Design of advanced control methods for space applications: harmonic control

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About myself

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- Researcher at DAER, Aerospace Systems & Control Laboratory – Prof. M. Lovera.

Education

- Master in space engineering (“modeling flexible
- Ph.D. in aerospace engineering with focus on nonlinear control theory («Geometric tracking control for underactuated and thrust-vectoring UAVs»)

Areas of research:

- Nonlinear control of underactuated UAVs
- Anti-windup compensator design
- Adaptive control

Plan of the activity

During the next four lectures we will cover the following application examples

- <u>Harmonic control for spacecraft with large rotating payloads</u>
- Detumbling control design for underactuated spacecraft
- Attitude tracking for inertially pointing spacecraft (SISO)
- Attitude tracking for nadir pointing momentum bias spacecraft (MIMO)

Motivation

In recent years observations about climate change in the Arctic regions has lead to some worrying conclusions:

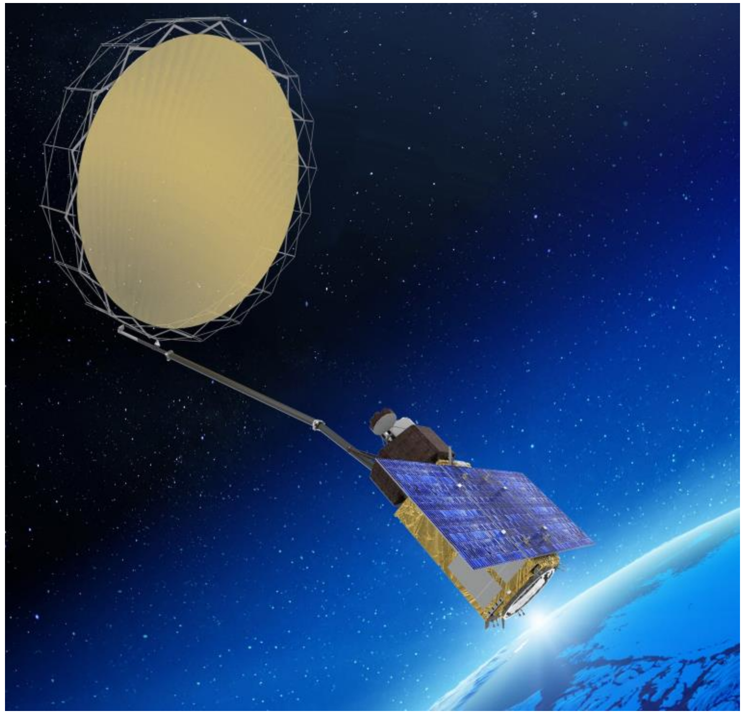
- Industrial activities have lead to the increase of greenhouse gasses that, in turn, contributed to the Arctic temperature rise;
- The melting of snow and ice cover causes a positive feedback loop by realising the endemic Arctic greenhouse gasses and by decreasing the surface albedo.

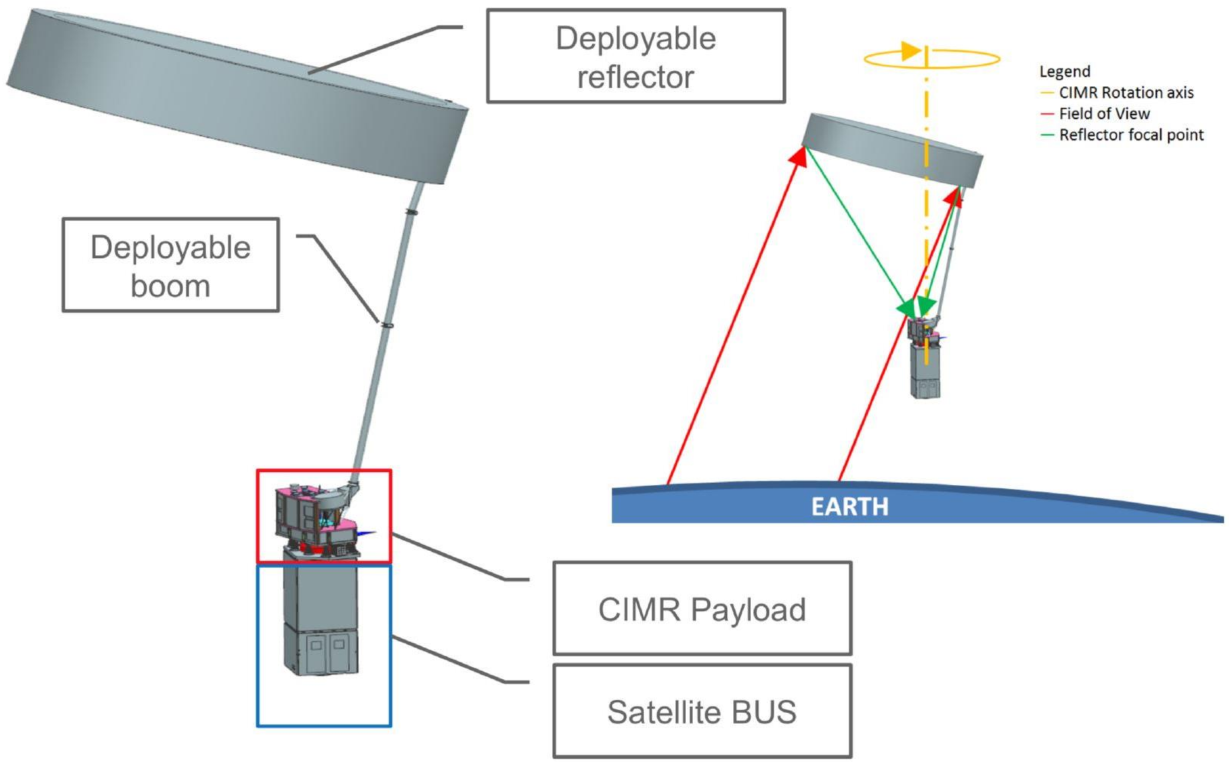
Consequently, it became part of the European Union climate efforts to develop strategies to safeguard the Arctic environment.

It is for this reason that the *Copernicus Imaging Microwave Radiometer* (CIMR) mission has been designed.

Introduction

The CIMR observatory is a dual-spinning, 3-axis stabilized spacecraft required to maintain a **nadir pointing** attitude while its payload assembly rotates at about 8 rpm.





- Altitude of about 820 km
- Inclination of about 98.7 deg
- Orbital period of about 6070 s

Introduction

Controlling such systems becomes particularly challenging when considering the presence of unavoidable inertial asymmetries in the rotating assembly.

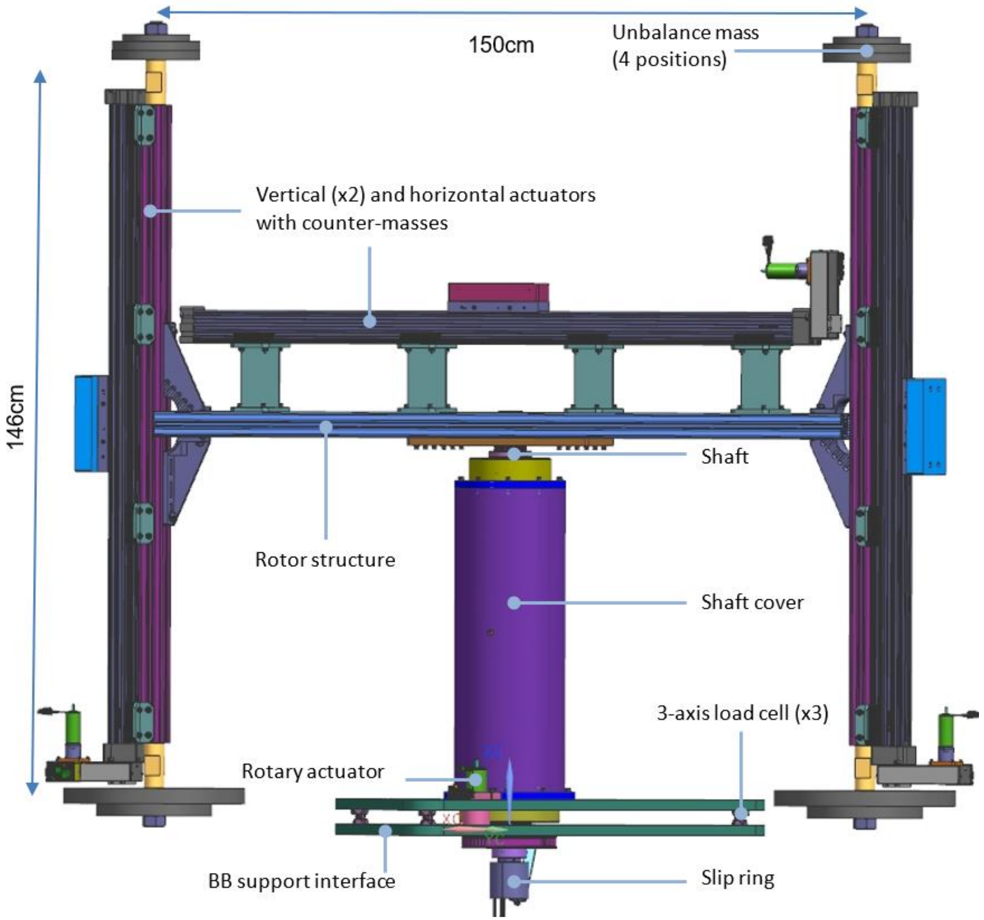
Active Balancing System (ABS) has been developed to compensate for inertial unbalances in the rotor.

