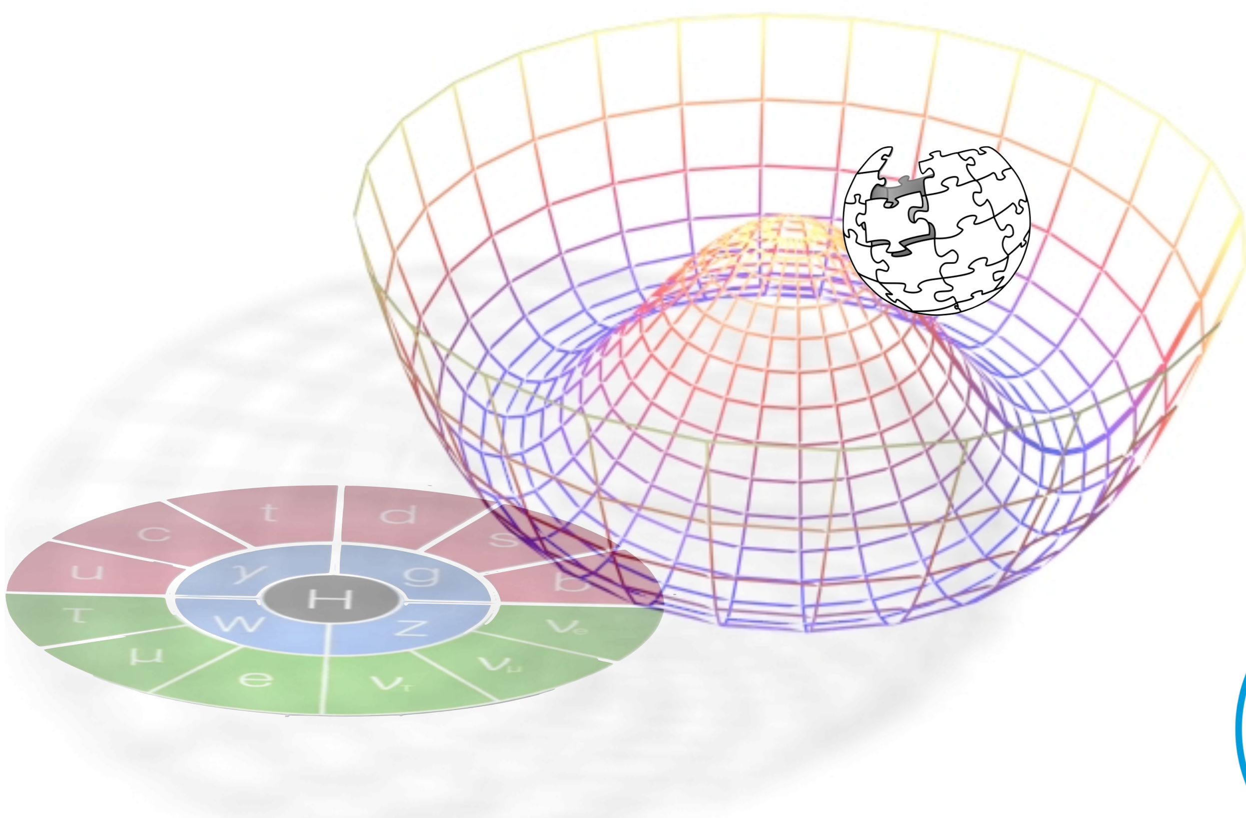
The Standard Model
of particle physics

CERN summer student lectures 2023

Lecture 5/5



Christophe Grojean
DESY (Hamburg)
Humboldt University (Berlin)

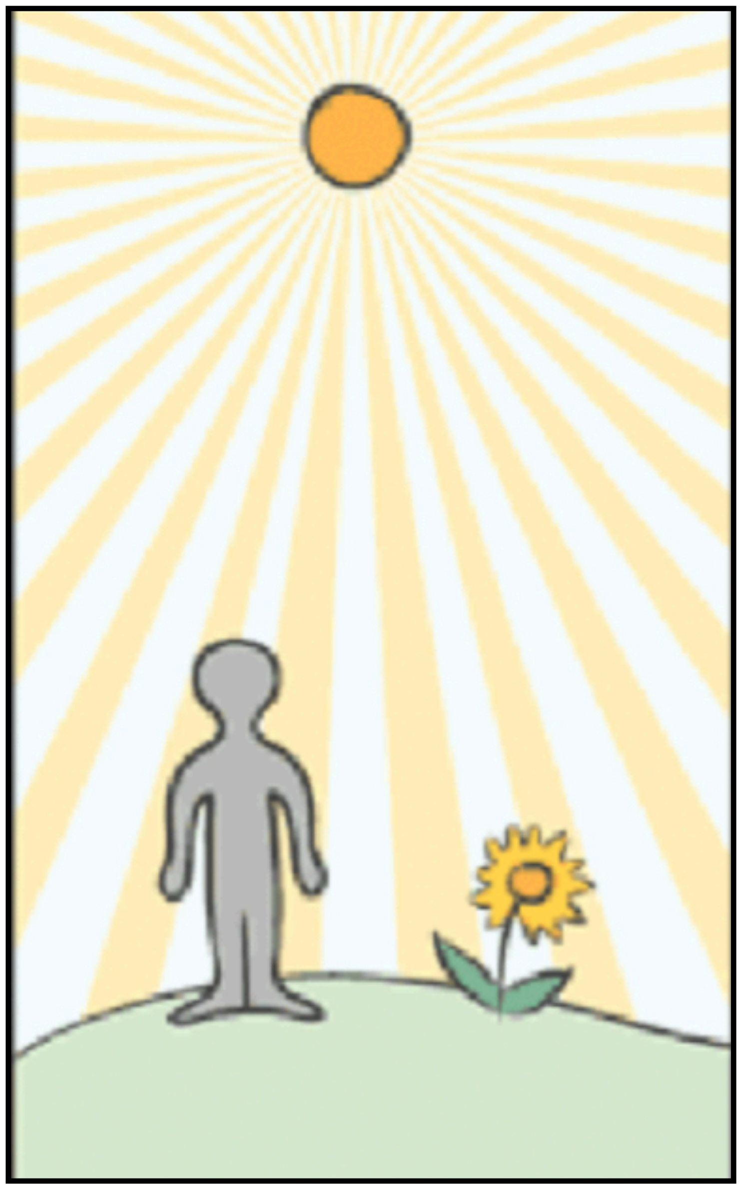
(christophe.grojean@desy.de)

Outline

- **Monday: symmetry**
  - Lagrangians
  - Lorentz symmetry - scalars, fermions, gauge bosons
  - Gauge/local symmetry as dynamical principle - Example: U(1) electromagnetism
- **Tuesday: SM symmetries**
  - Nuclear decay, Fermi theory and weak interactions: SU(2)
  - Dimensional analysis: cross-sections and life-time computations made simple
  - Strong interactions: SU(3)
- **Wednesday: chirality of weak interactions**
  - Chirality of weak interactions
  - Pion decay
- **Thursday: Higgs mechanism**
  - More about QCD
  - Spontaneous symmetry breaking and Higgs mechanism
  - Lepton and quark masses, quark mixings
  - Neutrino masses
- **Friday: quantum effects**
  - Running couplings
  - Asymptotic freedom of QCD
  - Anomalies cancelation

The Fundamental Interactions

Associated Symmetries



electromagnetic interactions

Photon γ

$\phi \rightarrow e^{iQ\theta} \phi$

change of phase of wavefunctions

weak interactions

weak bosons $W^{\pm}, Z^{0}$

$e \leftrightarrow \nu$ & $u \leftrightarrow d$

strong interactions

gluons g^{a}

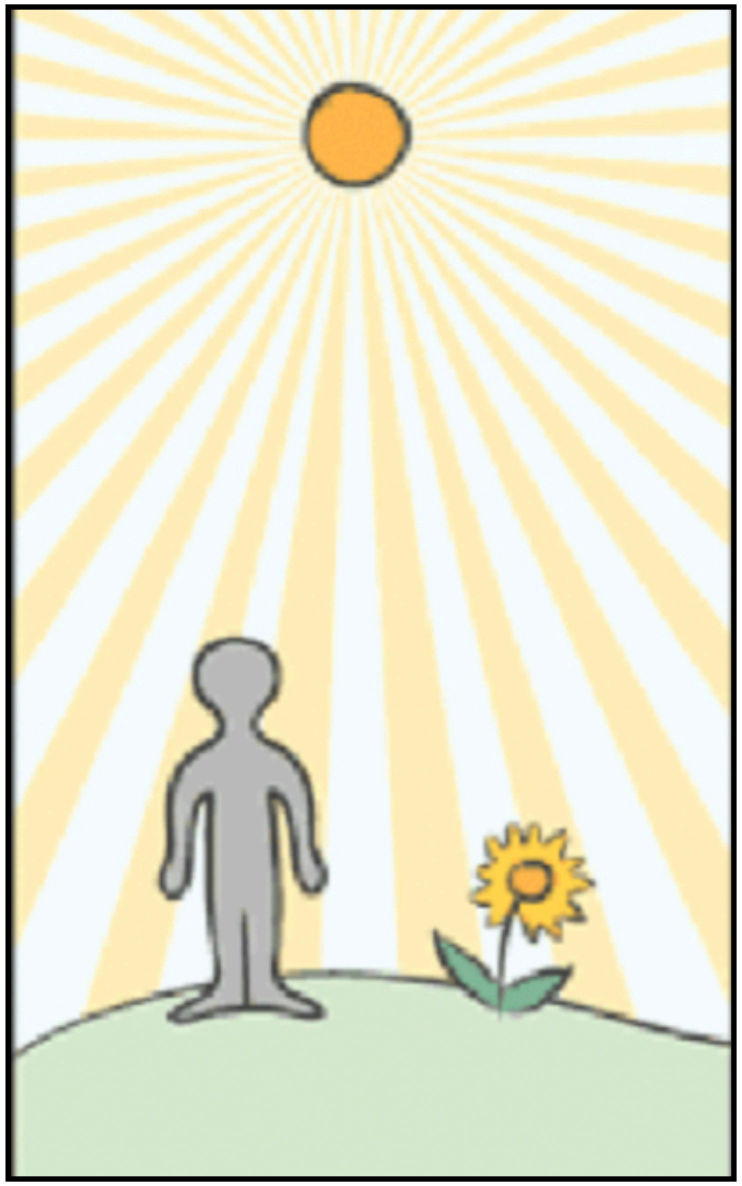
change of "colour" of quarks

(*gravity*)

