Co-evolutionary Algorithms: A Useful Computational Abstraction?

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Early examples (1990s):

- Hillis: evolve sorting networks
- Rosin: evolve competitive gameplayers
- Potter: evolve subcomponents
- …

The Co-EA hype:

- A powerful optimization tool
  - Competitive: infinite skyhook
  - Cooperative: dynamic decomposition
- A powerful modeling tool
  - Capture dynamics of multi-agent systems

The Co-EA reality:

- Difficult to design
- Difficult to tune
- Little guidance from standard EAs
- Little guidance from theory
- Complex dynamics:
  - Mediocre stable states, cycles …

Goal (2000s): better understanding

- Improved theory and improved empirical tools enabling:
  - Better insights
  - Improved designs
  - Improved applications

Today

- Modest impact on the field
- Fewer publications:
  - E.g., GECCO track demise
- Why?

So, what IS co-evolution?

- Minimal requirements to be co-evolution?
  - Biologist's view:
    - Ecology with interacting species
  - EC view:
    - Fitness via interactions with other individuals
    - Speciation?
    - Multiple populations?

Populations/species:

- Single population:
  - Single species:
    - competitive fitness
  - Multiple species:
    - Fixed species
    - Dynamic speciation
- Multiple populations:
  - Generally one species per population
  - Fixed/dynamic # of populations

Interacting fitness landscapes:

- Degree of Interaction
  - Full, partial mixing
- Who to interact with?
  - Yet another selection process
- Mode of interaction:
  - Competitive, cooperative
- Fitness aggregation:
  - Max, min, ave, …

Co-evolutionary Time Clocks:

- Asynchronous:
  - Interactions with moving targets

- Generational synchrony:
  - Freeze-thaw cycle
  - # generations/cycle?

Co-evolutionary Solutions:

- Convergence?
- To what?

Key issues:

- How do Co-EAs differ from EAs?
  - Internal time-varying fitness landscapes
  - Complex co-evolutionary dynamics
- What kinds of problems are Co-EAs good for?
  - Optimization?
  - Adaptation?
  - Complex systems?

Understanding Co-evolutionary Dynamics

- In practice, internal dynamic landscapes frequently result in:
  - Rapid convergence to uninteresting fixed points.
  - Endless dynamic mediocracy.
  - Arms races seldom observed.

Approaches to understanding co-evolutionary dynamics:

- Using evolutionary game theory (EGT)
- Using dynamical systems tools

Standard EGT:

- Extends traditional game theory via:
  - Infinite population of players
  - Finite number N of strategies (genotypes)
  - Population state: standard simplex notation

    $\mathbf{x} = \langle f_1, f_2, \ldots, f_n \rangle$

Standard EGT:

– Payoff matrix A specifies encounter outcome

– Strategy fitness: weighted average payoff

$$\mathbf{u} = A\mathbf{x}$$

– Fitness proportional selection:

$$x_i' = x_i \left(u_i / (\mathbf{x} \cdot \mathbf{u}) \right)$$

– No reproductive variation!!

- i.e, a replicator system

EGT focus:

- Nash equilibrium points
  - No incentive to change strategies
- Evolutionary stable strategies
  - Basin off attraction => can't be invaded by mutants

Extending standard EGT
(P. Wiegand)

- Multi-population models
  - Complicated even for two populations

    $$\mathbf{u} = A\mathbf{y} \quad \text{and} \quad \mathbf{w} = B\mathbf{x}$$

    - State space: cartesian product of two simplexes
- Adding reproductive variation
  - Lose most formal results => empirical studies

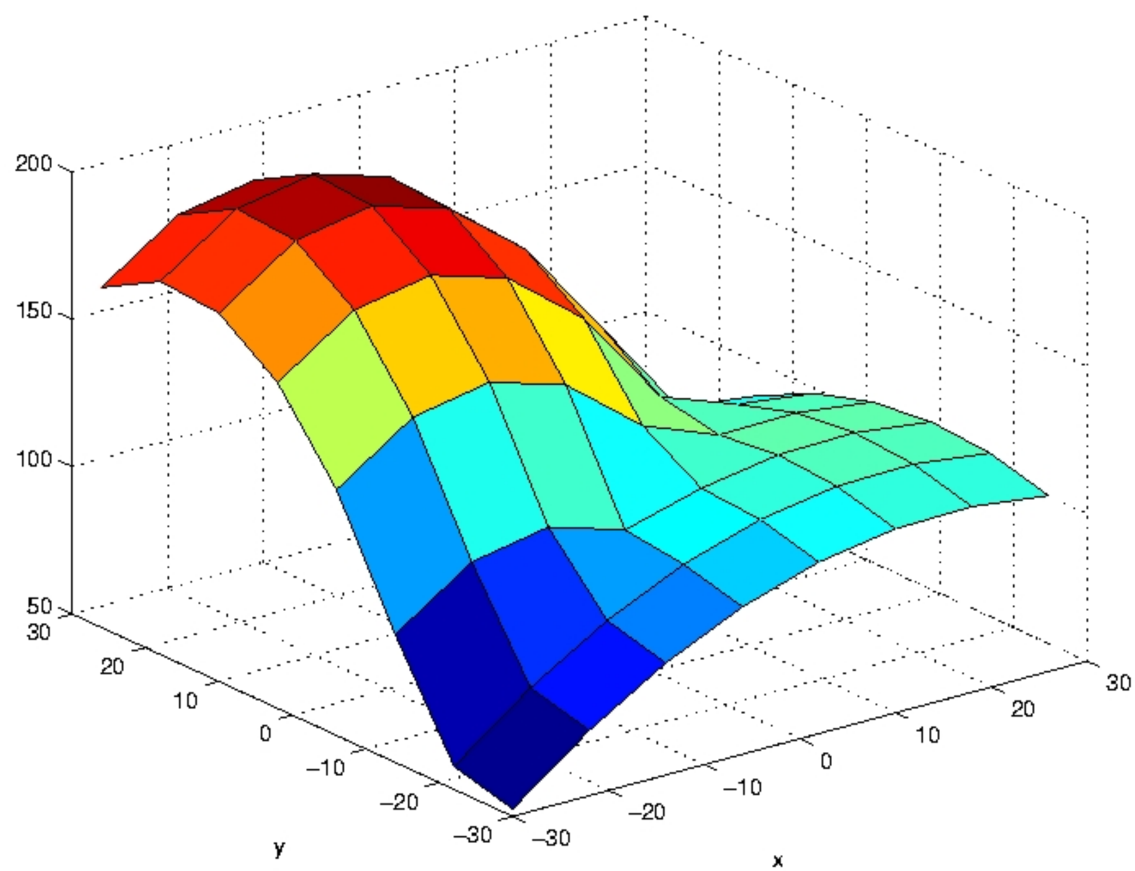
Examples of empirical tools:

- Rain gauge measures
  - Estimate likelihood of reaching particular fixed points by repeated iterating the model from uniform random initial configurations.
- Variational Distance Plots
  - Multi-dimensional trajectories to fixed points

Example: EGT analysis of cooperative Co-EAs

- Simpler dynamics
- Clearer interpretation for optimization

Example interaction function:



f_1

Payoff Matrix for f_1

52.5	72.18	88.02	100.02	108.18	112.5	112.98	109.62
57.78	77.46	93.3	105.3	113.46	117.78	118.26	114.9
93.4	111	115.8	107.8	114.9	119.22	119.7	116.34
133.4	151	155.8	147.8	127	116.82	117.3	113.94
160.6	178.2	183	175	154.2	120.6	111.06	107.7
175	192.6	197.4	189.4	168.6	135	100.98	97.62
176.6	194.2	199	191	170.2	136.6	90.2	83.7
165.4	183	187.8	179.8	159	125.4	79	65.94

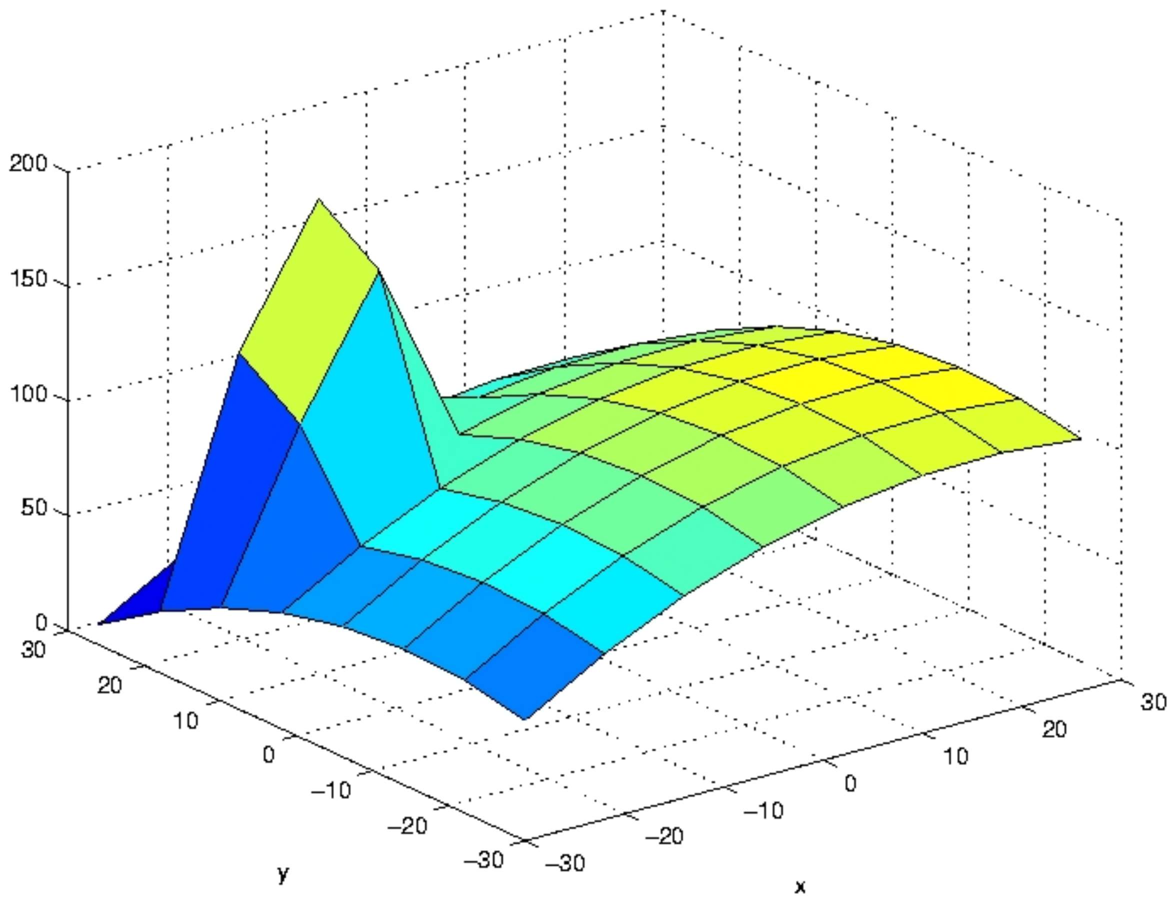
Rain Gauge Measures:

Percent of initial conditions leading to a particular fixed point.

f_1

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	**0**	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	**100**	0	0	0	0	0
0	0	0	0	0	0	0	0

Example interaction functions:



f_2

Payoff Matrix for f_2

52.5	72.18	88.02	100.02	108.18	112.5	112.98	109.62
57.78	77.46	93.3	105.3	113.46	117.78	118.26	114.9
59.22	78.9	94.74	106.74	114.9	119.22	119.7	116.34
56.82	76.5	92.34	104.34	112.5	116.82	117.3	113.94
50.58	70.26	86.1	98.1	106.26	110.58	111.06	107.7
40.5	111.2	168.8	88.02	96.18	100.5	100.98	97.62
26.58	130.4	188	92	82.26	86.58	87.06	83.7
8.82	28.5	53.6	56.34	64.5	68.82	69.3	65.94