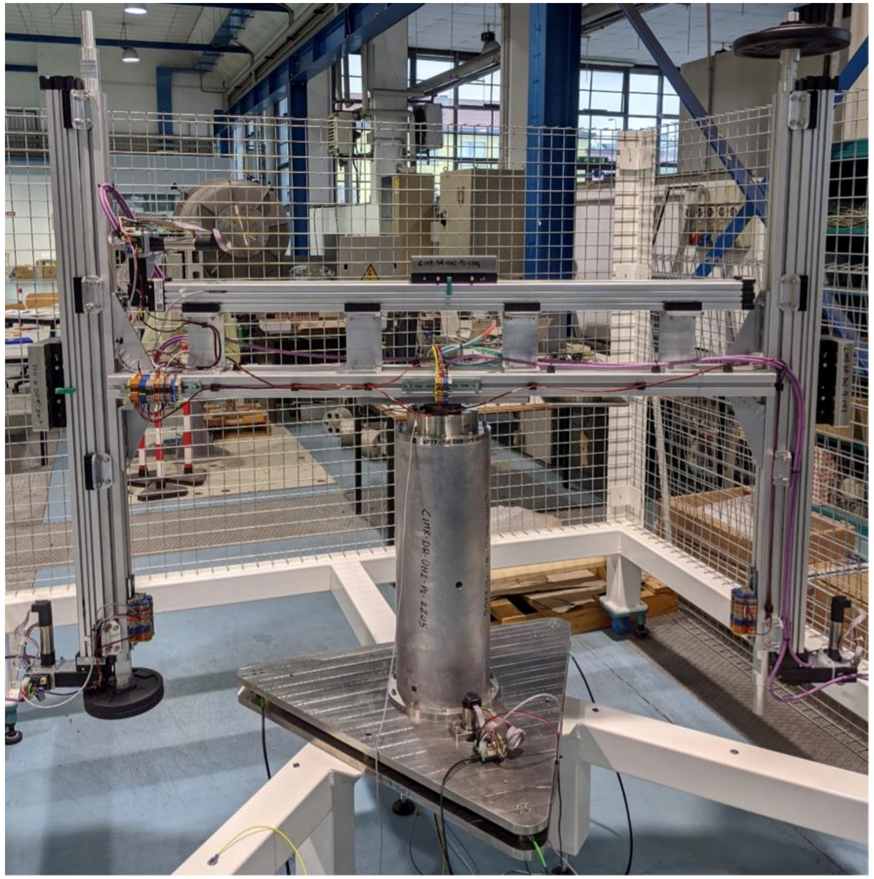
In particular, the ABS is composed of:

- N_m drive units, each of them composed by linear guides, balancing masses and linear actuators;
- a system of load cells to measure the forces and torques at the spacecraft rotor-bus interface;
- an electronic unit able to process data from the load cells and to command the actuators moving the balancing masses.

Breadboard

Breadboad developed by OHB Italy to validate the ABS concept on a simplified platform



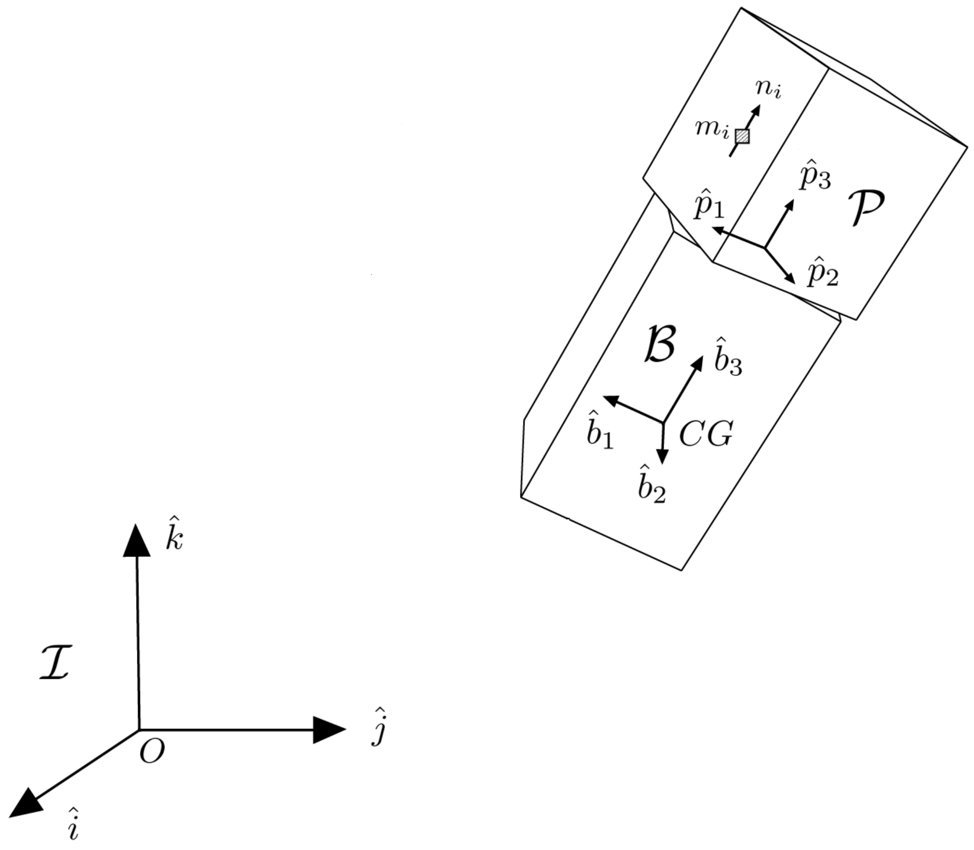


The multi-body spacecraft configuration is described by:

$$(R, x, q, s) \in SO(3) \times \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{R}^{N_m}$$

for a total of $7 + N_m$ degrees of freedoms.

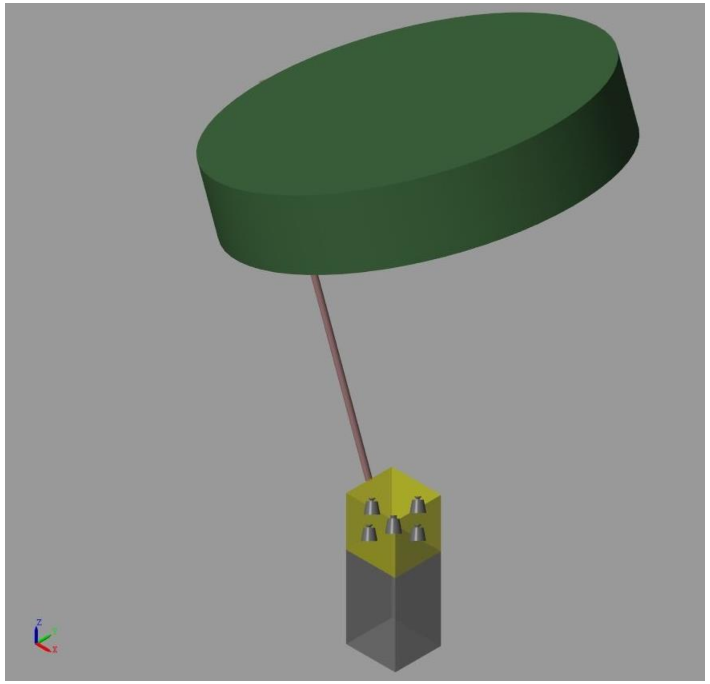
- The payload rotation (q) is imposed (constant rate);
- The motion of the balancing masses (s) is imposed by (position-controlled) linear actuators.



The Euler-Lagrange equations can be rearranged as follows:

$$\begin{bmatrix} M_{xx} & M_{xR}(R,Q,s) \\ M_{Rx}(R,Q,s) & M_{RR}(Q,s) \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \dot{\Omega} \end{Bmatrix} = \begin{Bmatrix} f_x(x,\Omega,\omega,R,Q,s) \\ f_R(x,\dot{x},\Omega,\omega,R,Q,s,\tau_c) \end{Bmatrix}$$

- The control torque τ_c is provided by a set of torque actuators on the spacecraft base.
- A modular simulator has been designed in Simscape to verify the accuracy of the analytical model and allow a systematic development of a more complex model (actuators and sensor dynamics, flexibility, etc.).



The first and second mass moment of inertia of the payload assembly are defined as