Higgs boson is not a gauge field (spin-0 not spin-1) → no gauge symmetry associated to it

The interactions among particles

The fact that the Sun shines since several billions years tells us that there are different forms/origins of energies



Sun = enormous source of energy

- 1 cm^3 of ice melts under the Sun in about 40 minutes
- A 1 cm-thick cap of ice surrounding the sun at a distance of 150 million kilometres also melts in 40 minutes

Latent heat of ice: 300kJ/kg → energy needed 10^{28}J
Heat capacity of oil: 30 MJ/litre
Sun would need to burn 10^{18} litres of oil every 40 mn
(volume of the Sun ~ 10^{30} litres)

If the energy of the Sun were due to EM (=chemistry) then sun life time would be only 10^7 years

The interactions among particles

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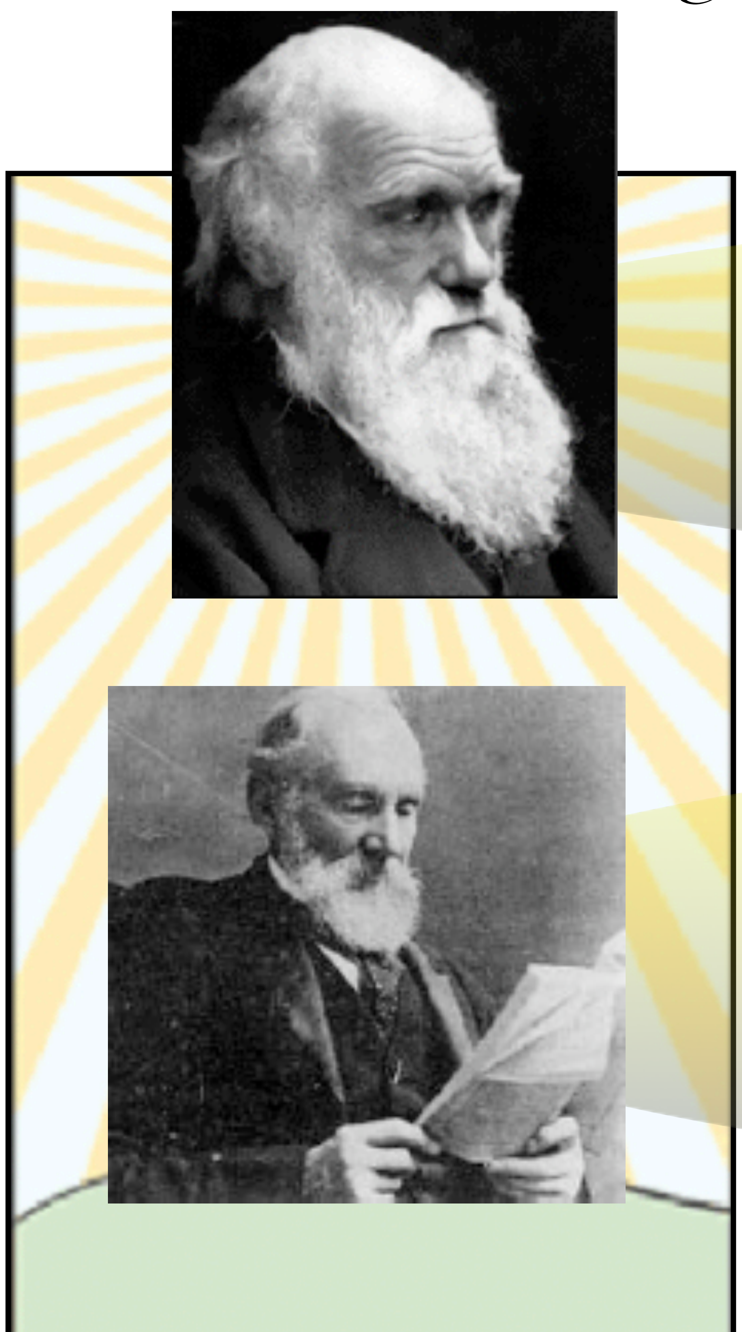


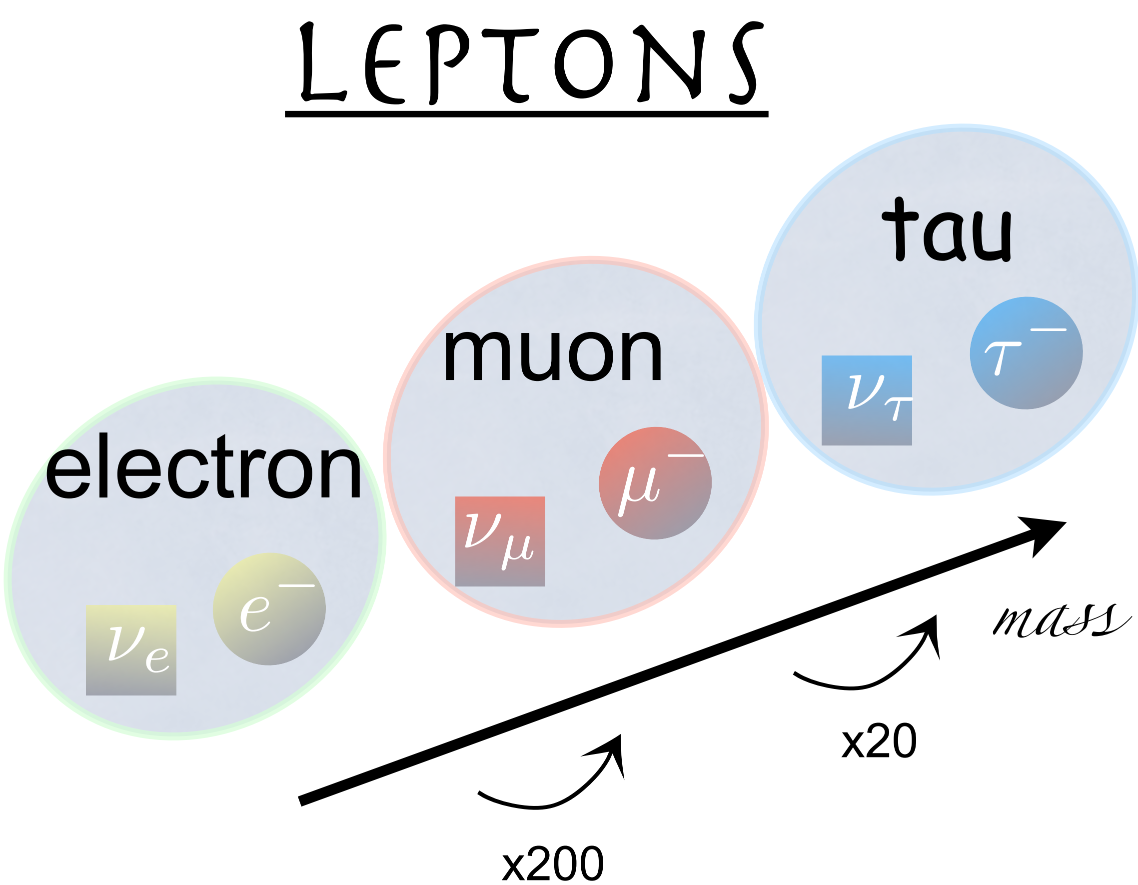
Darwin ("*On the origin of species by Means of Natural Selection*", 1st édition , 1859) estimated that the age of the Earth should be at least 300 millions years to give time for some hills to erode.

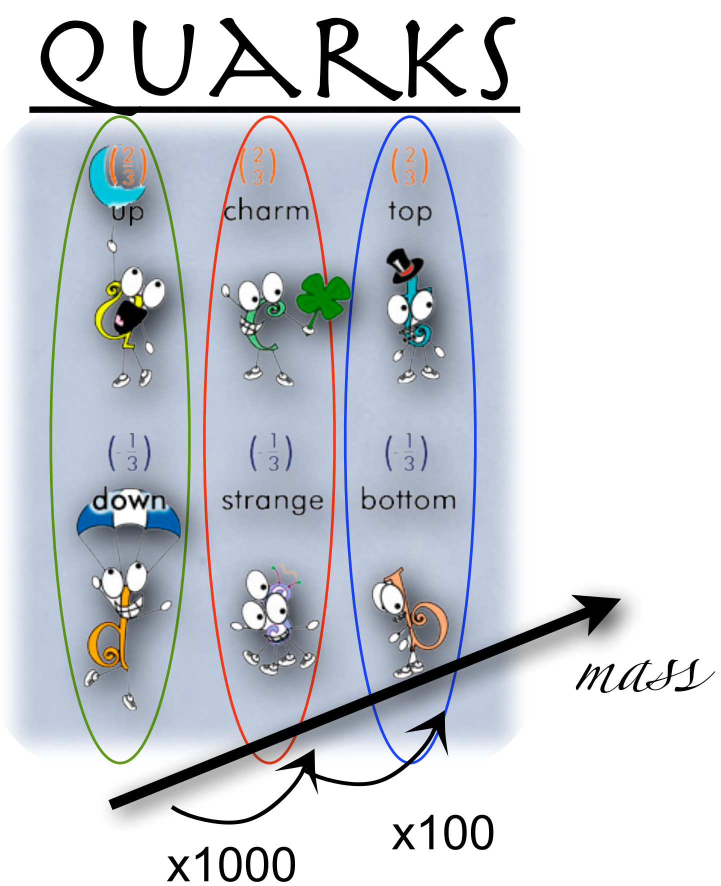
Thomson, Lord Kelvin
computed the gravitational/chemical energy of the Sun and, assuming that all this energy is covered into heat, concluded that the Sun cannot be older than 20 millions years

We know today that the Sun is about 4.7 billions years old

The Elementary Particles

(the genetic code of matter)





can exist as free particles
(at least for some time)

can only exist in bound states
(hadrons)