Rain Gauge Measures:

Percent of initial conditions leading to a particular basis vector fixed point.

f_2

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	**52**	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	**48**	0	0	0	0	0
0	0	0	0	0	0	0	0

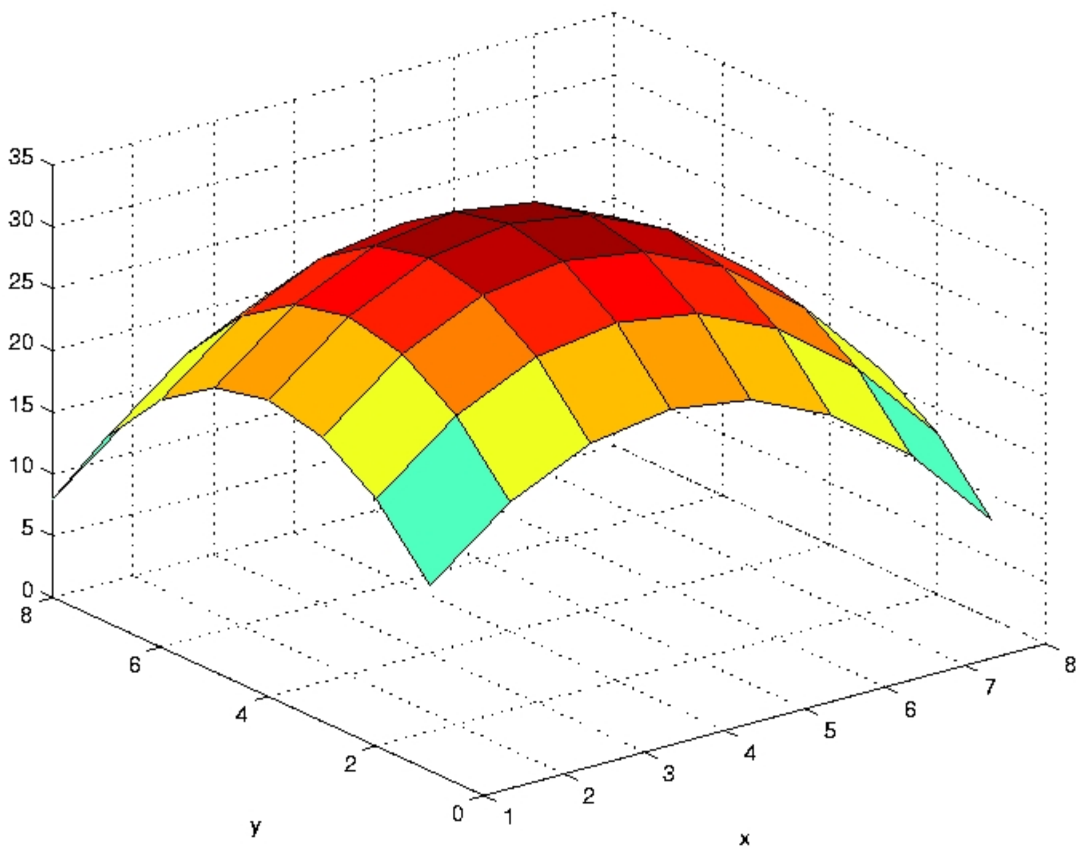
Incorporating Reproductive Variation

- Math gets messy fast
  - Significant non-linearities introduced.

- What can we say about the stability of fixed points?
  - More complicated.
  - Clear that variation affects the stability of the fixed points, but difficult to characterize precisely.

Incorporating Reproductive Variation

A simple unimodal function



Unimodal Function Values

15	20	23	24	23	20	15	8
20	25	28	29	28	25	20	13
23	28	31	32	31	28	23	16
24	29	32	33	32	29	24	17
23	28	31	32	31	28	23	16
20	25	28	29	28	25	20	13
15	20	23	24	23	20	15	8
8	13	16	17	16	13	8	1

Incorporating Reproductive Variation

Rain Gauge Measures for $\chi = 0.00$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	100	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Incorporating Reproductive Variation

Rain Gauge Measures: $\chi = 0.05$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	97.5	1.67	0	0	0
0	0	0	0.83	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Incorporating Reproductive Variation

Rain Gauge Measures: $\chi = 0.10$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	81.7	11.7	0	0	0
0	0	0	6.6	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Incorporating Reproductive Variation

Rain Gauge Measures: $\chi = 0.15$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	$\boxed{75.8}$	15.8	0	0	0
0	0	0	8.4	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Incorporating Reproductive Variation

Rain Gauge Measures: $\chi = 0.20$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	75	15.8	0	0	0
0	0	0	8.4	0.8	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Incorporating Reproductive Variation

Rain Gauge Measures: $\chi = 1.00$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	57.5	23.3	0	0	0
0	0	0	11.7	7.5	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

A Dynamical Systems Approach

(E. Popovici)

- Goal:

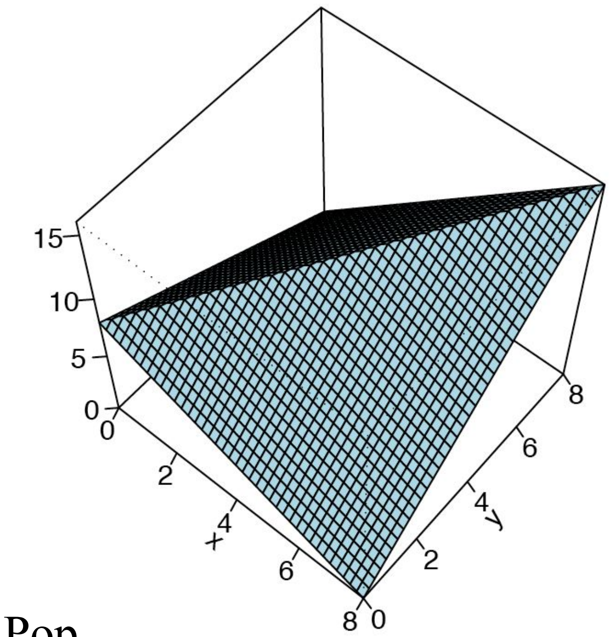
  A deeper understanding of 2-population Co-EA behavior.

