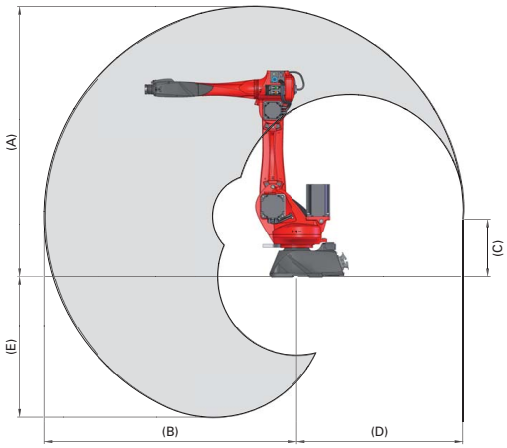
Robotics I

June 10, 2014

Consider the COMAU RACER 7-1.4 robot in Fig. 1. The robot has six revolute joints and a spherical wrist. As shown in the data sheet, each joint has a limited range (called 'stroke'), which is specified in terms of the *COMAU manufacturer convention* used for defining the joint variables.



MODEL		RACER 7-1.4
Structure / n° axes		Anthropomorphous / 6 axes
Maximum payload		7 kg
Stroke / (Speed)	Axis 1	+/- 165° (220 °/s)
	Axis 2	-85° / + 155° (250 °/s)
	Axis 3	0° / - 170° (300 °/s)
	Axis 4	+/- 210° (550 °/s)
	Axis 5	+/- 135° (550 °/s)
	Axis 6	+/- 2700° (600 °/s)
Maximum horizontal reach		1436 mm
Robot weight		160 kg
Repeatability		+/- 0,05 mm
Motors		AC brushless
Position measurement system		encoder
Total power installed		3KVA
Assembly position		Floor
		Ceiling
		Sloped (45° max)
Operating area	A	1716 mm
	B	1436 mm
	C	412 mm
	D	1134 mm
	E	801 mm

Figure 1: COMAU RACER 7-1.4 robot (taken from the data sheet)

Figure 2 (in the following page) shows a more detailed drawing of the robot workspace, with all relevant metric data (in mm or deg). For seven different Cartesian positions of the center W of the spherical wrist, a table reports the numerical values of the absolute coordinates X and Z of the point W and of the joint variables of axes 1 and 2. For the calibration position #7, the values of the complete joint configuration vector are reported.

- Assign the robot frames according to the *Denavit-Hartenberg convention* and define the numerical values in the associated table of DH parameters. On the provided extra sheet (a replica of Fig. 2), draw clearly the DH frames, put your name, and return the sheet with the rest of your solution.
- When comparing the joint variables according to the DH assignment with those used by the robot manufacturer, there may be some differences. Let $\boldsymbol{\theta} \in \mathbb{R}^6$ be the chosen DH variables, and $\boldsymbol{\theta}_C \in \mathbb{R}^6$ be the variables used by the COMAU manufacturer. Using the information in the data sheet, show that the two set of variables are related by the affine map

$$\boldsymbol{\theta}_C = \boldsymbol{\theta}_{C,0} + \boldsymbol{S}\,\boldsymbol{\theta}, \qquad \boldsymbol{S} = \text{diag}\{s_i,\, i = 1, \ldots, 6\}, \qquad s_i = \{+1, -1\}, \tag{1}$$

  where $\boldsymbol{\theta}_{C,0} \in \mathbb{R}^6$ is the value of the COMAU variables corresponding to $\boldsymbol{\theta} = \mathbf{0}$ (the zero configuration of the DH joint variables). The signs of the unitary elements s_i, $i = 1, \ldots, 6$, on the diagonal of matrix $\boldsymbol{S}$ are used to realign the sense of rotations in the two conventions. Find the actual values of vector $\boldsymbol{\theta}_{C,0}$ and of the diagonal elements of matrix $\boldsymbol{S}$.

