

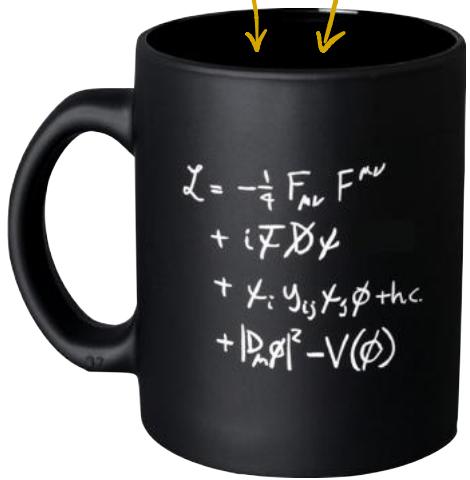
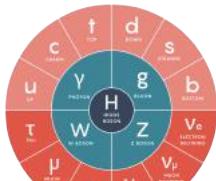
Making Predictions at Hadron Colliders

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CERN Summer Student Lecture Programme 2023

$SU(3)_c \times SU(2)_L \times U(1)_Y$



THEORY

frame work of QFT

\cong $QM \otimes SR$

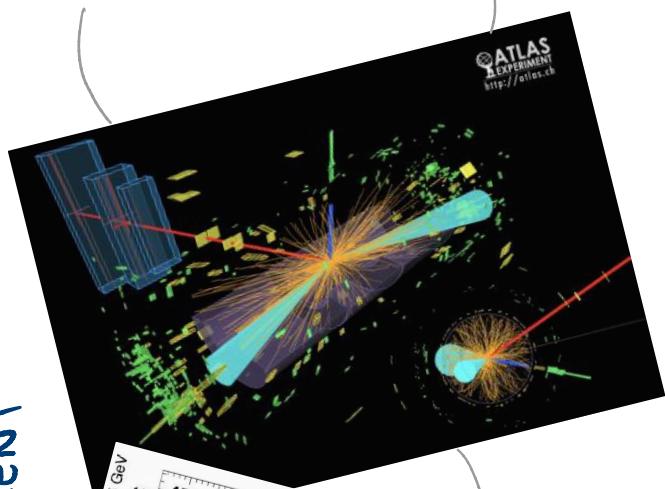
PHENOMENOLOGY



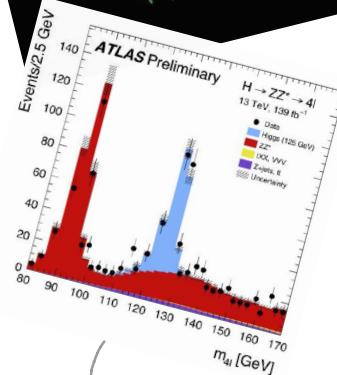
this lecture

Accelerators

Detectors



EXPERIMENT



statistics

data
aquisition/
analysis

Sources & Conventions

* all examples on: github.com/aykhuss/ssl23

(mainly python in org notebooks)

"making predictions"

* Conventions $[h] = [c] = 1$

$$\Rightarrow [\text{length}] = [\text{time}] = \text{eV}^{-1}$$

$$[\text{mass}] = [\text{energy}] = [\text{momentum}] = \text{eV}$$

(remember: $m_{\text{proton}} \sim 1 \text{ GeV}$)

* four vectors: $x^\mu = (t, x, y, z)^\top$

$$p^\mu = (E, p_x, p_y, p_z)^\top$$

\Rightarrow energy - momentum conservation:

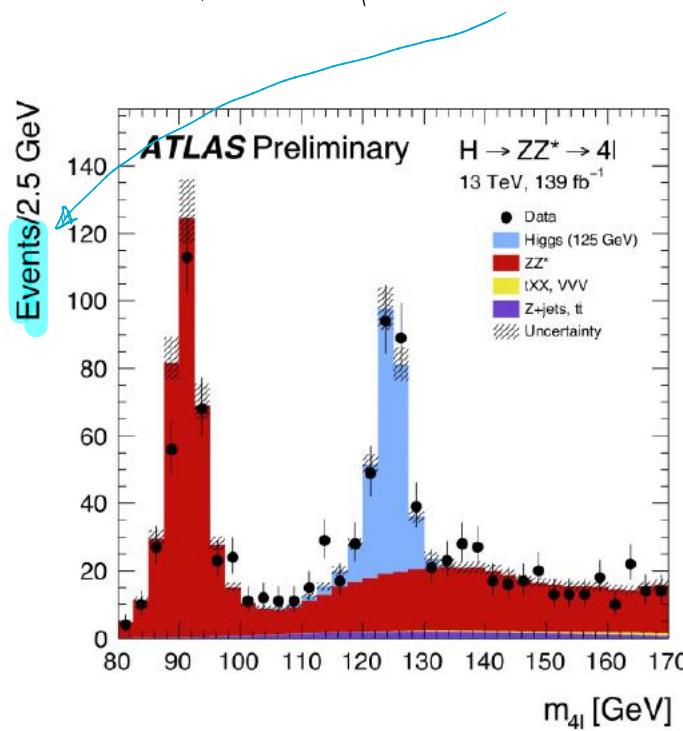
$$\delta^{(4)}(p_1 + p_2 - (p_a + p_b)) = \delta(E_a + E_b - E_1 - E_2) \delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{p}_a - \vec{p}_b)$$

The Plan

1. Event Rates, Cross Sections & Scattering Amplitudes
2. Warmup: Lepton Collider
3. Hadron Colliders – Parton Distribution Functions
4. The Drell-Yan Process
5. Higher-Order Corrections
6. QCD Jets, Parton Showers & MC Simulations

Event Rates

We ultimately measure **# Events**
for a specific process: $a+b \rightarrow 1+2+\dots+n$



$$dN = L d\sigma$$

Luminosity $\sim \# \text{ collisions}$

Cross section

- * σ_H (13 TeV) $\approx 50 \text{ pb}$
- $\int dt \mathcal{L}_{\text{Run 2}} \approx 150 \text{ fb}^{-1}$
- σ_Z (13 TeV) $\approx 50 \text{ nb}$
- σ_W (13 TeV) $\approx 200 \text{ nb}$
- \mathcal{L} (instantaneous) $\approx 0.02 \text{ pb}^{-1} \text{ s}^{-1}$
- $\left. \begin{array}{l} \sim 7 \text{ million} \\ \text{Higgs bosons} \\ \text{produced} \end{array} \right\}$
- $\left. \begin{array}{l} \sim 1000 \text{ Z} \\ \sim 4000 \text{ W's} \\ \text{every second!} \end{array} \right\}$

Calculating Cross Sections

Fermi's golden rule: $a+b \rightarrow 1+2+\dots+n$

$$d\sigma = \frac{1}{F} \langle |M|^2 \rangle$$

$$= \frac{1}{4(P_a P_b)} = \frac{1}{2 E_{cm}^2}$$

flux

$|amplitude|^2$

$$= \frac{1}{N_a N_b} \sum_{\text{dof.}} |M|^2$$

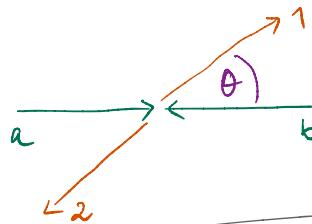
(degrees of freedom:
spin, colour)

$$d\Phi$$

phase space
(LIPS)

Special case $a+b \rightarrow 1+2$

$$d\Phi_2 = \frac{d \cos \theta}{16\pi} \quad (\text{massless})$$



$$d\Phi_n(p_1, \dots, p_n; p_a + p_b) = \frac{n}{T!} \frac{d^4 p_1}{(2\pi)^4} (2\pi) \delta(p_i^2 - m_i^2) \Theta(p_i^0)$$

$$\times (2\pi)^4 \delta^{(4)}(p_1 + \dots + p_n - (p_a + p_b))$$

on shell
 $E > 0$
energy-momentum
conservation

The Scattering Amplitude

\leftrightarrow evaluation of the path integral
(analogy to QM)

$$Z[J] = \int \mathcal{D}[\Phi] e^{i \int d^4x [L(\Phi, \partial\Phi) + J\Phi]} \quad \Phi \in \{\psi, \phi, A_\mu, \dots\}$$


