ΔA_{CP} formalism for $D^* \to D^0(h^-h^+)\pi_s$ decays

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1 Integrated asymmetries

Given a sample of reconstructed $D^{*+} \to D^0(h^-h^+)\pi_s^+$ and $D^{*-} \to \overline{D}^0(h^-h^+)\pi_s^-$ decays, to first order in $x\Gamma t$ and $y\Gamma t$ the D^{*+} and D^{*-} yields can be written as

$$
\begin{aligned}
N^{hh}_{D^{*\pm}} \quad \propto \quad & \int \mathrm{d}t\, \mathrm{d}\vec{r}_{\mathrm{PV}}\, \mathrm{d}\vec{r}_{\mathrm{SV}}\, \mathrm{d}\vec{p}_{D^*}\, \mathrm{d}\vec{p}_{\pi_s}\, \mathrm{d}\vec{p}_{h^+}\, \mathrm{d}\vec{p}_{h^-}\, \mathcal{N}_{D^{*\pm}} f_{D^{*\pm}}(\vec{p}_{D^*})\, |A^{\pm}_{hh}|^2\, e^{-\Gamma t}(1 + R^{\pm}_{hh}\Gamma t)\, \delta(t - t(\vec{r}_{\mathrm{PV}},\, \vec{r}_{\mathrm{SV}},\, \vec{p}_{h^+},\, \vec{p}_{h^-})) \\
& f_{\mathrm{PV}}(\vec{r}_{\mathrm{PV}})\, f_{\mathrm{SV}}(\vec{r}_{\mathrm{SV}})\, f_{hh\pi_s}(\vec{p}_{h^+},\, \vec{p}_{h^-},\, \vec{p}_{\pi_s}\,|\,\vec{p}_{D^*})\, \varepsilon_{hh}(\vec{r}_{\mathrm{PV}},\, \vec{r}_{\mathrm{SV}},\, \vec{p}_{h^+},\, \vec{p}_{h^-})\, \varepsilon_{\pi_s^\pm}(\vec{r}_{\mathrm{PV}},\, \vec{p}_{\pi_s}),
\end{aligned}
$$

where $\mathcal{N}_{D^{*\pm}}$ is the number of produced $D^{*\pm}$ mesons, A^+_{hh} and A^-_{hh} are the $D^0 \to h^-h^+$ and $\overline{D}^0 \to h^-h^+$ decay amplitudes, respectively, Γ is the mean D^0 lifetime, t is the D^0 decay time and $t(\vec{r}_{\mathrm{PV}},\, \vec{r}_{\mathrm{SV}},\, \vec{p}_{h^+}\, \vec{p}_{h^-})$ is its function of the various indicated quantities, $\vec{p}_X$ is the three-momentum of the given particle, $\vec{r}_{\mathrm{PV}}$ is the position of the primary vertex, $\vec{r}_{\mathrm{SV}}$ is the position of the secondary vertex, $f_{D^{*\pm}}(\vec{p}_{D^*})$ is the p.d.f. of $\vec{p}_{D^*}$, $f_{\mathrm{PV}}(\vec{r}_{\mathrm{PV}})$ is the p.d.f. of $\vec{r}_{\mathrm{PV}}$, $f_{\mathrm{SV}}(\vec{r}_{\mathrm{SV}})$ is the p.d.f. of $\vec{r}_{\mathrm{SV}}$, $f_{hh\pi_s}(\vec{p}_{h^+},\, \vec{p}_{h^-},\, \vec{p}_{\pi_s}\,|\,\vec{p}_{D^*})$ is the p.d.f. of $(\vec{p}_{h^+},\, \vec{p}_{h^-},\, \vec{p}_{\pi_s})$ given $\vec{p}_{D^*}$, $\varepsilon_{hh}(\vec{r}_{\mathrm{PV}},\, \vec{r}_{\mathrm{SV}},\, \vec{p}_{h^+},\, \vec{p}_{h^-})$ is the overall D^0 efficiency as a function of the various indicated quantities, and $\varepsilon_{\pi_s^\pm}(\vec{r}_{\mathrm{PV}},\, \vec{p}_{\pi_s})$ is the overall efficiency for soft pions as a function of three-momentum and primary-vertex position. The quantities $R^{\pm}_{hh}$ are given by

$$
R^+_{hh} \equiv -\eta^{CP}_{hh} \left|\frac{q}{p}\right| \left|\frac{A^-_{hh}}{A^+_{hh}}\right| (y\cos\phi_{hh} - x\sin\phi_{hh}),
$$

$$
R^-_{hh} \equiv -\eta^{CP}_{hh} \left|\frac{p}{q}\right| \left|\frac{A^+_{hh}}{A^-_{hh}}\right| (y\cos\phi_{hh} + x\sin\phi_{hh}),
$$

with

$$
\phi_{hh} \equiv \arg\left(-\eta^{CP}_{hh}\frac{q}{p}\frac{A^-_{hh}}{A^+_{hh}}\right).
$$

