Asset Allocation Final Project

Computational Finance

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Project D, Group 12

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Introduction

The aim of this work is to analyze and discuss several asset allocation strategies for specific asset classes and time horizons, relying on portfolios performances. We took our data from S&P100 companies, storing their daily closing prices in a table, which was used in order to extract historical parameters such as returns and covariance matrix. The whole dataset was split into two parts: a "training" set, from 11/05/2021 to 11/05/2022, on which we built our portfolios according to different metrics, and a "test" set, from 12/05/2022 to 12/05/2023, on which we evaluated their performances. We then compared the recorded performance on this last set with the one on the training set, in order to understand which allocation strategy could perform better.

1. Efficient Frontier under standard constraints

We started our work by computing the efficient frontier (in the Volatility - Expected Return plane) under the following standard constraints (no short positions):

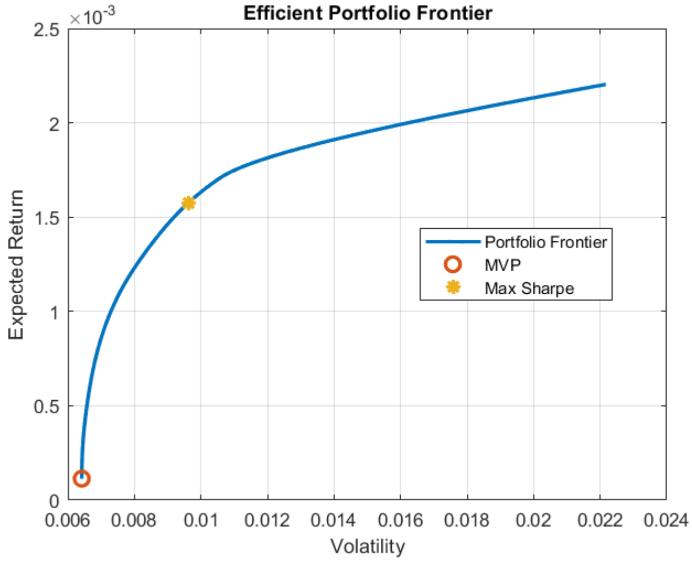
- $\sum_{i=1}^{N} w_i = 1$
- $0 \leq w_i \leq 1 \ \forall i \in [1, ..., N]$

Then, we computed the Minimum Variance Portfolio (Portfolio A), namely the particular asset allocation which minimizes the portfolio variance, and the Maximum Sharpe Ratio Portfolio (Portfolio B), whose objective is instead to maximize the Sharpe ratio (assuming a null risk free rate). Therefore, we solved the following two optimization problems:

$$\begin{gathered} \min w^T V w \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \leq 1 \end{gathered} \tag{1}$$

$$\begin{gathered} \max Sharpe \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \leq 1 \end{gathered} \tag{2}$$

Of course both of them are on the obtained efficient frontier.



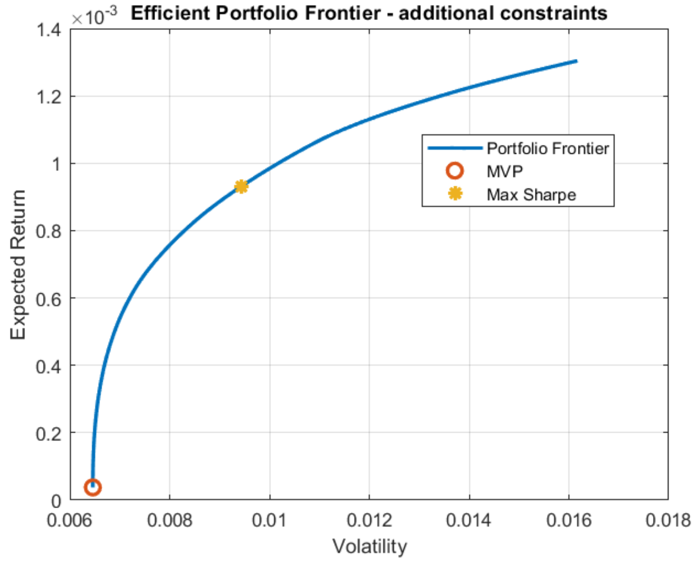
We saved the weights of each portfolio (A and B) in order to compare them with other strategies derived in the following steps, measuring metrics as Annualized Volatility, Annualized Return, Sharpe ratio, Max Drawdown, Calmar ratio, Entropy and Diversification.

2. Efficient Frontier under additional constraints

Moving forward in our work, we decided to upgrade our allocation strategies by introducing more sophisticated constraints. This was possible since we made use of another dataset in which the sector of each company was provided.
Hence, we considered the following constraints (all at once):

- Standard constraints
- The overall exposure of the companies belonging to the sector "Communication Services" has to be greater then 12%
- The overall exposure of the companies belonging to the sector "Utilities" has to be less then 10%
- The weights of the companies belonging to sectors that are composed by less than 5 companies has to be null.

We re-computed the efficient frontier under the above additional constraints.