\hookrightarrow extremely difficult to solve

except a **free** theory $\leftrightarrow L$ only has terms with at most two fields Φ

e.g. $\bar{\psi}(i\gamma^\mu - m)\psi$

Feynman rules for the free theory

very boring "scattering"
 $a+b \rightarrow a+b$
 $a \text{ — } a$
 $b \text{ — } b$

<u>incoming</u>	<u>outgoing</u>	<u>propagators</u>
$f \xrightarrow{p} \circ = u(p)$	$\circ \xrightarrow{p} f = \bar{u}(p)$	$\circ \xrightarrow{p} \not{f} = \frac{i}{p-m}$
$\bar{f} \xrightarrow{p} \circ = \bar{v}(p)$	$\circ \xleftarrow{p} \bar{f} = v(p)$	$\circ \xrightarrow{p} \not{f} = \frac{-ig^{\mu\nu}}{p^2 - M_Z^2 + iM_Z T_Z}$

Perturbation Theory

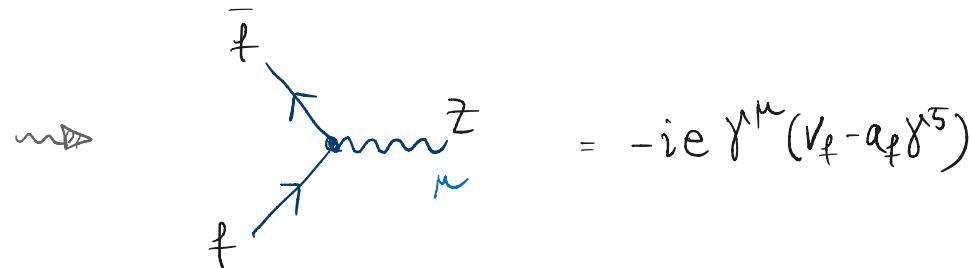
When interaction strength is small ($\lambda_{em} \sim 1/137$, $\alpha_s \sim 0.118$)

⇒ compute M perturbatively by expanding around free theory

Feynman rules for interactions — vertices

direct correspondence with terms in \mathcal{L}

$$-ie z_\mu \bar{\psi}_f \gamma^\mu (\nu_f - a_f \gamma^5) \psi \in \mathcal{L}$$



* more subtle than just "dropping" the fields
when derivatives (∂_μ) and/or identical particles

Warmup: Lepton Collider

Consider the process $e^+ e^- \rightarrow \mu^+ \mu^-$

At lowest order (tree level) there are two diagrams

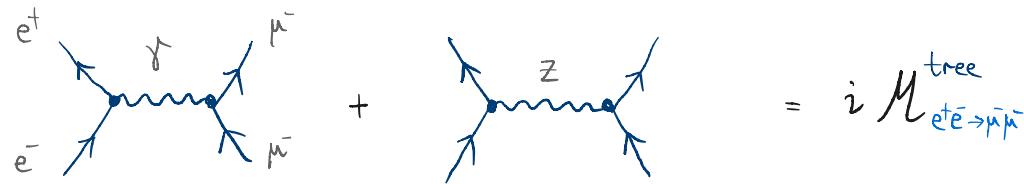
What are they? [demo: FeynGames]

Warmup: Lepton Collider

[demo: e^+e^-]

