



Advanced Aerospace Control Exam project AA 19/20



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Task: robust design of attitude control system for a multirotor UAV



- The ANT-R quadrotor is a small racing drone designed and built during the 18/19 edition of UAV Lab.
- Grey-box models of ANT-R have been identified from flight test data and are available for control law design.
- The project task is to design and verify a robust single-axis attitude control system for ANT-R.

Linear model for lateral dynamics

The lateral dynamics are given by:

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$$

where

$$\begin{aligned}u &= \delta_{lat}, & y &= \begin{bmatrix} p \\ \varphi \end{bmatrix} \\ x &= \begin{bmatrix} v \\ p \\ \varphi \end{bmatrix}\end{aligned}$$

and

$$A = \begin{bmatrix} Y_v & Y_p & g \\ L_v & L_p & 0 \\ 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} Y_\delta \\ L_\delta \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Lateral dynamics: parameters and uncertainties

The numerical values of the parameters are given by

Stability derivatives

$$Y_v = -0.264 \text{ 1/s (4.837\%)}$$

$$Y_p = 0 \text{ m/s rad (-)}$$

$$L_v = -7.349 \text{ rad s/m (4.927\%)}$$

$$L_p = 0 \text{ 1/s (-)}$$

Control derivatives

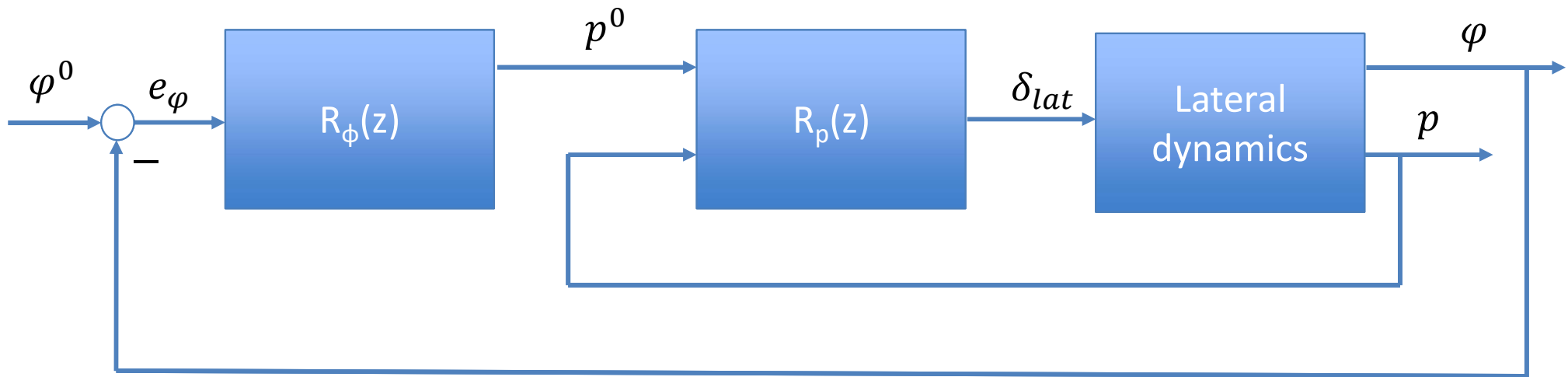
$$Y_d = 9.568 \text{ m/s}^2 \text{ (4.647\%)}$$

$$L_d = 1079.339 \text{ rad/s}^2 \text{ (2.762\%)}$$

where uncertainty is given in terms of standard deviations (provided as percentage of corresponding nominal values) assuming a Gaussian density for each parameter.

Lateral attitude control block diagram

The lateral attitude control system is represented in the block diagram.



In the diagram we already take into account the digital implementation of the roll rate ($R_p(z)$) and roll angle ($R_\phi(z)$) controllers (the sampling interval is $T_s = 0.004$ s).

Lateral control: controller structure

Controller $R_p(z)$ can be represented in state space form as:

$$\begin{aligned}x_p(k+1) &= A_p x_p(k) + B_p u_p(k) \\ \delta_{lat}(k) &= C_p x_p(k) + D_p u_p(k)\end{aligned}$$

where

$$u_p = \begin{bmatrix} p^0 \\ p \end{bmatrix}$$

and

$$A_p = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B_p = \begin{bmatrix} b & -b \\ 0 & 0.5 \end{bmatrix}, C_p = [c_1 \quad c_2], D_p = [d_1 \quad d_2]$$

Controller structure

Controller $R_\varphi(z)$ is given by

$$p^0(k) = D_\varphi e_\varphi(k)$$

where

$$e_\varphi = \varphi^0 - \varphi$$

Project tasks: nominal design

1. Build a design model for the feedback system using the available information and Matlab Control Design Blocks.
2. Analyse nominal and uncertain models in terms of poles, zeros, frequency response functions.
3. Nominal design: using the nominal model, tune the parameters of $R_p(z)$ and $R_\varphi(z)$ based on the NP design requirements given in the following slide.

4. Robust analysis and design:

- a) Build the uncertain model in $M - \Delta$ form using $\pm 3\sigma$ ranges as deterministic bounds to construct relative errors and verify that the obtained controller satisfies the RS condition.
- b) If not, redesign to achieve RS.

Project tasks: design requirements

Design requirements:

A. Nominal Performance:

- i. Response of φ to variations in φ^0 : equivalent to a second-order response with $\omega_n \geq 10 \text{ rad/s}$ and $\xi \geq 0.9$.
- ii. Control effort limitation: $|\delta_{lat}| \leq 3\%$ for a step change in φ^0 with 10 degrees amplitude.

B. Robust stability: ensure robustness of stability to model uncertainty associated with $\pm 3\sigma$ ranges on all parameters, treated as deterministic bounds.

Project tasks: verification

5. Verify if the obtained controller satisfies the RP condition.
6. Build an uncertain model for the feedback system taking into account the provided uncertainty on the model parameters.
7. Implement Monte Carlo simulation on the controlled linear model and verify RS using Gaussian densities for the uncertain parameters.
8. Build uncertain model using real Δ and verify RS using μ -analysis.

Presentation of followed design approach, code and results

When the tasks are complete prepare a Powerpoint presentation including

- An outline of the considered modelling approach
- An outline of the considered design approach
- An overview of the developed Matlab code
- A presentation of the results in terms of
 - Modelling approach
 - Selected weighting functions
 - Obtained closed-loop sensitivities
 - Frequency-domain analysis
 - Time-domain results

In-flight validation on real quadrotor



Following discussion of the presentation the controller tunings will be tested (if possible) on the real quadrotor.