Finally we stored once again the weights of the Minimum Variance Portfolio (Portfolio C) and Max Sharpe Ratio Portfolio (Portfolio D) for following comparisons.

3. Robust Frontier

The aim of this section is to construct a frontier which is less sensitive to changes in the inputs of the building procedure.
Since the frontier is derived from the particular historical realizations of the asset returns, we could observe significant variations in portfolios compositions for little modifications in asset returns realizations. In order to overcome this problem, we constructed the so called robust frontier. In particular, we used the resampling method.
The idea was to exploit the mean vector and the variance-covariance matrix of the historical logarithmic returns of the assets, assume the distribution as a multivariate Gaussian with those mean and variance-covariance matrix and sample all the returns. From the sample we could then derive the frontier based on that particular realization. We iterated the procedure N times in order to have N different frontiers. At this point, we extracted the sample mean of the portfolios.
We were interested in two portfolios, the Minimum Variance Portfolio (MVP) and the Max Sharpe Ratio Portfolio (MSRP), hence we took the sample mean of all N MV Portfolios and Max Sharpe Ratio Portfolios. We applied this methodology both to frontier under standard constraints and under additional constraints and we built:

- Portfolio E: MVP under standard constraints
- Portfolio F: MVP under additional constraints
- Portfolio G: MSRP under standard constraints
- Portfolio H: MSRP under additional constraints

4. The Black - Litterman Model

The Black-Litterman model fuses market equilibrium with investor views to estimate the asset returns. It begins with the market equilibrium expected returns (μ) and the covariance matrix of assets (Σ).

We first computed the prior distribution of the returns by relying on the implied equilibrium return vector, where the expected return of every asset is weighted for its capitalization (ω^T) and depends on the risk aversion coefficient(λ) :

$$\mu_{\text{market}} = \lambda \omega^T \mu \tag{3}$$

We then included investor views, expressed as expected returns (q) and uncertainty (P):

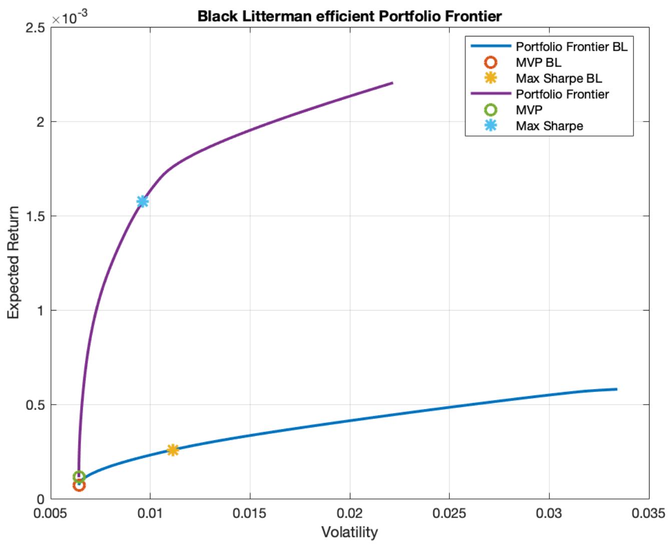
- The companies belonging to the sector "Industrials" will have an annual return of 3%,
- The companies belonging to the sector "Materials" will have an annual return of 5%,
- The companies belonging to the sector "Information Technology" will outperform the companies belonging to the sector "Consumer Staples" of 7%

We then combined our previous distributions to find the posterior distribution:

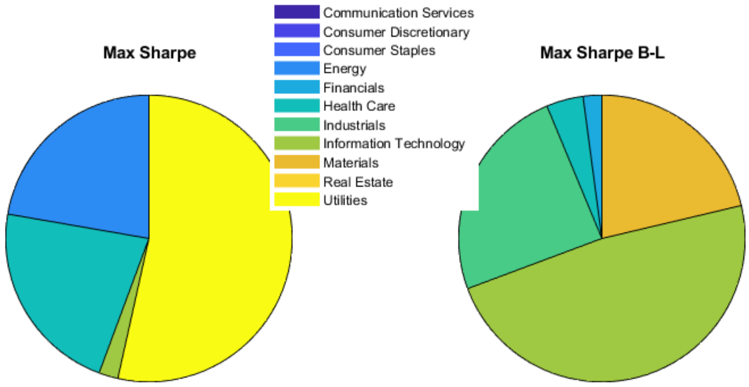
$$\mu_{BL} = \left((\tau \cdot \Sigma)^{-1} + P^T \Omega^{-1} P\right)^{-1} \left((\tau \cdot \Sigma)^{-1} \mu_{mkt} + P^T \Omega^{-1} q\right) \tag{4}$$

$$\Sigma_{BL} = \left((\tau \cdot \Sigma)^{-1} + P^T \Omega^{-1} P\right)^{-1} \tag{5}$$

Where τ is a scaling factor for the uncertainty in investor views.
We proceeded to find efficient frontier, minimum variance portfolio and maximum Sharpe ratio portfolio analogously as before and under the same constraints, using the new distribution parameters ($\mu_{BL}, \Sigma_{BL} + \Sigma$).
Computing the new portfolio frontier, we noticed that the investor views had a negative effect on the total return (lower return), without allowing to reach lower volatilities, as it can be observed in the next figure.