- baryons — proton p=(uud), neutron n=(udd)
- mesons — quark-antiquark

SM Summary

$W^{\pm}_\mu$ Z_μ A_μ

8 gluons g^A_μ 3 bosons W^a_μ 1 boson B_μ

color chirality hypercharge weak isospin electric charge effective coupling to Z boson

	SPIN	PARTICLES	$SU(3)_C$ ×	$SU(2)_L$ ×	$U(1)_Y$	T_{3_L}	$Q=T_{3_L}+Y$	g_{eff}	MEANING
LEPTONS	1/2	$L=\begin{pmatrix}\nu\\e\end{pmatrix}_L$	1	2	$\begin{pmatrix}-1/2\\-1/2\end{pmatrix}$	$\begin{pmatrix}1/2\\-1/2\end{pmatrix}$	$\begin{pmatrix}0\\-1\end{pmatrix}$	$\begin{pmatrix}1/2\\-1/2+\sin^2\theta_W\end{pmatrix}$	doublet under SU(2), singlet under SU(3)
		e_R	1	1	-1	0	-1	$\sin^2\theta_W$	singlet under SU(2) and SU(3)
QUARKS		$Q=\begin{pmatrix}u\\d\end{pmatrix}_L$	3	2	$\begin{pmatrix}1/6\\1/6\end{pmatrix}$	$\begin{pmatrix}1/2\\-1/2\end{pmatrix}$	$\begin{pmatrix}2/3\\-1/3\end{pmatrix}$	$\begin{pmatrix}1/2-\frac{2}{3}\sin^2\theta_W\\-1/2+\frac{1}{3}\sin^2\theta_W\end{pmatrix}$	doublet under SU(2), triplet under SU(3)
		u_R	3	1	2/3	0	2/3	$-\frac{1}{3}\sin^2\theta_W$	singlet under SU(2), triplet under SU(3)
		d_R	3	1	$-1/3$	0	$-1/3$	$\frac{1}{3}\sin^2\theta_W$	singlet under SU(2), triplet under SU(3)
HIGGS	0	$H=\begin{pmatrix}h^+\\h^0\end{pmatrix}$	1	2	$\begin{pmatrix}1/2\\1/2\end{pmatrix}$	$\begin{pmatrix}1/2\\-1/2\end{pmatrix}$	$\begin{pmatrix}1\\0\end{pmatrix}$	×	doublet under SU(2), singlet under SU(3)

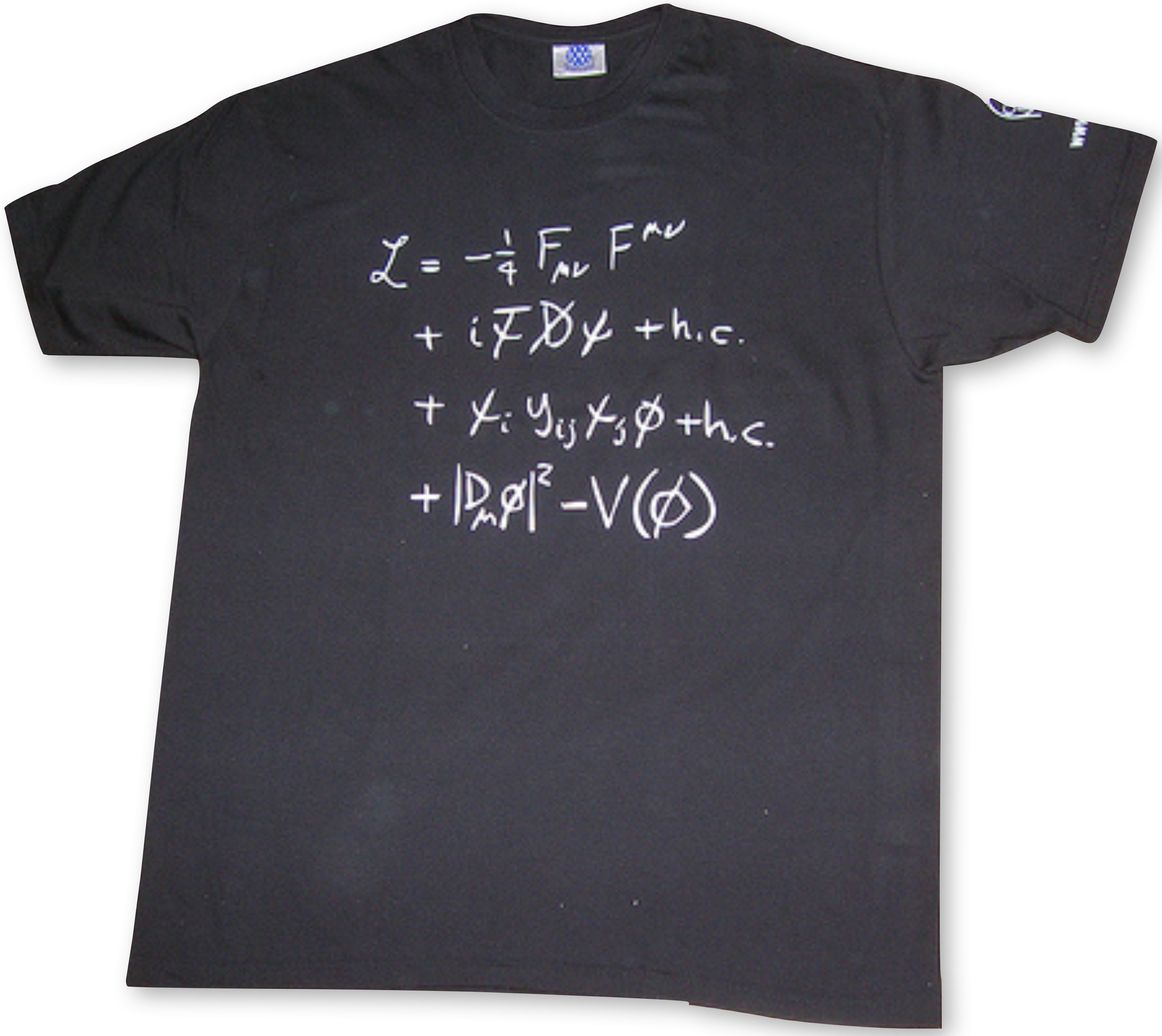
interactions from covariant derivatives

$$D_\mu H = \partial_\mu H - \tfrac{1}{2} g W^a_\mu \sigma^a H - \tfrac{1}{2} g' B_\mu H$$

$$D_\mu Q = \partial_\mu Q - g_s g^A_\mu \lambda^A Q - \tfrac{1}{2} g W^a_\mu \sigma^a Q - \tfrac{1}{12} g' B_\mu Q$$

The SM T-Shirt

Hopefully you now understand all what is written on the CERN T-shirt



and you can safely go to the beach with it without fearing any question

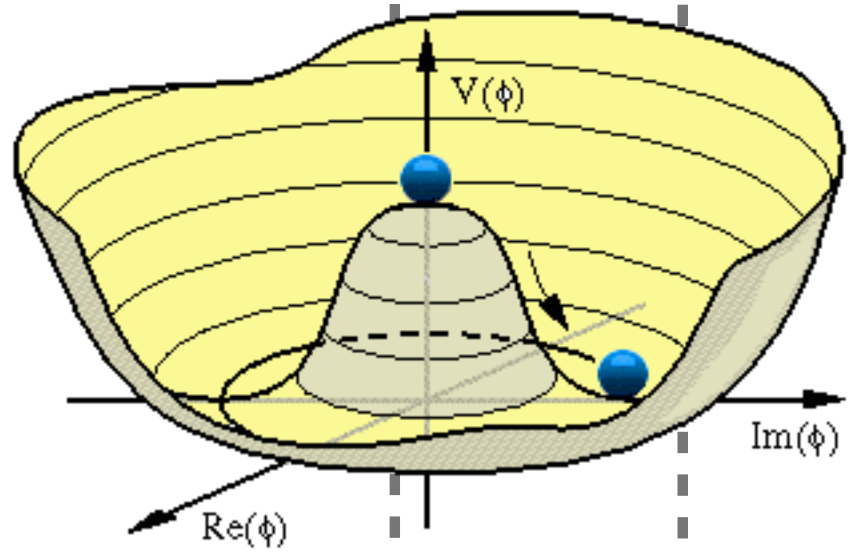
Gauge Bosons Masses

Symmetry of the Lagrangian

$$SU(2)_L \times U(1)_Y$$

Higgs Doublet

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$



Symmetry of the Vacuum

$$U(1)_{e.m.}$$

Vacuum Expectation Value

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad v \approx 246 \text{ GeV}$$

$$D_\mu H = \partial_\mu H - \frac{i}{2} \begin{pmatrix} g W^3_\mu + g' B_\mu & \sqrt{2} g W^+_\mu \\ \sqrt{2} g W^-_\mu & -g W^3_\mu + g' B_\mu \end{pmatrix} H \quad \text{with} \quad \mathrm{W}^\pm_\mu = \frac{1}{\sqrt{2}} (\mathrm{W}^1_\mu \mp \mathrm{W}^2_\mu)$$

$$|D_\mu H|^2 = \frac{1}{4}\, g^2 v^2\, W^+_\mu W^{-\,\mu} + \frac{1}{8}\,(W^3_\mu\ B_\mu) \begin{pmatrix} g^2 v^2 & -g g' v^2 \\ -g g' v^2 & g'^2 v^2 \end{pmatrix} \begin{pmatrix} W^{3\,\mu} \\ B^\mu \end{pmatrix}$$