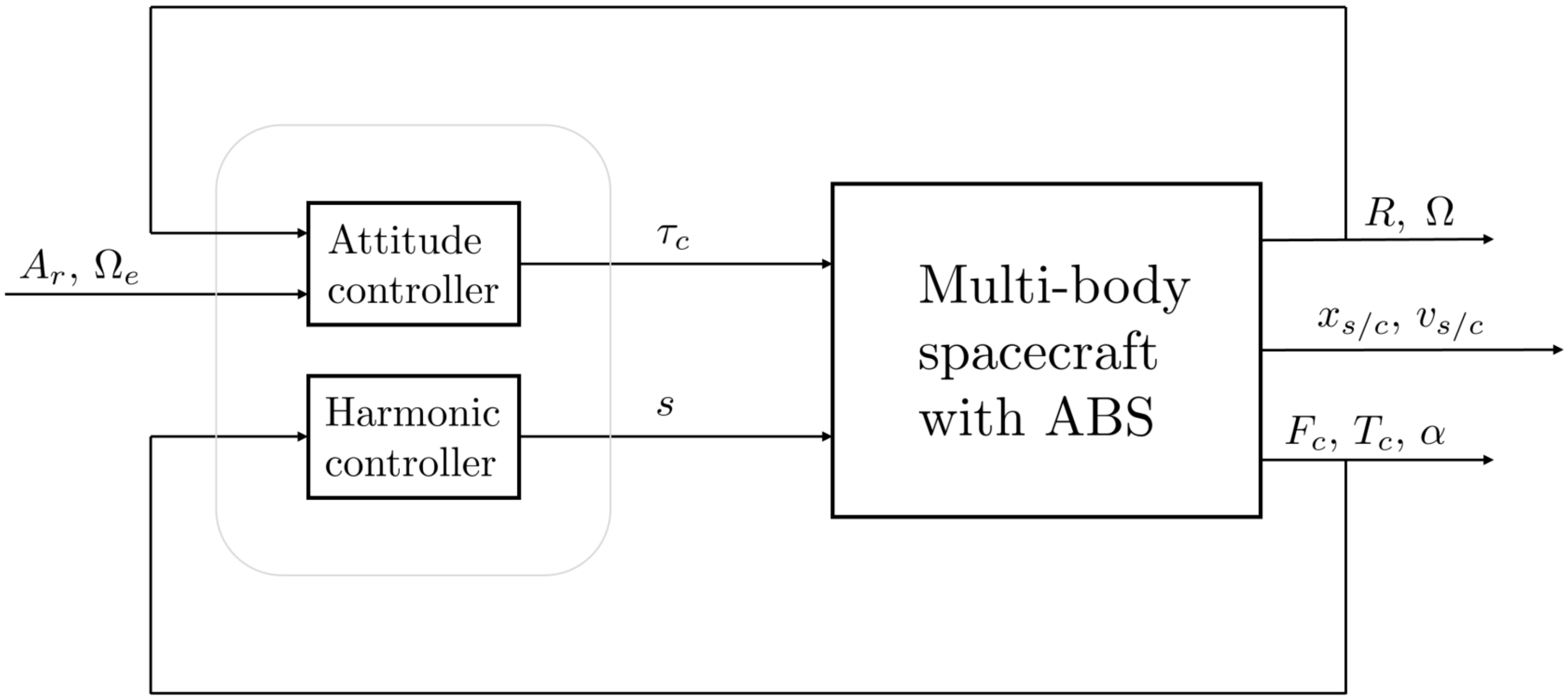
$$I_p(s) = \int_{\beta_p} \xi dm(\xi) + \sum_{i=1}^{N_m} \xi_i(s_i) m_i$$

$$J_p(s) = \int_{\beta_p} S(\xi)^T S(\xi) dm(\xi) + \sum_{i=1}^{N_m} S(\xi_i(s_i))^T S(\xi_i(s_i)) m_i$$

- the displacements of the ABS masses (s) can be used to modify the inertial properties of the payload and therefore to cancel unbalances.

Decoupled control architecture:

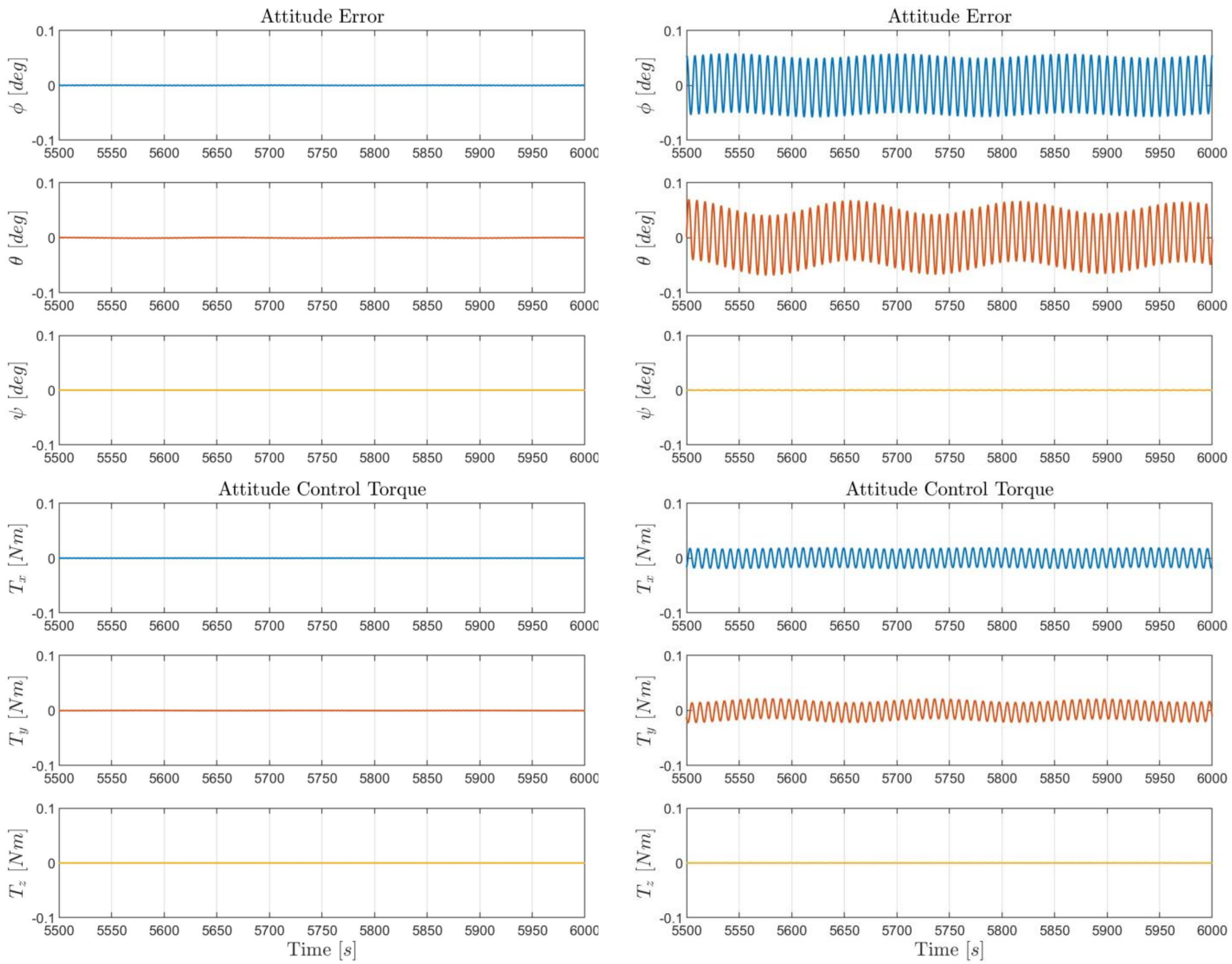
- the control torque τ_c is used for attitude control (nadir pointing);
- the displacement of the ABS masses (s) are used for rotor balancing.



Multibody Model: Controllers – Attitude

Simulation Results for Attitude Control

- Input data
  - Already at Nadir Pointing
  - Payload assembly at nominal velocity
  - Reaction Wheel included
- Differences
  - ***Left***: ABS masses at ideal position
  - ***Right***: ABS masses at rest position



Modeling for rotor balancing control

Assumption: the attitude control system is capable of keeping the desired attitude (the rotor is fixed to the ground)

Newton-Euler eqs.:

$$\tau_O^{s/c} = \frac{dh}{dt}, \qquad f_O^{s/c} = \frac{dq}{dt}$$

$$h := \mathcal{R}^\top J^r \omega + \sum_{i=1}^{N_m} r_i \times m_i v_i = \mathcal{R}^\top \left(J^r \omega + \sum_{i=1}^{N_m} m_i S(\bar{r}_i + s_i n_i) S^\top (\bar{r}_i + s_i n_i)\omega + \sum_{i=1}^{N_m} m_i S(\bar{r}_i) n_i \dot{s}_i \right)$$

$$q := m^r v_G + \sum_{i=1}^{N_m} m_i \dot{r}_i m^r \mathcal{R}^\top \omega \times r_G^r + R^\top \left(\sum_{i=1}^{N_m} m_i \dot{s}_i n_i + S(\omega)(\bar{r}_i + s_i n_i) \right)$$

In-plane components of the interface force

$$\begin{bmatrix} f_{O_1}^{s/c} \\ f_{O_2}^{s/c} \\ \tau_{O_1}^{s/c} \\ \tau_{O_2}^{s/c} \end{bmatrix} = \begin{bmatrix} e_1^\top f_{O_1}^{s/c} \\ e_2^\top f_{O_2}^{s/c} \\ e_1^\top \tau_{O_1}^{s/c} \\ e_2^\top \tau_{O_2}^{s/c} \end{bmatrix} = Z(t) \begin{bmatrix} f_{O_1}^r \\ f_{O_2}^r \\ \tau_{O_1}^r \\ \tau_{O_2}^r \end{bmatrix}, \qquad Z(t) := \mathrm{blkdiag}\left(\begin{bmatrix} \cos(\theta(t)) & -\sin(\theta(t)) \\ \sin(\theta(t)) & \cos(\theta(t)) \end{bmatrix}, \begin{bmatrix} \cos(\theta(t)) & -\sin(\theta(t)) \\ \sin(\theta(t)) & \cos(\theta(t)) \end{bmatrix} \right)$$