The minimum variance portfolio (Portfolio I) remained almost unchanged, while we saw a great difference in the maximum Sharpe ratio portfolio (Portfolio L), both in specific assets weights and in overall sectors exposure. In particular, we noticed a strong increase in the Industrials, Materials and Technology sectors, which were the ones we had positive views about.



Maximum Sharpe ratio portfolios exposure to sectors

5. Maximum Diversified and Maximum Entropy Portfolios

At this point, our aim is to compute two portfolios, the Maximum Diversified Portfolio (Portfolio M) and the portfolio maximizing the Entropy in Asset Volatility (Portfolio N), under the following set of constraints:

- Standard constraints, i.e. $\sum_i w_i = 1, \quad w_i \geq 0$
- The weights of the companies belonging to "Materials" sector satisfy $0.003 \leq w_i \leq 0.01$
- The weights of the companies belonging to "Energy" sector satisfy $0.001 \leq w_i \leq 0.03$

To compute the two portfolios, two similar maximization problems are to be solved:

$$\begin{aligned} &\arg\max\ f(w) \\ &0 \leq w_i \leq 1 \\ &\sum_i w_i = 1 \\ &0.003 \leq w_i \leq 0.01, \quad i \in \{M\} \\ &0.001 \leq w_i \leq 0.03, \quad i \in \{E\} \end{aligned} \tag{6}$$

where $\{M\}$ and $\{E\}$ are the sets of indices of companies belonging to the sectors *Materials* and *Energy*, respectively. $f(w)$, instead, is the diversification rate (7) or the entropy in asset volatility (8) for portfolio M and N, respectively.

Using the estimated covariance matrix Σ, the column vector of estimated volatilities σ and the volatility of the portfolio $\sigma_P = \sqrt{w^T \Sigma w}$, the (logarithm of the) diversification rate is given by:

$$f(w) = \ln \frac{w^T \sigma}{\sigma_P} \tag{7}$$

Using the same constants as above, the entropy in asset volatility is given by:

$$f(w) = -\sum_{i=1}^{N} \left(\frac{w_i^2 \sigma_i^2}{\sum_i^N w_i^2 \sigma_i^2} \right) \ln \left(\frac{w_i^2 \sigma_i^2}{\sum_i^N w_i^2 \sigma_i^2} \right) \tag{8}$$

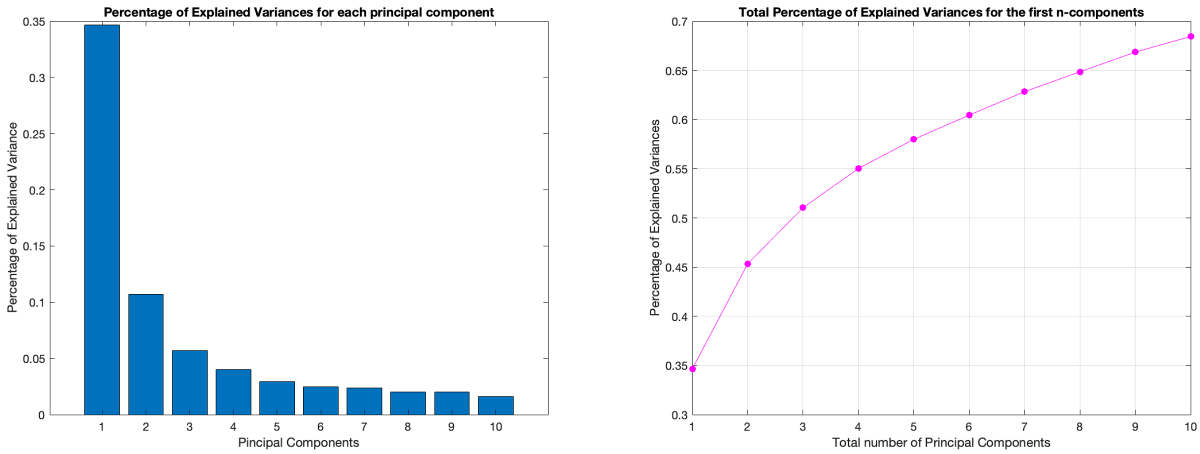
The maximization problems were both solved using the numerical solver **fmincon** in MATLAB® with two different initial guesses, one being the equally weighted portfolio, $w_i = 1/N$ $\forall i$, and the other a portfolio concentrated in one asset, $w_i = 0 \quad \forall i \notin M \cup E \cup \{1\}, w_{i \in M} = 0.003, w_{i \in E} = 0.001, w_1 = 1 - \Sigma_{i \in M \cup E} w_i$. Both these initial guesses yield results with negligible differences and the solution is thus satisfactory.

6. PCA

The goal of this section is to build a portfolio (Portfolio P), using the Principal Component Analysis (10 factors), that maximizes its expected return under the following two constraints:

- Standard Constraints
- The volatility of the portfolio has to be equal or less than a target volatility: $\sigma_{ptf} \leq \sigma_{tgt}$, with $\sigma_{tgt} = 0.007$.