By defining the operator $\mathcal{G}_{hh}[g]$, acting on the generic function g, as

$$
\mathcal{G}_{hh}[g] \equiv \mathcal{G}'_{hh}[g] / \mathcal{G}'_{hh}[1],
$$

with the operator $\mathcal{G}'_{hh}[g]$ given by

$$
\begin{aligned}
\mathcal{G}'_{hh}[g] \quad \equiv \quad & \int \mathrm{d}t\, \mathrm{d}\vec{r}_{\mathrm{PV}}\, \mathrm{d}\vec{r}_{\mathrm{SV}}\, \mathrm{d}\vec{p}_{D^*}\, \mathrm{d}\vec{p}_{\pi_s}\, \mathrm{d}\vec{p}_{h^+}\, \mathrm{d}\vec{p}_{h^-}\, e^{-\Gamma t}\, f_{D^*}(\vec{p}_{D^*})\, f_{\mathrm{PV}}(\vec{r}_{\mathrm{PV}})\, f_{\mathrm{SV}}(\vec{r}_{\mathrm{SV}})\, \delta(t - t(\vec{r}_{\mathrm{PV}},\, \vec{r}_{\mathrm{SV}},\, \vec{p}_{h^+},\, \vec{p}_{h^-})) \\
& f_{hh\pi_s}(\vec{p}_{h^+},\, \vec{p}_{h^-},\, \vec{p}_{\pi_s}\,|\,\vec{p}_{D^*})\, \varepsilon_{hh}(\vec{r}_{\mathrm{PV}},\, \vec{r}_{\mathrm{SV}},\, \vec{p}_{h^+},\, \vec{p}_{h^-})\, \varepsilon_{\pi_s}(\vec{r}_{\mathrm{PV}},\, \vec{p}_{\pi_s})\, g,
\end{aligned}
$$

where

$$
f_{D^*}(\vec{p}_{D^*}) \equiv \frac{\mathcal{N}_{D^{*+}} f_{D^{*+}}(\vec{p}_{D^*}) + \mathcal{N}_{D^{*-}} f_{D^{*-}}(\vec{p}_{D^*})}{\mathcal{N}_{D^{*+}} + \mathcal{N}_{D^{*-}}},
$$

$$
\varepsilon_{\pi_s}(\vec{r}_{\mathrm{PV}},\, \vec{p}_{\pi_s}) \equiv \frac{\varepsilon_{\pi_s^+}(\vec{r}_{\mathrm{PV}},\, \vec{p}_{\pi_s}) + \varepsilon_{\pi_s^-}(\vec{r}_{\mathrm{PV}},\, \vec{p}_{\pi_s})}{2},
$$

one obtains

$$
\begin{aligned}
A_{\rm raw}^{hh} &\equiv \frac{N_{D^{*+}}^{hh} - N_{D^{*-}}^{hh}}{N_{D^{*+}}^{hh} + N_{D^{*-}}^{hh}} = \\
&= \Big(\mathcal{G}_{hh}\left[A_{CP}^{hh} + A_{\rm P}(\vec{p}_{D^*}) + A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s}) + A_{CP}^{hh} A_{\rm P}(\vec{p}_{D^*})\, A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s})\right] + \\
&\quad + \Gamma\, R_{hh}\, \mathcal{G}_{hh}\left[t\left(A_{CP}^{hh} + A_{R_{hh}} + A_{\rm P}(\vec{p}_{D^*}) + A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s}) + A_{CP}^{hh}\, A_{R_{hh}}\, A_{\rm P}(\vec{p}_{D^*}) +\right.\right. \\
&\quad + A_{CP}^{hh}\, A_{R_{hh}}\, A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s}) + A_{CP}^{hh}\, A_{\rm P}(\vec{p}_{D^*})\, A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s}) + \\
&\quad \left.\left. + A_{R_{hh}}\, A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s})\, A_{\rm P}(\vec{p}_{D^*})\right)\right]\Big) \\
&\quad \Big(\mathcal{G}_{hh}\left[1 + A_{CP}^{hh}\, A_{\rm P}(\vec{p}_{D^*}) + A_{CP}^{hh}\, A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s}) + A_{\rm P}(\vec{p}_{D^*})\, A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s})\right] + \\
&\quad + \Gamma\, R_{hh}\, \mathcal{G}_{hh}\left[t\left(1 + A_{CP}^{hh}\, A_{R_{hh}} + A_{CP}^{hh}\, A_{\rm P}(\vec{p}_{D^*}) + A_{CP}^{hh}\, A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s}) +\right.\right. \\
&\quad + A_{R_{hh}}\, A_{\rm P}(\vec{p}_{D^*}) + A_{R_{hh}}\, A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s} + A_{\rm P}(\vec{p}_{D^*})\, A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s}) + \\
&\quad \left.\left. + A_{CP}^{hh}\, A_{R_{hh}}\, A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s})\, A_{\rm P}(\vec{p}_{D^*})\right)\right]\Big)^{-1},
\end{aligned}
$$

where

$$
\begin{aligned}
A_{CP}^{hh} &\equiv \frac{|A_{hh}^+|^2 - |A_{hh}^-|^2}{|A_{hh}^+|^2 + |A_{hh}^-|^2}, \\
A_{\rm P}(\vec{p}_{D^*}) &\equiv \frac{\mathcal{N}_{D^{*+}} f_{D^{*+}}(\vec{p}_{D^*}) - \mathcal{N}_{D^{*-}} f_{D^{*-}}(\vec{p}_{D^*})}{\mathcal{N}_{D^{*+}} f_{D^{*+}}(\vec{p}_{D^*}) + \mathcal{N}_{D^{*-}} f_{D^{*-}}(\vec{p}_{D^*})}, \\
A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s}) &\equiv \frac{\varepsilon_{\pi_s^+}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s}) - \varepsilon_{\pi_s^-}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s})}{\varepsilon_{\pi_s^+}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s}) + \varepsilon_{\pi_s^-}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s})}, \\
R_{hh} &\equiv \frac{R_{hh}^+ + R_{hh}^-}{2}, \\
A_{R_{hh}} &\equiv \frac{R_{hh}^+ - R_{hh}^-}{R_{hh}^+ + R_{hh}^-}.
\end{aligned}
$$