- Gauge boson spectrum
  - electrically charged bosons $\quad M_W^2 = \frac{1}{4} g^2 v^2$
  - electrically neutral bosons

$$Z_\mu = c W^3_\mu - s B_\mu$$
$$\gamma_\mu = s W^3_\mu + c B_\mu$$

Weak mixing angle

$$c = \frac{g}{\sqrt{g^2 + g'^2}} \qquad s = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2$$

$$M_\gamma = 0$$

Gauge Bosons Masses

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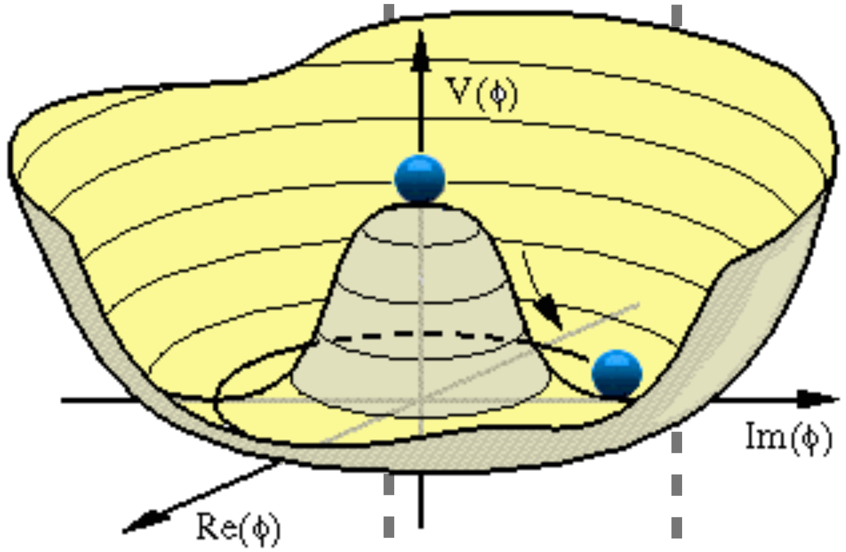

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Vacuum Expectation Value

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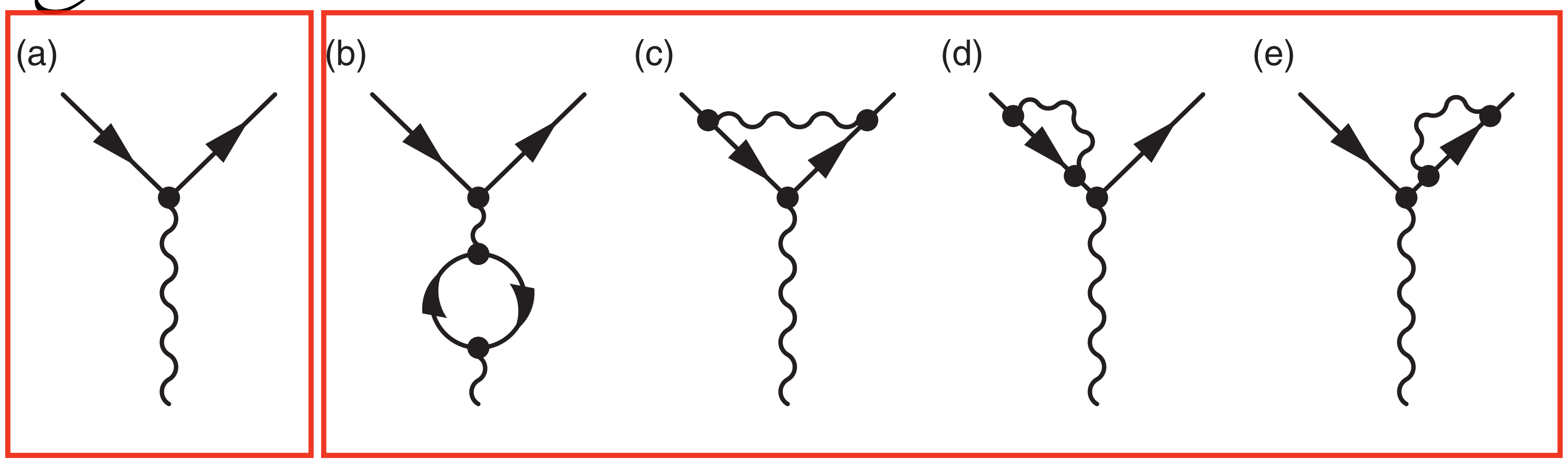
$$G_F = 2\sqrt{2}\frac{g^2}{m_W^2} = \frac{1}{\sqrt{2}v^2}$$

Evolution of Coupling Constants

Classical physics: the forces depend on distances

Quantum physics : the charges depend on distances



QED

classical interaction

one-loop quantum corrections

e

$$e^3 \int d^4k \, \frac{1}{k - m_e} \frac{1}{k - m_e} \frac{1}{k^2 - m_\gamma^2}$$

∞ ∞

finite (renormalised) electric charge
but it depends on the energy of the electron

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \alpha(\mu^2)\frac{1}{3\pi}\ln\left(\frac{q^2}{\mu^2}\right)},$$

$\alpha(m_e) = 1/137$ $\alpha(m_Z) = 1/128$

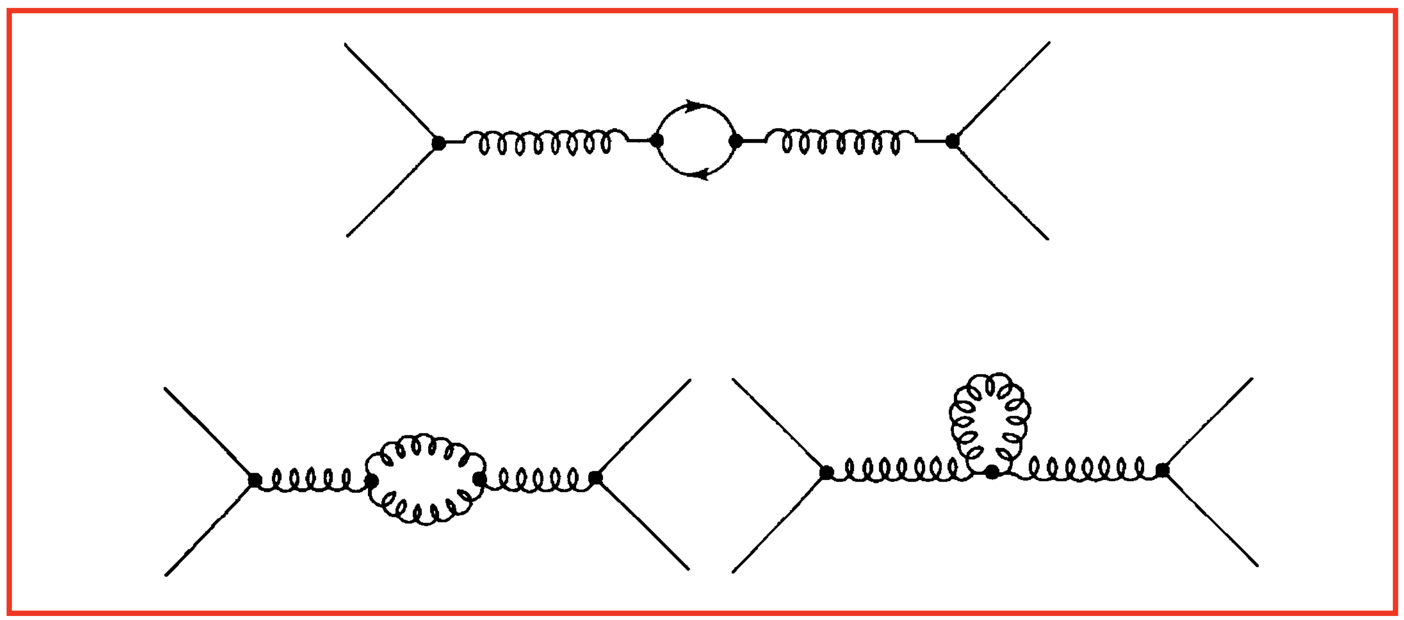
beautifully verified at LEP

Evolution of Coupling Constants

Classical physics: the forces depend on distances

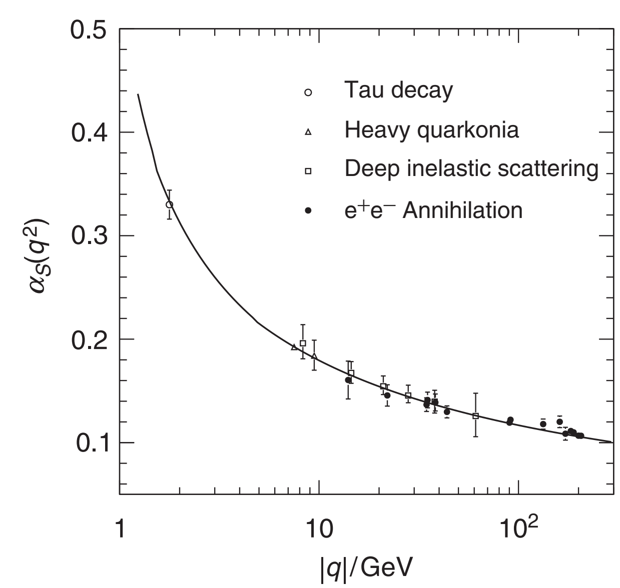
Quantum physics : the charges depend on distances

QCD



gluons interact with themselves

$$\alpha_s(|q^2|) = \frac{\alpha_s(\mu^2)}{1 + (\alpha_s(\mu^2)/12\pi)(11n - 2f)\ln(|q^2|/\mu^2)}$$



Evolution of Coupling Constants

Classical physics: the forces depend on distances

Quantum physics : the charges depend on distances

QED

virtual particles screen
the electric charge: $\alpha \searrow$ when d $\nearrow$