By applying the **pca** MATLAB® function (with 10 principal components) to the log-returns and computing the explained variances and cumulative explained variances, we got the following results.



(a) Explained Variance

(b) Cumulative Explained Variance

Figure 1: Explained Variance and Cumulative Explained Variance

Once we found the factor loadings, the factor returns and the eigenvalues, we used them in order to solve the following optimization problem:

$$\begin{aligned} \max \mu_a^T w_a \\ 0 \leq w_i \leq 1 \\ \sum_i w_i = 1 \\ \sigma_{ptf} \leq \sigma_{tgt} \end{aligned} \tag{9}$$

Where σ_{ptf} is the portfolio volatility, σ_{tgt} is the target volatility mentioned above, w_i are the weights and μ_a are the asset returns.
The optimization (9) was performed via the **fmincon** MATLAB® function.
With such a restriction in the portfolio volatility, we expect to observe a poor performance compared to the equally weighted portfolio (in the out of sample case). Further details can be found in next sections.

7. Maximum Expected Shortfall - Modified Sharpe Ratio Portfolio

We proceeded in our work seeking to compute, under standard constraints, the portfolio which maximizes the Expected Shortfall-modified Sharpe ratio. This is an application of the Sharpe ratio whereby the risk in the formula is replaced by the Expected Shortfall.

We had to solve :

$$\begin{aligned} \max \frac{\omega^T e}{ES_p} \\ 0 \leq \omega_i \leq 1 \\ \sum_i \omega_i = 1 \end{aligned} \tag{10}$$

where ω is the array of weights, e the array of expected returns, and ES_p the Expected Shortfall at confidence level p.
Here we chose a 95% confidence level ($p = 0.95$).

We computed ES_p by taking the mean of the expected portfolio daily returns of the p% worst cases.

This reads mathematically as:

$$ES_p = -\frac{1}{N - Np} \sum_{i=1}^{N-Np} R_{(i)} \tag{11}$$

where $R_{(i)}$ represents the i-th worst log return in the sorted sequence of log returns, and N is the total number of observations.

Then, the MATLAB® function **fmincon** was used to find the portfolio weighting (Portfolio Q) optimizing this modified Sharpe ratio.

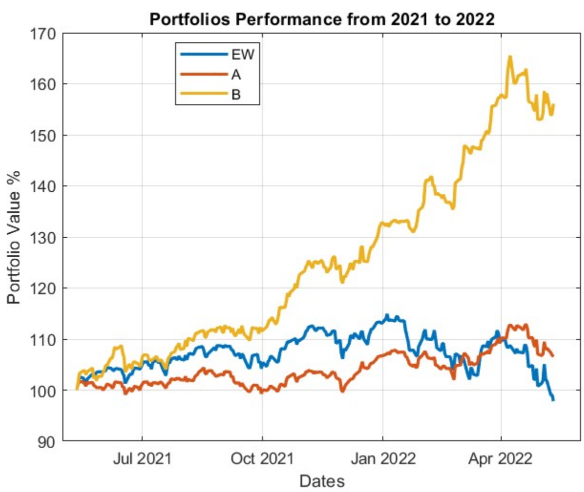
8. In-Sample Comparisons

In this section we used the portfolio allocations computed before in order to evaluate the performance in the period from 11/05/2021 to 11/05/2022, i.e. we did an in-sample evaluation. In order to assess the portfolios, we computed the performance metrics and we made a plot of the equity curves.

Metrics	AnnRet	AnnVol	Sharpe Ratio	MaxDD	Calmar Ratio	Entropy	Diversification Ratio
EW	-0,02	0,15	-0,14	-0,15	-0,14	-131020,63	1,79
A	0,06	0,10	0,62	-0,06	1,12	-125999,91	2,09
B	0,54	0,15	3,54	-0,08	7,10	-158257,22	1,50
C	0,04	0,10	0,42	-0,06	0,74	-124662,58	2,07
D	0,30	0,15	1,98	-0,11	2,78	-148050,03	1,56
E	0,05	0,10	0,52	-0,06	0,89	-125691,96	2,11
F	0,03	0,10	0,30	-0,06	0,50	-124275,40	2,08
G	0,35	0,13	2,76	-0,07	5,14	-147214,11	1,90
H	0,19	0,12	1,57	-0,07	2,68	-138921,74	1,94
I	0,06	0,10	0,62	-0,06	1,12	-125989,57	2,09
L	0,07	0,18	0,39	-0,11	0,60	-136091,31	1,70
M	0,07	0,11	0,61	-0,08	0,88	-130263,69	2,47
N	0,00	0,14	-0,01	-0,11	-0,02	-129354,42	1,80
P	0,29	0,11	2,54	-0,05	5,63	-139539,22	1,88
Q	0,37	0,14	2,63	-0,07	5,25	-151741,38	1,81