- Approach:

  Trajectories of best response individuals

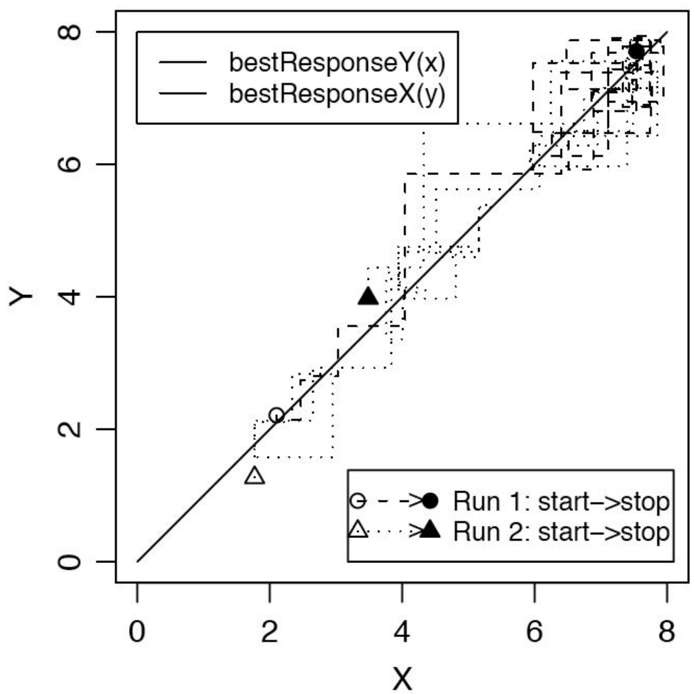
  - Fixed points / periodic orbits
  - Stability
  - Chaotic behavior

Cooperative Setting

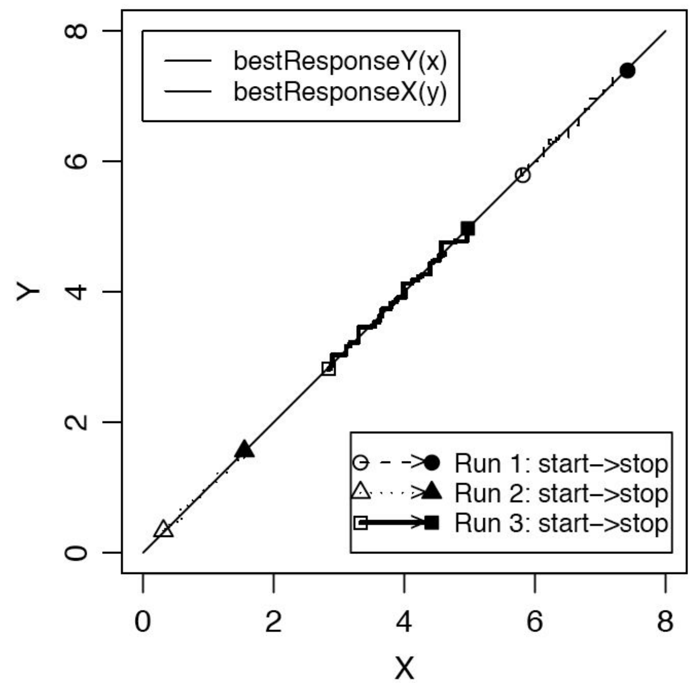


Effects of population size:

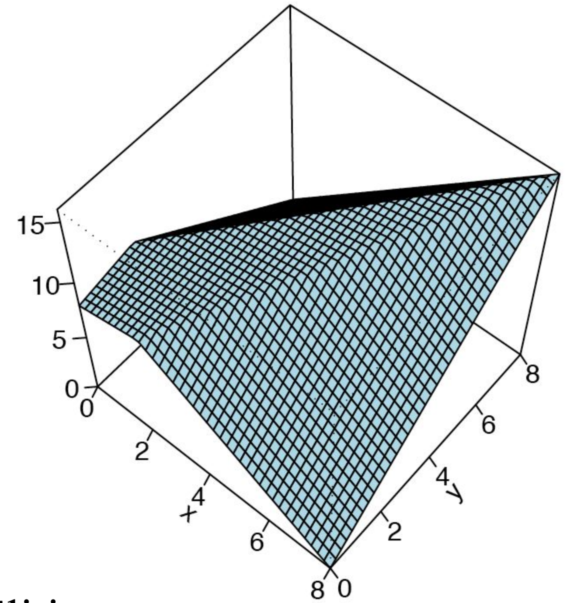
Small Pop



Large Pop

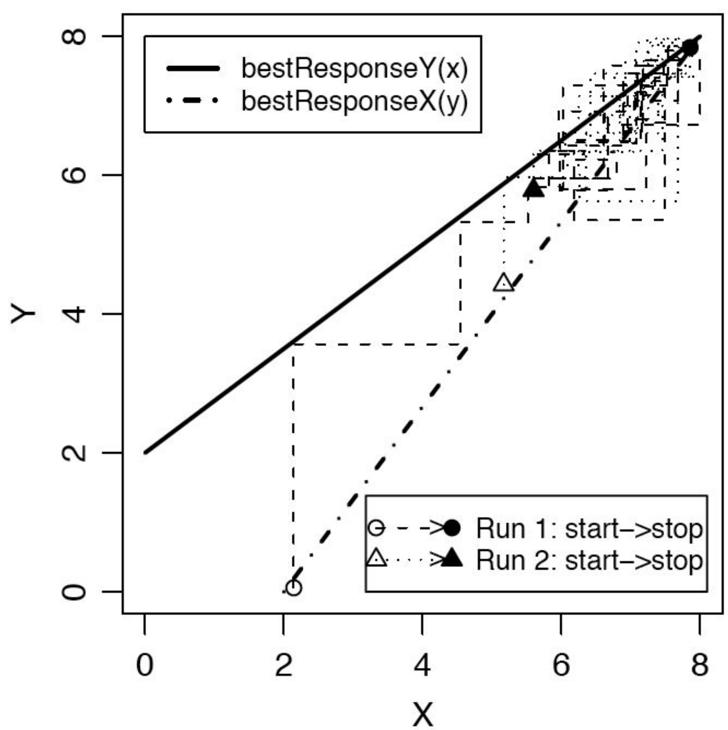


Cooperative Setting

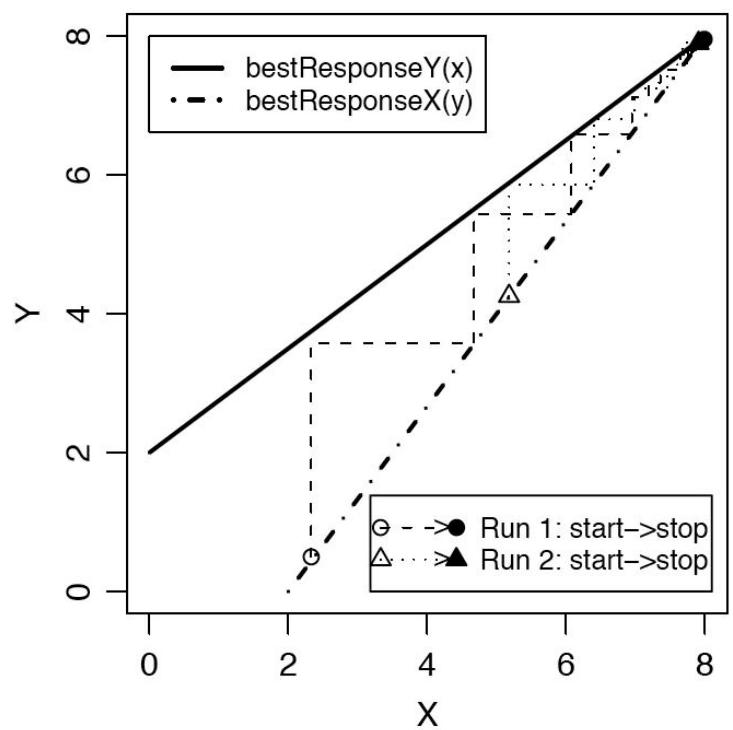


Effects of elitism:

