UNIVERSITÀ DEGLI STUDI
DI TRENTO

Final project
Design of an active suspension system

Automatic Control

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1 Introduction

The project concerns a wheel-suspension assembly of a car. When there is no control action on the system the suspension is called "passive", while it is "active" when there is a force, provided by an actuator placed between the wheel center and the car body, that acts as the control input for the system. My objective has been to solve a simplified version of the problem in order to reduce the impact of an uneven road on the vertical acceleration of the car body.

2 Answers

1. By considering the passive suspension system (i.e. $u = 0$), we can verify if the system is stable with the eigenvalues test: all eigenvalues of the matrix A have negative real part ($Re(\lambda_i) < 0$), so it is stable.

   The convergence rate α is smaller than 1.5 because the biggest eigenvalue of the matrix A is -1.327. It must be verified the inequality $Re(\lambda_i(A)) < -\alpha$.

   The $\mathcal{L}_2$ gain of a system is the maximum value of the ratio between energy of the output and energy of the noise. I can compute this value using the function `getPeakGain`. The result is: $\mathcal{L}_2 = 21.4026878512419$.
   In the image below we can see the Bode plot of the transfer function of the passive suspension system. The red horizontal line is the value of the $\mathcal{L}_2$ gain obtained before, and it corresponds exactly to the peak of the transfer function.

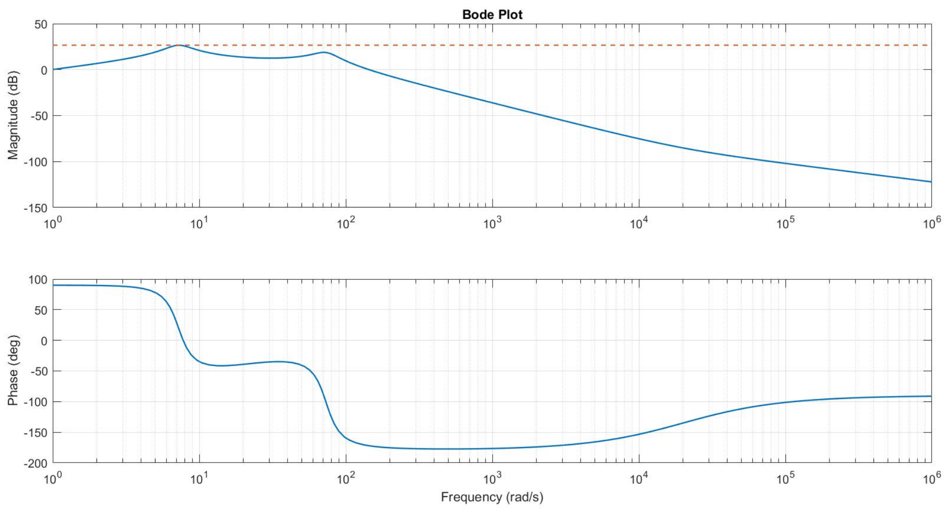

Figure 1: Bode plot of the open loop system

2. Before synthesizing a full-state feedback controller $u = Kx$ for the active suspension system, we have to know if we can control it, so we check the controllability of the system.
   Since the controllability matrix is full rank I can place the real parts of the eigenvalues of the closed-loop system wherever I want in the negative half plane.

The constraints of the LMI problem are:

$$\begin{cases} W \geq \rho I_{nxn} \\ \rho \geq 0 \\ \begin{bmatrix} He(AW+BX) & E & (CW+DX)^{\mathrm{T}} \\ E^{\mathrm{T}} & -\gamma \cdot I_{dxd} & F^{\mathrm{T}} \\ (CW+DX) & F & -\gamma \cdot I_{mxm} \end{bmatrix} \leq 0 \\ \begin{bmatrix} k \cdot \rho \cdot I_{nxn} & X^{\mathrm{T}} \\ X & k \cdot \rho \cdot I_{pxp} \end{bmatrix} \geq 0 \end{cases} \tag{1}$$

The matrix K needs to have norm k smaller than $2 \cdot 10^5$, so I fix this value and I search for the desired controller and its parameters: by means of the solver I find the optimal solution for γ, and then K with the corresponding matrix M and X.

- $\rightarrow K_a = \begin{bmatrix} 12879.7774 & 30131.0741 & -52.2964489 & 94.2500777 \end{bmatrix}$
- $\rightarrow \alpha_a = 1.3828$
- $\rightarrow \gamma_a = 9.2275$

As you can see, we confirm that it is possible to obtain a $\mathcal{L}_2$ smaller than 10. The following picture shows the Bode plot of the closed loop system.