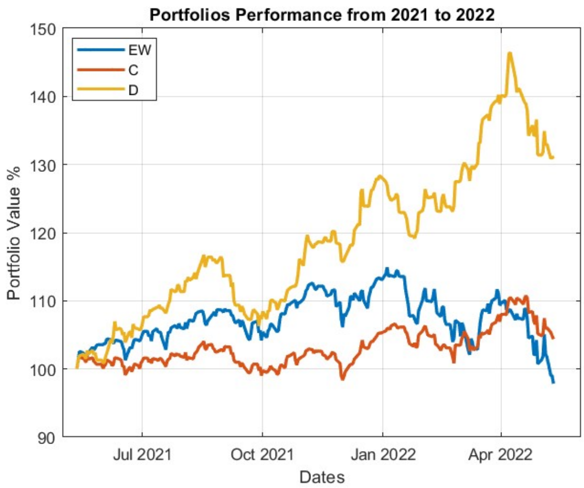
From this table we can notice that portfolio B has the best annual performance, with an annual return equal to 54%, which is a surprisingly high return, and a really strong Sharpe ratio of 3.54. This result is coherent with the fact that portfolio B maximizes the Sharpe ratio (hence tries to obtain a great return with a reduced volatility). However, we see that the volatility of this portfolio is quite high (it's the second highest one), which means that a great value of the Sharpe ratio is mainly explained by a great return. For this reason, we can consider that portfolio B embeds a significant risk and its valuable performance could be highly dependent on the specific time horizon. Indeed, in part B we will see a completely different behaviour for this portfolio.

8.1. EX1



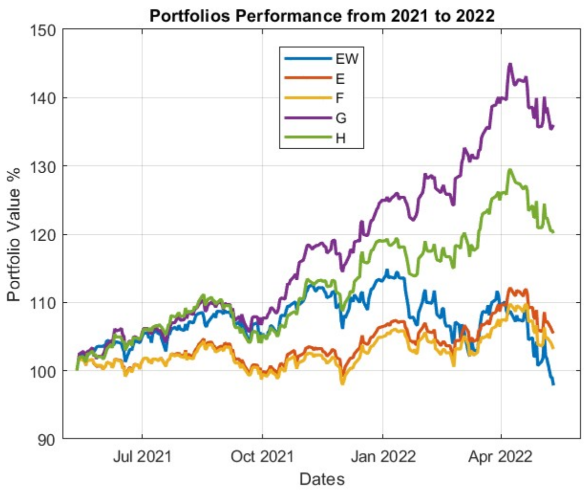
We can notice that portfolio A overtook the EW one only in the final part of the time window, with a 6% profit. Instead, portfolio B, outperformed significantly the benchmark, especially in the second half of the considered time horizon, leading to a stunning 54% annualized return with a reduced DrawDown, which causes a very high Calmar ratio as well. As already said, this allocation is the best one for this particular historical period and the considered asset class.

8.2. EX2



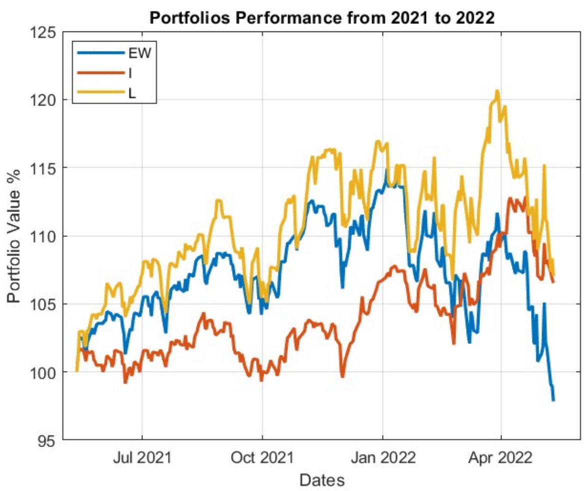
Portfolio C delivered a better performance with respect to the EW one, but we would prefer something more than just a 4% annualized return. On the other hand, portfolio D outperformed the benchmark with a significant 30% annualized return, however a Max DrawDown of 11% could begin to be relevant, reason why we should prefer another allocation with fewer variations. Moreover, portfolio D has a lower diversification ratio.

8.3. EX3



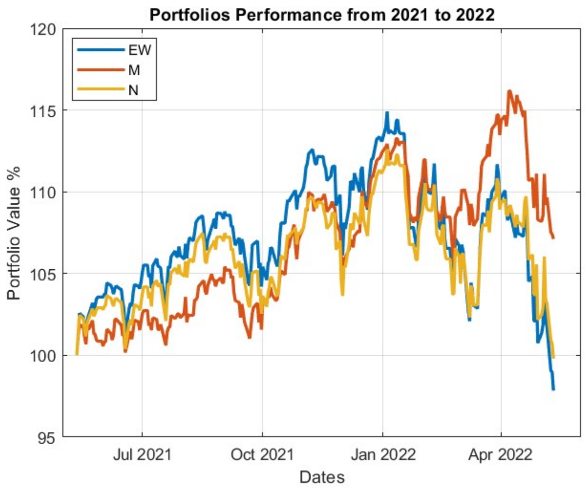
Portfolios E and F didn't close far from the EW one, even if they delivered a final positive annualized return. More relevant are instead portfolios H and G, with annualized returns of 19% and 35% respectively, which outperformed the benchmark. Moreover, we can notice that DrawDowns are limited for every portfolio in this section. Both of them have a high Calmar ratio, but this evidence is particularly remarkable in portfolio G, which realize an outstanding performance.