QCD

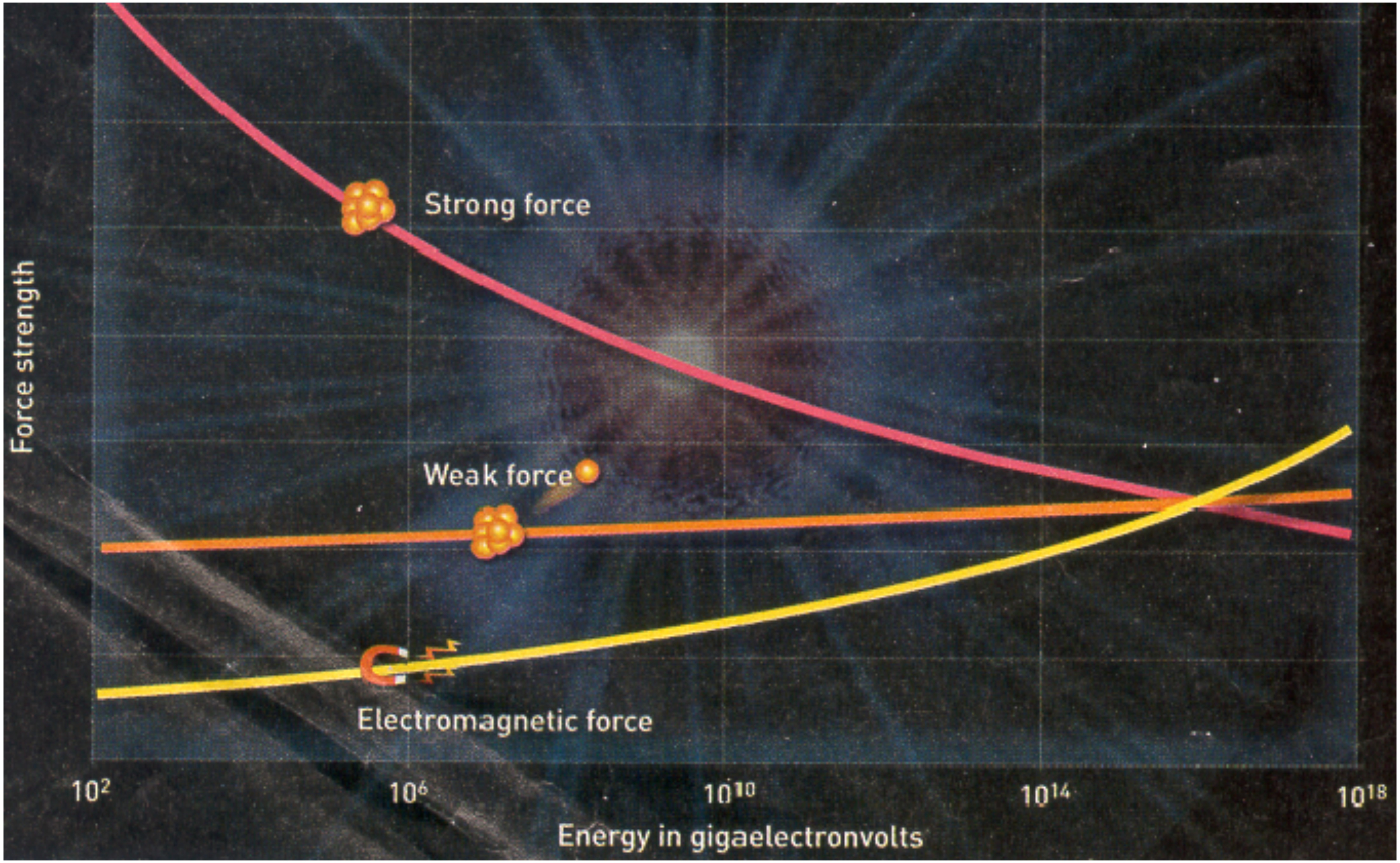
virtual particles (quarks and *gluons*) screen
the strong charge: $\alpha_s \nearrow$ when d $\nearrow$

'asymptotic freedom'

$$\frac{\partial \alpha_s}{\partial \log E} = \beta(\alpha_s) = \frac{\alpha_s^2}{\pi}\left(-\frac{11N_c}{6} + \frac{N_f}{3}\right)$$

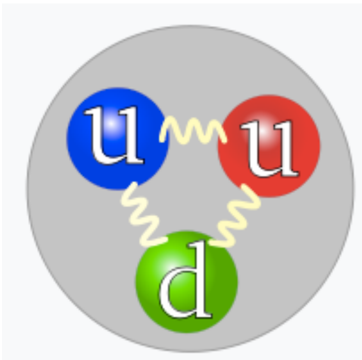
α_s becomes infinite at long distance: the quarks cannot escape →"confinement"

Grand Unified Theories



A single form of matter
A single fundamental interaction

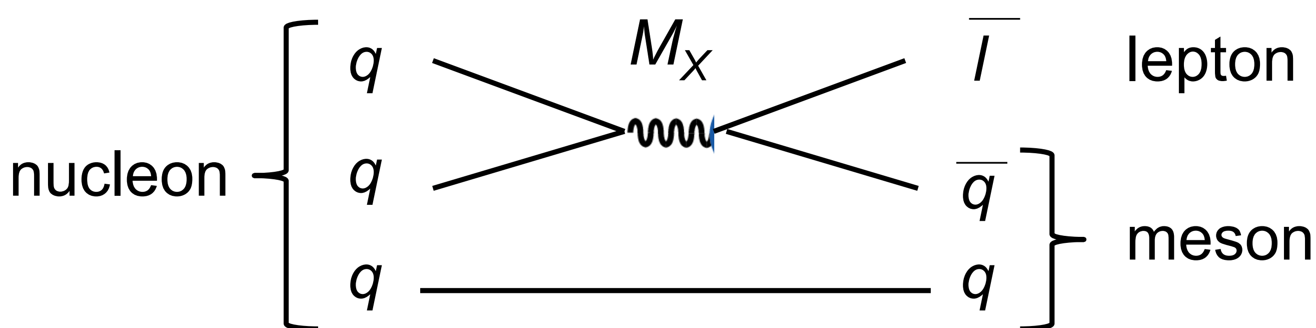
Proton Decay



938.2720813(58) MeV

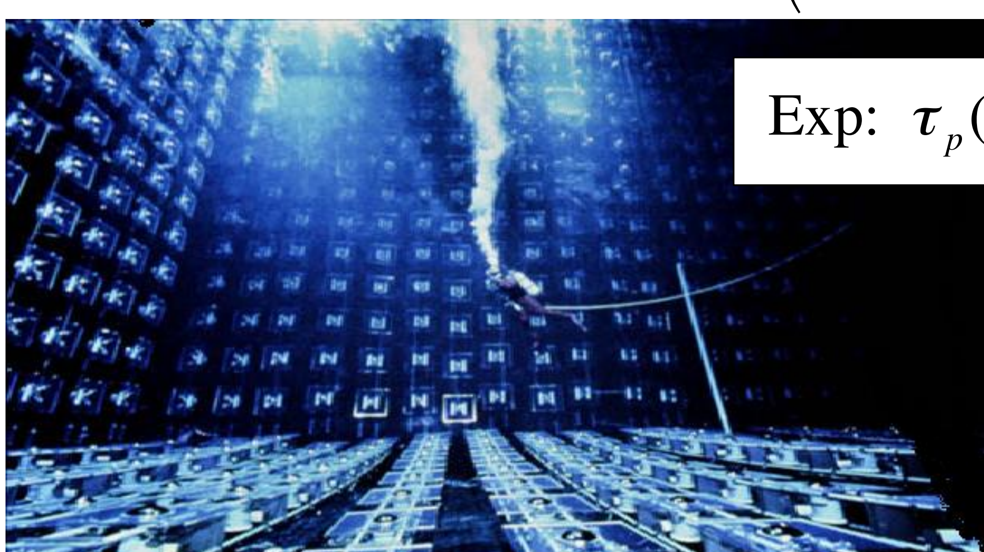
Why is the proton stable?

Baryon number conservation?
It turns out to be an accidental symmetry at low energy:
with the SM matter content, one cannot write
gauge invariant interaction that breaks this symmetry



in GUT, “matter” is unstable
decay of proton mediated by new
SU(5)/SO(10) gauge bosons

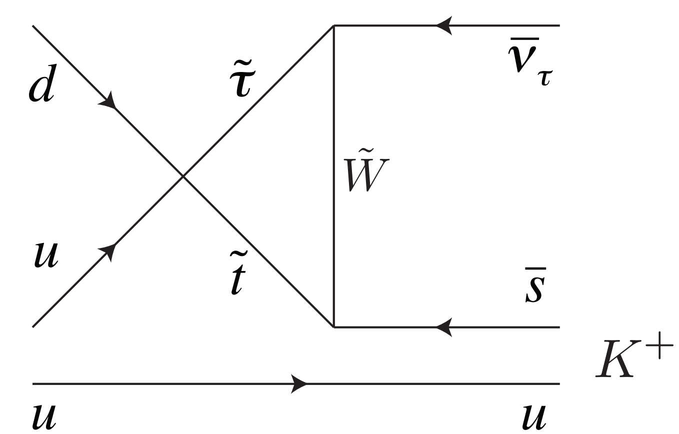
$$\text{GUT:}\quad \tau_p(p \to e^+\pi^0) = \left(\frac{M_X}{10^{15}\ \text{GeV}}\right)^4 10^{31-32}\ \text{yr}$$



$$\text{Exp:}\quad \tau_p(p \to e^+\pi^0) > 8.2\times 10^{33}\ \text{yr}$$

other decay mode:

$$p^+ \to K^+\overline{\nu}$$



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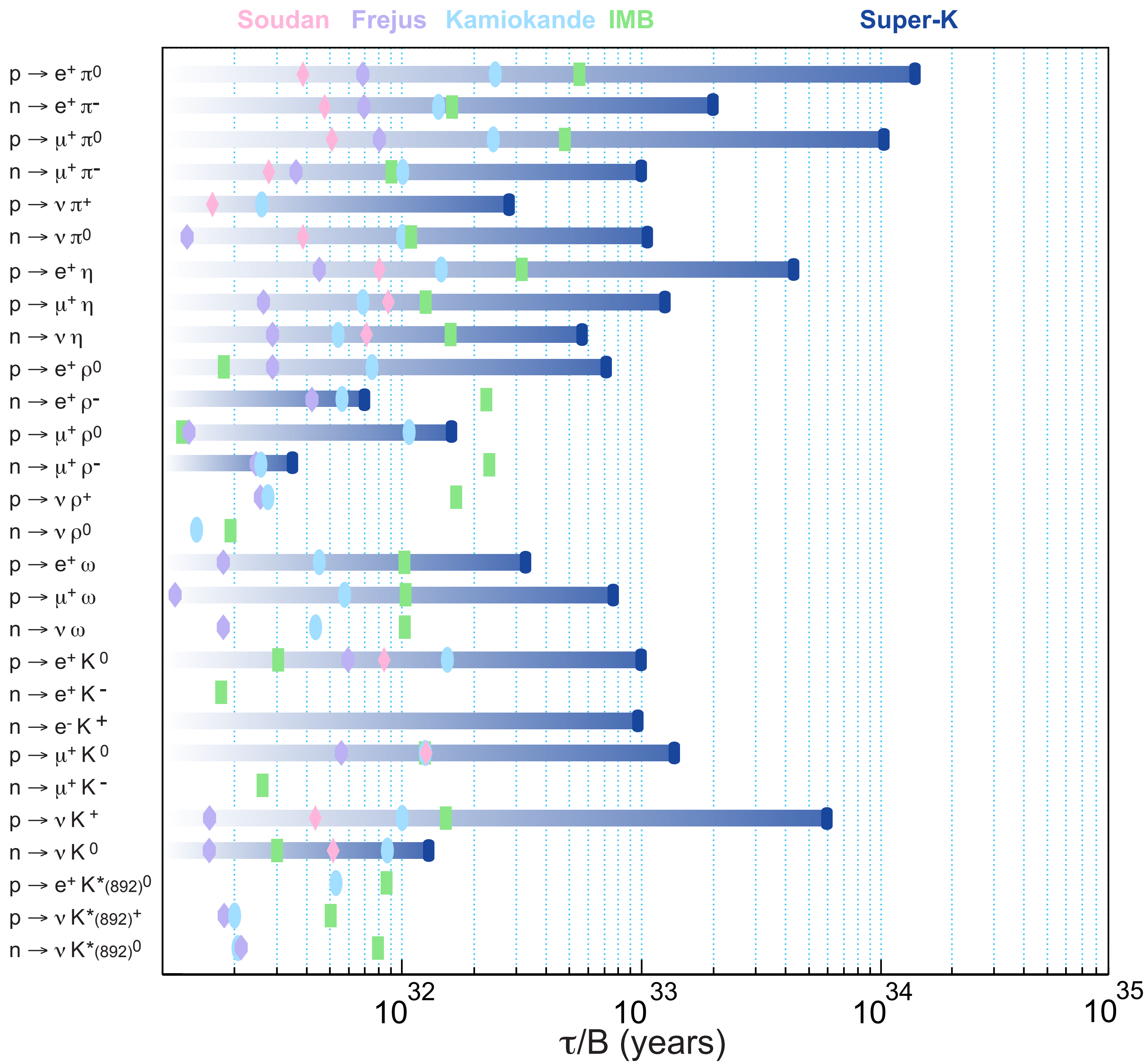
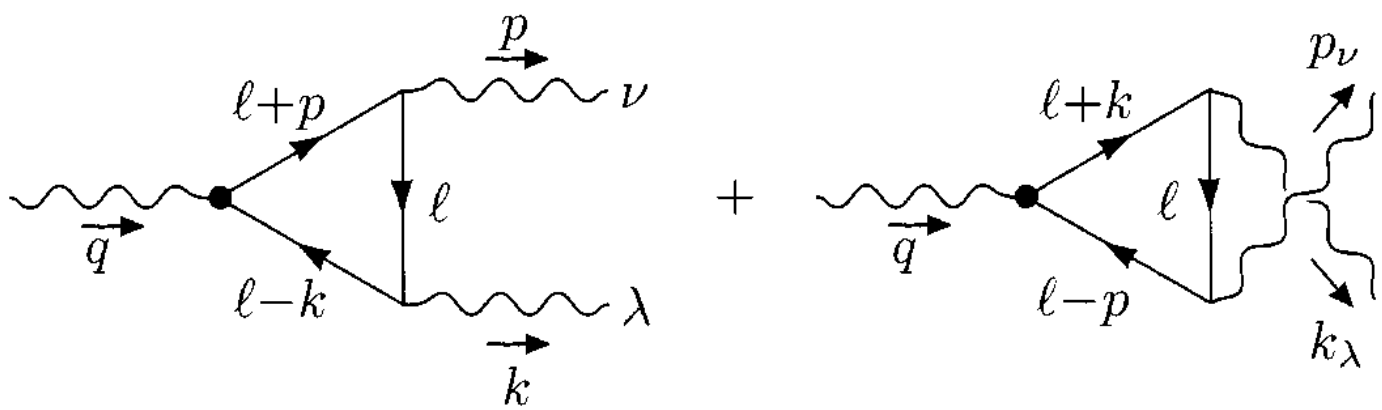


Babu et al '13

Anomaly Cancelation

Do quantum effects break the SM symmetries of the SM?



classically $\partial_\mu J^\mu = 0$

quantum mechanically $$\partial_\mu J^\mu = \boxed{C}\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

— only possibly non-zero for chiral symmetry —
chiral anomaly coefficient
(depends on the charges of the particles in the loop)

$$C_{U(1)^3_Y} = 3\cdot\left(\underbrace{3\cdot 2\cdot\left(\frac{1}{6}\right)^3}_{Q_L} + \underbrace{2\cdot\left(-\frac{1}{2}\right)^3}_{L_L}\right) - 3\cdot\left(\underbrace{3\cdot\left(\frac{2}{3}\right)^3}_{u_R} + \underbrace{3\cdot\left(-\frac{1}{3}\right)^3}_{d_R} + \underbrace{(-1)^3}_{e_R}\right) = 0$$

Similarly cancelations hold for all the other anomaly coefficients.
Delicate cancelations among quarks and leptons, LH and RH degrees of freedom.

— SM is consistent at the quantum level —

The Hierarchy Problem

only a few electrons are enough to lift your hair (~ 10^{25} mass of e^-)

the electric force between 2 e^- is 10^{43} times larger than their gravitational interaction

we don’t know why gravity is so weak?

we don’t know why the masses of particles are so small?

Several theoretical hypothesis

new dynamics? new symmetries? new space-time structure?

modification of special relativity? of quantum mechanics?

Conclusion

Once upon a time…

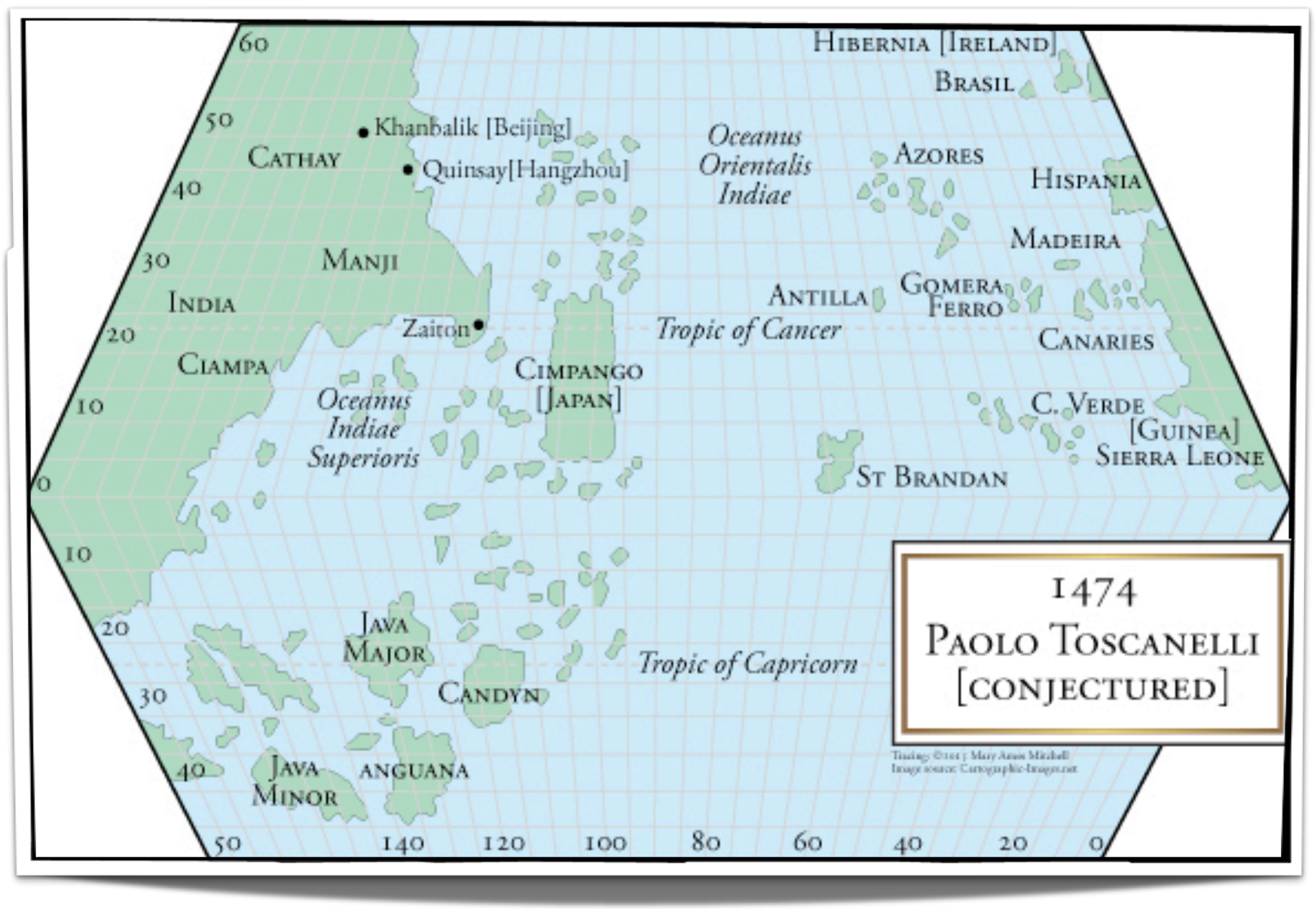
Columbus had a great proposal: “reaching India by sailing to the West”

He had a theoretical model

- the Earth is round,
- Eratosthenes of Cyrene first estimated its circumference to be 250’000 stadia
- other measurements later found smaller values ☞Toscanelli’s map
- lost in unit-conversion or misled by post-truth statements, Columbus thought it was only 70’000 stadia, so he believed he could reach India in 4 weeks

He had the right technology

- Caravels were the only ships at that time to sail against the wind, necessary tool to fight the prevailing winds, aka Alizée. Actually, the Vikings had the right technology too but the knowledge was lost



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His proposal was scientifically rejected twice (by Portuguese's & Salamanca U.) but fortunately the decision was overruled by Isabel ... and America became great (already)

Moral(s)

"if your proposal is rejected, submit it again"

"you need the right technology to beat your competitors"

"theorists don't need to be right!
but progress needs theoretical models to motivate exploration"

Thank you for your attention.
Good luck for your studies!