Neglecting all higher order terms, the expression of $A_{\rm raw}^{hh}$ simplifies to

$$
A_{\rm raw}^{hh} = \frac{\mathcal{G}_{hh}\left[A_{CP}^{hh} + A_{\rm P}(\vec{p}_{D^*}) + A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s})\right] + \Gamma\, R_{hh}\, \mathcal{G}_{hh}\left[t\left(A_{CP}^{hh} + A_{R_{hh}} + A_{\rm P}(\vec{p}_{D^*}) + A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s})\right)\right]}{1 + \Gamma\, R_{hh}\, \mathcal{G}_{hh}[t]}.
$$

Note that this expression of the integrated raw asymmetry is valid if the integrated values of the various asymmetries are small, independently of their variations over phase space. Expanding further one obtains

$$
A_{\rm raw}^{hh} = A_{CP}^{hh} - A_{\Gamma}^{hh}\, \Gamma\langle t\rangle_{hh} + A_{\rm P}^{hh} + A_{\rm D}^{hh} + \Gamma R_{hh}\left(\langle t\, A_{\rm P}\rangle_{hh} - \langle t\rangle_{hh}\, A_{\rm P}^{hh} + \langle t\, A_{\rm D}\rangle_{hh} - \langle t\rangle_{hh}\, A_{\rm D}^{hh}\right),
$$

where

$$
\begin{aligned}
A_{\Gamma}^{hh} &\equiv -A_{R_{hh}} R_{hh}, \\
A_{\rm P}^{hh} &\equiv \mathcal{G}_{hh}\left[A_{\rm P}(\vec{p}_{D^*})\right], \\
A_{\rm D}^{hh} &\equiv \mathcal{G}_{hh}\left[A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s})\right], \\
\langle t\rangle_{hh} &\equiv \mathcal{G}_{hh}\left[t\right], \\
\langle t\, A_{\rm P}\rangle_{hh} &\equiv \mathcal{G}_{hh}\left[t\, A_{\rm P}(\vec{p}_{D^*})\right], \\
\langle t\, A_{\rm D}\rangle_{hh} &\equiv \mathcal{G}_{hh}\left[t\, A_{\rm D}(\vec{r}_{\rm PV},\, \vec{p}_{\pi_s})\right].
\end{aligned}
$$

Finally, taking $\langle t\, A_{\rm P}\rangle_{hh} \simeq \langle t\rangle_{hh}\, A_{\rm P}^{hh}$ and $\langle t\, A_{\rm D}\rangle_{hh} \simeq \langle t\rangle_{hh} A_{\rm D}^{hh}$, one obtains the familiar expression

$$
A_{\rm raw}^{hh} = A_{CP}^{hh} - A_{\Gamma}^{hh}\, \Gamma\langle t\rangle_{hh} + A_{\rm P}^{hh} + A_{\rm D}^{hh}.
$$

2 Equalisation of K^-K^+ and $\pi^-\pi^+$ distributions

In the following, the notation will be simplified by neglecting the small effect of time evolution and the dependence of the efficiencies on origin and decay vertices.

2.1 Weighting in D^* and π_s kinematics

By introducing the decay rate as a function of D^* and π_s momenta

$$\Gamma^{hh}_{D^{*\pm}}(\vec{p}_{D^*},\vec{p}_{\pi_s}) \equiv \frac{\mathrm{d}N^{hh}_{D^{*\pm}}}{\mathrm{d}\vec{p}_{D^*}\,\mathrm{d}\vec{p}_{\pi_s}} \propto \int \mathrm{d}\vec{p}_{h^+}\,\mathrm{d}\vec{p}_{h^-}\,\mathcal{N}_{D^{*\pm}} f_{D^{*\pm}}(\vec{p}_{D^*})\,|A^{\pm}_{hh}|^2\, f_{hh\pi_s}(\vec{p}_{h^+},\vec{p}_{h^-},\vec{p}_{\pi_s}\,|\,\vec{p}_{D^*})\,\varepsilon_{hh}(\vec{p}_{h^+},\vec{p}_{h^-})\,\varepsilon_{\pi_s^\pm}(\vec{p}_{\pi_s}),$$

one can define the weighting function

$$W(\vec{p}_{D^*},\vec{p}_{\pi_s}) \equiv \frac{\Gamma^{\pi\pi}_{D^{*+}}(\vec{p}_{D^*},\vec{p}_{\pi_s})+\Gamma^{\pi\pi}_{D^{*-}}(\vec{p}_{D^*},\vec{p}_{\pi_s})}{\Gamma^{KK}_{D^{*+}}(\vec{p}_{D^*},\vec{p}_{\pi_s})+\Gamma^{KK}_{D^{*-}}(\vec{p}_{D^*},\vec{p}_{\pi_s})}.$$