Interface forces (rotor frame)

$$f^r_{O_1} = -\dot{\theta}^2 \underline{\left(m^r x^r_G + \sum_{i=1}^{N_m} m_i(\bar{x}_i + s_i e_1^\top n_i) \right)} + 2\dot{\theta} \sum_{i=1}^{N_m} m_i e_2^\top n_i \dot{s}_i + \sum_{i=1}^{N_m} m_i e_1^\top n_i \ddot{s}_i \tag{1}$$

$$f^r_{O_2} = -\dot{\theta}^2 \underline{\left(m^r y^r_G + \sum_{i=1}^{N_m} m_i(\bar{y}_i + s_i e_2^\top n_i) \right)} - 2\dot{\theta} \sum_{i=1}^{N_m} m_i e_1^\top n_i \dot{s}_i + \sum_{i=1}^{N_m} m_i e_2^\top n_i \ddot{s}_i \tag{2}$$

$$\begin{aligned} \tau^r_{O_1} = {} & -\dot{\theta}^2 \underline{\left(J^r_{23} - \sum_{i=1}^{N_m} m_i(\bar{y}_i + s_i e_2^\top n_i)(\bar{z}_i + s_i e_3^\top n_i) \right)} + \sum_{i=1}^{N_m} m_i \begin{bmatrix} 0 & -\bar{z}_i & \bar{y}_i \end{bmatrix} n_i \ddot{s}_i + \dot{\theta} \sum_{i=1}^{N_m} m_i \Big(\begin{bmatrix} -\bar{z}_i & 0 & \bar{x}_i \end{bmatrix} n_i + \\ & + e_1^\top S(n_i) \begin{bmatrix} -\bar{y}_i - s_i e_2^\top n_i & \bar{x}_i + s_i e_1^\top n_i & 0 \end{bmatrix}^\top + \begin{bmatrix} 0 & -\bar{z}_i - s_i e_3^\top n_i & \bar{y}_i + s_i e_2^\top n_i \end{bmatrix} S^\top(n_i) e_3 \Big) \dot{s}_i \end{aligned} \tag{3}$$

$$\begin{aligned} \tau^r_{O_2} = {} & \dot{\theta}^2 \underline{\left(J^r_{13} - \sum_{i=1}^{N_m} m_i(\bar{x}_i + s_i e_1^\top n_i)(\bar{z}_i + s_i e_3^\top n_i) \right)} + \sum_{i=1}^{N_m} m_i \begin{bmatrix} \bar{z}_i & 0 & -\bar{x}_i \end{bmatrix} n_i \ddot{s}_i + \dot{\theta} \sum_{i=1}^{N_m} m_i \Big(\begin{bmatrix} 0 & -\bar{z}_i & \bar{y}_i \end{bmatrix} n_i + \\ & + e_2^\top S(n_i) \begin{bmatrix} -\bar{y}_i - s_i e_2^\top n_i & \bar{x}_i + s_i e_1^\top n_i & 0 \end{bmatrix}^\top + \begin{bmatrix} \bar{z}_i + s_i e_3^\top n_i & 0 & -\bar{x}_i - s_i e_1^\top n_i \end{bmatrix} S^\top(n_i) e_3 \Big) \dot{s}_i. \end{aligned} \tag{4}$$

Problem statement

For constant perturbations, balanced equilibrium conditions can be obtained provided that there is a sufficient number of (suitably) placed balancing masses.

Specifically, a basic requirement is that the system of equations

$$
\begin{aligned}
&\sum_{i=1}^{M} m_i e_1^T n_i s_i = -\Delta S_1 \qquad && \sum_{i=1}^{M} m_i \left((e_2^T n_i)(e_3^T n_i) s_i^2 \bar{y}_i (e_3^T n_i) + \bar{z}_i (e_2^T n_i) s_i\right) = \Delta J_{23} \\
&\sum_{i=1}^{M} m_i e_2^T n_i s_i = -\Delta S_2 \qquad && \sum_{i=1}^{M} m_i \left((e_1^T n_i)(e_3^T n_i) s_i^2 \bar{x}_i (e_3^T n_i) + \bar{z}_i (e_1^T n_i) s_i\right) = \Delta J_{13},
\end{aligned}
$$

admits at least one solution.

In the following we will assume such conditions to hold.

Problem statement

Assuming linear dynamics for

- the servo-actuators moving the balancing masses (state x_a)
- the sensors measuring joint force and torque (state x_s)

a complete model for the system can be written as

$$\dot{x} = A(t)x + B_u(t)u + B_d(t)d \qquad y = \begin{bmatrix} 0 & C_s \end{bmatrix} x$$

where

$$x = \begin{bmatrix} x_a & x_s \end{bmatrix}^T \qquad d = \begin{bmatrix} \Delta S_1 & \Delta S_2 & \Delta J_{13} & \Delta J_{23} \end{bmatrix}^T$$

and

$$A(t) = \begin{bmatrix} A_a & 0 \\ B_s \bar{R}(\theta(t)) C_{abs} C_a & A_s \end{bmatrix} \quad B_u(t) = \begin{bmatrix} B_a \\ B_s \bar{R}(\theta(t)) C_{abs} D_a \end{bmatrix} \quad B_d(t) = \begin{bmatrix} 0 \\ B_s \bar{R}(\theta(t)) D_{abs} \end{bmatrix}$$

Computation of the *T* matrix

- In the case under study the dynamics is time-periodic (actuators in the rotating frame, sensors in the fixed-frame).

- Assuming linearity, the dynamics is then given by the linear time-periodic (LTP) model

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \qquad y(t) = C(t)x(t) + D(t)u(t)$$

- Therefore, the so-called Harmonic Transfer Function (HTF) of LTP systems can be used

Computation of the T matrix

The HTF is defined as $$s\mathcal{X} = (\mathcal{A}-\mathcal{N})\mathcal{X} + \mathcal{B}\mathcal{U}, \qquad \mathcal{Y} = \mathcal{C}\mathcal{X} + \mathcal{D}\mathcal{U}$$

where $$\mathcal{G}(s) = \mathcal{C}[s\mathcal{I} - (\mathcal{A}-\mathcal{N})]^{-1}\mathcal{B} + \mathcal{D}$$

$$x(t) = e^{st}\sum_{n=-\infty}^{\infty} x_n e^{jn\Omega t} \quad \Rightarrow \quad \mathcal{X} = \begin{bmatrix} \vdots \\ x_{-2} \\ x_{-1} \\ x_0 \\ x_1 \\ x_2 \\ \vdots \end{bmatrix}$$

$$A(t) = \sum_{m=-\infty}^{\infty} A_m e^{jm\Omega t} \quad \Rightarrow \quad \mathcal{A} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & A_0 & A_{-1} & A_{-2} & A_{-3} & A_{-4} & \cdots \\ \cdots & A_1 & A_0 & A_{-1} & A_{-2} & A_{-3} & \cdots \\ \cdots & A_2 & A_1 & A_0 & A_{-1} & A_{-2} & \cdots \\ \cdots & A_3 & A_2 & A_1 & A_0 & A_{-1} & \cdots \\ \cdots & A_4 & A_3 & A_2 & A_1 & A_0 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Computation of the T matrix

For constant inputs, $u(t) = u_0$, *i.e.*, $\mathcal{U}^T = [\cdots\ 0\ 0\ u_0^T\ 0\ 0\ \cdots]$. the Harmonic Transfer function matrix (HTM) is derived

$$\begin{bmatrix} y_{Nc} \\ y_{Ns} \end{bmatrix} = 2 \begin{bmatrix} \mathrm{Real}[G_{N,0}] \\ \mathrm{Imag}[G_{N,0}] \end{bmatrix} u_0, \qquad T = 2 \begin{bmatrix} \mathrm{Real}[G_{N,0}] \\ \mathrm{Imag}[G_{N,0}] \end{bmatrix}$$

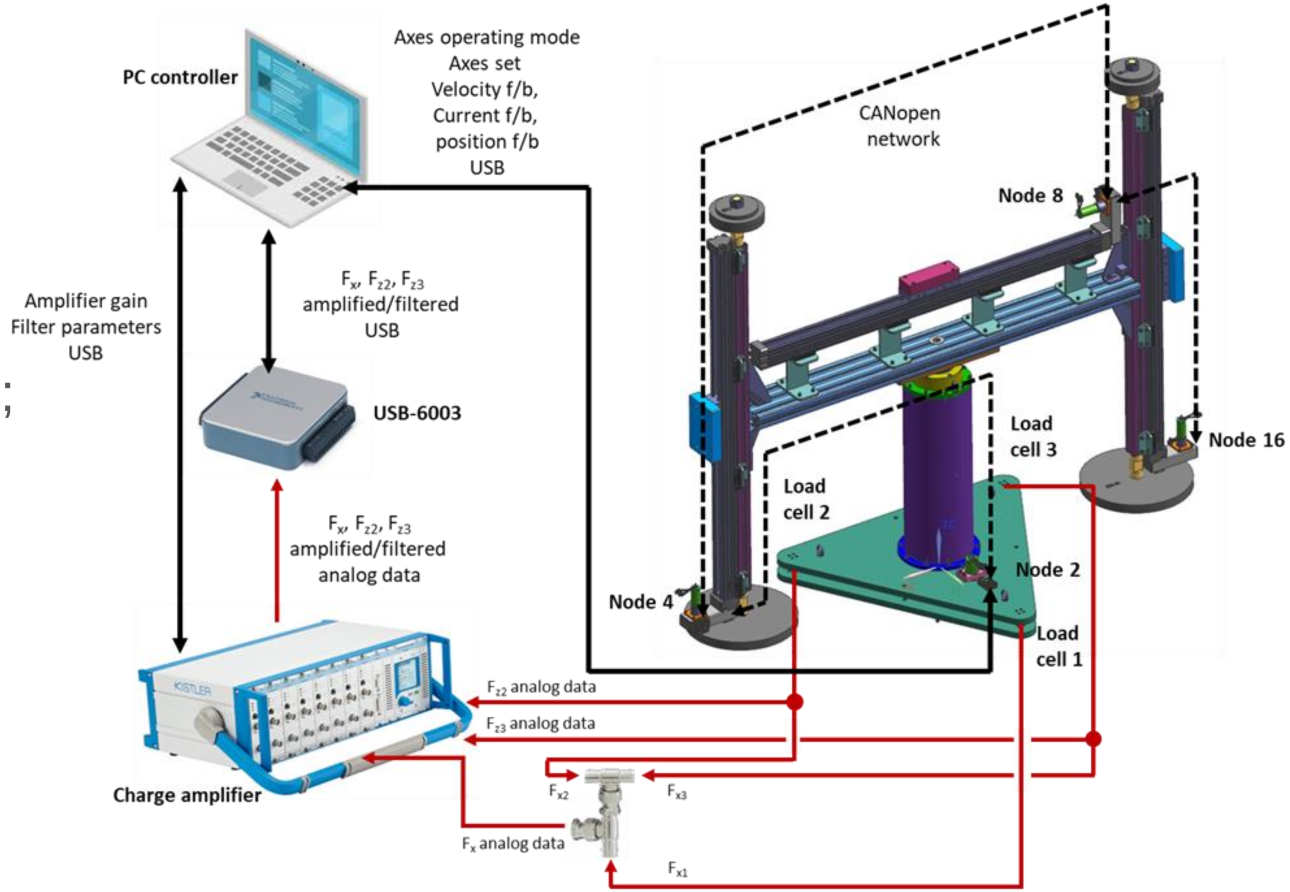
where $G_{N,0} = G_{N,0}$ (s=0) is obtained from the first harmonic of the steady state response