if you have question/want to know more

do not hesitate to send me an email

christophe.grojean@desy.de

Technical Details for Advanced Students

Dimensionality of π

In HEP natural units, we set c=ħ=1, such that [length]=[time]=[mass]$^{-1}$=[energy]$^{-1}$
But these fundamental constants are dimensionful. And it might be useful to keep track of the ħ-dimensions in addition to the mass dimension of any physical quantity

		M^n	$\hbar^n$
scalar field	ϕ	1	1/2
fermion field	ψ	3/2	1/2
vector field	A_μ	1	1/2
mass	m	1	0
gauge coupling	g	0	−1/2
quartic coupling	λ	0	−1
Yukawa coupling	y_f	0	−1/2

$$\mathcal{S} = \int d^4x \left(\mathcal{L}_0 + \hbar\mathcal{L}_1 + \hbar^2\mathcal{L}_2 + \ldots\right)$$

$[\mathcal{L}_0]_\hbar = 1$ $\quad [\mathcal{L}_1]_\hbar = 0$ $\quad [\mathcal{L}_2]_\hbar = -1$

$[\mathcal{L}_0]_M = 4$ $\quad [\mathcal{L}_1]_M = 4$ $\quad [\mathcal{L}_2]_M = 4$

example:
tree-level generated operator

$[\cdot]_\hbar = -1$ $\quad [\cdot]_\hbar = 2$

$$\frac{1}{M^2} g_*^2 \left(\partial^\mu |H|^2\right)^2$$

example:
one-loop generated operator

$$\frac{1}{M^2}\frac{g^2}{16\pi^2} g'^2 |H|^2 B_{\mu\nu}B^{\mu\nu}$$

The factors of π are very often associated to loop factors which are counting the ħ-dimension
Remember the normalisation of the states in QFT: $d^4k/(2\pi)^4$

SU(5) GUT: Gauge Group Structure

SU(3)$_c$xSU(2)$_L$xU(1)$_Y$: SM Matter Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad u_R^c = (\bar{3},1)_{-2/3}, \quad d_R^c = (\bar{3},1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1,2)_{-1/2}, \quad e_R^c = (1,1)_1$$

How can you ever remember all these numbers?

SU(3)$_c$xSU(2)$_L$xU(1)$_Y$ ⊂ SU(5)

SU(5)
Adjoint rep.

$$\left(\begin{array}{c|c} SU(2) & \\ \hline & SU(3) \end{array}\right)$$

$$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$$

additional U(1) factor that commutes with SU(3)xSU(2)

$$T^{12} = \sqrt{\frac{3}{5}} \left(\begin{array}{cc|ccc} 1/2 & & & & \\ & 1/2 & & & \\ \hline & & -1/3 & & \\ & & & -1/3 & \\ & & & & -1/3 \end{array}\right)$$

$$\boxed{\begin{array}{c} \bar{5} = (1,2)_{-\frac{1}{2}\sqrt{\frac{3}{5}}} + (\bar{3},1)_{\frac{1}{3}\sqrt{\frac{3}{5}}} \\ \bar{5} = L + d_R^c \end{array}}$$

$$\boxed{\begin{array}{c} 10 = (5 \times 5)_A = (\bar{3},1)_{-\frac{2}{3}\sqrt{\frac{3}{5}}} + (3,2)_{\frac{1}{6}\sqrt{\frac{3}{5}}} + (1,1)_{\sqrt{\frac{3}{5}}} \\ 10 = u_R^c + Q_L + e_R^c \end{array}}$$

$$T^{12} = \sqrt{\frac{3}{5}} Y \qquad g_5 \sqrt{\frac{3}{5}} = g' \qquad g_5 = g = g_s$$

$$g_5 T^{12} = g' Y$$

$\sin^2\theta_W = \frac{3}{8}$ @ M$_{GUT}$

SU(5) GUT: low energy consistency condition

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} - \frac{b_i}{4\pi} \ln \frac{M_{GUT}^2}{M_Z^2} \qquad i = SU(3), SU(2), U(1)$$

$\alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z) \longleftarrow$ experimental inputs

$b_3, b_2, b_1 \longleftarrow$ predicted by the matter content

3 equations & 2 unknowns (α_{GUT}, M_{GUT})

one consistency relation on low energy parameters

$$\epsilon_{ijk} \frac{b_j - b_k}{\alpha_i(M_Z)} = 0 \quad \Rightarrow \quad \boxed{\sin^2\theta_W = \frac{3(b_3 - b_2)}{8b_3 - 3b_2 - 5b_1} + \frac{5(b_2 - b_1)}{8b_3 - 3b_2 - 5b_1} \frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)}}$$

$$\alpha_{em}(M_Z) \approx \frac{1}{128} \qquad \alpha_s(M_Z) \approx 0.1184 \pm 0.0007$$

$$\Downarrow$$

$\sin^2\theta_W \approx 0.207$ not bad… (observed value: 0.23)
Even better in MSSM

SU(5) GUT: low energy consistency condition

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} - \frac{b_i}{4\pi} \ln \frac{M_{GUT}^2}{M_Z^2} \qquad i = SU(3), SU(2), U(1)$$

$\alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z)$ ⟵ experimental inputs

b_3, b_2, b_1 ⟵ predicted by the matter content

3 equations & 2 unknowns (α_{GUT}, M_{GUT})

one consistency relation on low energy parameters

$$M_{GUT} = M_Z \exp\left(2\pi \frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)}\right) \approx 7 \times 10^{14}\ \text{GeV}$$

$$\alpha_{GUT}^{-1} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 41.5$$

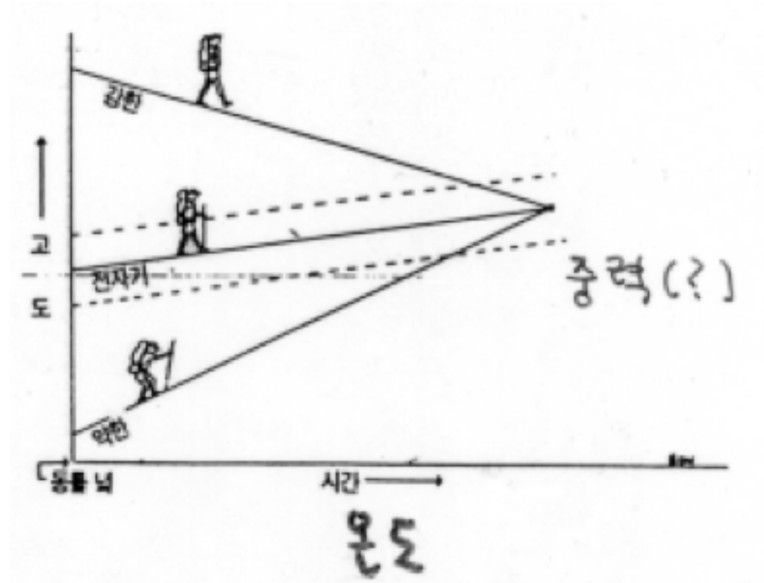
self-consistent computation:

- $M_{GUT} << M_{Pl}$ safe to neglect quantum gravity effects
- $\alpha_{GUT} << 1$ perturbative computation valid

SU(5) GUT: SM β fcts

g, g' and g_s are different but this is a low energy artefact!

$$\beta = \frac{dg}{d\log\mu} = -\frac{1}{16\pi^2} b g^3 + \dots$$



$$\frac{1}{g^2(Q)} = \frac{1}{g^2(Q_0)} + \frac{b}{16\pi^2}\ln\frac{Q^2}{Q_0^2}$$

$$b = \frac{11}{3} T_2(\texttt{spin-1}) - \frac{2}{3} T_2(\texttt{chiral spin-1/2}) - \frac{1}{3} T_2(\texttt{complex spin-0})$$

$$\mathrm{Tr}\left(T^a(R)T^b(R)\right) = T_2(R)\delta^{ab} \qquad T_2(\texttt{fund}) = \frac{1}{2} \qquad T_2(\texttt{adj}) = N$$

$$b_{SU(3)} = \overset{g}{\frac{11}{3}\times 3} - \frac{2}{3}\left(\overset{Q_L}{\frac{1}{2}\times 2\times 3} + \overset{u_R^c}{\frac{1}{2}\times 1\times 3} + \overset{d_R^c}{\frac{1}{2}\times 1\times 3}\right) = 7$$

$$b_{SU(2)} = \overset{W^\pm, Z}{\frac{11}{3}\times 2} - \frac{2}{3}\left(\overset{Q_L}{\frac{1}{2}\times 3\times 3} + \overset{L}{\frac{1}{2}\times 1\times 3}\right) - \overset{H}{\frac{1}{3}\times\frac{1}{2}} = \frac{19}{6}$$

$$b_Y = -\frac{2}{3}\left(\overset{Q_L}{\left(\frac{1}{6}\right)^2 3\times 2\times 3} + \overset{u_R^c}{\left(-\frac{2}{3}\right)^2 3\times 3} + \overset{d_R^c}{\left(\frac{1}{3}\right)^2 3\times 3} + \overset{L}{\left(-\frac{1}{2}\right)^2 2\times 3} + \overset{e_R^c}{(1)^2\times 3}\right) - \overset{H}{\frac{1}{3}\left(\frac{1}{2}\right)^2\times 2} = -\frac{41}{6} \Rightarrow b_{T^{12}} = -\frac{41}{10}$$

SU(5) GUT: SM vs MSSM β fcts

chiral superfield

complex spin-0
Weyl spin-1/2
in same representation of gauge group

vector superfield

Weyl spin-1/2
real spin-1
in same representation of gauge group

$$b = \frac{11}{3} T_2(\texttt{vector}) - \frac{2}{3} T_2(\texttt{vector}) - \frac{2}{3} T_2(\texttt{chiral}) - \frac{1}{3} T_2(\texttt{chiral}) = 3\, T_2(\texttt{vector}) - T_2(\texttt{chiral})$$