In principle this is the idea adopted in the ΔA_{CP} paper.[1] The weighted KK raw asymmetry becomes

$$A'^{KK}_{\mathrm{raw}} = \frac{\int \mathrm{d}\vec{p}_{D^*}\,\mathrm{d}\vec{p}_{\pi_s}\,\left[\Gamma^{KK}_{D^{*+}}(\vec{p}_{D^*},\vec{p}_{\pi_s})-\Gamma^{KK}_{D^{*-}}(\vec{p}_{D^*},\vec{p}_{\pi_s})\right]W(\vec{p}_{D^*},\vec{p}_{\pi_s})}{\int \mathrm{d}\vec{p}_{D^*}\,\mathrm{d}\vec{p}_{\pi_s}\,\left[\Gamma^{\pi\pi}_{D^{*+}}(\vec{p}_{D^*},\vec{p}_{\pi_s})+\Gamma^{\pi\pi}_{D^{*-}}(\vec{p}_{D^*},\vec{p}_{\pi_s})\right]},$$

and thus

$$\begin{aligned}\Delta A'_{\mathrm{raw}} &\equiv A'^{KK}_{\mathrm{raw}} - A^{\pi\pi}_{\mathrm{raw}} = \\ &= \frac{\int \mathrm{d}\vec{p}_{D^*}\,\mathrm{d}\vec{p}_{\pi_s}\,\left\{\left[\Gamma^{KK}_{D^{*+}}(\vec{p}_{D^*},\vec{p}_{\pi_s})-\Gamma^{KK}_{D^{*-}}(\vec{p}_{D^*},\vec{p}_{\pi_s})\right]W(\vec{p}_{D^*},\vec{p}_{\pi_s})-\left[\Gamma^{\pi\pi}_{D^{*+}}(\vec{p}_{D^*},\vec{p}_{\pi_s})-\Gamma^{\pi\pi}_{D^{*-}}(\vec{p}_{D^*},\vec{p}_{\pi_s})\right]\right\}}{\int \mathrm{d}\vec{p}_{D^*}\,\mathrm{d}\vec{p}_{\pi_s}\,\left[\Gamma^{\pi\pi}_{D^{*+}}(\vec{p}_{D^*},\vec{p}_{\pi_s})+\Gamma^{\pi\pi}_{D^{*-}}(\vec{p}_{D^*},\vec{p}_{\pi_s})\right]}.\end{aligned}$$

To simplify the notation further, it is convenient to rewrite the decay rate in a more compact form

$$\Gamma^{hh}_{D^{*\pm}}(\vec{p}_{D^*},\vec{p}_{\pi_s}) \propto \vartheta^{hh}(\vec{p}_{D^*},\vec{p}_{\pi_s})\left[1\pm A^{hh}_{CP}\right]\left[1\pm A_{\mathrm{D}}(\vec{p}_{\pi_s})\right]\left[1\pm A_{\mathrm{P}}(\vec{p}_{D^*})\right],$$

with

$$\vartheta^{hh}(\vec{p}_{D^*},\vec{p}_{\pi_s}) \equiv f_{D^*}(\vec{p}_{D^*})\varepsilon_{\pi_s}(\vec{p}_{\pi_s})\int \mathrm{d}\vec{p}_{h^+}\,\mathrm{d}\vec{p}_{h^-}\, f_{hh\pi_s}(\vec{p}_{h^+},\vec{p}_{h^-},\vec{p}_{\pi_s}\,|\,\vec{p}_{D^*})\,\varepsilon_{hh}(\vec{p}_{h^+},\vec{p}_{h^-}).$$

After some algebra, the following expression for $\Delta A'_{\mathrm{raw}}$ can be obtained

$$\Delta A'_{\mathrm{raw}} = K\,\Delta A_{CP},$$

with

$$K = \frac{\int \mathrm{d}\vec{p}_{D^*}\,\mathrm{d}\vec{p}_{\pi_s}\,\vartheta^{\pi\pi}(\vec{p}_{D^*},\vec{p}_{\pi_s})\left[1-A^2_{\mathrm{D}}(\vec{p}_{\pi_s})\right]\left[1-A^2_{\mathrm{P}}(\vec{p}_{D^*})\right]\left[1+A^{KK}_{CP}A_{\mathrm{D}}(\vec{p}_{\pi_s})+A^{KK}_{CP}A_{\mathrm{P}}(\vec{p}_{D^*})+A_{\mathrm{D}}(\vec{p}_{\pi_s})A_{\mathrm{P}}(\vec{p}_{D^*})\right]^{-1}}{\int \mathrm{d}\vec{p}_{D^*}\,\mathrm{d}\vec{p}_{\pi_s}\,\vartheta^{\pi\pi}(\vec{p}_{D^*},\vec{p}_{\pi_s})\left[1+A^{\pi\pi}_{CP}A_{\mathrm{D}}(\vec{p}_{\pi_s})+A^{\pi\pi}_{CP}A_{\mathrm{P}}(\vec{p}_{D^*})+A_{\mathrm{D}}(\vec{p}_{\pi_s})A_{\mathrm{P}}(\vec{p}_{D^*})\right]}.$$

