ADVANCED AEROSPACE CONTROL

Exam project AA 2019/2020

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Meet our team

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Abstract

The aim of the project is to design the control system for the lateral dynamics of an ANT-R quadrotor starting from grey-box model. After a brief plant analysis, the nominal design is carried out and robustness of the obtained controller is verified both in terms of stability and performance.

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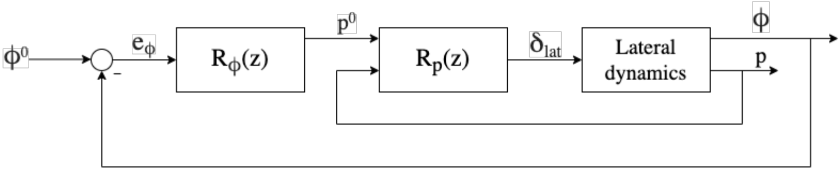
Presentation of the model

Scheme of the model



$$\dot{x} = Ax + Bu \qquad u = \delta_{lat}; \quad x = \begin{bmatrix} v & p & \phi \end{bmatrix}'$$

$$y = Cx + Du \qquad y = \begin{bmatrix} p & \phi \end{bmatrix}'$$

$$A = \begin{bmatrix} Y_v & Y_p & g \\ L_v & L_p & 0 \\ 0 & 1 & 0 \end{bmatrix}; \; B = \begin{bmatrix} Y_\delta \\ L_\delta \\ 0 \end{bmatrix}; \; C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \; D = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

where:

Stability derivatives

$Y_v = -0.264 s^{-1} (4.837\%)$

$Y_p = 0 m/s$

$L_p = 0 s^{-1}$

$L_v = -7.349 rad \cdot s/m (4.927\%)$

Control derivatives

$Y_d = 9.568 m/s^2 (4.674\%)$

$L_d = 1079.339 rad/s^2 (2.762\%)$

The controllers are digitally implemented and the sampling time is set at $T_s = 0.004s$.

$$R_p(z): \quad \begin{aligned} x_p(k+1) = &\quad A_p x_p(k) + B_p u_p(k) \\ \delta_{lat}(k) = &\quad C_p x_p(k) + D_p u_p(k) \end{aligned}$$

where:

$$u_p = \begin{bmatrix} p^o \\ p \end{bmatrix}$$

$$A_p = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \; B_p = \begin{bmatrix} b & -b \\ 0 & 0.5 \end{bmatrix} \; C_p = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \; D_p = \begin{bmatrix} d_1 & d_2 \end{bmatrix}$$

Outer loop controller

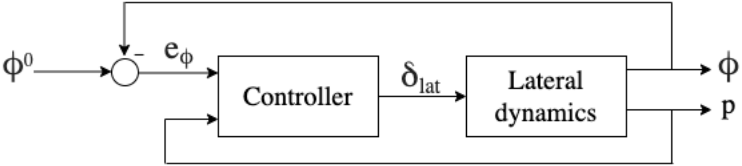
$R_\phi(z)$: $\quad p^o(k) = \quad D_\phi e_\phi(k)$

where :

$e_\phi(k) = \phi^o(k) - \phi(k)$

Scheme of the re-defined model

Controllers are merged into one:



Re-definition of the controller

The state space matrices are:

$$A_R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_R = \begin{bmatrix} b & -b \cdot D_\phi \\ 0 & 0.5 \end{bmatrix}$$

$$C_R = \begin{bmatrix} c_1 & c_2 \end{bmatrix}$$

$$D_R = \begin{bmatrix} d_1 & d_2 \cdot D_\phi \end{bmatrix}$$

Matlab implementation(1)

Lateral dynamics

- Definition of uncertain parameters, for example:

```
Y_v = ureal('Y_v',-0.264,'Percentage',3*4.837);
```

- Build of matrices `A,B,C,D`
- Definition of the uncertain plant:

```
G = uss(A,B,C,D);
G.InputName = 'Delta_lat';
G.OutputName = {'p';'phi'};
```

Matlab implementation(2)

Controller

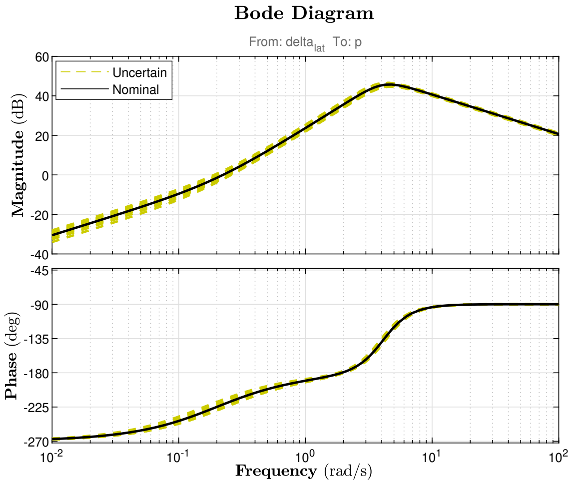
- Definition tunable parameters, for example:

```
b = realp('b',1);
```

