambda = 10\ Km = 10000\ m$

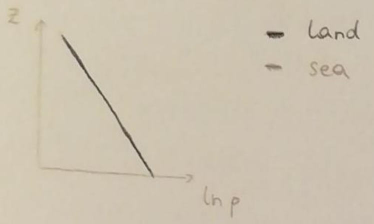
$$\lambda = \frac{U}{f} \Rightarrow U = \lambda f \quad \left(f = \frac{1}{\tau}\right)$$

$$= \frac{\lambda}{\tau} = \frac{10000}{495.5} = 20.2\ m/s = 72.65\ Km/h$$

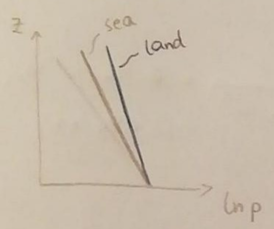
Exercise 7

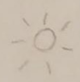
(7) Consider a coastline with initially $T_{LAND} = T_{SEA}$, $P_{LAND} = P_{SEA}$

$C_{SEA} > C_{LAND}$



a)

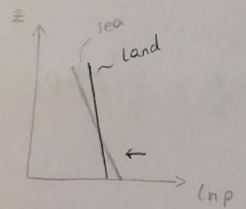




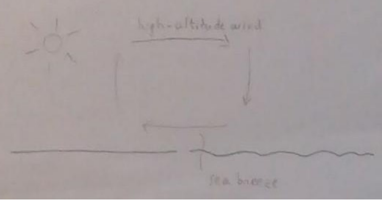
air warms up

Both scale heights increase since temperature increases. Over land this effect is more evident for the larger heat capacity of the sea. Greater scale height means pressure decreases exponentially more slowly with altitude. And the result is a high-altitude flow from land to sea

b)



The high-altitude wind produces a decreasing of the mass of the air column over land, and so a decreasing of surface pressure too, since it depends on the weight of the atmospheric column just above it. At this point surface pressure is greater over the sea and this produces the famous sea breeze. (Most probably pressure over the sea increases due to movement of air mass. The changes of scale heights and surface pressure/temperature are correlated and happen together).

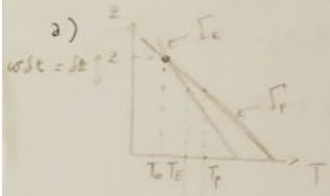


c) During the night the opposite cycle occurs: due to the larger heat capacity of the sea, $T_{SEA} > T_{LAND}$. Thus, for the same reason of a) a high-altitude wind will flow to the land, the pressure at the sea surface will decrease and a nighttime land breeze will blow

Exercise 8

(8) $\Gamma_E = 7\ K/Km$

a)



Show that after δt

$T_P = T_E + w S \delta t$

w subsidence velocity

$$S = \frac{dT_E}{dz} + \frac{g}{C_p} = -7 + 9.81 = 2.81\ K/Km$$

$T_E(\delta t) = T_0 + \Gamma_E w \delta t$

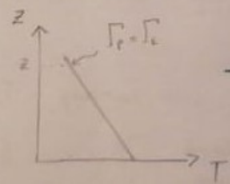
$T_P(\delta t) = T_0 + \Gamma_d w \delta t$

$$T_P - T_E = \cancel{T_0} + \Gamma_d w \delta t - \cancel{T_0} - \Gamma_E w \delta t = \left(\frac{dT_E}{dz} + \frac{g}{C_p}\right) w \delta t$$

$$\Rightarrow T_P = T_E + \left(\frac{dT_E}{dz} + \frac{g}{C_p}\right) w \delta t = T_E + w S \delta t$$

b) displacement not adiabatic

$\Gamma_P = \Gamma_E$



→ local rate of change of temperature

$$\frac{T_P - T_E}{\delta t} = \frac{\Delta T}{\delta t} = w S$$

$$\dot{q} = C_p \frac{DT_E}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt} \longrightarrow \dot{q} = C_p w S$$

I do not consider pressure changes

It should be negative since it's a loss of energy, but I'll consider it positive

$\rho\, dz = -\frac{dp}{g}$

rate of energy loss per unit volume $\frac{\dot{q}}{V} = \rho C_p w S$ → net radiative loss to space per unit horizontal area

$$\frac{\dot{q}}{A} = \int_0^\infty \rho C_p w S\, dz = \int_{p_s}^{0} -\frac{C_p}{g} w S\, dp$$

$$\Rightarrow = \frac{C_p}{g} w S p_s$$

c) $20\ \frac{W}{m^2} = \frac{C_p}{g} w S p_s \longrightarrow$

$$w = \frac{20\frac{W}{m^2} \cdot 9.8\ m/s^2}{1005\frac{J}{K\,kg} \cdot 2.81\frac{K}{Km} \cdot 100000\frac{N}{m^2} \cdot 10^{-3}} = 6.94 \cdot 10^{-4}\ m/s = 0.694\ mm/s$$

↑ vertically-averaged and annually averaged subsidence velocity