Aerospace Control Systems
Exam project AA 20/21

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AEROSPACE SYSTEMS & CONTROL LABORATORY

Task: robust design of attitude control system for a multirotor UAV

- The ANT-R quadrotor is a small racing drone designed and built during the 18/19 edition of UAV Lab.
- Grey-box models of ANT-R have been identified from flight test data and are available for control law design.
- The project task is to design and verify a robust single-axis attitude control system for ANT-R.

Linear model for lateral dynamics

The lateral dynamics are given by:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where

$$u = \delta_{lat}, \qquad y = \begin{bmatrix} p \\ \varphi \end{bmatrix}$$
$$x = \begin{bmatrix} v \\ p \\ \varphi \end{bmatrix}$$

and

$$A = \begin{bmatrix} Y_v & Y_p & g \\ L_v & L_p & 0 \\ 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} Y_\delta \\ L_\delta \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Lateral dynamics: parameters and uncertainties

The numerical values of the parameters are given by ($g = 9.81$)

Stability derivatives

$Y_v \;= -0.264 \;\; 1/s \;(4.837\%)$

$Y_p \;= \; 0 \;\; m/\,s\,rad \;(-)$

$L_v \;= -7.349 \;\; rad\; s/m \;(4.927\%)$

$L_p \;= \; 0 \;\; 1/s \;(-)$

Control derivatives

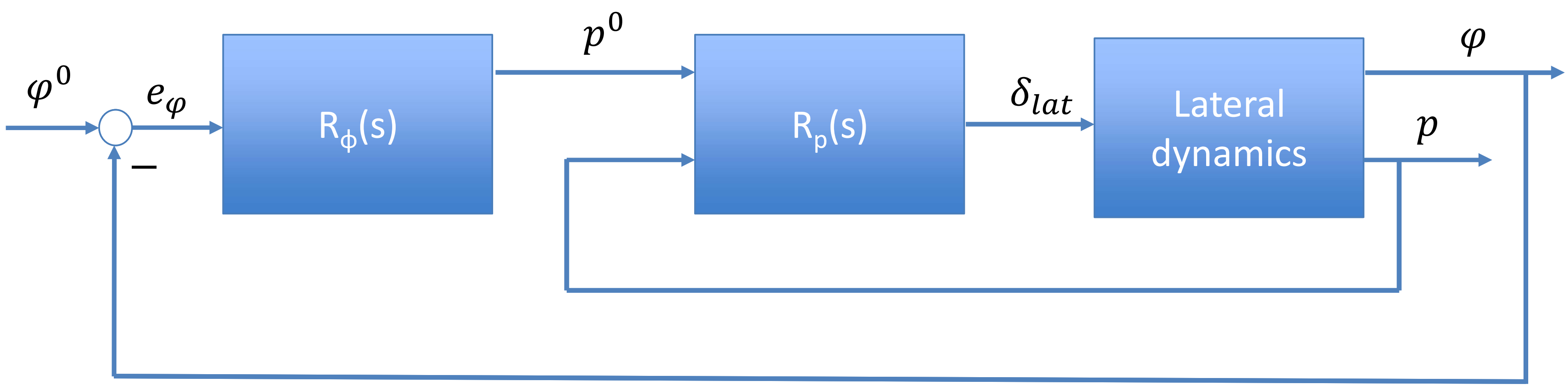
$Y_d \;= 9.568 \;\; m/s^2 \;(4.647\%)$

$L_d \;= 1079.339 \;\; rad/s^2 \;(2.762\%)$

where uncertainty is given in terms of standard deviations (provided as percentage of corresponding nominal values) assuming a Gaussian density for each parameter.

Lateral attitude control block diagram

The lateral attitude control system is represented in the block diagram.



The roll rate ($R_p(s)$) and roll angle ($R_\varphi(s)$) controllers are, respectively, a PID and a P controller.

Lateral control: controller structure

Controller $R_p(s)$:

$$\delta_{lat} = K_{p,P} e_p + \frac{K_{p,I}}{s} e_p + \frac{K_{p,D} s}{1 + s T_p} e_p,$$
$$e_p = p^0 - p$$

Controller $R_\varphi(s)$:

$$p^0 = K_\varphi e_\varphi,$$
$$e_\varphi = \varphi^0 - \varphi$$

Project tasks: nominal design

1. Build a design model for the quadrotor dynamics using the available information and the MATLAB Robust Control Toolbox for uncertain syste representation.