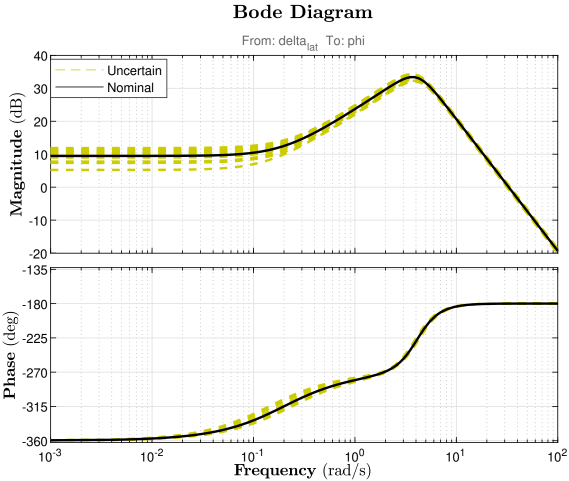
- Build of matrices of the controller A_R,B_R,C_R,D_R
- Definition of the controller:

```
R0 = ss(A_R,B_R,C_R,D_R,Ts);
R0.InputName = {'e_phi';'p'};
R0.OutputName = 'Delta_lat';
```

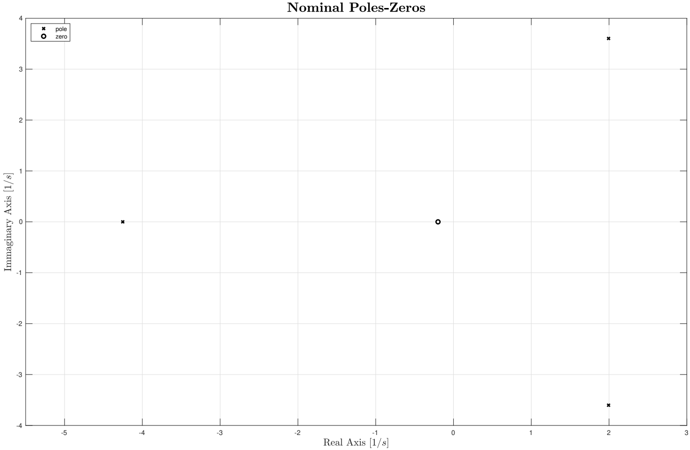
Bode plots of G(1)



Bode plots of G(2)

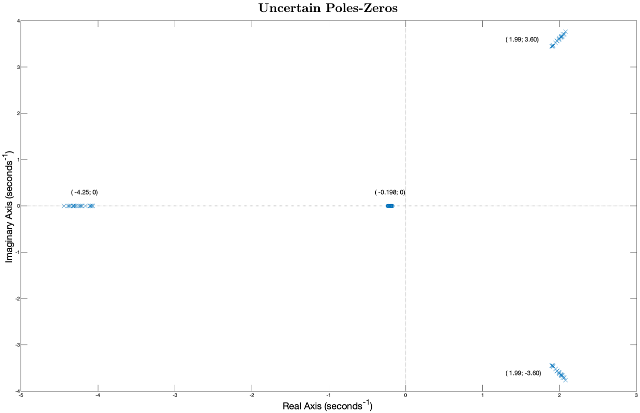


Nominal poles and zeros of G



Uncertain poles and zeros of G


```
pzplot(G)
```



Nominal design - H_∞

Tuning with H_∞

The tuning of the controller is carried out using H_∞ and the nominal values of the plant.

Weight functions

The performance requirement is of this type

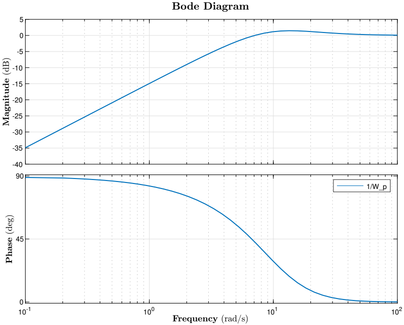
$$\frac{1}{W_p(s)} = \frac{s^2 + 2\xi\omega_b s}{s^2 + 2\xi\omega_b s + \omega_b^2}$$

where

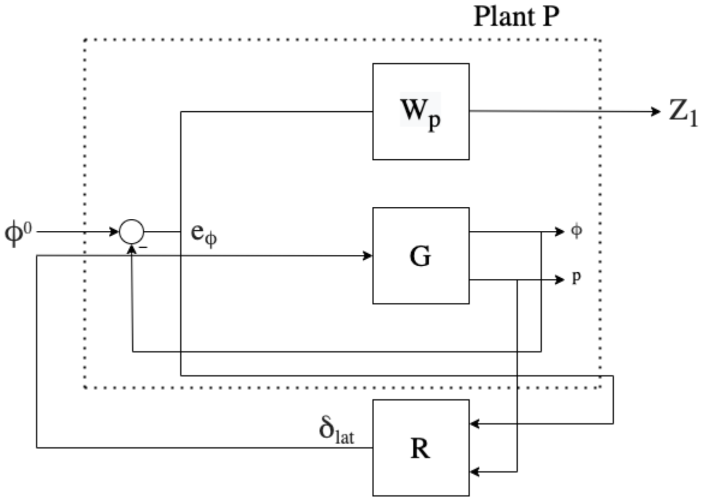
- ξ : minimum damping ratio.
- ω_b: lower bound of crossover frequency.