$$\begin{bmatrix} \vdots \\ y_{-2N} \\ y_{-N} \\ y_0 \\ y_N \\ y_{2N} \\ \vdots \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & G_{-2N,-2N} & G_{-2N,-N} & G_{-2N,0} & G_{-2N,N} & G_{-2N,2N} & \cdots \\ \cdots & G_{-N,-2N} & G_{-N,-N} & G_{-N,0} & G_{-N,N} & G_{-N,2N} & \cdots \\ \cdots & G_{0,-2N} & G_{0,-N} & G_{0,0} & G_{0,N} & G_{0,2N} & \cdots \\ \cdots & G_{N,-2N} & G_{N,-N} & G_{N,0} & G_{N,N} & G_{N,2N} & \cdots \\ \cdots & G_{2N,-2N} & G_{2N,-N} & G_{2N,0} & G_{2N,N} & G_{2N,2N} & \cdots \\ \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ 0 \\ 0 \\ u_0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

Breadboard results: platform

Components

- the rotor structure, with the possibility of applying a known unbalance (static and dynamic) through the placement of four masses at the corners;
- three guides, each endowed with a linear actuator, to move the counter-mass;
- three load cell sensors in an equilateral triangle configuration forming a dynamometer;
- the stator structure;
- the rotary actuator, composed of motor, gearbox, and a differential digital encoder to rotate the rotor shaft;
- the slip ring, to guarantee the electrical connection between rotor and stator;
- the power supply, acquisition system, conditioners, and controllers.



Breadboard results: equations of motion

Interface loads

$$f^r_{O_1} = -\dot{\theta}^2 \left(\Delta S_1 + m_1 s_1\right) + m_1 \ddot{s}_1 - f^r_{e_1} \tag{1}$$

$$\tau^r_{O_2} = \dot{\theta}^2 \left(\Delta J^r_{13} - m_1 \bar{z}_1 s_1 - m_2 \bar{x}_2 s_2 - m_3 \bar{x}_3 s_3\right) + \sum_{i=1}^{3} m_i \left[\, \bar{z}_i \;\; 0 \;\; -\bar{x}_i \,\right] n_i \ddot{s}_i - \tau^r_{e_2} \tag{2}$$

$$\Delta S_1 = \textstyle\sum_{i=1}^4 m_i^s \bar{x}_i^s, \; \Delta J^r_{13} = \sum_{i=1}^4 -m_i^s \bar{x}_i^s \bar{z}_i^s$$

Disturbance loads

$$\tau_{orb} = c_1 + c_2 \sin(2\dot{\theta} t)$$

$$\tau^r_g := \sum_{i=1}^{4} m_i^s g x_i + m^r g x^r_G + m_1 g s_1,$$

$$f^{s/c}_{e_1} = A_e \sin(\dot{\theta} t + \phi_e), \text{ with } A_e = 0.21N, \; \phi_e = 1.93 rad$$

Breadboard results: equations of motion

LTP system

$$\dot{x} = A(t)x + B_u(t)u + B_d(t)d \qquad y = \begin{bmatrix} 0 & C_s \end{bmatrix} x$$

where

$$Z(t) = \begin{bmatrix} \cos(\theta(t)) & 0 \\ 0 & \cos(\theta(t)) \end{bmatrix}$$

$$A(t) = \begin{bmatrix} A_a & 0 \\ B_s Z(t) C_{abs} C_a & A_s \end{bmatrix}, \quad B_u(t) = \begin{bmatrix} B_a \\ B_s Z(t) C_{abs} D_a \end{bmatrix},$$

$$B_d(t) = \begin{bmatrix} 0 \\ B_s Z(t) D_{abs} \end{bmatrix}.$$

$$C^1_{abs} = m_1 \begin{bmatrix} -\dot{\theta}^2 & 0 & 1 \\ -\dot{\theta}^2 \bar{z}_1 & 0 & \bar{z}_1 \end{bmatrix}, \quad C^2_{abs} = -m_2 \begin{bmatrix} 0 & 0 & 1 \\ \dot{\theta}^2 \bar{x}_2 & 0 & \bar{x}_2 \end{bmatrix},$$

$$C^2_{abs} = -m_3 \begin{bmatrix} 0 & 0 & 0 \\ \dot{\theta}^2 \bar{x}_3 & 0 & \bar{x}_3 \end{bmatrix}, \quad D_{abs} := \begin{bmatrix} -\dot{\theta}^2 & 0 \\ 0 & \dot{\theta}^2 \end{bmatrix}.$$

Harmonic transfer functions

$$\mathcal{G}_u(s) = \mathcal{C}[s\mathcal{I} - (\mathcal{A} - \mathcal{N})]^{-1}\mathcal{B}_u,$$

$$\mathcal{G}_d(s) = \mathcal{C}[s\mathcal{I} - (\mathcal{A} - \mathcal{N})]^{-1}\mathcal{B}_d,$$

LTP truncated model

$$\mathcal{A} = \begin{bmatrix} A_0 & A_{-1} & 0 \\ A_1 & A_0 & A_{-1} \\ 0 & A_1 & A_0 \end{bmatrix} \qquad \mathcal{B}_u = \begin{bmatrix} B_{u_0} & 0 & 0 \\ 0 & B_{u_0} & 0 \\ 0 & 0 & B_{u_0} \end{bmatrix}$$

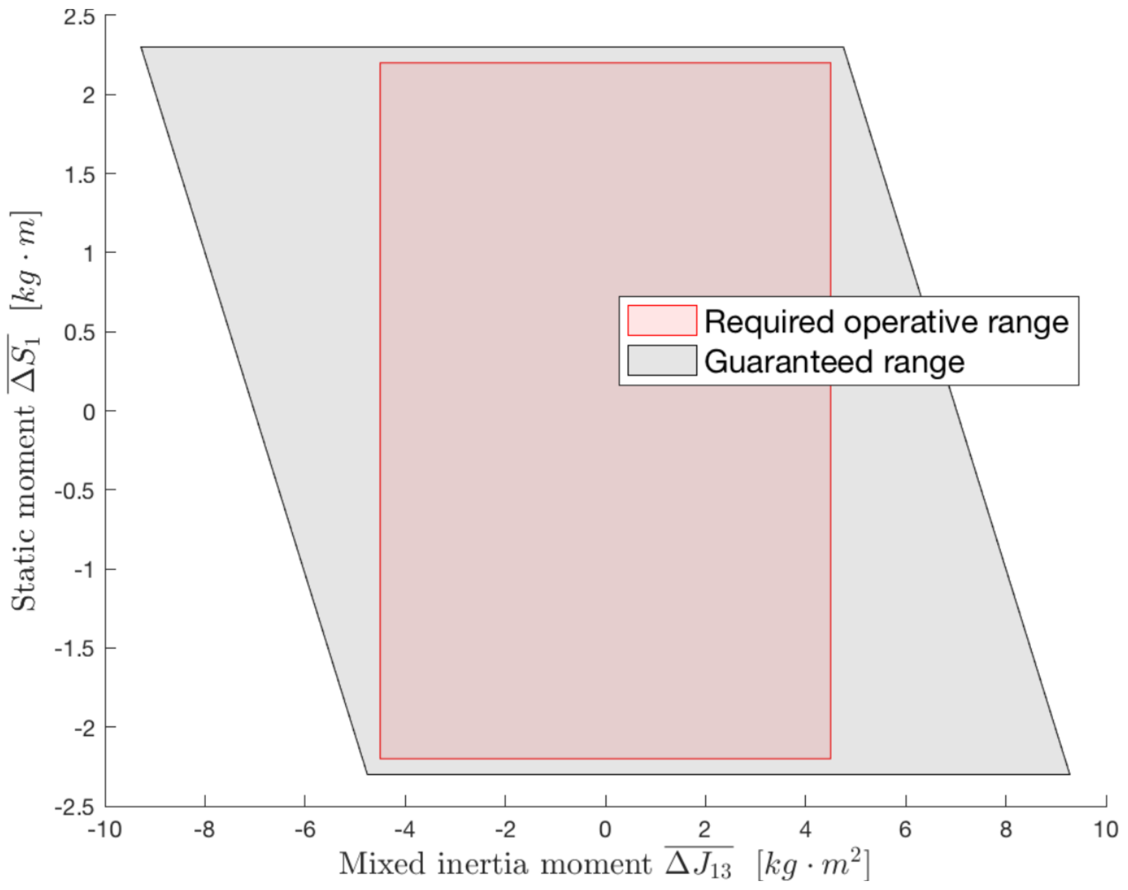
$$\mathcal{B}_d = \begin{bmatrix} B_{d_0} & B_{d_{-1}} & 0 \\ B_{d_1} & B_{d_0} & B_{d_{-1}} \\ 0 & B_{d_1} & B_{d_0} \end{bmatrix} \qquad \mathcal{C} = \begin{bmatrix} C_0 & 0 & 0 \\ 0 & C_0 & 0 \\ 0 & 0 & C_0 \end{bmatrix}$$

Breadboard results: ABS sizing capabilities

The ABS is well defined since the system

$$\begin{bmatrix} m_1 & 0 & 0 \\ m_1\bar{z}_1 & m_2\bar{x}_2 & m_3\bar{x}_3 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} -\Delta S_1 \\ \Delta J_{13} \end{bmatrix},$$

admits solutions for admissible unbalances.



