14. Si $\nabla^2 \underline{E} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\underline{E} = 0$ y $\underline{E}(\underline{r},t) = Re\{\underline{\tilde{E}}(\underline{r})e^{i\omega t}\}$ entonces

$$\nabla^2 \underline{\tilde{E}} + \omega^2 \mu_0 \epsilon \underline{\tilde{E}} = 0,$$

demostrar esto último. Es notación usual usar $\underline{U}$ para el fasor, $\underline{U} = \underline{\tilde{E}}$.

Para esto usamos identidades vectoriales:

$\nabla^2 \underline{E} = \nabla^2(\underline{\tilde{E}}e^{i\omega t}) = \underline{\tilde{E}}\nabla^2 e^{i\omega t} + 2(\nabla e^{i\omega t}\cdot\nabla)\underline{\tilde{E}} + e^{i\omega t}\nabla^2\underline{\tilde{E}}$, dado que $e^{i\omega t}$ no depende de $\underline{r}$ entonces $\nabla e^{i\omega t} = 0$, de modo que $\nabla^2 \underline{E} = e^{i\omega t}\nabla^2\underline{\tilde{E}}$.

Luego $\frac{\partial^2}{\partial t^2}\underline{E} = \underline{\tilde{E}}\frac{\partial^2}{\partial t^2}e^{i\omega t} = -\omega^2\underline{\tilde{E}}e^{i\omega t}$, así pues

$\nabla^2\underline{\tilde{E}} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\underline{E} = e^{i\omega t}\nabla^2\underline{\tilde{E}} + \frac{1}{v^2}\omega^2\underline{\tilde{E}}e^{i\omega t} = e^{i\omega t}(\nabla^2\underline{\tilde{E}} + \omega^2\mu\epsilon\underline{\tilde{E}})$,

al estar considerando materiales no magnéticos tenemos $\mu = \mu_0$, entonces

$$\nabla^2\underline{\tilde{E}} + \omega^2\mu_0\epsilon\underline{\tilde{E}} = 0.$$