Weight functions - choice of parameters

- ξ : 0.9
- ω_b: 10 [rad/s]



Definition of the augmented plant



Definition of the augmented plant

The summing junction can be created with

```
S = sumblk('%e_phi=phi0-%phi',R0.InputName(1), ...
G.OutputName(2));
```

W_p is defined:

```
F_w = omega_w^2/(s^2+2*csi_w*omega_w*s+omega_w^2);
S_w = 1-F_w;
Wp = c2d(1/S_w,Ts);
Wp.InputName = 'e_phi';
Wp.OutputName ='z1';
```

The augmented plant is finally obtained through

```
P = connect(S,G,Wp,{'phi0','Delta_{lat}'}, ...
{'z1';'e_phi';'p'});
P.InputGroup.U1 =[1];
P.InputGroup.U2 =[2];
P.OutputGroup.Y1 = [1];
P.OutputGroup.Y2 = [2,3];
```

Application of H_∞

Applying the command

```
opt = hinfstructOptions('Display','final', ...
'RandomStart',5);
R = hinfstruct(P,R0,opt);
```

it's possible to find the structured controller which minimizes the Hinf norm of the closed-loop transfer function.

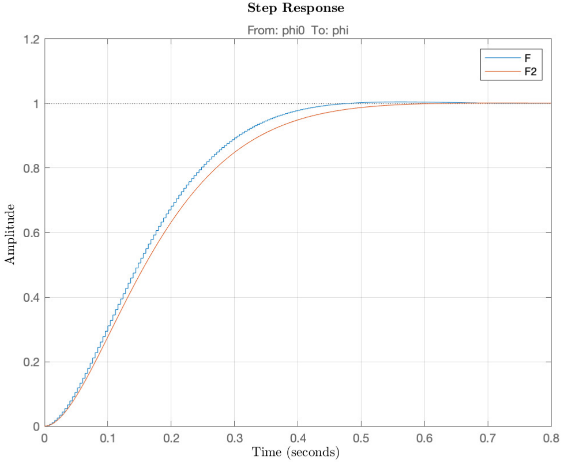
Typical output in command window:

```
Final: Peak gain = 1, Iterations = 60
```

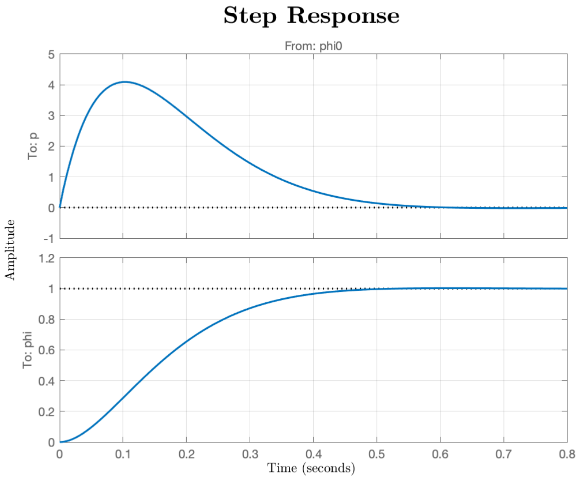
Results


```
D_phi = 0.1705;
b = 1.6208;
c1 = 0.08620;
c2 = 0.1032;
d1 = 0.1947;
d2 = -2.4833;
```

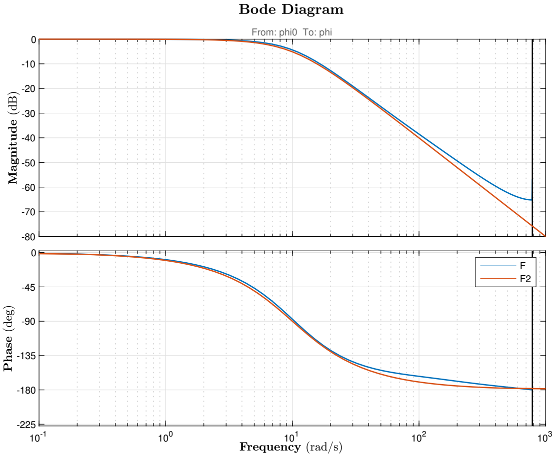
Response to step function (1)



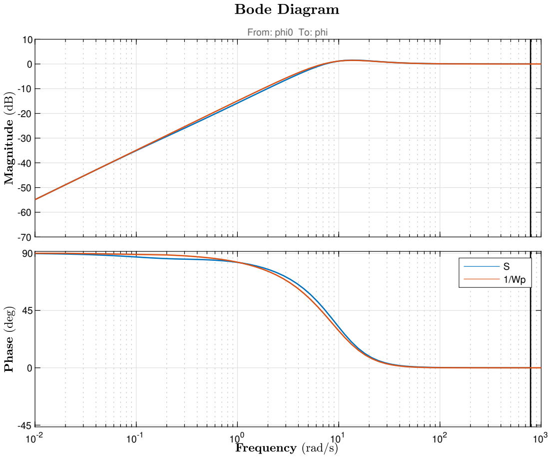
Response to step function (2)