The Harmonic control law is

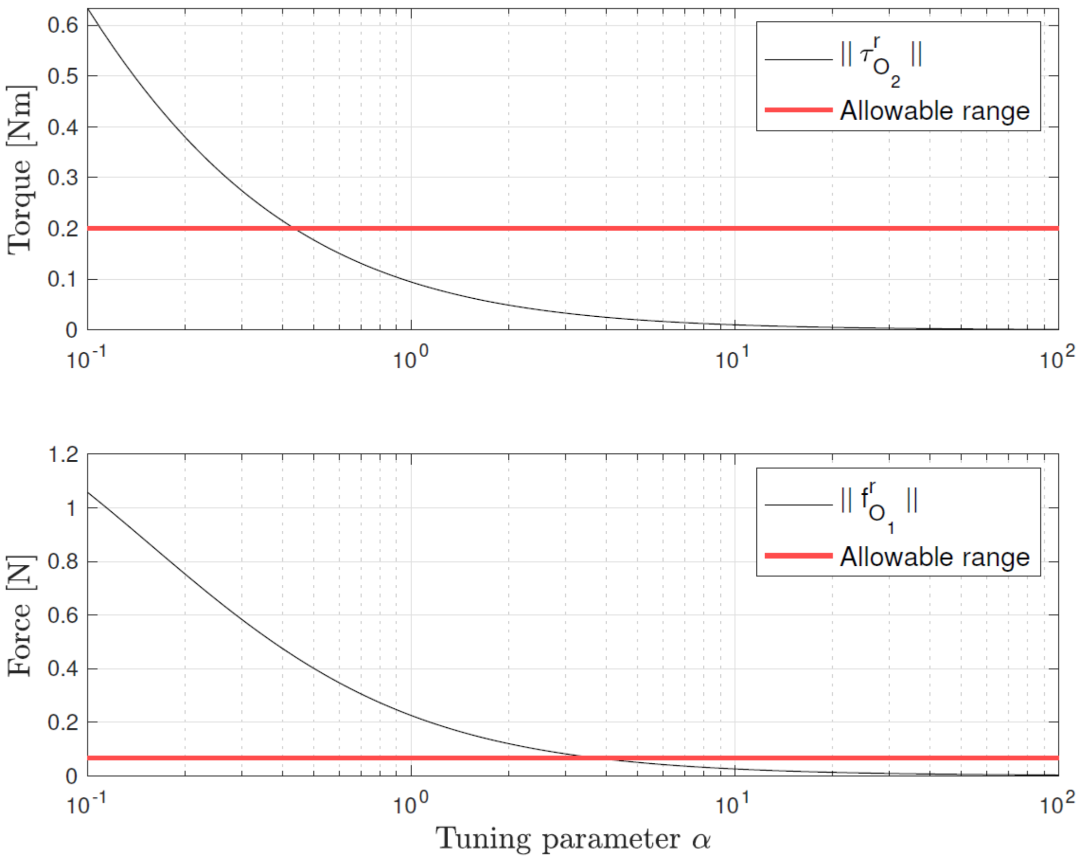
$$u(k+1) = KTu(k) - Ky_N(k)$$

where $K = (T^\top QT + R)^{-1}T^\top Q$

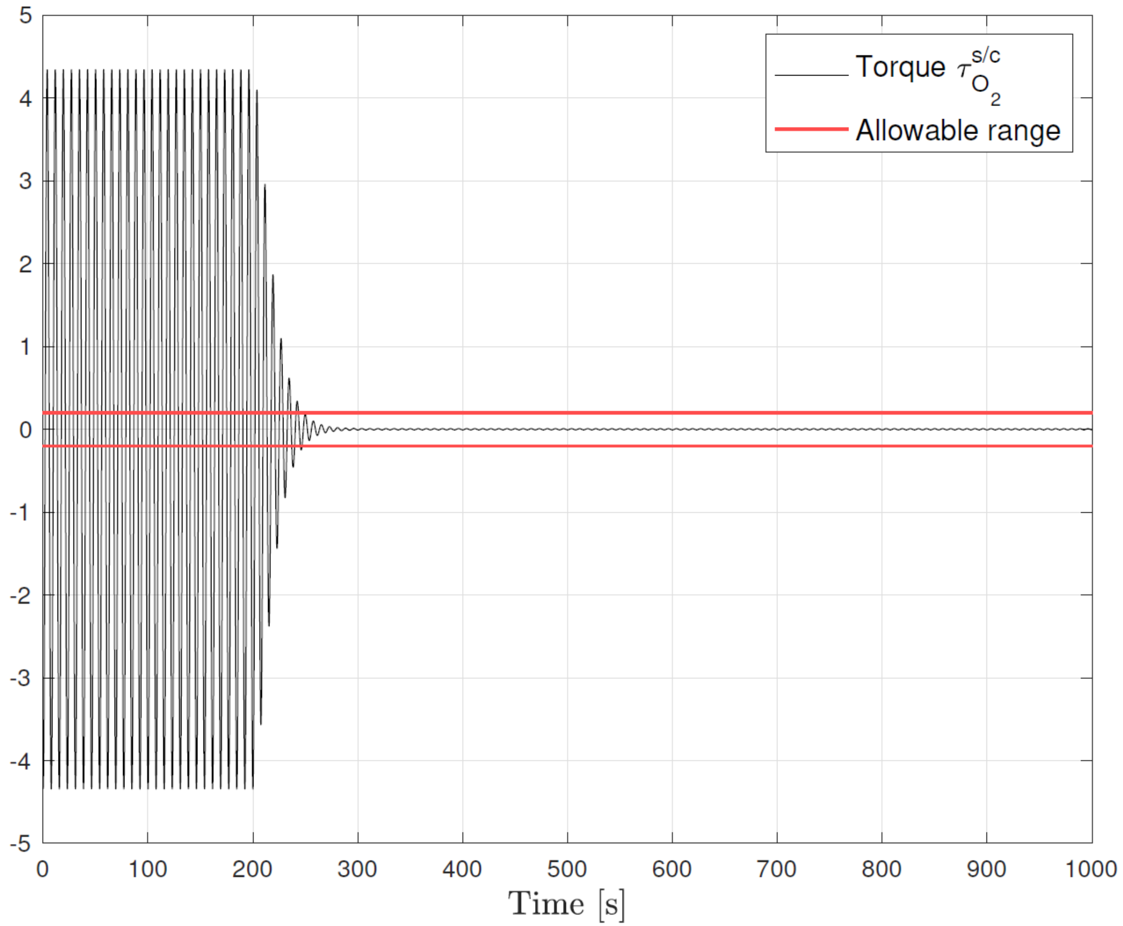
is the gain matrix.

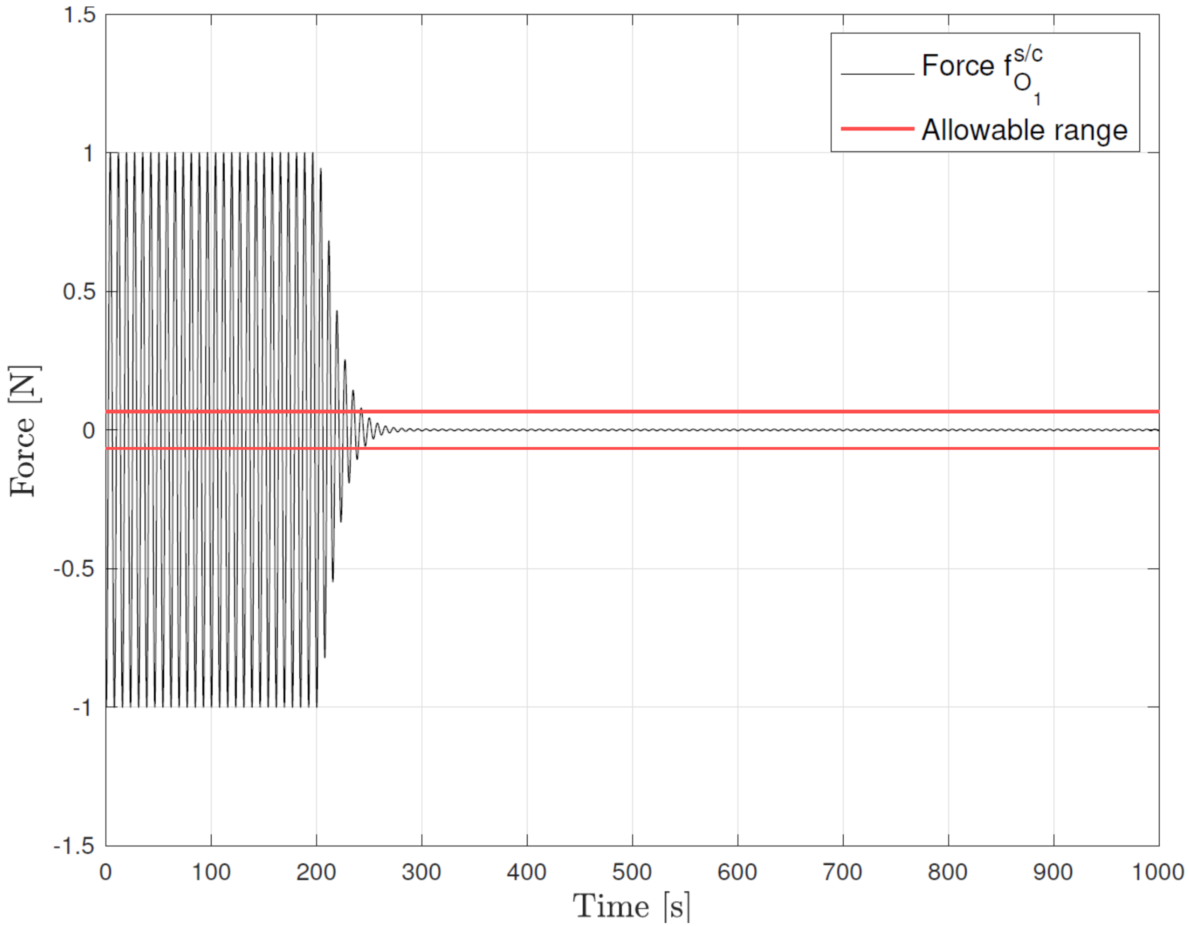
Tuning trade-off between robustness and performance