8.4. EX4



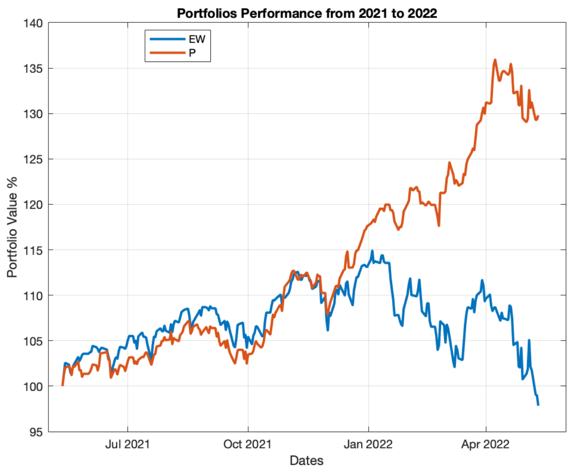
Both portfolios I and L performed very similarly in terms of annualized return, delivering a better performance than the EW one. However, portfolio L has a higher volatility (18%) if compared to portfolio I (10%), which translates into more risk associated with this particular asset allocation. Indeed, Max DrawDown of portfolio L exceeds 10%, meaning that this strategy could be very sensitive to market's future trend. Anyway, this allocation strategies may experience some problems due to the inaccuracy in the market view during the portfolio construction.

8.5. EX5



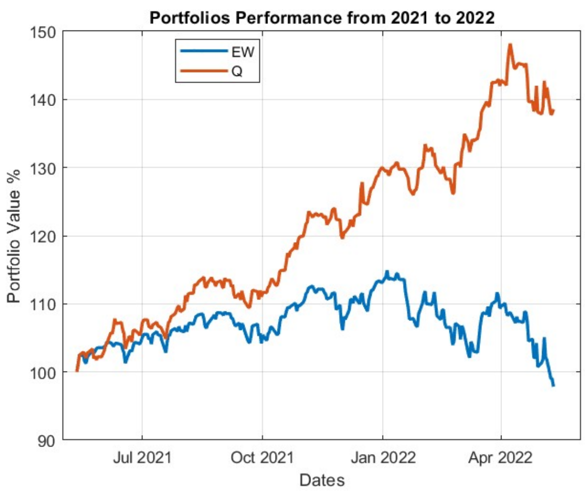
Portfolio N followed the EW one for most of the time window, closing a little higher than the benchmark, with a null annualized return. On the other hand, portfolio M performed worse in the first half of the time horizon and then it recovered and accumulated profits. Another aspects one can notice is the high diversification ratio of the portfolio M, which was expected due to the fact that we are talking about the maximum diversified portfolio.

8.6. EX6



As expected, we can observe that the PCA portfolio outperformed the equally weighted one in-sample, since we maximized the expected return. Moreover, the annualized volatility is relatively low since we bound it in our optimization. However, even if we found a good performance in sample, we expect a poor performance out of sample, due to the volatility constraint of 0.7%, which is very low.

8.7. EX7



Portfolio Q, being optimized for this period of time, naturally outperformed the equally weighted portfolio. A 37% annualized return is impressive. The same holds also for its Sharpe ratio and for its Max DrawDown, which is not so wide. Furthermore, the Calmar ratio shows an outstanding performance. Indeed, it is among the highest ones.

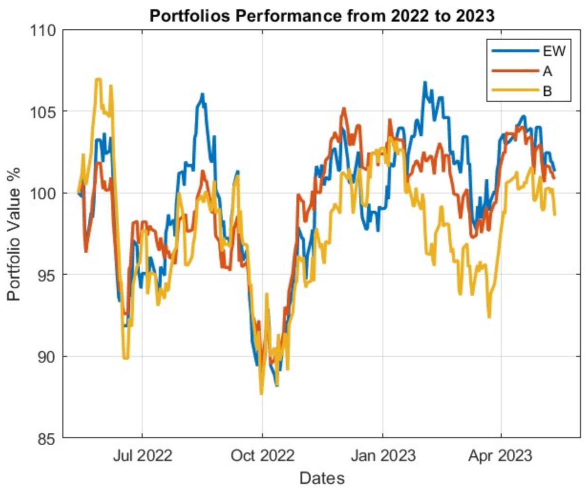
9. PART B: Out-of-Sample Comparisons

Finally, we used the portfolio allocations computed in the PART A in order to evaluate the performance in the period from 12/05/2022 to 12/05/2023, i.e. we did an out-of-sample evaluation.
In order to assess the portfolios, we computed again the performance metrics and we made a plot of the equity curves in the same way as in PART A, but in a different time period.