Consider the process $e^+e^- \rightarrow \mu^+\mu^-$

At lowest order (tree level) there are two diagrams



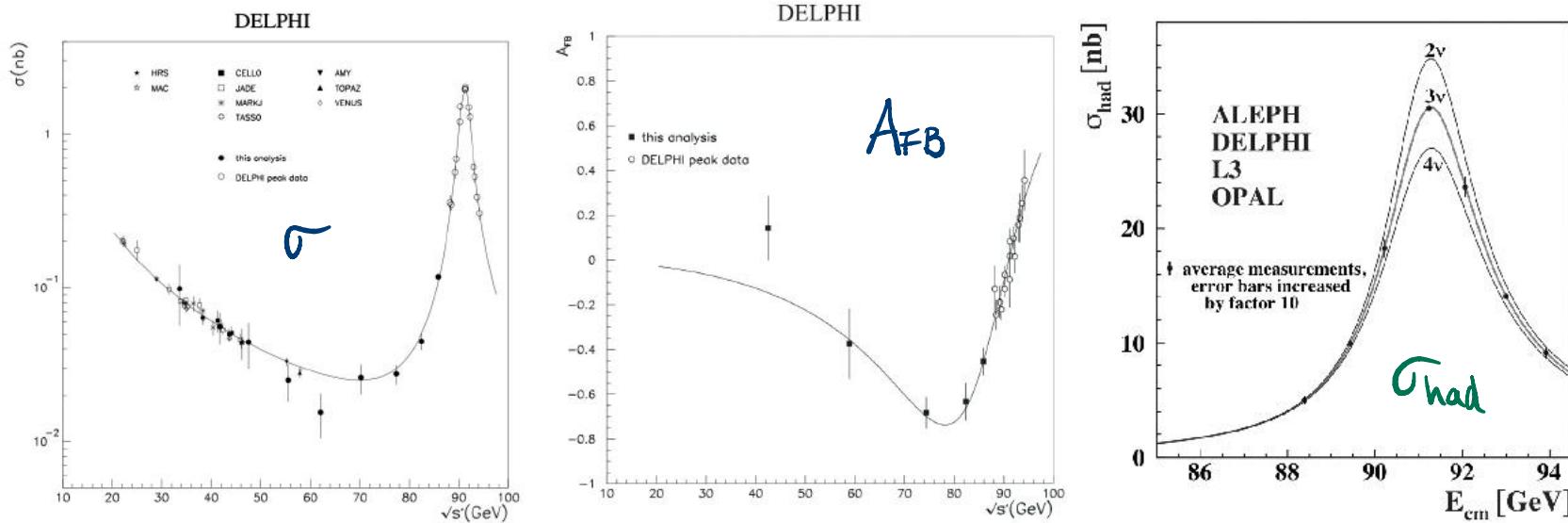
\Rightarrow inserting into Fermi's golden rule $[S = E_{cm}^2; P_a \cdot P_1 = P_a^M P_{1,m} = E_{cm}^2 (1 - \cos\theta)]$

$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha^2 \pi}{2S} \left[(1 + \cos^2\theta) G_1(s) + 2 \cos\theta G_2(s) \right]$$

$$G_1(s) = 1 + 2 V_e^2 \operatorname{Re} \left\{ \frac{s}{s - M_Z^2 + i M_Z T_Z} \right\} + (V_e^2 + a_e^2)^2 \left| \frac{s}{s - M_Z^2 + i M_Z T_Z} \right|^2$$

$$G_2(s) = 0 + 2 a_e^2 \operatorname{Re} \left\{ \frac{s}{s - M_Z^2 + i M_Z T_Z} \right\} + 4 V_e^2 \cdot a_e^2 \left| \frac{s}{s - M_Z^2 + i M_Z T_Z} \right|^2$$

"Comparison" with data



- * In principle, you now can use the predictions to fit M_Z & $\sin^2 \Theta_W$ from the data (at leading order)
- * σ_{had} is the hadronic cross section: @ LO: $e^+ e^- \rightarrow q \bar{q}$
(what changes w.r.t. $\mu^+ \mu^-$?)

Hadron Colliders: The parton model

At the LHC we collide protons \neq elementary

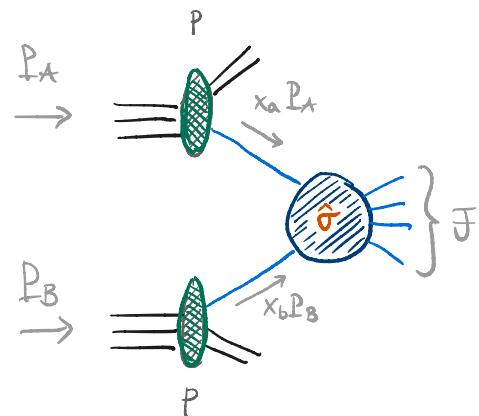
→ hadronic cross section = sum over scattering of partons ($q \& g$)

$$d\sigma_{A+B \rightarrow F+X}^{(P_A, P_B)} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b f_a(x_a) f_b(x_b) d\hat{\sigma}_{a+b \rightarrow F+X}^{(x_a P_A, x_b P_B)}$$

fractional momentum

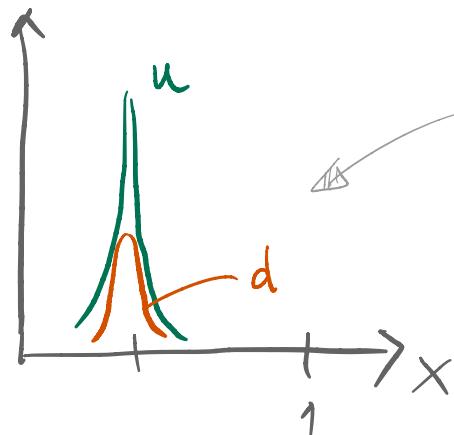
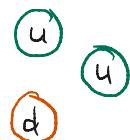
parton distribution function

