

43. Obtener el espectro en frecuencias de la intensidad generada por un campo $t_a(\xi, \eta) = |\cos(2\pi f \xi)|$, tanto para luz coherente como incoherente. Utilizar una pupila rectangular $H(f_x, f_y) = \text{rect}(f_x/2f_0) \text{rect}(f_y/2f_0)$, $f < f_0 < 2f$.

• Luz coherente:

En este caso tenemos $\mathcal{F}\{I_i\} = (HG_g) * (HG_g)$ con G_g la TF de t_a . Se hará sobre el eje f_x , así

$$G_g(f_x) = \int_{-\infty}^{\infty} |\cos(2\pi f \xi)| e^{-i2\pi f_x \xi} d\xi, \text{ por otro lado tenemos que}$$

$$\cos(2\pi f \xi) \geq 0 \Leftrightarrow \frac{1}{f} \left(n - \frac{1}{4}\right) \leq \xi \leq \frac{1}{f} \left(n + \frac{1}{4}\right)$$

$$\cos(2\pi f \xi) \leq 0 \Leftrightarrow \frac{1}{f} \left(n + \frac{1}{4}\right) \leq \xi \leq \frac{1}{f} \left(n + \frac{3}{4}\right),$$

para $n \in \mathbb{Z}$, de modo que

$$G_g(f_x) = \sum_{n=-\infty}^{\infty} \left[\underbrace{\int_{(n-1/4)/f}^{(n+1/4)/f} \cos(2\pi f \xi) e^{-i2\pi f_x \xi} d\xi}_{I_1(f_x; n)} - \underbrace{\int_{(n+1/4)/f}^{(n+3/4)/f} \cos(2\pi f \xi) e^{-i2\pi f_x \xi} d\xi}_{I_2(f_x; n)} \right],$$

$$\Rightarrow I_1(f_x; n) = \frac{1}{i4\pi} \left[\frac{1}{f-f_x} e^{i2\pi(f-f_x)\xi} \Big|_{(n-1/4)/f}^{(n+1/4)/f} - \frac{1}{f+f_x} e^{-i2\pi(f+f_x)\xi} \Big|_{(n+1/4)/f}^{(n+3/4)/f} \right]$$

$$= \frac{1}{2\pi} e^{-i2\pi n f_x / f} \left[\frac{1}{f-f_x} \sin\left(\frac{\pi}{2} \left(1 - \frac{f_x}{f}\right)\right) + \frac{1}{f+f_x} \sin\left(\frac{\pi}{2} \left(1 + \frac{f_x}{f}\right)\right) \right]$$

$$= \frac{1}{2\pi} e^{-i2\pi n f_x / f} \cos\left(\frac{\pi}{2} \frac{f_x}{f}\right) \left(\frac{1}{f-f_x} + \frac{1}{f+f_x}\right), \text{ entonces}$$

$$I_1(f_x; n) = \frac{f}{\pi(f^2 - f_x^2)} \cos\left(\frac{\pi}{2} \frac{f_x}{f}\right) e^{-i2\pi n f_x / f},$$

luego con el cambio de variable $\xi' = \xi - 1/2f$ uno encuentra que $I_2(f_x; n) = -e^{-i\pi f_x / f} I_1(f_x; n)$, de modo que

$$G_g(f_x) = (1 + e^{-i\pi f_x/\bar{f}}) \sum_{n=-\infty}^{\infty} I_1(f_x, n) \\ = \frac{\bar{f}}{\pi(\bar{f}^2 - f_x^2)} \cos\left(\frac{\pi}{2} \frac{f_x}{\bar{f}}\right) (1 + e^{-i\pi f_x/\bar{f}}) \underbrace{\sum_{n=-\infty}^{\infty} e^{i2\pi n f_x/\bar{f}}}_{\text{comb}(f_x/\bar{f})}$$