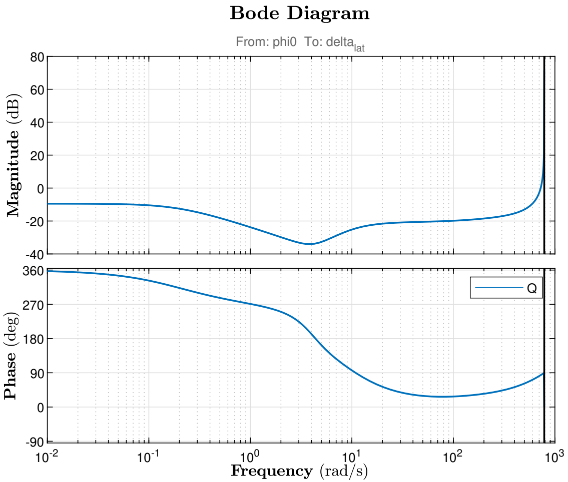
Comparison between F and F2



Comparison between S and Wp



Control sensitivity function



Nominal design - Systune

Application of Systune


```
SoftReqs = [req1];
HardReqs = [];
opt = systuneOptions('Display','final', ...
'RandomStart',5);
[F,fSoft,gHard,info] = ...
systune(F0,SoftReqs,HardReqs,opt);
```

Tuning goal

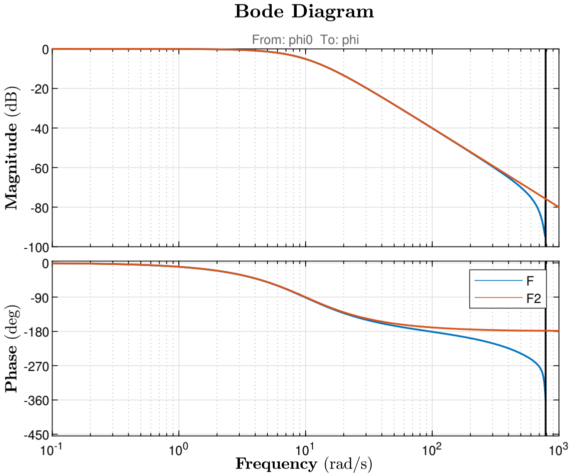

```
req1 = ...
TuningGoal.Transient(F0.InputName,F0.OutputName,F2);
```

$$F2 = \frac{\omega_b^2}{(s^2 + 2\xi\omega_b s + \omega_b^2)};$$

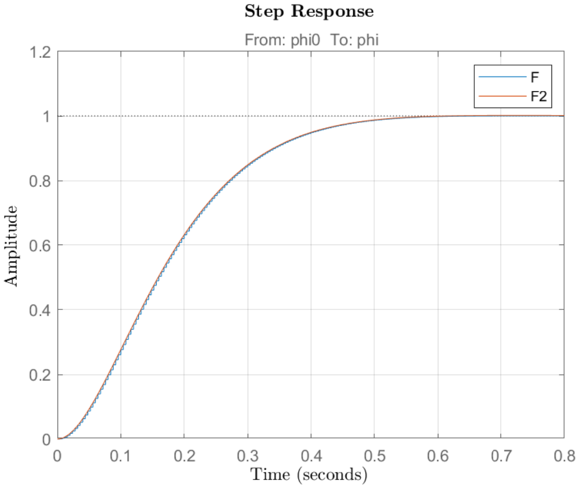
Where:

$\omega_b = 10[rad/s];$
$\xi = 0.9;$

Presentation of results (1)



Presentation of results (2)



Selection of method

H_∞ vs Systune

Robust stability analysis

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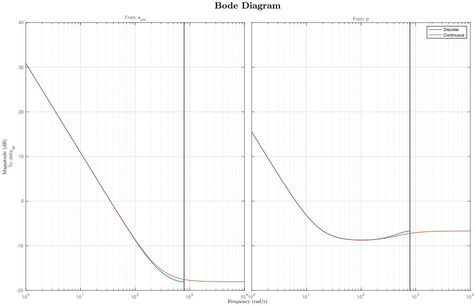
Conversion of R

From discrete to continuous time using Tustin method.

```
R = d2c(R,'tustin');
```



The maximum considered frequency: $\omega \approx 10^3$ [rad/s]

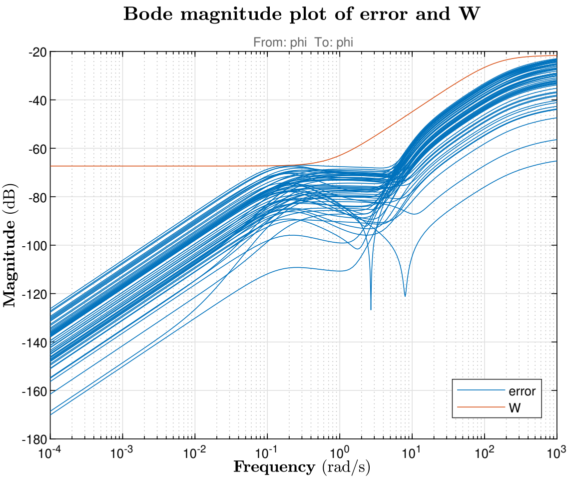
Complementary sensitivity function


```
S1 = sumblk('%e_phi = phi0-%phi', R.InputName(1), ...
G.OutputName(2));

F_nominal = connect(G.NominalValue,R,S1, ...
S1.InputName(1),{'p';'phi'});

F_uncertain = ...
connect(G,R,S1,S1.InputName(1),{'p';'phi'});
```