[180 minutes; open books]

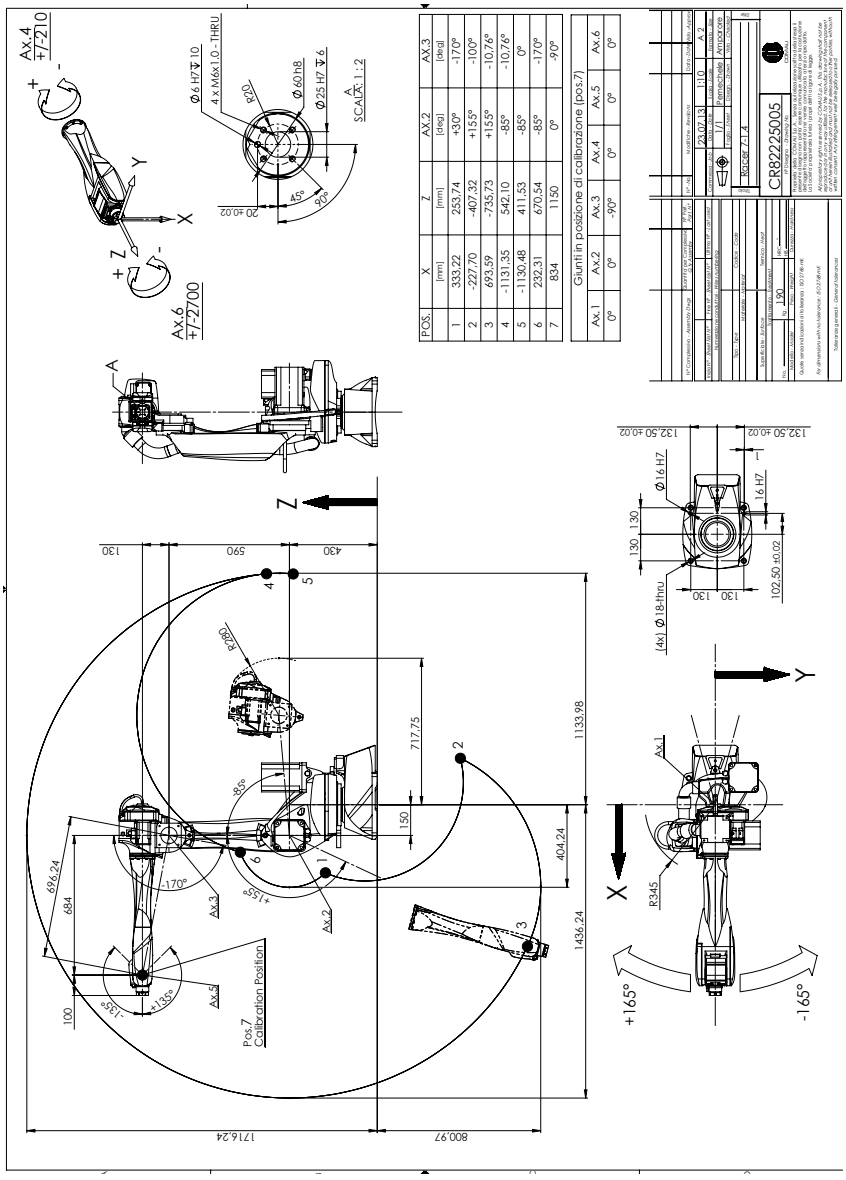


Figure 2: The workspace of the COMAU RACER 7-1.4, with the data associated to the wrist center positions (labeled from 1 to 7)

Solution

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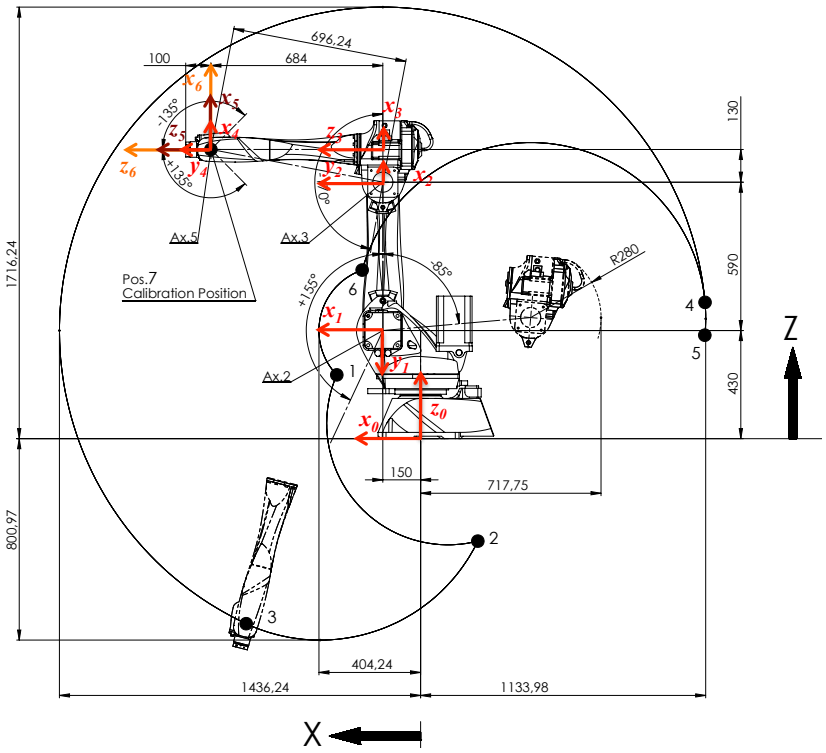