$$\therefore G_g(f_x) = \frac{\bar{f}}{\pi(\bar{f}^2 - f_x^2)} \cos\left(\frac{\pi}{2} \frac{f_x}{\bar{f}}\right) (1 + e^{-i\pi f_x/\bar{f}}) \text{comb}(f_x/\bar{f}),$$

entonces al multiplicar por $H(f_x, 0) = \text{rect}(f_x/2f_0)$ se tiene que

$$(HG_g)(f_x) = \frac{\bar{f}^2}{\pi(\bar{f}^2 - f_x^2)} \cos\left(\frac{\pi}{2} \frac{f_x}{\bar{f}}\right) (1 + e^{-i\pi f_x/\bar{f}}) [\delta(f_x + \bar{f}) + \delta(f_x) + \delta(f_x - \bar{f})] \\ = \frac{2}{\pi} \delta(f_x)$$

$$\therefore \mathcal{F}\{I_i\} = (HG_g) * (HG_g) = \frac{4}{\pi^2} \delta(f_x)$$

• Luz incoherente:

Necesitamos $\mathcal{F}\{I_i\} = (H * H)(G_g * G_g)$; ya sabemos que $H * H$ es la función triángulo con máximo 1 y que va de $-2f_0$ a $2f_0$. Otra forma de escribir G_g , dado que la función comb es suma de δ , es

$$G_g(f_x) = \frac{2}{\pi} \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{1-4k^2} \delta(f_x - 2k\bar{f}),$$

así

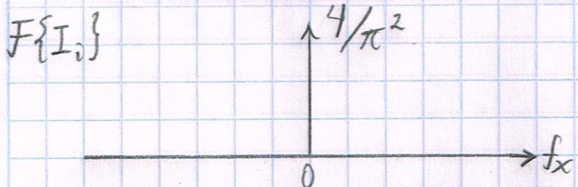
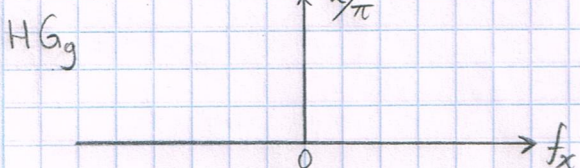
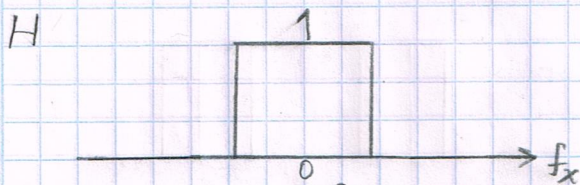
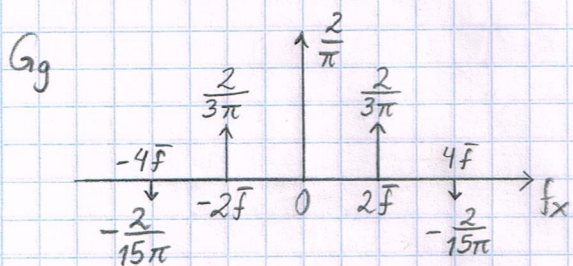
$$(G_g * G_g)(f_x) = \frac{4}{\pi^2} \sum_{k,l=-\infty}^{\infty} \frac{(-1)^{k+l}}{(1-4k^2)(1-4l^2)} \int_{-\infty}^{\infty} \delta(t - 2k\bar{f}) \delta(f_x - t - 2l\bar{f}) dt$$

$$\Rightarrow (G_g * G_g)(f_x) = \frac{4}{\pi^2} \sum_{k,l=-\infty}^{\infty} \frac{(-1)^{k+l}}{(1-4k^2)(1-4l^2)} \delta(f_x - 2(k+l)/\bar{f}), \text{ entonces}$$

$$(H * H)(G_g * G_g)(f_x) = (H * H)(f_x) \left[\frac{1}{4} \delta(f_x + 2\bar{f}) + \frac{1}{2} \delta(f_x) + \frac{1}{4} \delta(f_x - 2\bar{f}) \right]$$

de manera que:

Coherente



Incoherente