2. Analyse nominal and uncertain models in terms of poles, zeros, frequency response functions.

3. Build a design model for the feedback system using the available information and the MATLAB Robust Control Toolbox for uncertain system representation.

4. Nominal design: using the nominal model, tune the parameters of $R_p(s)$ and $R_\varphi(s)$ based on the NP design requirements given in the following slides «design requirements».

Project tasks: design

4. Robust analysis and design:
   a) Build the uncertain model in $M - \Delta$ form using $\pm 3\sigma$ ranges as deterministic bounds to construct relative errors and verify that the obtained controller satisfies the RS condition.
   b) If not, redesign to achieve RS.

Project tasks: design requirements (1)

Design requirements:

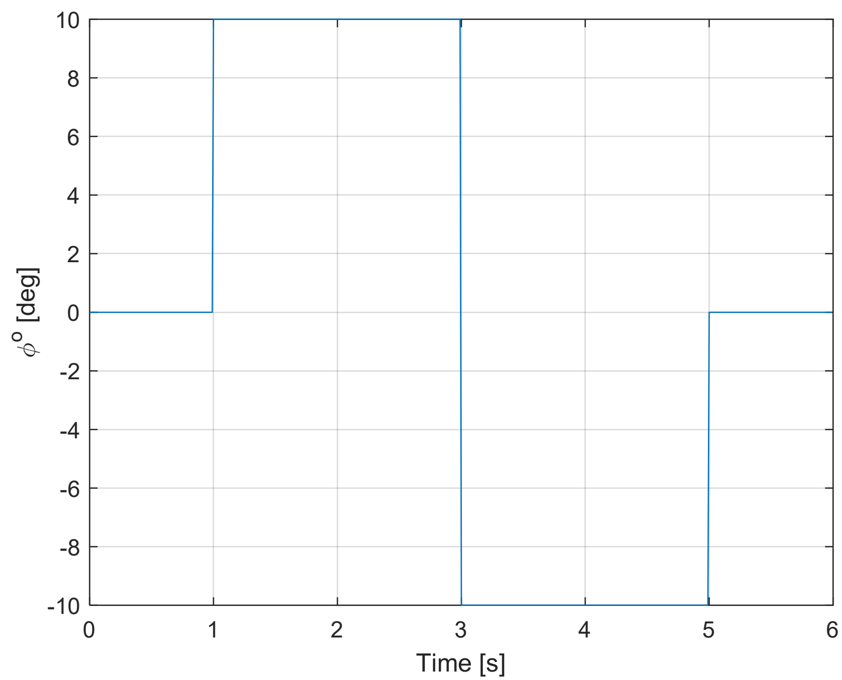
A. Nominal Performance:

Response of φ to variations in φ^0: equivalent to a second-order response with $\omega_n \geq 10\, rad/s$ and $\xi \geq 0.9$.

Project tasks: design requirements (2)

Design requirements:

B. Control effort limitation: $|\delta_{lat}| \leq 5$ for the doublet change in φ^0 (with 10 degrees amplitude) given by



Project tasks: design requirements (3)

Design requirements:

C. Robust stability: ensure robustness of stability to model uncertainty associated with $\pm 3\sigma$ ranges on all parameters, treated as deterministic bounds.

Project tasks: verification

5. Verify if the obtained controller satisfies the RP condition.

6. Build an uncertain model for the feedback system taking into account the provided uncertainty on the model parameters.

7. Implement Monte Carlo simulation on the controlled linear model and verify RS using Gaussian densities for the uncertain parameters.

8. Build uncertain model using real Δ and verify RS using μ-analysis.

Presentation of followed design approach, code and results

When the tasks are complete prepare a Powerpoint presentation including

- An outline of the considered modelling approach
- An outline of the considered design approach
- An overview of the developed Matlab code
- A presentation of the results in terms of
  - Modelling approach
  - Selected weighting functions
  - Obtained closed-loop sensitivities
  - Frequency-domain analysis
  - Time-domain results