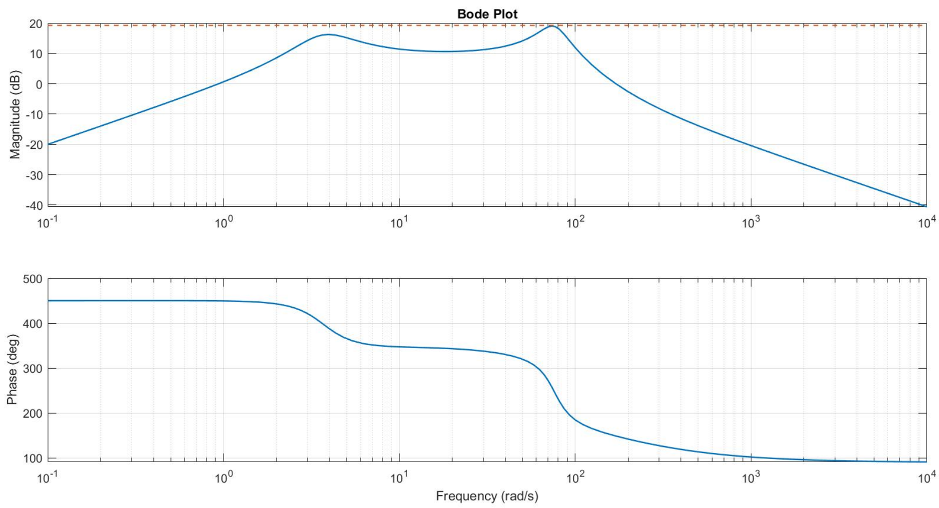

Figure 2: Bode plot of the closed loop system ($k = 2 \cdot 10^5$)

3. In order to obtain a convergence rate α greater than 1.5, I have to find a new matrix K such that $Re(\lambda_i(A_{cl})) < -1.5$. I can get it by solving the previous LMI (I keep the norm value k fixed) with an additional constraint: $\mathrm{He}(AW + BX) \leq -2\alpha^* W$ where $\alpha^* = 1.5$.

   Finally I obtain these results:

   - $\rightarrow K_b = \begin{bmatrix} 11691.6768 & 37832.7639 & -875.028917 & 78.4479637 \end{bmatrix}$
   - $\rightarrow \alpha_b = 2.4143$
   - $\rightarrow \gamma_b = 10.3393$

   The new matrix A_{cl} has all the eigenvalues with real part smaller than $-\alpha_b$.
   The Bode plot of the new transfer function of the system is shown below.

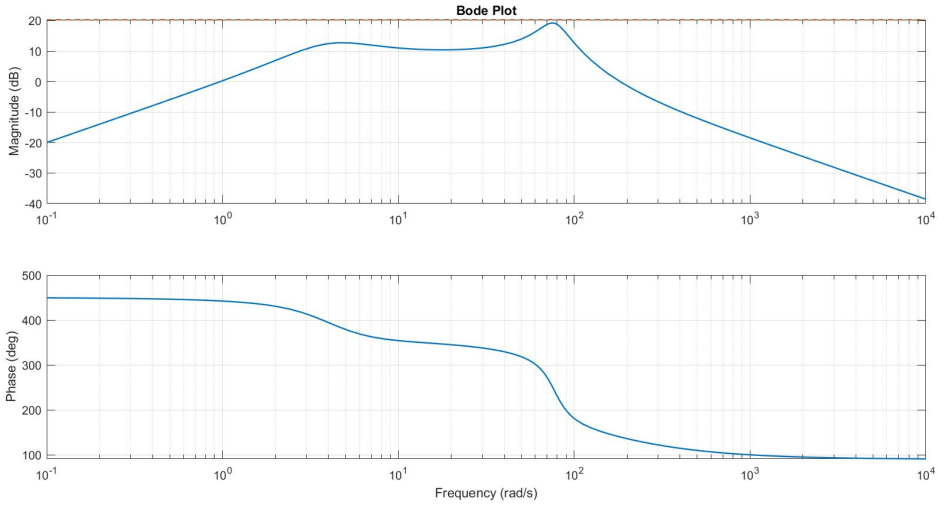

Figure 3: Bode plot of the closed loop system ($k = 2 \cdot 10^5$ and $\alpha > 1.5$)

Despite the results of the two cases I think it is possible to obtain a controller which satisfies both requirements ($\gamma < 10$ and $\alpha > 1.5$).

4. If you use an optimization solver with both requirements as input, you get the following gain matrix K:

$$K_c = \begin{bmatrix} 12674 & 31347 & -168.3 & 92.5 \end{bmatrix} \tag{2}$$

and the corresponding $\mathcal{L}_2$ gain and convergence rate α that satisfy the conditions ($\gamma_c = 9.35$ and $\alpha_c = 1.53$).
With my LMI formulation I didn't obtain this result because I am looking for a single Lyapunov function that satisfies two different conditions, but it is wrong. I should use two functions with different matrix variables, but then the problem would become nonconvex, and I can't solve it with LMI anymore.

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