Homework 2 — Giorgio Checola

Exercise 1

(1) N homogeneous layers in radiative equilibrium

albedo α

$\varepsilon = 1$

Giorgio Checola

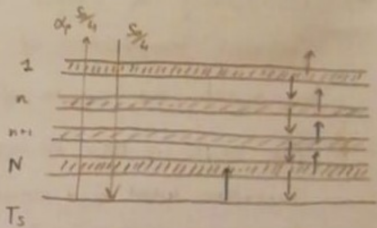
a) radiative energy balance

top of the atmosphere $\quad \frac{S_0}{4}(1-\alpha_p) = \sigma T_1^4$

each atmospheric layer n $\quad \sigma T_{n+1}^4 + \sigma T_{n-1}^4 = 2\sigma T_n^4 \qquad n = 1, \dots, N$

$T_0 = 0$

$T_{N+1} = T_s$



b) LAYER 1: $\sigma T_2^4 = 2\sigma T_1^4$

LAYER 2: $\sigma T_3^4 + \sigma T_1^4 = 2\sigma T_2^4 = 4\sigma T_1^4$

$\rightarrow \sigma T_3^4 = 3\sigma T_1^4$

$\vdots$

for each layer and surface $\quad \sigma T_n^4 = n\sigma T_1^4$

$\Rightarrow T_n = n^{\frac{1}{4}} T_1 \qquad n = 1, \dots, N+1$

$T_{N+1} = T_s$

considering
$\alpha_p = 0.3$, $S_0 = 1367\ W/m^2$

	$N = 1$	$N = 2$	$N = 4$	$N = 8$
$T_1 =$	$\left[\frac{S_0}{4\sigma}(1-\alpha_p)\right]^{\frac{1}{4}} = 254.86\,K$	$\left[\frac{S_0}{4\sigma}(1-\alpha_p)\right]^{\frac{1}{4}} = 254.86\,K$	$\left[\frac{S_0}{4\sigma}(1-\alpha_p)\right]^{\frac{1}{4}} = 254.86\,K$	$\left[\frac{S_0}{4\sigma}(1-\alpha_p)\right]^{\frac{1}{4}} = 254.86\,K$
$T_s =$	$T_2 = 2^{\frac{1}{4}} T_1 = \left[\frac{S_0}{2\sigma}(1-\alpha_p)\right]^{\frac{1}{4}} = 303.08\,K$	$T_3 = 3^{\frac{1}{4}} T_1 = \left[\frac{3}{4}\frac{S_0}{\sigma}(1-\alpha_p)\right]^{\frac{1}{4}} = 335.42\,K$	$T_5 = 5^{\frac{1}{4}} T_1 = \left[\frac{5}{4}\frac{S_0}{\sigma}(1-\alpha_p)\right]^{\frac{1}{4}} = 381.11\,K$	$T_9 = 9^{\frac{1}{4}} T_1 = \left[\frac{9}{4}\frac{S_0}{\sigma}(1-\alpha_p)\right]^{\frac{1}{4}} = 441.43\,K$

c) The temperature of the top layer doesn't depend on the number of atmospheric layers since the atmosphere is optically thick ($\varepsilon = 1$) and so, there is not the component originating from surface in the energy balance at the top of atmosphere. From a physical point of view, since the layers emit as blackbodies the lowest temperature of the top layer is the one only due to the radiation incoming from the Sun.

Instead, for the surface it's different : the greater the number of atmospheric layers the higher the temperature of the surface. It happens always for the same reason : the atmospheric layers behave as black bodies, both in absorption and emissivity thus each layer emits energy towards the surface increasing the temperature beneath itself until reaching the surface whom emitted radiation is in equilibrium with the incoming SW from the Sun and the energy re-emitted by the layer adjacent to the surface.

d) Considering a simple model like this one, there will be a finite difference between temperature of the surface and temperature of the adjacent layer, no matter the number of layers or its thickness.

The cause is that we are not taking into account other minor fluxes as turbulent latent and sensible heat fluxes or convective heat fluxes, necessary to redistribute energy in the vertical direction.
In this way we would have a more realistic model which we can add constraints to, in order to keep the lapse rate at fixed value, for example, and study the different changes of temperature due to modifications in convective fluxes.