No elitism



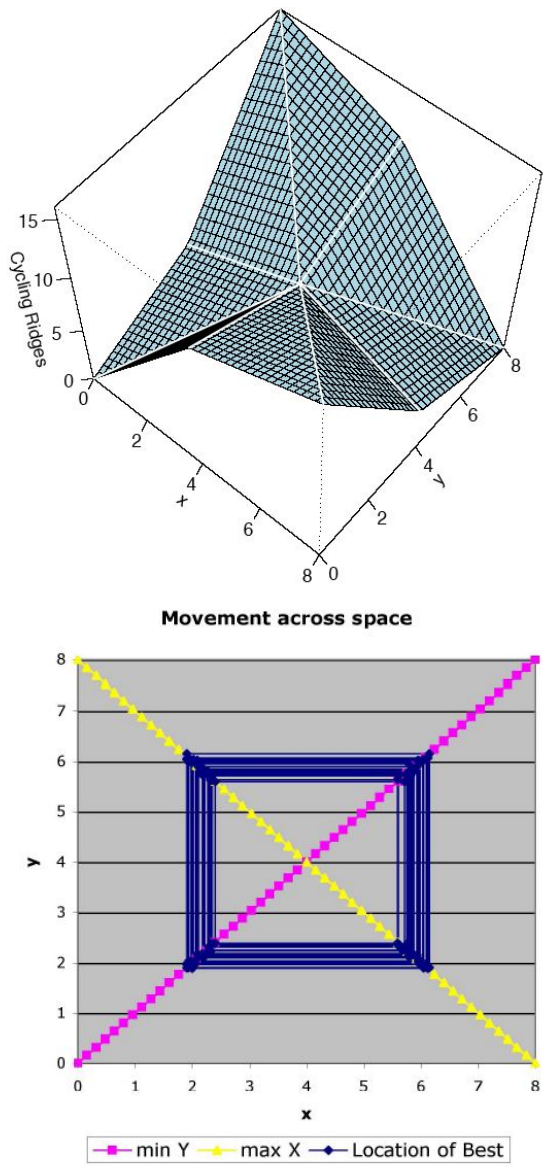
Elitism

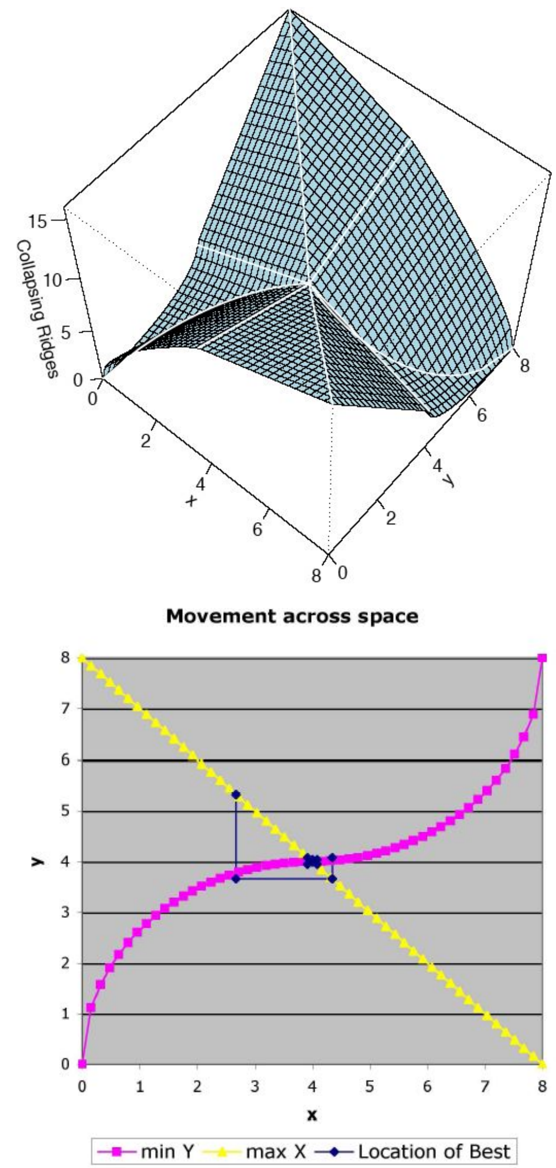


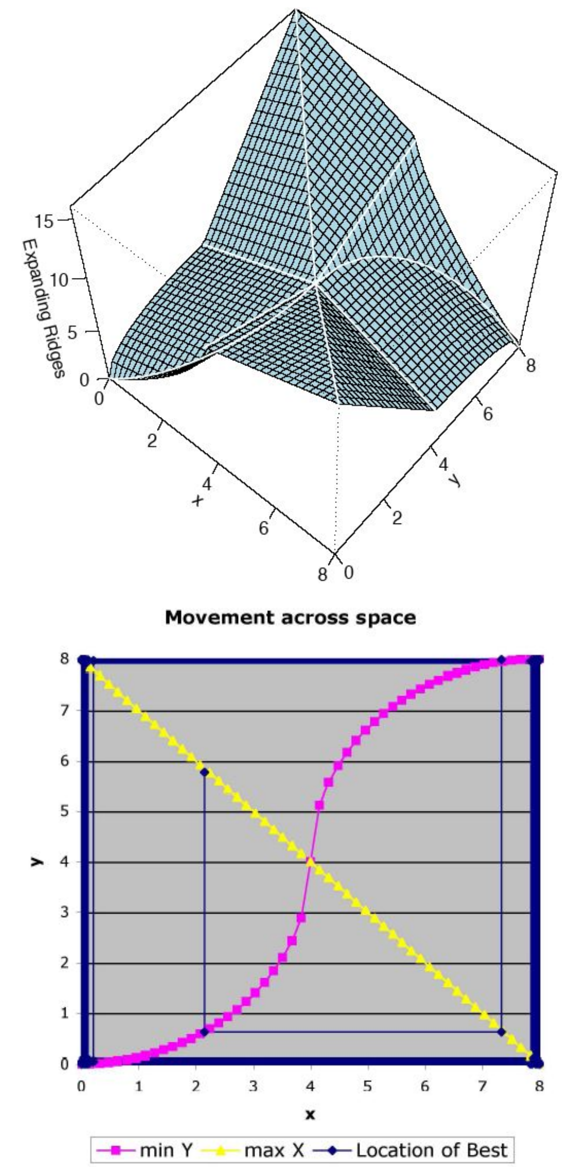
Cooperative Setting

- Importance of population diversity
- Importance of interaction diversity

Competitive Setting







Competitive Setting

- Critical role of interaction landscape
- Subtle changes affect dynamics

Co-EA Extensions:

- Archive methods (Ficici, ...)
  - External memory for emerging objective function
  - Require interactions with archive members
    - How?
    - When?

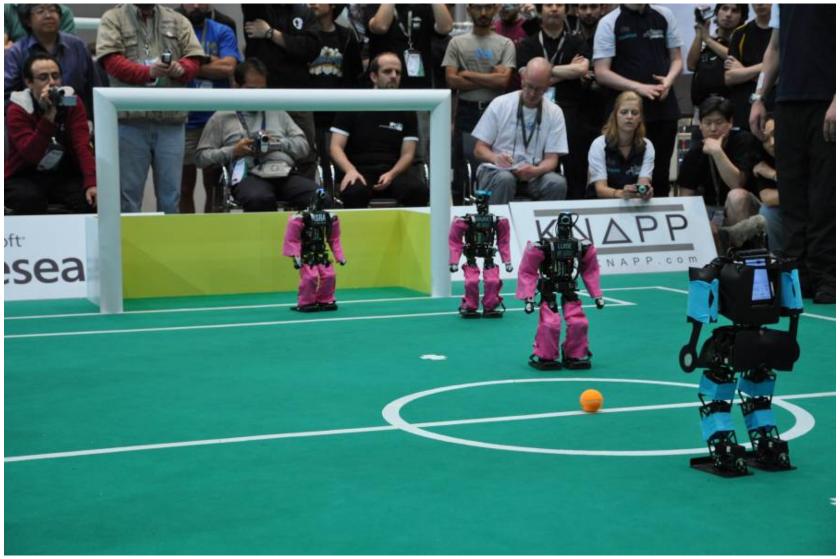
Co-EA Extensions:

- Spatial Co-EAs (Mitchell, ...)
  - Use spatial EAs at population level
  - Use spatially-constrained interactions
  - Result:
    - Positive impact on diversity
    - Damping impact on dynamics

Applications:

- What kinds of problems are they good for?
  - Optimization?
    - Example: No external objective fitness function
      - Fitness defined by internal interactions
      - Classic case: Game playing
      - Typical approach:
        - » Single population with competitive fitness
        - » Use an archive for emerging objective function

Applications:



Example: No external objective fitness function

- More difficult: team games
  - » Cooperation and competition
- Subpopulations (species) for evolving team members/roles
- Collectively compete against other teams

Applications:

- What kinds of problems are they good for?
  - Optimization?
    - Example: Decomposable objective functions
      - One population for each decomposable part
      - Fitness obtained by composing parts
      - Improve diversity via spatial EAs
      - Dynamic decomposition?