If the detection and production asymmetries, $A_{\mathrm{D}}(\vec{p}_{\pi_s})$ and $A_{\mathrm{P}}(\vec{p}_{D^*})$, are small over the full phase space, the factor K approximates to unity, and $\Delta A'_{\mathrm{raw}}$ measures ΔA_{CP}. For example, taking $A_{\mathrm{D}}(\vec{p}_{\pi_s})$ and $A_{\mathrm{P}}(\vec{p}_{D^*})$ to be $\mathcal{O}(10^{-2})$ uniformly over the full phase space, K is equal to unity up to a few parts in 10^{-4}, and considering the present size of the statistical uncertainty on $\Delta A'_{\mathrm{raw}}$, that is order of 20%, such a difference would turn out to be entirely negligible. Even with a detection asymmetry as large as 20% uniformly over the full phase space, the factor K would differ from unity by about 4%, *i.e.* still providing a sub-dominant systematic uncertainty to the measurement. By contrast, phase space regions with detection asymmetries at ±100% would not give any contribution to the measurement: if only regions with $A_{\mathrm{D}}(\vec{p}_{\pi_s}) = \pm 1$ were considered, the value of $\Delta A'_{\mathrm{raw}}$ would be identically zero. Such regions should be excluded by means of fiducial requirements, but there is no reason to stay too far from them, provided that the raw asymmetry does not exceed order of 20% or so (with present statistical uncertainties).

[1] For simplicity, in the ΔA_{CP} paper only the D^* kinematics were weighted, exploiting the high correlation between D^* and π_s kinematics to have a good automatic weighting also in the π_s kinematics. However this is not ideal: although the correlation is certainly high, it is not 100%. Formally, one would have had to perform a six-dimensional weighting of D^* and π_s kinematics for a rigorous application of the method described in this section. Although not reported here for brevity, using similar calculations it can be shown that weighting in D^* kinematics only leads to a rigorous cancellation of the D^* production asymmetry in ΔA_{CP}, but not to a rigorous cancellation of the π_s detection asymmetry, and vice versa weighting in π_s kinematics leads to a rigorous cancellation of the π_s detection asymmetry but not to a rigorous cancellation of the D^* production asymmetry.

2.2 Weighting in D^0 kinematics

The decay rate as a function of the D^0 momentum, $\vec{p}_{D^0} \equiv \vec{p}_{D^*} - \vec{p}_{\pi_s}$, can be written as

$$\Gamma^{hh}_{D^{*\pm}}\left(\vec{p}_{D^0}\right) \equiv \int \mathrm{d}\vec{p}_{D^*}\, \mathrm{d}\vec{p}_{\pi_s} \Gamma^{hh}_{D^{*\pm}}\left(\vec{p}_{D^*}, \vec{p}_{\pi_s}\right) \delta\left(\vec{p}_{D^0} - \vec{p}_{D^*} + \vec{p}_{\pi_s}\right).$$

By introducing the weighting function

$$H\left(\vec{p}_{D^0}\right) \equiv \frac{\Gamma^{\pi\pi}_{D^{*+}}\left(\vec{p}_{D^0}\right) + \Gamma^{\pi\pi}_{D^{*-}}\left(\vec{p}_{D^0}\right)}{\Gamma^{KK}_{D^{*+}}\left(\vec{p}_{D^0}\right) + \Gamma^{KK}_{D^{*-}}\left(\vec{p}_{D^0}\right)},$$

the weighted raw-asymmetry difference is given by

$$\begin{aligned}\Delta A'_{\mathrm{raw}} &\equiv A'^{KK}_{\mathrm{raw}} - A^{\pi\pi}_{\mathrm{raw}} = \\ &= \frac{\int \mathrm{d}\vec{p}_{D^0}\, \left\{\left[\Gamma^{KK}_{D^{*+}}\left(\vec{p}_{D^0}\right) - \Gamma^{KK}_{D^{*-}}\left(\vec{p}_{D^0}\right)\right] H\left(\vec{p}_{D^0}\right) - \left[\Gamma^{\pi\pi}_{D^{*+}}\left(\vec{p}_{D^0}\right) - \Gamma^{\pi\pi}_{D^{*-}}\left(\vec{p}_{D^0}\right)\right]\right\}}{\int \mathrm{d}\vec{p}_{D^0}\, \left[\Gamma^{\pi\pi}_{D^{*+}}\left(\vec{p}_{D^0}\right) + \Gamma^{\pi\pi}_{D^{*-}}\left(\vec{p}_{D^0}\right)\right]}.\end{aligned}$$

After some algebra and neglecting higher order terms it can be shown that

$$\Delta A'_{\mathrm{raw}} = \Delta A_{CP} + \frac{\int \mathrm{d}\vec{p}_{D^0}\, \mathcal{F}_{KK}\left[A_{\mathrm{D}}\left(\vec{p}_{\pi_s}\right) + A_{\mathrm{P}}\left(\vec{p}_{D^*}\right)\right] \mathcal{F}_{\pi\pi}[1]/\mathcal{F}_{KK}[1] - \mathcal{F}_{\pi\pi}\left[A_{\mathrm{D}}\left(\vec{p}_{\pi_s}\right) + A_{\mathrm{P}}\left(\vec{p}_{D^*}\right)\right]}{\int \mathrm{d}\vec{p}_{D^0}\, \mathcal{F}_{\pi\pi}[1]},$$

where the operator $\mathcal{F}_{hh}$ acting on the generic function g is defined as

$$\mathcal{F}_{hh}[g] \equiv \int \mathrm{d}\vec{p}_{D^*}\, \mathrm{d}\vec{p}_{\pi_s} \vartheta^{hh}\left(\vec{p}_{D^*}, \vec{p}_{\pi_s}\right) \delta\left(\vec{p}_{D^0} - \vec{p}_{D^*} + \vec{p}_{\pi_s}\right) g.$$