Metrics	AnnRet	AnnVol	Sharpe Ratio	MaxDD	Calmar Ratio	Entropy	Diversification Ratio
EW	0,01	0,20	0,06	-0,17	0,07	-119584,98	1,52
A	0,01	0,14	0,06	-0,12	0,07	-119081,99	1,67
B	-0,01	0,20	-0,07	-0,18	-0,08	-116852,04	1,40
C	0,00	0,14	0,03	-0,13	0,03	-118143,40	1,66
D	0,11	0,17	0,64	-0,09	1,15	-123534,44	1,52
E	0,01	0,15	0,06	-0,12	0,07	-119218,20	1,69
F	0,00	0,14	0,00	-0,13	0,00	-117909,53	1,69
G	0,02	0,17	0,12	-0,14	0,15	-119084,96	1,62
H	0,05	0,16	0,30	-0,11	0,42	-119739,25	1,65
I	0,01	0,14	0,06	-0,12	0,07	-119086,84	1,67
L	0,02	0,23	0,10	-0,24	0,09	-113235,88	1,38
M	0,02	0,17	0,14	-0,14	0,16	-121563,50	1,70
N	0,00	0,18	0,02	-0,16	0,03	-118869,58	1,54
P	-0,01	0,15	-0,07	-0,13	-0,08	-116185,05	1,60
Q	0,02	0,18	0,11	-0,14	0,13	-120868,61	1,58

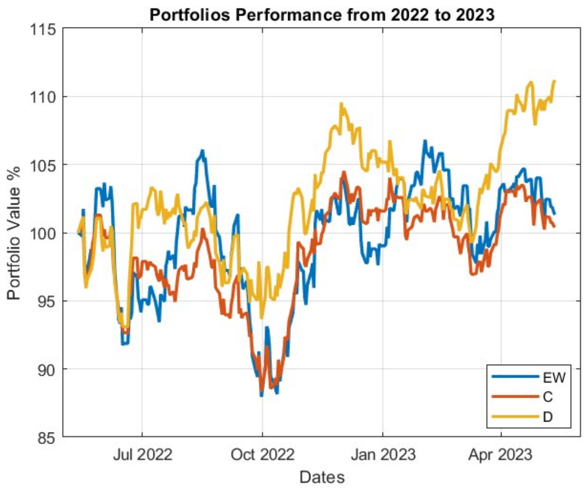
As we can see from the above table, the portfolio that performed best during the second time window is portfolio D, namely the Max Sharpe Ratio portfolio under additional constraints coming from exercise 2. We can also notice that this portfolio outperformed the previous best portfolio, portfolio B, in the current time period. Moreover, portfolio B passes from being the best portfolio to being the worst one in terms of performance in the new time window (and the only one with negative annual return). This confirms how the performance of each portfolio is highly sensitive with respect to the market's trend in the specific time horizon. However, in both time windows, the best portfolio is a Max Sharpe Ratio one, which makes sense according to the definition of the Sharpe ratio itself.

9.1. EX1



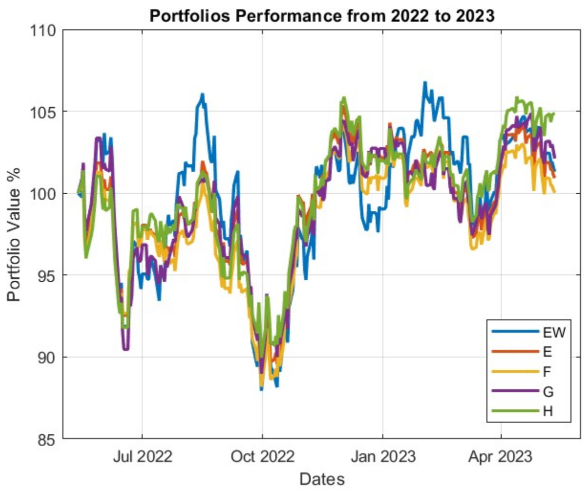
Portfolios A and B tended to follow the EW one, however portfolio B closed a little lower than the benchmark, leading to a negative annualized return for the previous best portfolio. Drawdowns are significant in these 3 portfolios, hence we would prefer to look for other allocations, since we aim to less oscillating portfolios.

9.2. EX2



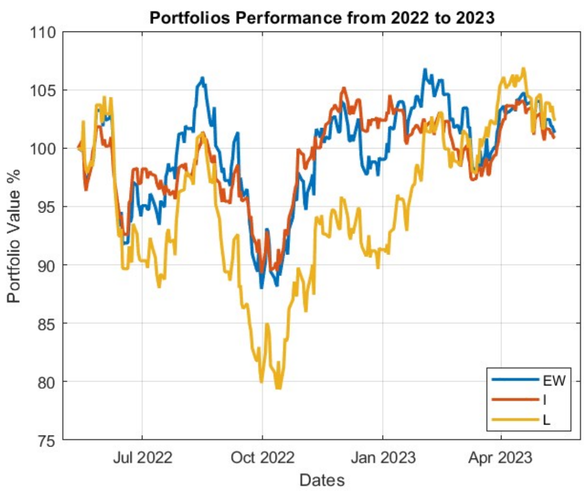
Portfolio C didn't perform very well at the beginning, however it closed its performance near to the benchmark. Instead, portfolio D outperformed the EW one, especially in the last part of the time window, leading to a final annualized return above 10% for this strategy. Furthermore, Maximum DrawDown is less than 10%, which makes this allocation acceptable also in terms of risk. This implies a much higher Calmar ratio if compared to the other portfolios.