$$R = I_3, \qquad Q = \alpha I_3$$



Breadboard: results





Multibody spacecraft: control design

- **Attitude controller** based on a non-linear PD-like control law of the form

$$\tau_c = -k_R \gamma_R(A_r) - K_\Omega \Omega_e$$

- **Active Balancing System** control based on a harmonic controller
  - Harmonic analysis (online Fast Fourier Transform) of the interface loads;
  - LQ-based discrete time control law

$$s(k+1) = s(k) - K y_N(k)$$

$$K = \left(T^T \bar{Q} T + \bar{R}\right)^{-1} T^T \bar{Q}$$

Assumption: the harmonic control is updated slowly enough that transient dynamics of the ABS masses have a minimal impact on the system dynamics.

Multibody spacecraft: control design

- If the interface torque $\tau_O \neq 0$ (unbalanced rotor), the proposed attitude controller guarantees boundedness of the attitude states;

- If the inertial asymmetries of the payload assembly are balanced out then, even if the rotor keeps a significant angular velocity, τ_O will converge to zero;
  - In this condition

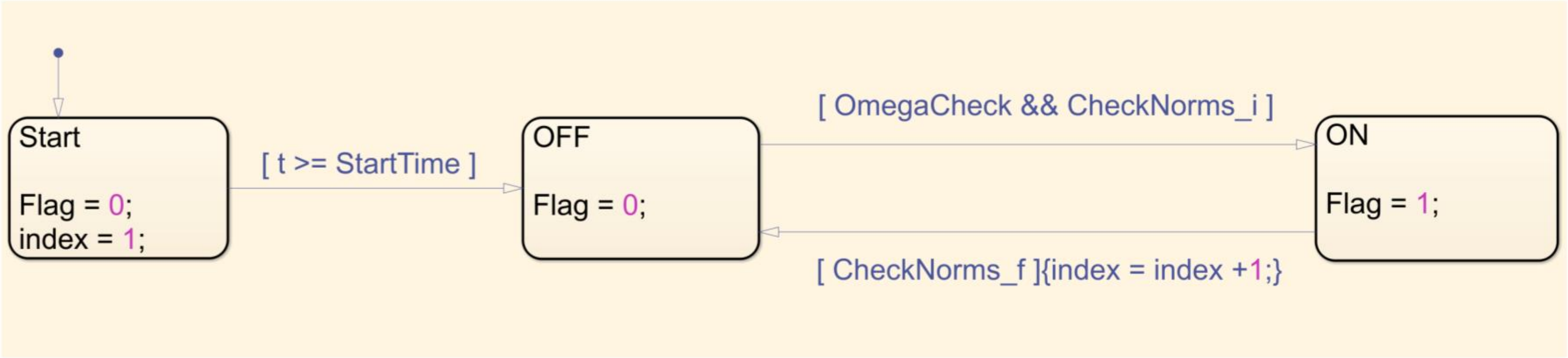
Multibody spacecraft: control design

The harmonic controller works only when the payload rotation rate is constant.

- To spin-up the rotor while keeping under control the interface loads, we propose a procedure consisting several accelerations steps separated by balancing phases.

The control logic has been implemented using the state-flow environment in Simulink:

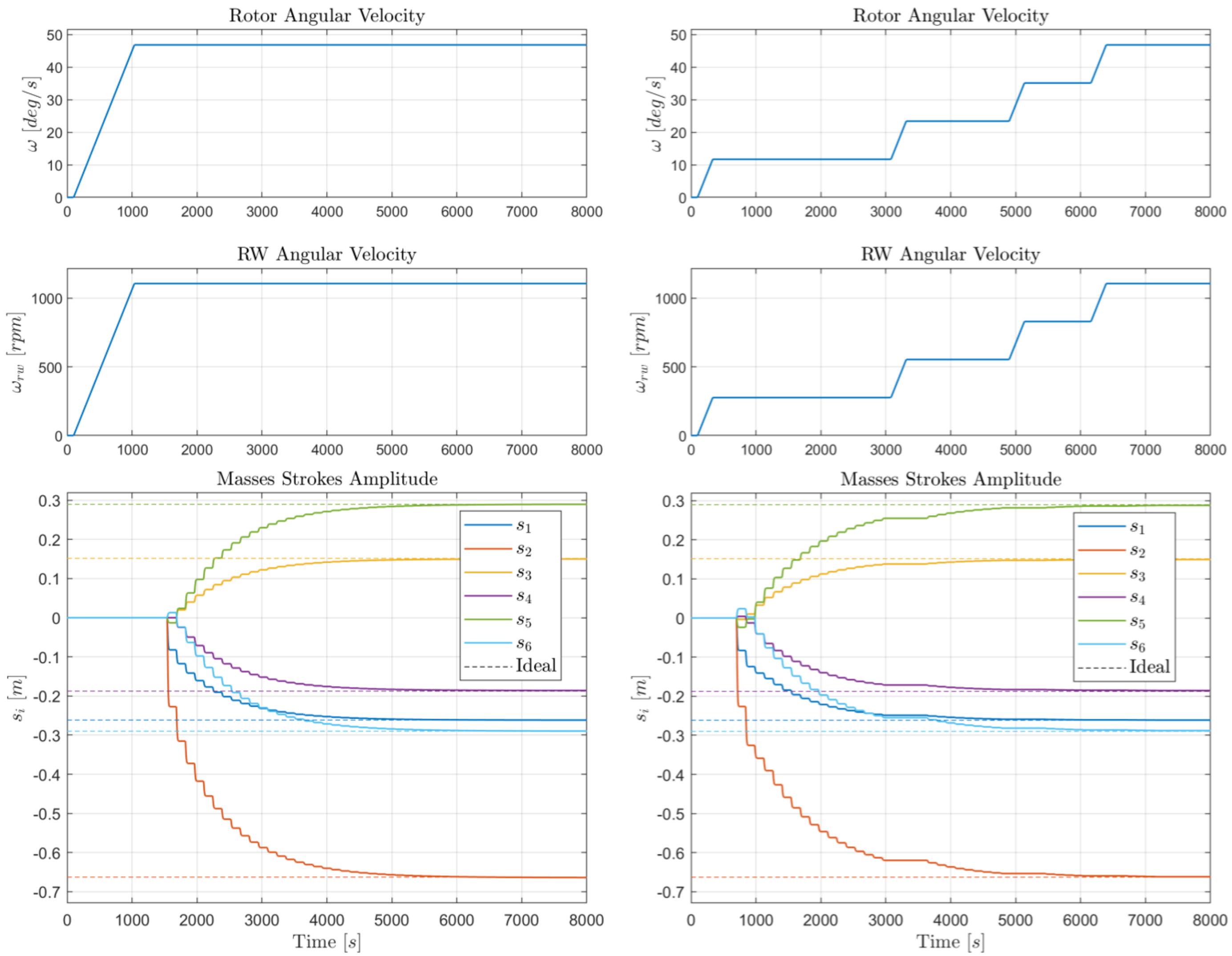
- The rotor acceleration is constant and applied only when the interface loads are under a given threshold;
- The ABS masses are updated only when the rotor acceleration is paused;



Multibody spacecraft: simulation results

Spin-up procedure

- Input data
  - Near nadir pointing initial attitude
  - ABS masses starting from rest pos.
- Main orbital parameters
  - Altitude of about 820 km
  - Inclination of about 98.7 deg
  - Orbital period of about 6070 s
- Differences
  - ***Left***: Single spin-up procedure
  - ***Right***: Multiple spin-up procedure

