Random Walk Matrix

(i) Transition Matrix M

M is the transition matrix. If there is an edge from $i \to j$ then $M_{ij} = \frac{1}{d_i}$. The matrix then becomes:

$$M = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 1/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(ii) Stationary Distribution r

Let $r = [a, b, c, d]^T$, we need to solve $Mr = r$ or, $(M - I)r = 0$. This will give us the following system of linear equations:

$$\begin{aligned} -a + b/2 + d/3 &= 0 \\ a/2 - b + d/3 &= 0 \\ -c + d/3 &= 0 \\ a/2 + b/2 + c - d &= 0 \end{aligned}$$

Solving this we can get $r = d \cdot [2/3, 2/3, 1/3, 1]^T$. However, we also know $r^T r = 1$ (r is normalized), this gives $d = \frac{1}{\sqrt{2}}$. So the final answer is:

$$r = \begin{bmatrix} \sqrt{2}/3 \\ \sqrt{2}/3 \\ 1/(3\sqrt{2}) \\ 1/\sqrt{2} \end{bmatrix}$$