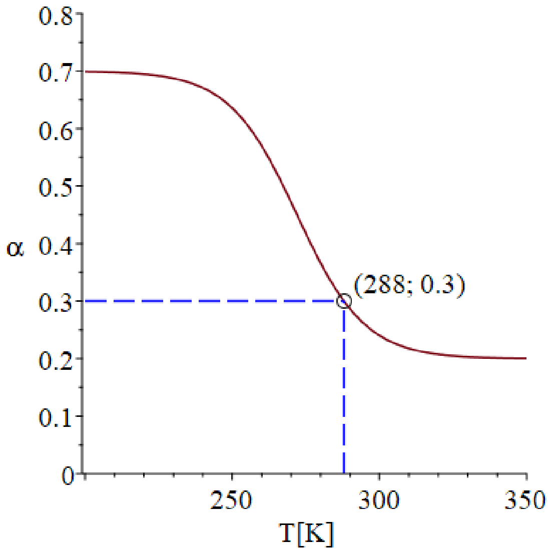
Exercise 2

Homework 3

Giorgio Checola

a) Plotting the Albedo as a function of temperature

```
> display(plot(alpha(T),T=200..350, alpha = 0..(0.8),labels=["T[K]", alpha]),
         plot(0.3, T=200..288, color = blue, linestyle = dash),
         plot([288,alpha, alpha =0..0.3], color = blue, linestyle = dash),
         plots[textplot]([310,0.33,typeset("(288; 0.3)"),font=[15]]),
         pointplot([288,0.3], color = black, symbol=circle, symbolsize=20));
```



Solving for the equilibrium solution

```
> EBM1 := (T) -> C*diff(T(t),t) = S[0]/4*(1-alpha(T))-(sigma*T^4)/(1+tau1);
  data_a  := [S[0] = 1367, sigma = 5.67*1e-8, diff(T(t),t)=0, tau1 = 0.63];
  EBM1_0  := subs(data_a, EBM1(T));
  sol_EBM1  := fsolve(EBM1_0,T, T=0...1000, maxsols = 3): <%>;
```

$$EBM1 := T \to C\left(\frac{d}{dt}T(t)\right) = \frac{1}{4}S_0\left(1-\alpha(T)\right) - \frac{\sigma T^4}{1+\tau 1}$$

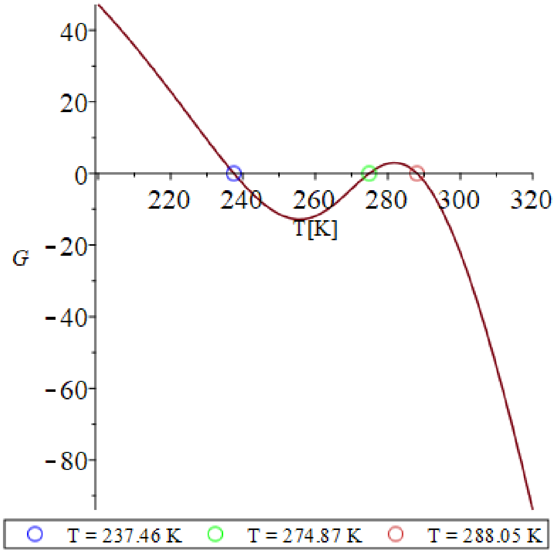
$$data\_a := \left[S_0 = 1367, \sigma = 5.67\ 10^{-8}, \frac{d}{dt}T(t) = 0, \tau 1 = 0.63\right]$$

$$EBM1\_0 := 0 = 187.9625000 + 85.43750000 \tanh\left(\frac{T}{23} - \frac{272}{23}\right) - 3.478527607\ 10^{-8}\, T^4$$

$$\begin{bmatrix} 237.4612833 \\ 274.8665469 \\ 288.0514351 \end{bmatrix}$$

Plotting the right hand side of the globally energy balance equation

```
> display(plot(rhs(EBM1_0), T=200...320, labels=["T[K]", G]),
          pointplot([237.4612833, 0], color = blue,   symbol=circle, symbolsize=20, legend = "T = 237.46 K"),
          pointplot([274.8665470, 0], color = green,  symbol=circle, symbolsize=20, legend = "T = 274.87 K"),
          pointplot([288.0514351, 0], color = orange, symbol=circle, symbolsize=20, legend = "T = 288.05 K") );
```



You can see that there are 3 real and positive solutions: the ice free and the totally frozen state solutions are stable since the derivative of the function is negative, while the partially ice-covered solution, the intermediate one, is unstable (positive slope)

b)