Considering that

$$f_{hh\pi_s}(\vec{p}_{h^+}, \vec{p}_{h^-}, \vec{p}_{\pi_s} \,|\, \vec{p}_{D^*}) = f_{hh}(\vec{p}_{h^+}, \vec{p}_{h^-} \,|\, \vec{p}_{D^*} - \vec{p}_{\pi_s}) f_{\pi_s}(\vec{p}_{\pi_s} \,|\, \vec{p}_{D^*}) = f_{hh}(\vec{p}_{h^+}, \vec{p}_{h^-} \,|\, \vec{p}_{D^0}) f_{\pi_s}(\vec{p}_{\pi_s} \,|\, \vec{p}_{D^*}),$$

the following relation holds

$$\mathcal{F}_{KK}\left[A_{\mathrm{D}}\left(\vec{p}_{\pi_s}\right) + A_{\mathrm{P}}\left(\vec{p}_{D^*}\right)\right] \mathcal{F}_{\pi\pi}[1]/\mathcal{F}_{KK}[1] = \mathcal{F}_{\pi\pi}\left[A_{\mathrm{D}}\left(\vec{p}_{\pi_s}\right) + A_{\mathrm{P}}\left(\vec{p}_{D^*}\right)\right],$$

and so (in the limit of small asymmetries) $\Delta A'_{\mathrm{raw}} = \Delta A_{CP}$. In a few words, although equivalent, it is more convenient to perform event weighting in $\vec{p}_{D^0}$ rather than in $\vec{p}_{D^*}$ and $\vec{p}_{\pi_s}$, as it is technically simpler to implement three-dimensional weighting than working in six dimensions.

2.3 Could we do anything better?

It is interesting to note that there is even a better weighting function, *i.e.*

$$Q\left(\vec{p}_{D^*}, \vec{p}_{\pi_s}\right) \equiv \frac{\vartheta^{\pi\pi}\left(\vec{p}_{D^*}, \vec{p}_{\pi_s}\right)}{\vartheta^{KK}\left(\vec{p}_{D^*}, \vec{p}_{\pi_s}\right)} = \frac{\int \mathrm{d}\vec{p}_{h^+}\, \mathrm{d}\vec{p}_{h^-}\, f_{\pi\pi\pi_s}(\vec{p}_{h^+}, \vec{p}_{h^-}, \vec{p}_{\pi_s} \,|\, \vec{p}_{D^*})\, \varepsilon_{\pi\pi}(\vec{p}_{h^+}, \vec{p}_{h^-})}{\int \mathrm{d}\vec{p}_{h^+}\, \mathrm{d}\vec{p}_{h^-}\, f_{KK\pi_s}(\vec{p}_{h^+}, \vec{p}_{h^-}, \vec{p}_{\pi_s} \,|\, \vec{p}_{D^*})\, \varepsilon_{KK}(\vec{p}_{h^+}, \vec{p}_{h^-})}.$$

In such a case, $\Delta A'_{\mathrm{raw}}$ would measure ΔA_{CP} independently of how large the detection asymmetry is in any phase space region, provided that the average is small, as one has

$$K = \frac{\left(1 + A^{\pi\pi}_{\mathrm{DP}}\right)^2 - \left(A^{\pi\pi}_{\mathrm{D}} + A^{\pi\pi}_{\mathrm{P}}\right)^2}{\left(1 + A^{\pi\pi}_{CP} A^{\pi\pi}_{\mathrm{D}} + A^{\pi\pi}_{CP} A^{\pi\pi}_{\mathrm{P}} + A^{\pi\pi}_{\mathrm{DP}}\right)\left(1 + A^{KK}_{CP} A^{\pi\pi}_{\mathrm{D}} + A^{KK}_{CP} A^{\pi\pi}_{\mathrm{P}} + A^{\pi\pi}_{\mathrm{DP}}\right)},$$

where

$$A^{\pi\pi}_{\mathrm{D}} \equiv \frac{\int \mathrm{d}\vec{p}_{D^*}\, \mathrm{d}\vec{p}_{\pi_s}\, \vartheta^{\pi\pi}\left(\vec{p}_{D^*}, \vec{p}_{\pi_s}\right) A_{\mathrm{D}}(\vec{p}_{\pi_s})}{\int \mathrm{d}\vec{p}_{D^*}\, \mathrm{d}\vec{p}_{\pi_s}\, \vartheta^{\pi\pi}\left(\vec{p}_{D^*}, \vec{p}_{\pi_s}\right)},$$

$$A^{\pi\pi}_{\mathrm{P}} \equiv \frac{\int \mathrm{d}\vec{p}_{D^*}\, \mathrm{d}\vec{p}_{\pi_s}\, \vartheta^{\pi\pi}\left(\vec{p}_{D^*}, \vec{p}_{\pi_s}\right) A_{\mathrm{P}}(\vec{p}_{D^*})}{\int \mathrm{d}\vec{p}_{D^*}\, \mathrm{d}\vec{p}_{\pi_s}\, \vartheta^{\pi\pi}\left(\vec{p}_{D^*}, \vec{p}_{\pi_s}\right)},$$