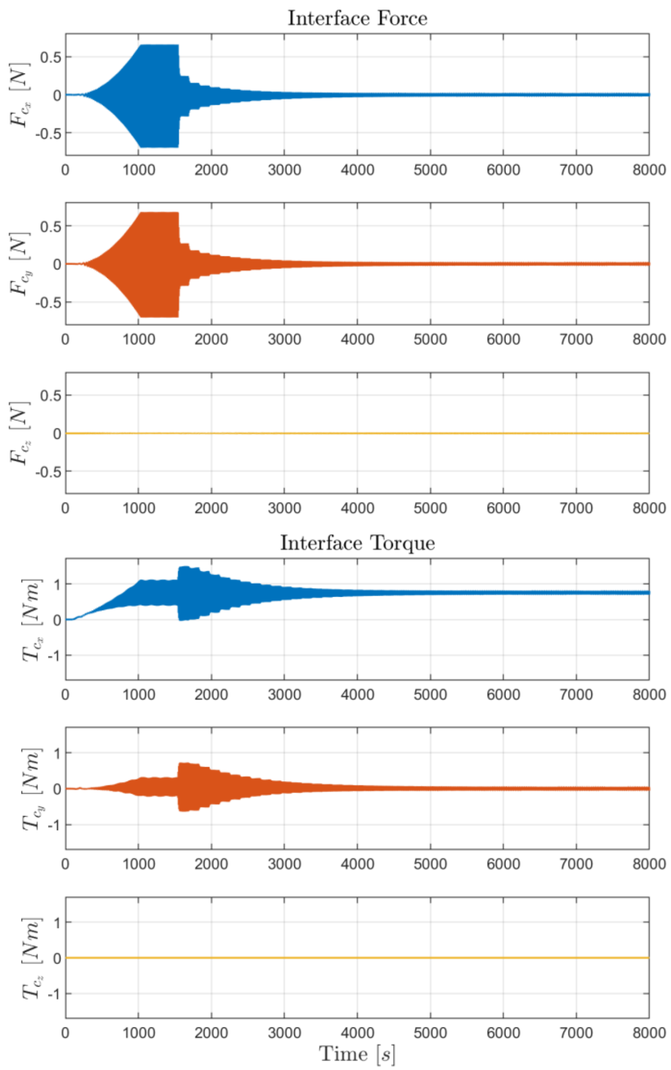


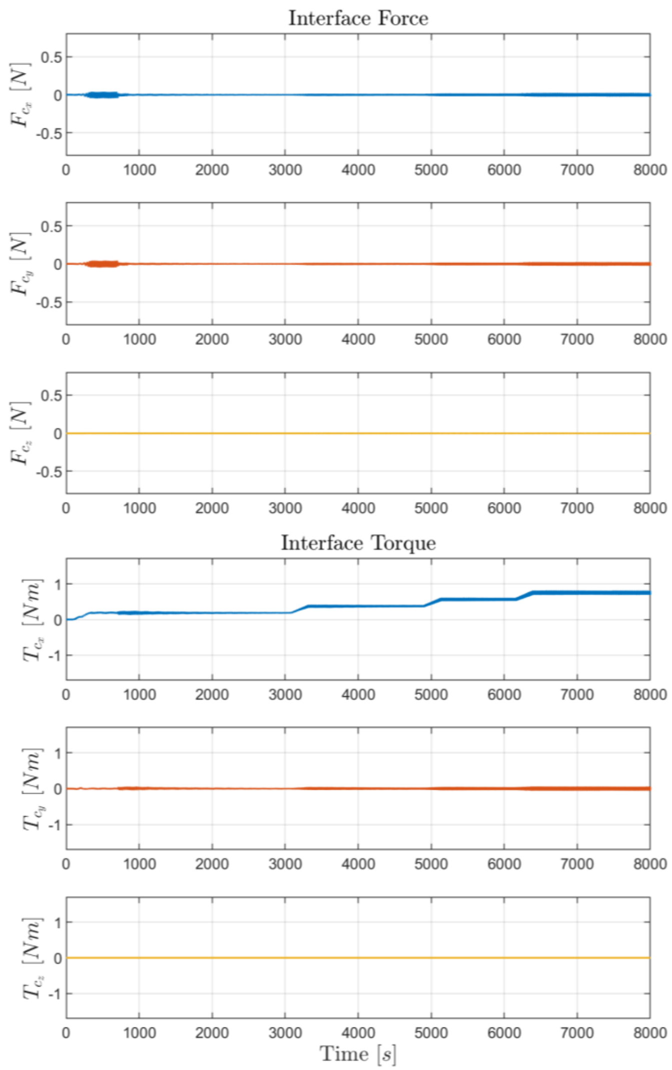
Multibody spacecraft: simulation results

Interface loads

- Input data
  - Near nadir pointing initial attitude
  - ABS masses starting from rest pos.
- Differences
  - ***Left***: <u>Single</u> spin-up procedure
  - ***Right***: <u>Multiple</u> spin-up procedure

	Single Step	Safe Start Up	Difference
Max Abs. Force	0.7032 N	0.0476 N	- 93.23%
Max Abs. Torque	1.4773 Nm	0.8084 Nm	- 45.28%

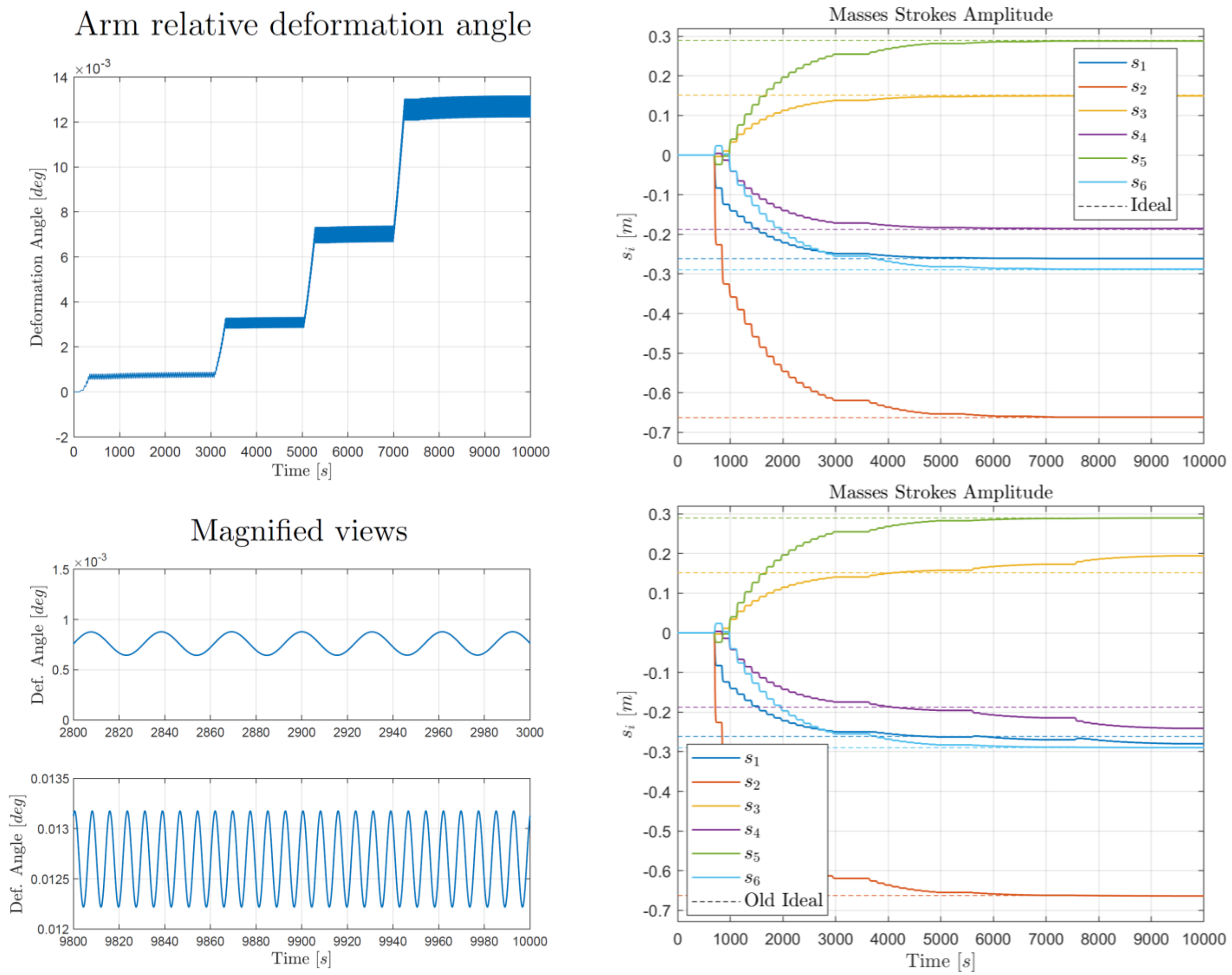




Multibody spacecraft: simulation results

The control design has been carried out under the rigid body assumption.

- The controller performance has been assessed when accounting for flexibility effects, which might play a significant role due to the slenderness of the arm at which the CIMR reflector is fixed.
    - ➢ A simplified lumped model including a torsional spring at the rotor-arm hinge has been considered.



Multibody spacecraft: simulation results

Monte-Carlo simulation to test the robustness of our control architecture to the perturbation of different parameters.

Parameter	Mean value	Standard dev.
Initial angular velocity [rad/s]	$[0, 10^{-3}, 0]$	$0.25 \cdot 10^{-3}$
Initial Euler angles [deg]	$[8.702, 0, -180]$	4.3
Spring stiffness [Nm/rad]	$7.9 \cdot 10^4$	$3 \cdot 10^3$

	Mean Value	Standard Dev.
s_1 [m]	-0.279663	$7.43433 \cdot 10^{-4}$
s_2 [m]	-0.663744	$7.42215 \cdot 10^{-6}$
s_3 [m]	0.194292	$1.76495 \cdot 10^{-3}$
s_4 [m]	-0.240958	$2.18886 \cdot 10^{-3}$
s_5 [m]	0.28969	$5.85404 \cdot 10^{-6}$
s_6 [m]	-0.28969	$5.85405 \cdot 10^{-6}$

RMS error	Mean Value (Steady state)	Standard Dev.
ϕ_e [deg]	$3.0780 \cdot 10^{-3}$	$1.7713 \cdot 10^{-3}$
θ_e [deg]	$2.8771 \cdot 10^{-3}$	$1.7072 \cdot 10^{-3}$
ψ_e [deg]	$2.9546 \cdot 10^{-5}$	$9.5320 \cdot 10^{-6}$
Ω_{e_x} [deg/s]	$7.7195 \cdot 10^{-4}$	$2.1135 \cdot 10^{-5}$
Ω_{e_y} [deg/s]	$8.0082 \cdot 10^{-4}$	$2.2636 \cdot 10^{-5}$
Ω_{e_z} [deg/s]	$1.2008 \cdot 10^{-5}$	$2.6411 \cdot 10^{-6}$

Multibody spacecraft: simulation results