MSSM Chiral Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad U = (\bar{3},1)_{-2/3}, \quad D = (\bar{3},1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1,2)_{-1/2}, \quad E = (1,1)_1, \ H_u = (1,2)_{1/2}, \quad H_d = (1,2)_{-1/2}$$

$$b_{SU(3)} = \overset{g}{3 \times 3} - \left(\overset{Q_L}{\frac{1}{2} \times 2 \times 3} + \overset{U}{\frac{1}{2} \times 1 \times 3} + \overset{D}{\frac{1}{2} \times 1 \times 3} \right) = 3$$

$$b_{SU(2)} = \overset{W^\pm, Z}{3 \times 2} - \left(\overset{Q_L}{\frac{1}{2} \times 3 \times 3} + \overset{L}{\frac{1}{2} \times 1 \times 3} \right) - \overset{H_u}{\frac{1}{2}} - \overset{H_d}{\frac{1}{2}} = -1$$

$$b_Y = -\left(\overset{Q_L}{\left(\frac{1}{6}\right)^2 3 \times 2 \times 3} + \overset{U}{\left(-\frac{2}{3}\right)^2 3 \times 3} + \overset{D}{\left(\frac{1}{3}\right)^2 3 \times 3} + \overset{L}{\left(-\frac{1}{2}\right)^2 2 \times 3} + \overset{E}{(1)^2 \times 3} \right) - \overset{H_u}{\left(\frac{1}{2}\right)^2 \times 2} - \overset{H_d}{\left(\frac{1}{2}\right)^2 \times 2} = -11 \quad \Rightarrow \quad b_{T^{12}} = -\frac{33}{5}$$

SU(5) GUT: MSSM GUT

$$b_3 = 3, \quad b_2 = -1, \quad b_1 = -33/5$$

low-energy consistency relation for unification

$$\sin^2\theta_W = \frac{3(b_3 - b_2)}{8b_3 - 3b_2 - 5b_1} + \frac{5(b_2 - b_1)}{8b_3 - 3b_2 - 5b_1}\frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)} \approx 0.23$$

squarks and sleptons form complete SU(5) reps ➙ they don't improve unification!
gauginos and higgsinos are improving the unification of gauge couplings

GUT scale predictions

$$M_{GUT} = M_Z \exp\left(2\pi \frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)}\right) \approx 2 \times 10^{16} \text{ GeV}$$

$$\alpha_{GUT}^{-1} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 24.3$$

Proton Decay

	Mode	Partial mean life (10^{30} years)	Confidence level
	Antilepton + meson		
τ_1	$N \to e^+\pi$	> 2000 (n), > 8200 (p)	90%
τ_2	$N \to \mu^+\pi$	> 1000 (n), > 6600 (p)	90%
τ_3	$N \to \nu\pi$	> 1100 (n), > 390 (p)	90%
τ_4	$p \to e^+\eta$	> 4200	90%
τ_5	$p \to \mu^+\eta$	> 1300	90%
τ_6	$n \to \nu\eta$	> 158	90%
τ_7	$N \to e^+\rho$	> 217 (n), > 710 (p)	90%
τ_8	$N \to \mu^+\rho$	> 228 (n), > 160 (p)	90%
τ_9	$N \to \nu\rho$	> 19 (n), > 162 (p)	90%
τ_{10}	$p \to e^+\omega$	> 320	90%
τ_{11}	$p \to \mu^+\omega$	> 780	90%
τ_{12}	$n \to \nu\omega$	> 108	90%
τ_{13}	$N \to e^+K$	> 17 (n), > 1000 (p)	90%
τ_{14}	$p \to e^+K^0_S$		
τ_{15}	$p \to e^+K^0_L$		
τ_{16}	$N \to \mu^+K$	> 26 (n), > 1600 (p)	90%
τ_{17}	$p \to \mu^+K^0_S$		
τ_{18}	$p \to \mu^+K^0_L$		
τ_{19}	$N \to \nu K$	> 86 (n), > 5900 (p)	90%
τ_{20}	$n \to \nu K^0_S$	> 260	90%
τ_{21}	$p \to e^+K^*(892)^0$	> 84	90%
τ_{22}	$N \to \nu K^*(892)$	> 78 (n), > 51 (p)	90%
	Antilepton + mesons		
τ_{23}	$p \to e^+\pi^+\pi^-$	> 82	90%
τ_{24}	$p \to e^+\pi^0\pi^0$	> 147	90%
τ_{25}	$n \to e^+\pi^-\pi^0$	> 52	90%
τ_{26}	$p \to \mu^+\pi^+\pi^-$	> 133	90%
τ_{27}	$p \to \mu^+\pi^0\pi^0$	> 101	90%
τ_{28}	$n \to \mu^+\pi^-\pi^0$	> 74	90%
τ_{29}	$n \to e^+K^0\pi^-$	> 18	90%

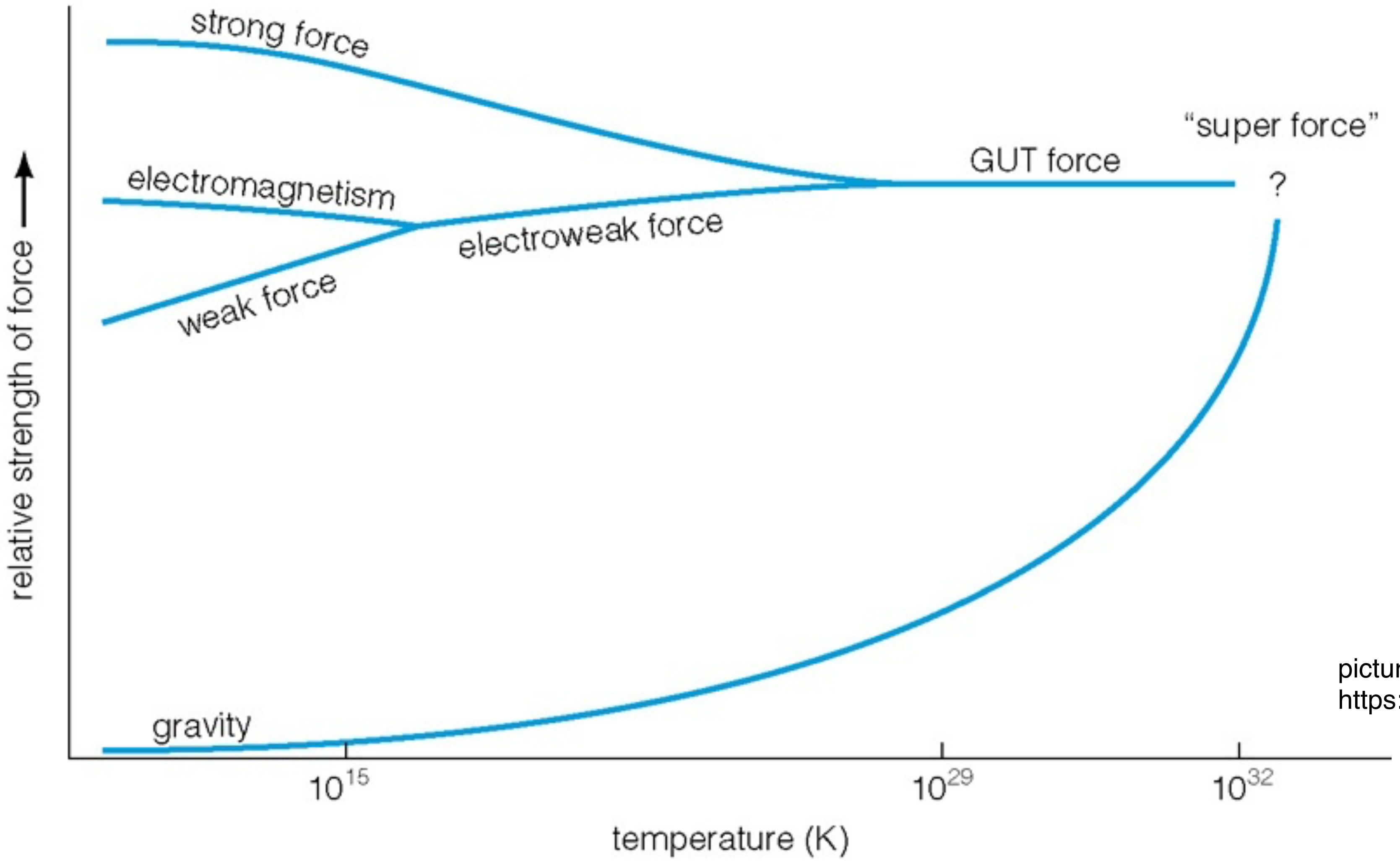
ΔB=ΔL=1 decay bounds

	Mode	Partial mean life (10^{30} years)	Confidence level
	Lepton + meson		
τ_{30}	$n \to e^-\pi^+$	> 65	90%
τ_{31}	$n \to \mu^-\pi^+$	> 49	90%
τ_{32}	$n \to e^-\rho^+$	> 62	90%
τ_{33}	$n \to \mu^-\rho^+$	> 7	90%
τ_{34}	$n \to e^-K^+$	> 32	90%
τ_{35}	$n \to \mu^-K^+$	> 57	90%
	Lepton + mesons		
τ_{36}	$p \to e^-\pi^+\pi^+$	> 30	90%
τ_{37}	$n \to e^-\pi^+\pi^0$	> 29	90%
τ_{38}	$p \to \mu^-\pi^+\pi^+$	> 17	90%
τ_{39}	$n \to \mu^-\pi^+\pi^0$	> 34	90%
τ_{40}	$p \to e^-\pi^+K^+$	> 75	90%
τ_{41}	$p \to \mu^-\pi^+K^+$	> 245	90%

ΔB=-ΔL=1 decay bounds

SM & Gravity

It is actually possible to couple the SM to gravity and to quantise the graviton. The issue is that gravity is not renormalisable and to get ride of infinities in loop computation, one needs to add more and more counter-terms that are not present originally in the classical GR Lagrangian. At most gravity can be treated as an effective field theory and there are arguments that show that its UV completion is unlikely to be a quantum field theory but rather a theory of more complicated objects like matrices or strings. There is an important difference between gauge (spin-1) interactions and gravity: the gauge couplings of the former exhibit a logarithmic evolution with the energy of the process, while the strength of gravity grows like E^2. An important question is to figure out the scale of quantum gravity: is it M_{Planck}~10^{19}GeV? it could be lower down to few TeVs if there are (large or highly curved) extra dimensions. In that case, totally new phenomena could be observed at colliders… see the BSM lectures



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