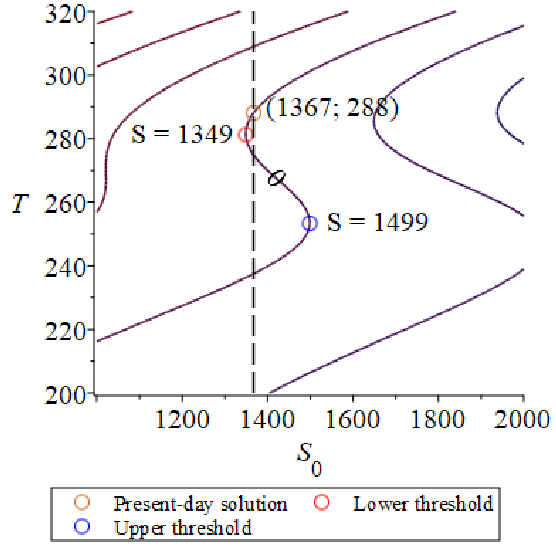
```
> data_b  := [sigma = 5.67*1e-8, diff(T(t),t)=0, tau1 = 0.63];
  EBM2     := subs(data_b, EBM1(T));
```

$$data\_b := \left[\sigma = 5.67\ 10^{-8},\ \frac{d}{dt}\ T(t) = 0,\ \tau 1 = 0.63\right]$$

$$EBM2 := 0 = \frac{S_0\left(0.55 + 0.25\tanh\left(\frac{T}{23} - \frac{272}{23}\right)\right)}{4} - 3.478527607\ 10^{-8}\ T^4 \quad (1.3)$$

Doing a contour plot of the globally energy balance equation with the total solar irradiance which varies

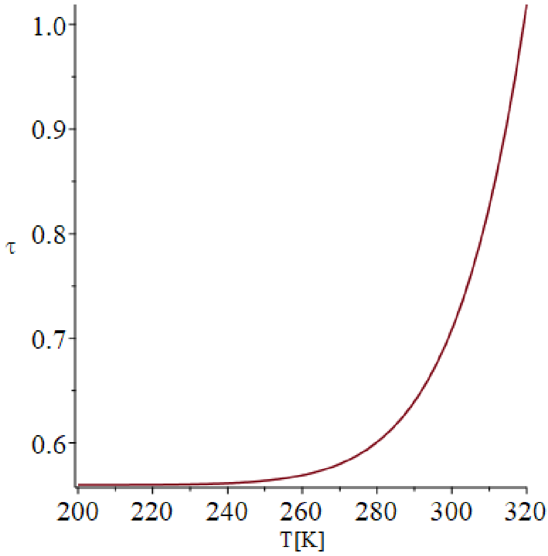
```
> display(contourplot(rhs(EBM2),S[0]=1000...2000, T=200...320, contours= [-150, -100, -50, 0, 50, 100, 150], grid
  = [100, 100]),
         plots[textplot]([1420,267,typeset("0"),font=[15], rotation = -4]),
         pointplot([1367,288.0514351], color = "Chocolate", symbol=circle, symbolsize=20, legend = "Present-day
  solution"),
         pointplot([1349,281.2], color =red, symbol=circle, symbolsize=20, legend = "Lower threshold"),
         pointplot([1498.6, 253.3], color = blue, symbol=circle, symbolsize=20, legend = "Upper threshold"),
         plots[textplot]([1550,290,typeset("(1367; 288)"),font=[30]]),
         plots[textplot]([1200,281.5,typeset("S = 1349"),font=[30]]),
         plots[textplot]([1660,254,typeset("S = 1499"),font=[30]]),
         plot([1367,T, T =200..320], color = black, linestyle = dash));
```



In the plot I have indentified the present-day Earth's climate (warm solution) by the intersection of the contour at the energy equilibrium and a straight line with S = 1367. If S was slightly reduced below 1349, the Earth would fall in the snowball state. The total solar irradiance should increase to 1499 W/m^2 to get back to the warm Earth state

c) Plotting tau as a function of temperature

```
> plot(tau(T), T = 200...320, labels=["T[K]", tau])
```



Solving for the equilibrium solution

```
> EBM3 := (T) -> C*diff(T(t),t) = S[0]/4*(1-alpha)-(sigma*T^4)/(1+tau(T)) ;
  data3 := [alpha = 0.3, S[0]=1367,sigma = 5.67*1e-8, diff(T(t),t)=0];
  EBM3_0  := subs(data3, EBM3(T));
  #y_3 := [EBM3_0=0];
  sol_EBM3_1 := fsolve(rhs(EBM3_0),T, T=0..1000, maxsols = 2): <%>;
```