$$A^{\pi\pi}_{\mathrm{DP}} \equiv \frac{\int \mathrm{d}\vec{p}_{D^*}\, \mathrm{d}\vec{p}_{\pi_s}\, \vartheta^{\pi\pi}\left(\vec{p}_{D^*}, \vec{p}_{\pi_s}\right) A_{\mathrm{D}}(\vec{p}_{\pi_s}) A_{\mathrm{P}}(\vec{p}_{D^*})}{\int \mathrm{d}\vec{p}_{D^*}\, \mathrm{d}\vec{p}_{\pi_s}\, \vartheta^{\pi\pi}\left(\vec{p}_{D^*}, \vec{p}_{\pi_s}\right)}.$$

A sample with phase-space regions characterised by +100% detection asymmetries in the left-hand side of the detector and −100% in the right-hand side (or vice versa) would be perfectly usable, as the large detection asymmetry in one region would counterbalance that in the other region. In other words, there is no need to apply fiducial requirements. By considering again that

$$f_{hh\pi_s}(\vec{p}_{h^+},\, \vec{p}_{h^-},\, \vec{p}_{\pi_s}\,|\, \vec{p}_{D^*}) = f_{hh}(\vec{p}_{h^+},\, \vec{p}_{h^-}\,|\, \vec{p}_{D^*} - \vec{p}_{\pi_s}) f_{\pi_s}(\vec{p}_{\pi_s}\,|\, \vec{p}_{D^*}),$$

it follows that the Q weighting function can be simplified to

$$Q\,(\vec{p}_{D^*},\, \vec{p}_{\pi_s}) = \frac{\int \mathrm{d}\vec{p}_{h^+}\, \mathrm{d}\vec{p}_{h^-}\, f_{\pi\pi}(\vec{p}_{h^+},\, \vec{p}_{h^-}\,|\, \vec{p}_{D^*} - \vec{p}_{\pi_s})\, \varepsilon_{\pi\pi}(\vec{p}_{h^+},\, \vec{p}_{h^-})}{\int \mathrm{d}\vec{p}_{h^+}\, \mathrm{d}\vec{p}_{h^-}\, f_{KK}(\vec{p}_{h^+},\, \vec{p}_{h^-}\,|\, \vec{p}_{D^*} - \vec{p}_{\pi_s})\, \varepsilon_{KK}(\vec{p}_{h^+},\, \vec{p}_{h^-})}.$$

The rates of selected prompt D^0 and $\overline{D}^0$ mesons decaying to a hh pair as a function of D^0 momentum are

$$\Gamma^{hh}_{D^0}(\vec{p}_{D^0}) \propto \mathcal{N}_{D^0} f_{D^0}(\vec{p}_{D^0}) |A^+_{hh}|^2 \int \mathrm{d}\vec{p}_{h^+}\, \mathrm{d}\vec{p}_{h^-}\, f_{hh}(\vec{p}_{h^+},\, \vec{p}_{h^-}\,|\, \vec{p}_{D^0})\, \varepsilon_{hh}(\vec{p}_{h^+},\, \vec{p}_{h^-}),$$

$$\Gamma^{hh}_{\overline{D}^0}(\vec{p}_{D^0}) \propto \mathcal{N}_{\overline{D}^0} f_{\overline{D}^0}(\vec{p}_{D^0}) |A^-_{hh}|^2 \int \mathrm{d}\vec{p}_{h^+}\, \mathrm{d}\vec{p}_{h^-}\, f_{hh}(\vec{p}_{h^+},\, \vec{p}_{h^-}\,|\, \vec{p}_{D^0})\, \varepsilon_{hh}(\vec{p}_{h^+},\, \vec{p}_{h^-}),$$

where $\mathcal{N}_{D^0}$ and $\mathcal{N}_{\overline{D}^0}$ are the numbers of produced D^0 and $\overline{D}^0$ mesons, and $f_{D^0}(\vec{p}_{D^0})$ and $f_{\overline{D}^0}(\vec{p}_{D^0})$ are the p.d.f.s of $\vec{p}_{D^0}$. Hence

$$\begin{aligned}\Gamma^{hh}_{D^0}(\vec{p}_{D^0}) + \Gamma^{hh}_{\overline{D}^0}(\vec{p}_{D^0}) \quad \propto \quad & [\mathcal{N}_{D^0} f_{D^0}(\vec{p}_{D^0}) + \mathcal{N}_{\overline{D}^0} f_{\overline{D}^0}(\vec{p}_{D^0})] \left(|A^+_{hh}|^2 + |A^-_{hh}|^2\right) \left[1 + A^{D^0}_{\mathrm{P}}(\vec{p}_{D^0}) A^{hh}_{CP}\right] \\ & \int \mathrm{d}\vec{p}_{h^+}\, \mathrm{d}\vec{p}_{h^-}\, f_{hh}(\vec{p}_{h^+},\, \vec{p}_{h^-}\,|\, \vec{p}_{D^0})\, \varepsilon_{hh}(\vec{p}_{h^+},\, \vec{p}_{h^-}),\end{aligned}$$

where

$$A^{D^0}_{\mathrm{P}}(\vec{p}_{D^0}) \equiv \frac{\mathcal{N}_{D^0} f_{D^0}(\vec{p}_{D^0}) - \mathcal{N}_{\overline{D}^0} f_{\overline{D}^0}(\vec{p}_{D^0})}{\mathcal{N}_{D^0} f_{D^0}(\vec{p}_{D^0}) + \mathcal{N}_{\overline{D}^0} f_{\overline{D}^0}(\vec{p}_{D^0})}.$$