Applications:

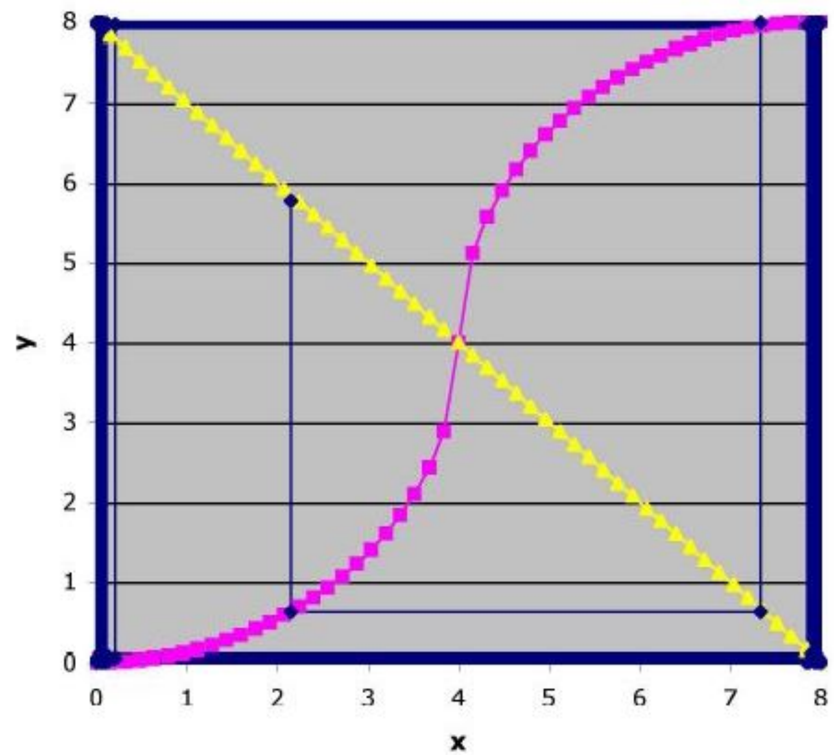
- What kinds of problems are they good for?
  - Optimization?
    - Example: Complex Production Control Problems
      - Production described as a DAG.
      - Nodes are individual local processes.
      - Local EA on each node optimizing in context of neighboring nodes.

Applications:

- What kinds of problems are they good for?
  - Adaptation?
    - Control dynamics to achieve goals
      - Useful mediocre, stable states?

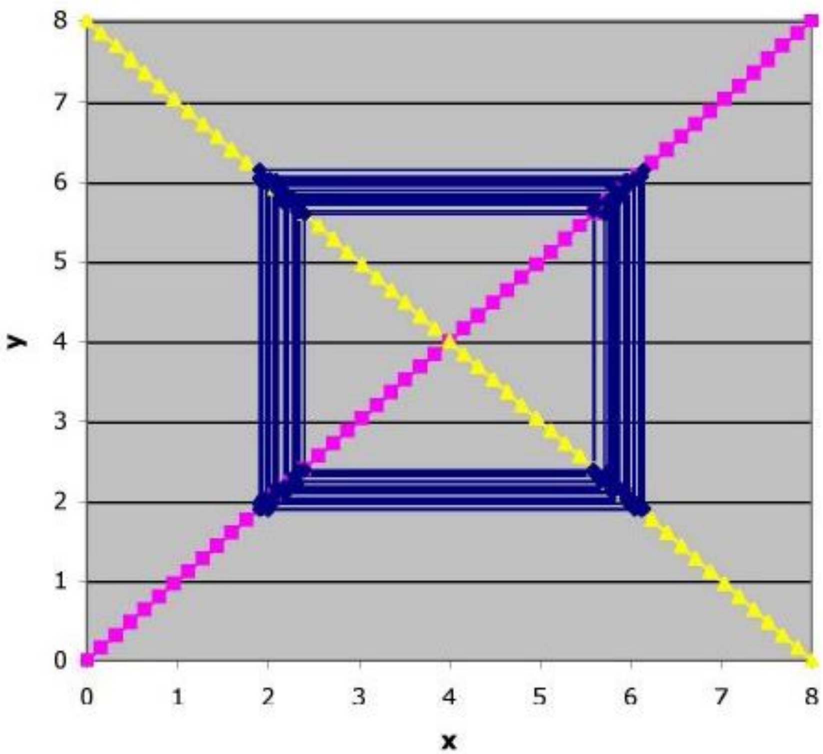
Undesired behavior

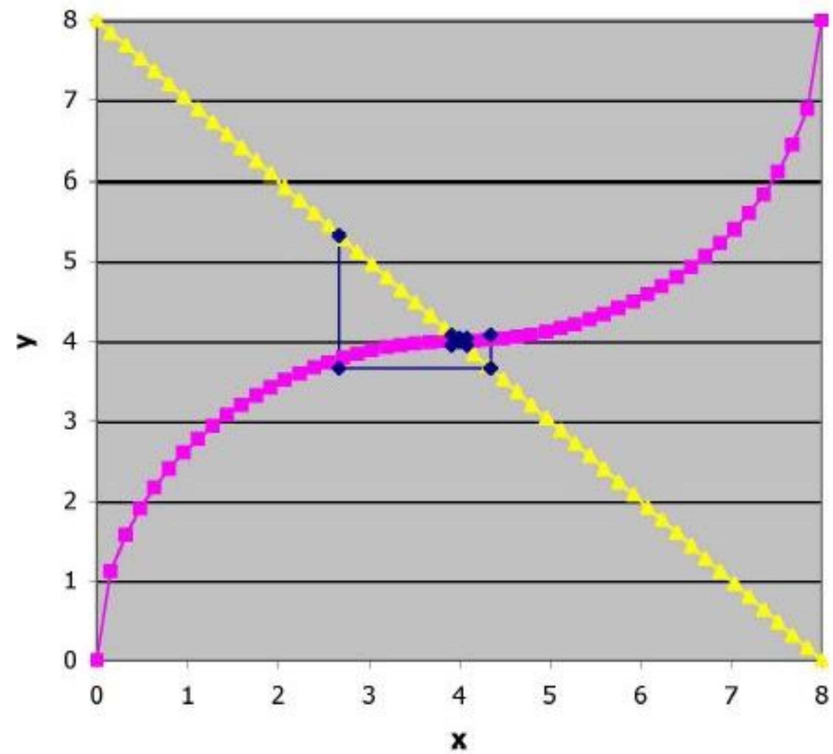
- Arms race: hackers keep getting better