Figure 3: Assignment of Denavit-Hartenberg frames for the COMAU RACER 7-1.4 robot

i	α_i	a_i	d_i	θ_i
1	$-\pi/2$	$a_1 > 0$	$d_1 > 0$	θ_1
2	0	$a_2 > 0$	0	θ_2
3	$-\pi/2$	$a_3 > 0$	0	θ_3
4	$\pi/2$	0	$d_4 > 0$	θ_4
5	$-\pi/2$	0	0	θ_5
6	0	0	0	θ_6

Table 1: Denavit-Hartenberg parameters associated to the frame assignments in Fig. 3

A possible DH frame assignment is shown in Fig. 3, with the associated parameters given in Table 1. The actual numerical values are (in mm):

$$a_1 = 150, \quad d_1 = 430, \quad a_2 = 590, \quad a_3 = 130, \quad d_4 = 684.$$

The configuration $\boldsymbol{\theta} = \mathbf{0}$ of the chosen DH convention corresponds to the COMAU angles (in deg):

$$\boldsymbol{\theta}_{C,0} = \begin{pmatrix} 0 & +90 & -90 & 0 & 0 & 0 \end{pmatrix}^T.$$

The diagonal elements matrix $\boldsymbol{S}$ are:

$$\boldsymbol{S} = \mathrm{diag}\{-1, +1, -1, -1, +1, -1\}.$$

One can also reverse eq. (1), by multiplying it by $\boldsymbol{S}$ and noting that $\boldsymbol{S}^2 = \boldsymbol{I}$. We obtain

$$\boldsymbol{\theta} = -\boldsymbol{S}\,\boldsymbol{\theta}_{C,0} + \boldsymbol{S}\,\boldsymbol{\theta}_C = \boldsymbol{\theta}_0 + \boldsymbol{S}\,\boldsymbol{\theta}_C, \qquad \boldsymbol{S} = \mathrm{diag}\{s_i,\, i = 1, \ldots, 6\}, \qquad s_i = \{+1, -1\}. \tag{2}$$

Thus, the value $\boldsymbol{\theta}_0 \in \mathbb{R}^6$ of the DH joint variables corresponding to $\boldsymbol{\theta}_C = \mathbf{0}$, the zero configuration of the COMAU variables, is given by:

$$\boldsymbol{\theta}_0 = -\boldsymbol{S}\,\boldsymbol{\theta}_{C,0} = \begin{pmatrix} 0 & -90 & -90 & 0 & 0 & 0 \end{pmatrix}^T. \tag{3}$$

Finally, Table 2 compares the feasible joint ranges (in deg) in the two notations. For $\boldsymbol{\theta}$, instead of reporting the min and max values (in increasing order), we preferred to use the notation $\lim_1$ and $\lim_2$ (corresponding, respectively, to the lower and upper limits of θ_C). In this way, we take into account the possible reverse sense of rotation of the joints (i.e., when $s_i = -1$ for joint i) in the two conventions. Note that almost all limits in the fourth and fifth columns of this table can be computed directly using eqs. (2) and (3). Joint 5 is an exception: a $+90°$ should be added because of the special placement of the $\boldsymbol{x}_5$ axis in the $\theta_5 = 0$ configuration w.r.t. the actual joint range.

i	min $\theta_{C,i}$	max $\theta_{C,i}$	$\lim_1 \theta_i$	$\lim_2 \theta_i$
1	$-165°$	$165°$	$165°$	$-165°$
2	$-85°$	$155°$	$-175°$	$65°$
3	$-170°$	$0°$	$80°$	$-90°$
4	$-210°$	$210°$	$210°$	$-210°$
5	$-135°$	$135°$	$-45°$	$225°$
6	$-2700°$	$2700°$	$2700°$	$-2700°$

Table 2: Joint ranges in the COMAU and DH conventions

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