It then follows that

$$Q\,(\vec{p}_{D^*},\, \vec{p}_{\pi_s}) = \frac{\Gamma^{\pi\pi}_{D^0}(\vec{p}_{D^*} - \vec{p}_{\pi_s}) + \Gamma^{\pi\pi}_{\overline{D}^0}(\vec{p}_{D^*} - \vec{p}_{\pi_s})}{\Gamma^{KK}_{D^0}(\vec{p}_{D^*} - \vec{p}_{\pi_s}) + \Gamma^{KK}_{\overline{D}^0}(\vec{p}_{D^*} - \vec{p}_{\pi_s})} \frac{|A^+_{KK}|^2 + |A^-_{KK}|^2}{|A^+_{\pi\pi}|^2 + |A^-_{\pi\pi}|^2} \frac{1 + A^{D^0}_{\mathrm{P}}(\vec{p}_{D^*} - \vec{p}_{\pi_s}) A^{KK}_{CP}}{1 + A^{D^0}_{\mathrm{P}}(\vec{p}_{D^*} - \vec{p}_{\pi_s}) A^{\pi\pi}_{CP}}.$$

The ratio of CP-averaged branching fractions is a constant, and since a global scale factor in the weighting function plays no role, it can be ignored. With a D^0 production asymmetry at the percent level, with excellent accuracy the Q function can be written simply as

$$Q\,(\vec{p}_{D^*},\, \vec{p}_{\pi_s}) \simeq \frac{\Gamma^{\pi\pi}_{D^0}(\vec{p}_{D^*} - \vec{p}_{\pi_s}) + \Gamma^{\pi\pi}_{\overline{D}^0}(\vec{p}_{D^*} - \vec{p}_{\pi_s})}{\Gamma^{KK}_{D^0}(\vec{p}_{D^*} - \vec{p}_{\pi_s}) + \Gamma^{KK}_{\overline{D}^0}(\vec{p}_{D^*} - \vec{p}_{\pi_s})}.$$

This function can be determined from data, counting the yields of $D^0 \to \pi^-\pi^+$ and $D^0 \to K^-K^+$ candidates in bins of D^0 momentum before the D^* reconstruction takes place.

Hence, the best approach would be to adopt this (approximated) Q function to weight KK events without applying any fiducial requirement, provided that the average detection and production asymmetries are small. If the average π_s detection asymmetry and D^0 and D^* production asymmetries are order of percent, it can be shown that $\Delta A'_{\mathrm{raw}}$ measures ΔA_{CP} up to a (relative) systematic uncertainty of $\mathcal{O}(10^{-4})$. In presence of a relatively large average π_s detection asymmetry, say at the 20% level, $\Delta A'_{\mathrm{raw}}$ would measure $\Delta A_{CP} \times [1 - (A^{\pi\pi}_{D})^2] \simeq \Delta A_{CP} \times [1 - (A^{\pi\pi}_{\mathrm{raw}})^2]$, corresponding to a relative systematic uncertainty of 4%, still sub-dominant with present statistical uncertainties and even correctable at the price of a small reduction in sensitivity. Unfortunately, D^0 candidates were not saved along in the Run-2 Turbo stream, unless they were associated to a π_s to form a D^*, and so the Q function cannot be determined from Run-2 data. In principle, there would be the possibility to use the full stream instead, but the price to pay would be that of a greatly reduced trigger efficiency in HLT2, which would spoil the statistical sensitivity in a significant way.

3 Conclusions

In this note a more rigorous formalism for the experimental determination of ΔA_{CP} is presented. The main conclusions are

- The approach to event weighting used to equalise the K^-K^+ and $\pi^-\pi^+$ distributions in the ΔA_{CP} paper is not ideal (Sec. 2.1). A better approach would be to weight in D^0 kinematics rather than in D^* and π_s kinematics (Sec. 2.2).
- In any case, there is no need to apply fiducial requirements which are too tight, as with present statistical uncertainties even if we were in presence of raw asymmetries as large as 20% over the full phase space, the additional systematic uncertainty to ΔA_{CP} would be as low as a relative 4%. For this reason, the fiducial requirements can be relaxed to approach the kinematic regions where the raw asymmetries start to raise abruptly, provided that they stay below order of 20%. Good part of the statistics discarded by the fiducial requirements adopted in the ΔA_{CP} paper could be reused to improve the statistical uncertainty of the measurement.
- There is even a better weighting approach using the D^0 kinematics of reconstructed mesons prior to the D^* reconstruction (Sec. 2.3). With this approach, there is no longer the need to apply fiducial requirements, allowing the full available statistics to be used. Unfortunately, this is not possible with the Turbo Run-2 stream, since D^0 candidates prior to the D^* reconstruction were entirely discarded during data taking. It is then recommended that, in Run 3, prescaled (but large enough) samples of $D^0 \to K^-K^+$ and $D^0 \to \pi^-\pi^+$ candidates are persisted.

Experimental findings will be shown at forthcoming charm working group meetings.