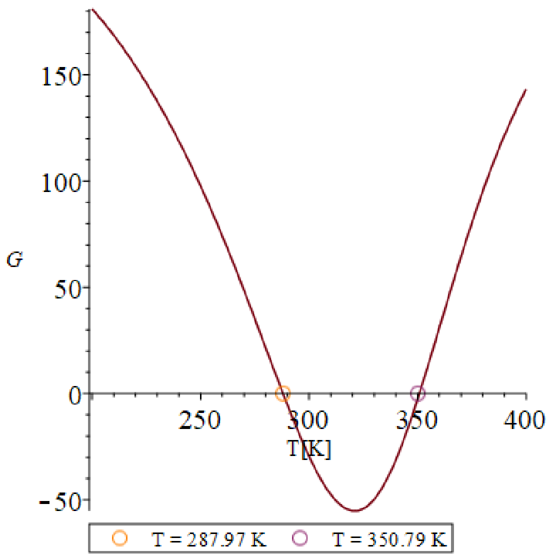
$$EBM3 := T \to C\left(\frac{d}{dt} T(t)\right) = \frac{1}{4} S_0 (1-\alpha) - \frac{\sigma T^4}{1+\tau(T)}$$

$$data3 := \left[\alpha = 0.3, S_0 = 1367, \sigma = 5.67\, 10^{-8}, \frac{d}{dt} T(t) = 0\right]$$

$$EBM3\_0 := 0 = 239.2250000 - \frac{5.67\, 10^{-8}\, T^4}{1.56 + 0.07\, e^{-\frac{5417.118093}{T} + 18.80943782}}$$

$$\begin{bmatrix} 287.9670126 \\ 350.7881620 \end{bmatrix}$$

```
> display(plot(rhs(EBM3_0), T = 200...400, labels=["T[K]", G]),
        pointplot([287.97, 0], color = coral, symbol=circle, symbolsize=20, legend = "T = 287.97 K"),
        pointplot([350.1, 0], color = maroon, symbol=circle, symbolsize=20, legend = "T = 350.79 K"));
```



The first solution corresponds to the present-day Earth's climate and it's stable. Instead, the other one is much higher and unstable, since the slope is positive

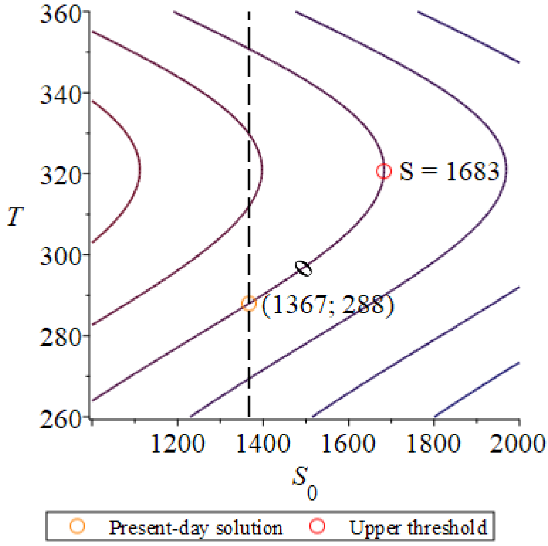
d)

```
> data4 := [alpha = 0.3, sigma = 5.67*1e-8, diff(T(t),t)=0];
  EBM4  := subs(data4, EBM3(T));
```

$$data4 := \left[\alpha = 0.3, \sigma = 5.67\,10^{-8}, \frac{d}{dt}\,T(t) = 0\right]$$

$$EBM4 := 0 = 0.1750000000\,S_0 - \frac{5.67\,10^{-8}\,T^4}{1.56 + 0.07\,e^{-\frac{5417.118093}{T} + 18.80943782}} \qquad (1.5)$$

```
> display(contourplot(rhs(EBM4),S[0]=1000...2000, T=260...360, contours= [-150, -100, -50, 0, 50, 100, 150], grid
  = [100, 100]),
          plots[textplot]([1500,296,typeset("0"),font=[15], rotation = 4]),
          pointplot([1367,287.9670126], color =coral, symbol=circle, symbolsize=20, legend = "Present-day
  solution"),
          pointplot([1682.9,320.6], color =red, symbol=circle, symbolsize=20, legend = "Upper threshold"),
          plot([1367,T, T=260..360], color = black, linestyle = dash),
          plots[textplot]([1550,288,typeset("(1367; 288)"),font=[30]]),
          plots[textplot]([1840,321,typeset("S = 1683"),font=[30]]));
```



As we can see from the plot, both equilibrium solutions cease to exist if S exceeds 1683 W/m^2