Keeping hackers under control

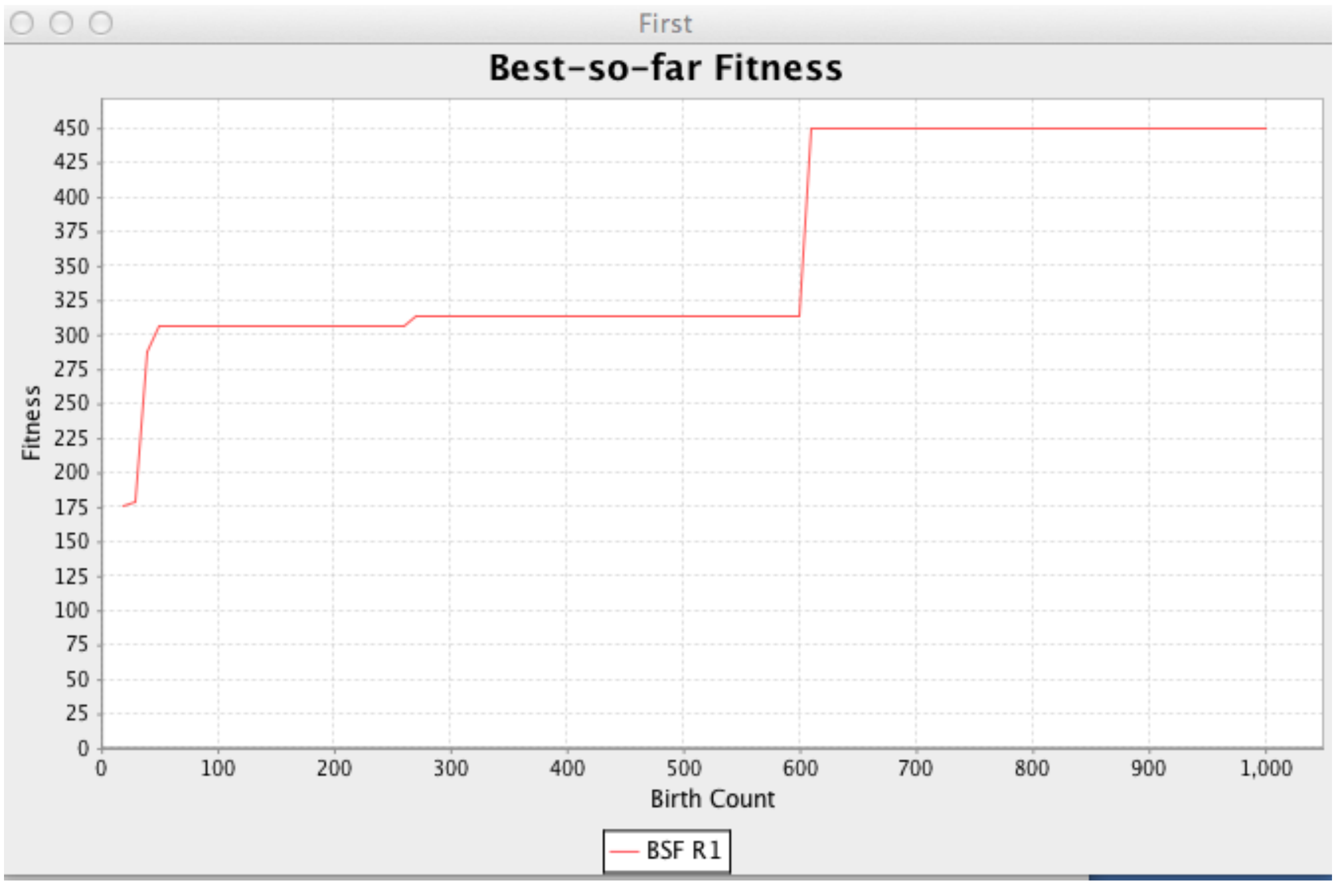
- Cycling
- Attract to stable state





Network Intrusion Example

- Hackers evolving against static security



Network Intrusion Example

- Hackers evolving against co-ev security

Applications:

- What kinds of problems are they good for?
  - Understanding complex adaptive systems
    - ICOSystem examples
  - E.g., understanding the implications of rule changes
    - Gov’t regulations
      - Airlines (FAA)
      - Stock exchanges (SEC)

Applications:

- What kinds of problems are they good for?
  - Abstract approach: (S. Ficici, A. Bucci, ...)
    - Solution concepts:

      Be more precise about desired solutions:

      » Maximize mean performance

      » Minimize worst-case performance

      » …
    - Monotonicity:

      Theorems about monotonicity of solution concepts

      » Local interaction fitness => global solutions

Lots of open issues:

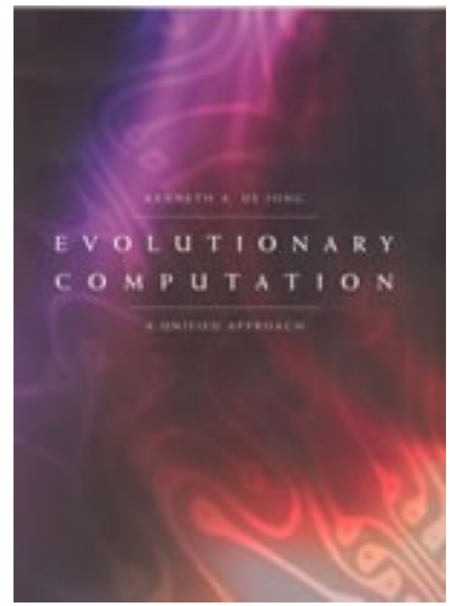
- EGT Analysis
  - Reproductive variation
  - Finite population models
  - Partial mixing
- Dynamical systems analysis
  - Spatial models
  - Archives
- More than two populations

Conclusions:

- Co-EAs behaviorally different from EAs.
- Increasing understanding of those differences.
- A number of successful applications.
- Lots of opportunities for:
  - Improved analysis
  - Additional applications

More information:

- Journals:
  - Evolutionary Computation (MIT Press)
  - Trans. on Evolutionary Computation (IEEE)
  - Genetic Programming & Evolvable Hardware
- Conferences:
  - GECCO, CEC, PPSN, FOGA, …
- Internet:
  - www.cs.gmu.edu/~eclab
- My book:



  - Evolutionary Computation: A Unified Approach
    - MIT Press, 2006