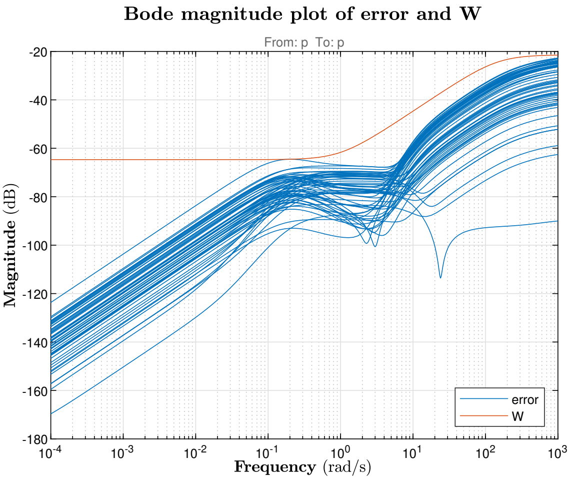
Creation of uncertainty function W (2)


```
F_array = usample(F_uncertain,60);
error = (F_nominal-F_array)/F_nominal;
[F,Info] = ucover(F_array,F_nominal,1);
W = Info.W1;
```

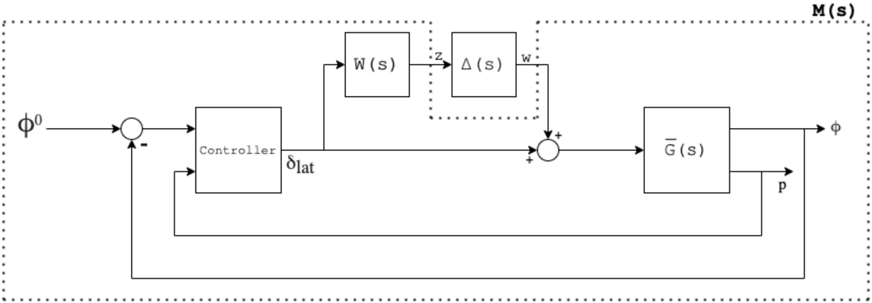
Presentation of results (1)



Presentation of results (2)



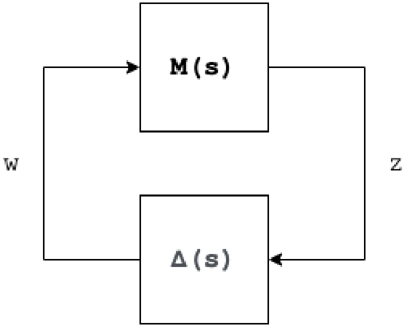
Model in M-Δ form



Creation of M


```
G_nominal = G.NominalValue;
G_nominal.InputName = 'Delta_lat2';
G_nominal.OutputName = {'p','phi'};

R.InputName = {'e_phi','p'};
R.OutputName = 'Delta_lat';

W.InputName = 'Delta_lat';
W.OutputName = 'z';

S1 = sumblk('%e_phi = phi0-%phi', R.InputName(1), ...
G.OutputName(2));
S2 = sumblk('%Delta_lat2 = %Delta_lat + w', ...
G_nominal.InputName, R.OutputName);

M = connect(G_nominal,R,W,S1,S2,'w','z');
```

Model in M-Δ form

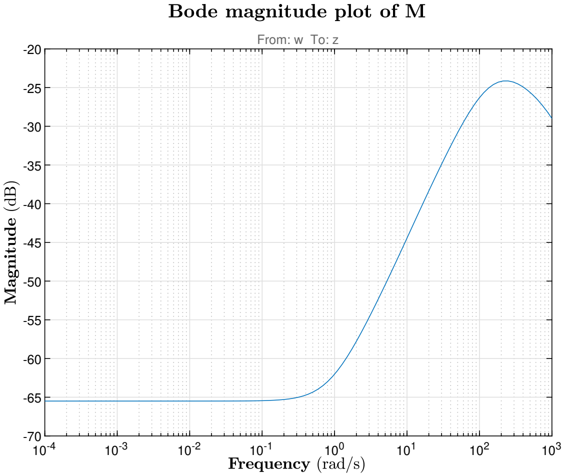


Condition

The condition of robust stability in M-Δ form is verified by the inequality:

$$|M(j\omega)| \leq 1, \quad \forall\omega$$

Presentation of results



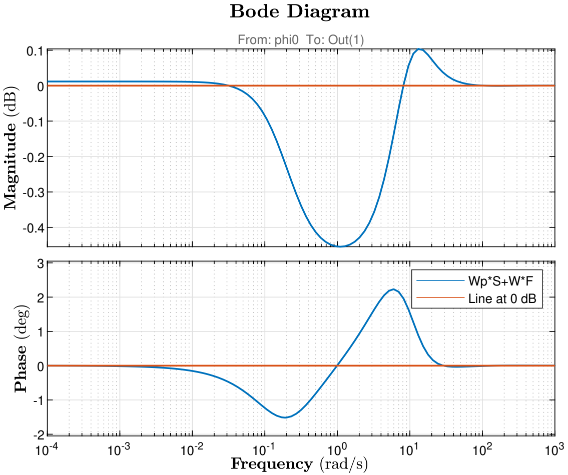
Verification of robust performance

Condition

The condition of robust performance is verified by the inequality:

$$|W_p(j\omega)\overline{S}(j\omega)| + |W(j\omega)\overline{F}(j\omega)| \leq 1 \quad \forall\omega$$