9.3. EX3



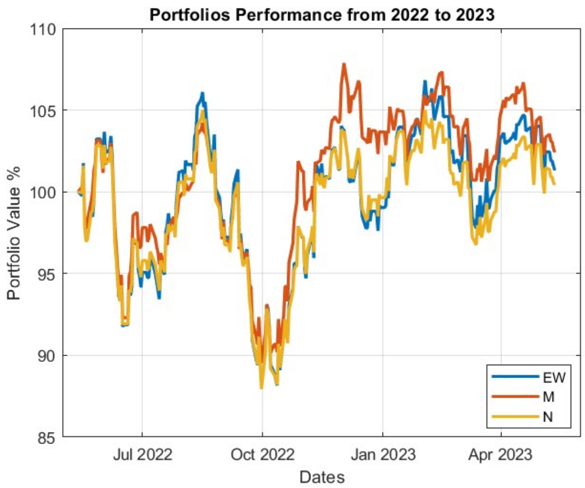
Each portfolio in this section seem to follow the EW one, even staying below its performance in the first half of the time horizon. However, in the final part of the observed period, portfolio H performed better than the other ones, leading to a 5% annualized return, which allows it to outperform the benchmark. In terms of diversification, they have among the highest diversification ratios.

9.4. EX4



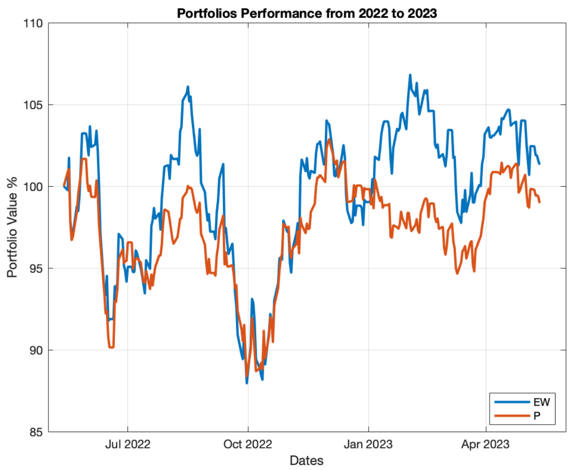
In this case we do not have any exceptional performance, since portfolio I's annualized return is the same as the EW one, while portfolio L closed a little above that level. However, since this difference is very small, it does not justify the significant DrawDown of this portfolio, which is at 24%, namely too risky for our ideal strategy. Thus, we have to find better asset allocations than these ones. Once again we are observing the problems linked to the wrong view during the portfolio construction.

9.5. EX5



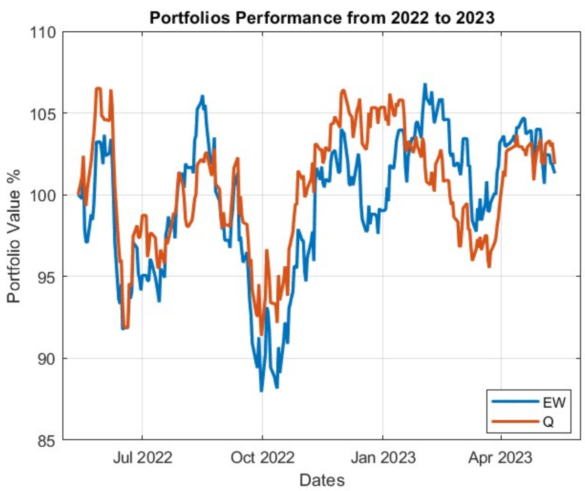
There are no remarkable differences between portfolio M, N and the EW one. Even if portfolio M accumulated a decent return in the second half of the time window, it concluded with a total annualized return close to the benchmark, since it lost most of its profits in the last part of the time horizon. On top of that, DrawDowns are very significant, especially in the first half of the dates set. However, also in this case it is evident the higher diversification of the portfolio M compared to the others.

9.6. EX6



As discussed in the previous section, we did not expect an outstanding performance of the PCA portfolio compared to the equally weighted one, due to the volatility restriction. Indeed, as we can infer from the performance metrics table, there is no evidence to say that portfolio P outperformed portfolio EW, but at least it has a lower annualized volatility.

9.7. EX7



Portfolio Q outperformed the equally weighted portfolio in a larger portion of out of sample dates, with on average: higher return, lower volatility, and a lower max drawdown. This proves the efficacy of using the Expected Shortfall Modified Sharpe Ratio to optimise the weightings.