= number density for parton a
to carry momentum fraction $[x, x+dx]$
of parent hadron



Parton Distribution Functions

* just free quarks? ($p = uud$)

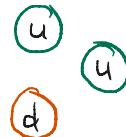


$$f_a(x) = \delta_{au} 2 \delta(x - \frac{1}{3}) + \delta_{ad} 1 \delta(x - \frac{2}{3})$$

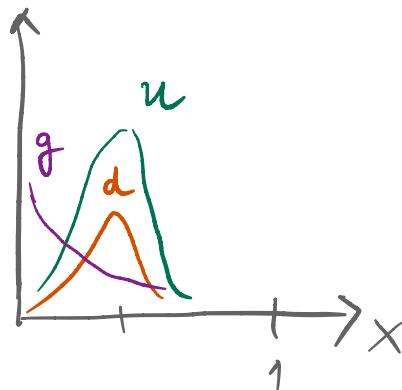
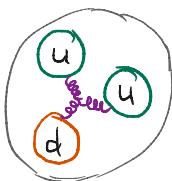
+ some smearing

Parton Distribution Functions

* just free quarks? ($p = uud$)



* bound by gluons?



naive parton model:

\hookrightarrow composition of point particles

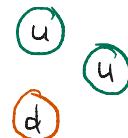
\rightsquigarrow zoom in ($Q^2 \uparrow$) \leftrightarrow same composition

scaling (PDFs independent on scale
at which it is probed,
as long as $Q^2 \gg m_p^2$)

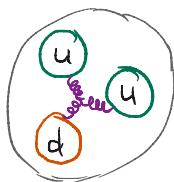
Parton Distribution Functions

[demo: PDF]

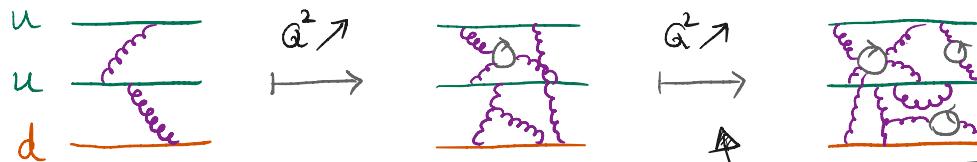
* just free quarks? ($p = uud$)



* bound by gluons?



* QCD-improved parton model



\Rightarrow predominantly shifts partons from high- x to low- x

evolution perturbatively calculable!

The Drell-Yan process

[demo: DY]

$$d\sigma_{DY} = \int dx_a \int dx_b f_a(x_a) f_b(x_b) d\hat{\sigma}_{ab \rightarrow e^+e^-} \quad (\text{only } (a,b) \in \{(q,\bar{q}), (\bar{q},q)\} \text{ @ LO})$$

- * After integrating out $Z \rightarrow l^+l^-$ decay we'll look at the observables of the intermediate gauge boson $q^\mu = (p_1 + p_2)^\mu = (p_A + p_B)^\mu$

$$M_{ll} = \sqrt{q^2} \quad ; \quad Y_{ll} = \frac{1}{2} \ln \left(\frac{q^0 + q^3}{q^0 - q^3} \right)$$

rapidity: $Y \rightarrow Y + \frac{1}{2} \ln(\gamma/\gamma_2)$

$$\Rightarrow \boxed{\frac{d^2 \sigma_{DY}}{d M_{ll} d Y_{ll}} = f_a(x_a) f_b(x_b) \frac{2 M_{ll}}{E_{cm}} \hat{\sigma}_{ab} \Big|_{x_a x_b = \frac{M_{ll}}{E_{cm}} e^{\pm Y_{ll}}}}$$