Presentation of results

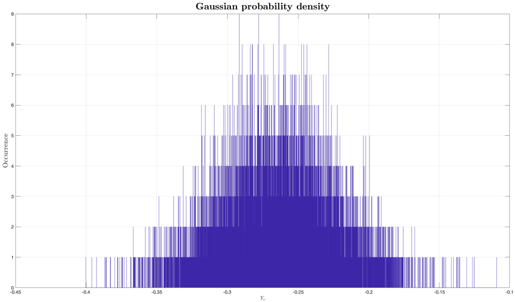


Montecarlo analysis

Re-sampling of uncertain parameters

An array of parameters of the plant can be created using a Gaussian probability distribution, for example:

```
Y_v_mc(n)= Y_v.NominalValue + Y_v.PlusMinus(2)* ...
randn(1,1);
```



Procedure

For each re-sampled parameter (N=100):

- Creation of the state space matrices.
- Creation of the state space model.
- Creation of the open loop transfer function.
- Computation of Gain and Phase margins.

```
[Gm(n),Pm(n)] = margin(L(n));
```

- Computation of step response and its information (settling time and overshoot).

```
y = step(minreal(L(n)/(1+L(n))),t);
S = stepinfo(y,t,1);
Sett(n)= S.SettlingTime;
Over(n)= S.Overshoot;
```

Presentation of results - Step response

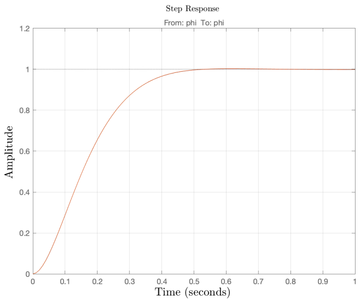


Figure: Step response

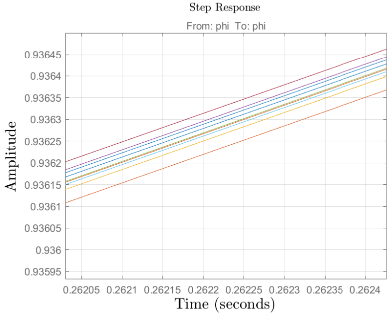
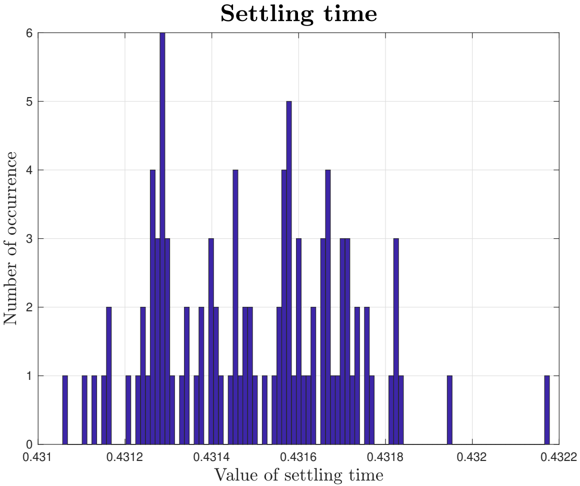
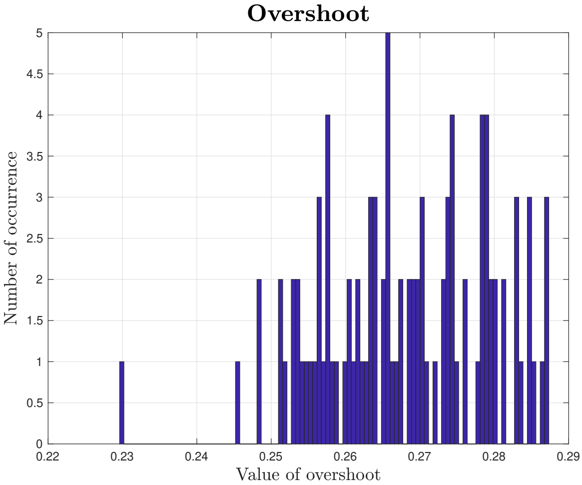


Figure: Zoom on step response

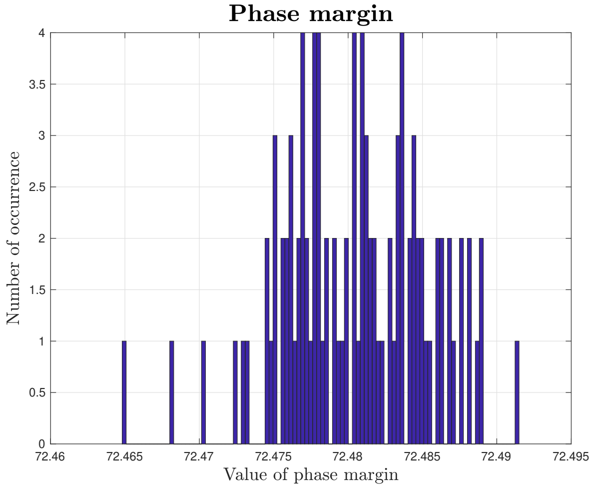
Presentation of results - Step Info (1)



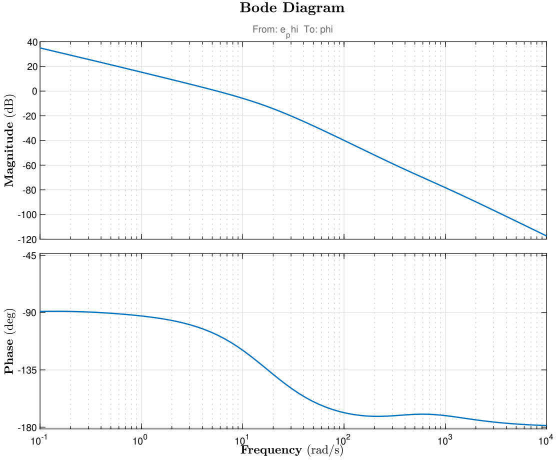
Presentation of results - Step Info (2)



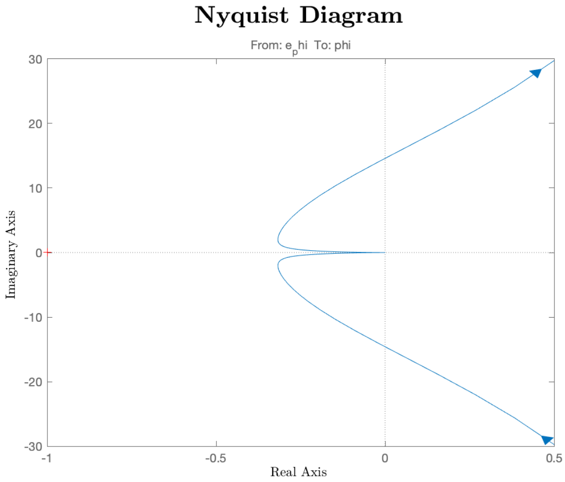
Presentation of results - Bode Info



Presentation of results - Bode Info



Presentation of results - Bode Info



Convergence of Montecarlo method

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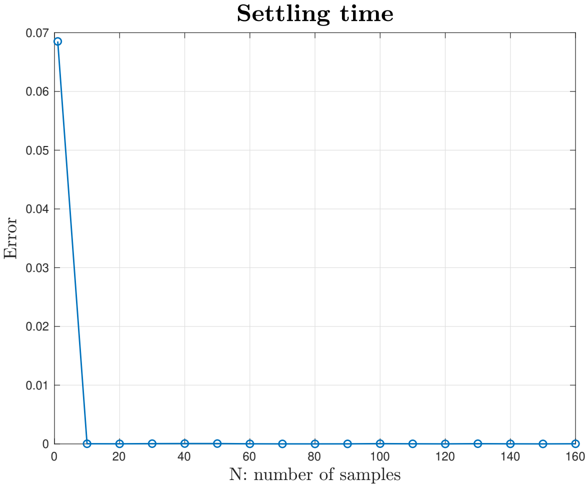
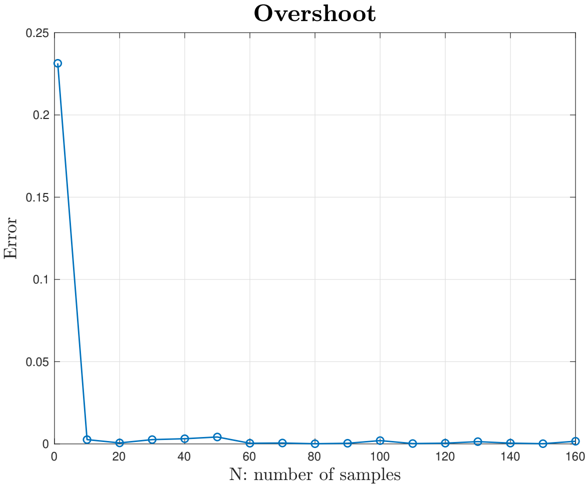
What is the effect of increase the number of samples in Montecarlo analysis?

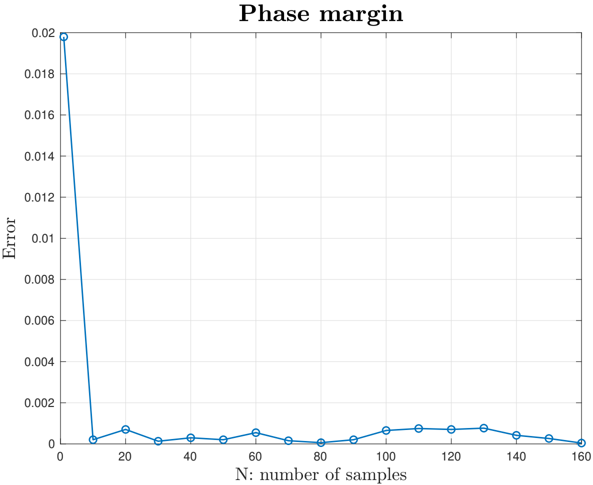
Presentation of results



Presentation of results



Presentation of results



μ-analysis

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Structured Singular Value

Computation of the the array of the frequencies:

```
omega = logspace(−2,3,500);
```

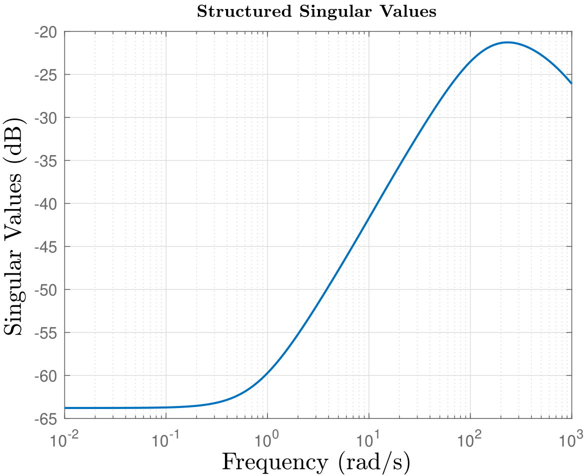
Computation of the bounds:

```
bounds = mussv(frd(M,omega),[1,0]);
```

The condition for RS is:

$$\mu(M(j\omega)) < 1, \quad \forall\omega$$

Presentation of results



Critical observations on Control

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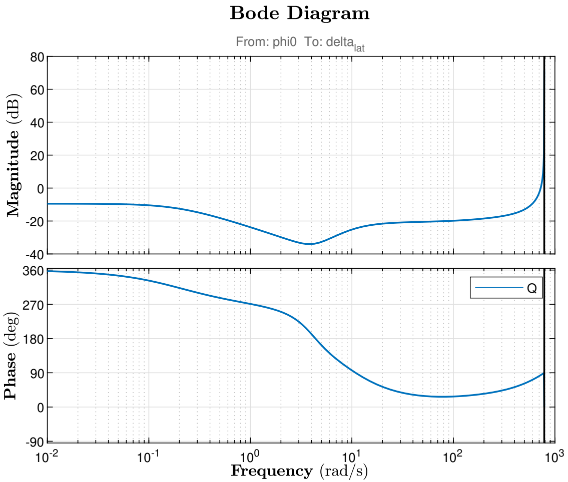
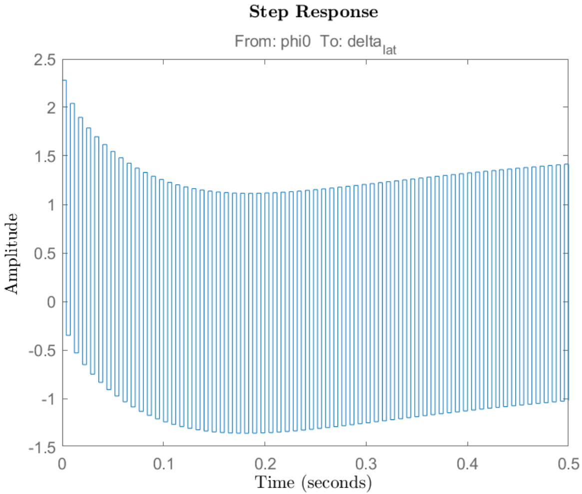
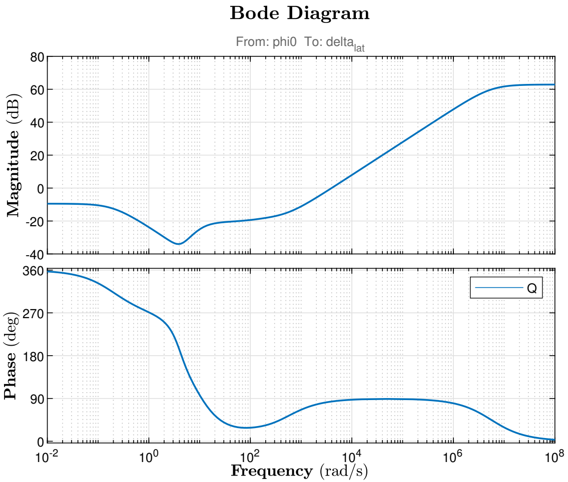
Control sensitivity function (1)



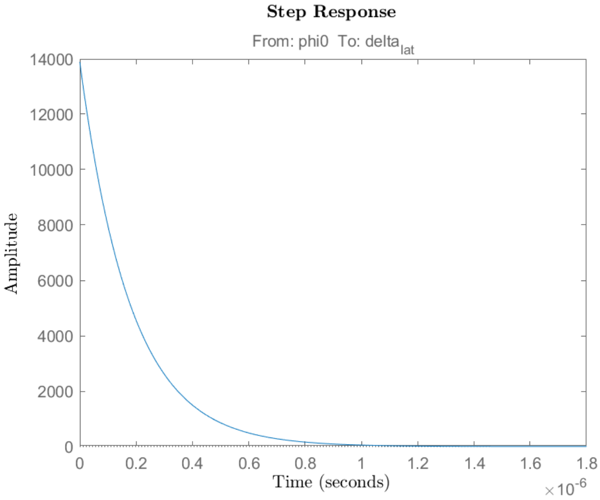
Control sensitivity function (2)



Control sensitivity function (3)



Control sensitivity function (4)



Thanks for your attention!