Recombination of shower partons in fragmentation processes

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We develop the notion of shower partons and determine their distributions in jets in the framework of the recombination model. The shower parton distributions obtained render a good fit of the fragmentation functions. We then illustrate the usefulness of the distributions in a problem where a jet is produced in the environment of thermal partons as in heavy-ion collisions. The recombination of shower and thermal partons is shown to be more important than fragmentation. Application to the study of two-particle correlation in a jet is also carried out.

I. INTRODUCTION

The theoretical description of hadron production at large transverse momentum (p_T) in either hadronic or nuclear collisions at high energies is traditionally framed in a two-step process that involves first a hard scattering of partons, followed by the fragmentation of the scattered parton to the detected hadron [1,2]. The first part is calculable in perturbation QCD, while the second part makes use of fragmentation functions that are determined phenomenologically. Such a production mechanism has recently been found to be inadequate for the production of particles at intermediate p_T in heavy-ion collisions [3–5]. Instead of fragmentation it is the recombination of partons that is shown to be the more appropriate hadronization process, especially when the soft partons are involved. Although at extremely high p_T fragmentation is still dominant, it is desirable to have a universal description that can be applied to any p_T, based on the same hadronization scheme. To achieve that goal it is necessary that the fragmentation process can be treated as the result of recombination of shower partons in a jet. The purpose of this paper is to take that first step, namely, to introduce the notion of shower partons and to determine their distributions in order to represent the phenomenological fragmentation functions in terms of recombination.

The subject matter of this work is primarily of interest only to high-energy nuclear collisions because hadronization in such processes is always in the environment of soft partons. Semihard shower partons initiated by a hard parton can either recombine among themselves or recombine with soft partons in the environment. In the former case the fragmentation function is reproduced, and nothing new is achieved. It is in the latter case that a component emerges in heavy-ion collisions, one that has escaped theoretical attention thus far. It should be an important hadronization process in the intermediate p_T region. Our main objective here is to quantify the properties of shower partons and to illustrate the importance of their recombination with thermal partons. The actual application of the shower parton distributions (SPD) developed here to heavy-ion collisions will be considered elsewhere [6].

The concept of shower partons is not new, since attempts have been made to generate such partons in perturbative

QCD (pQCD) processes as far as is permitted by the validity of the procedure. Two notable examples of such attempts are the work of Marchesini and Webber [7] and Geiger [8]. However, since pQCD cannot be used down to the hadronization scale, the branching or cascading processes terminate at the formation of color-singlet prehadronic clusters, which cannot easily be related to our shower partons and their hadronization. We shall discuss in more detail at the end of Secs. III and IV the similarities and differences in the various approaches.

II. RECOMBINATION MODEL FOR FRAGMENTATION

The fragmentation of a parton to a hadron is not a process that can be calculated in pQCD, although the Q^2 evolution of the fragmentation function (FF) is calculable. The FF's are usually parametrized by fitting the data from e^+e^- annihilations [9–11] as well as from $p\bar{p}$ and $e^{\pm}p$ collisions [11]. Although the QCD processes of generating a parton shower by gluon radiation and pair creation cannot be tracked by perturbative methods down to low virtuality, we can determine the SPD's phenomenologically in much the same way that the FF's themselves are, except that we fit the FF's, whereas the FF's are determined by fitting the data. An important difference is that both the shower partons and their distributions are defined in the context of the recombination model, which is the key link between the shower partons (inside the black box called FF) and the observed hadron (outside the black box).

In the recombination model the generic formula for a hadronization process is [12]

$$xD(x) = \int_0^x \frac{dx_1}{x_1} \int_0^x \frac{dx_2}{x_2} F_{q\bar{q}'}(x_1, x_2) R(x_1, x_2, x), \quad (1)$$

where $F_{q\bar{q}'}(x_1, x_2)$ is the joint distribution of a quark q at momentum fraction x_1 and an antiquark $\bar{q}'$ at x_2, and $R(x_1, x_2, x)$ is the recombination function (RF) for the formation of a meson at x. We have written the left-hand side (LHS) of Eq. (1) as $xD(x)$, the invariant FF, but the right-hand side (RHS) would have the same form if the equation were written for the inclusive distribution, xdN/dx, of a me-

son produced in a collisional process. In the former case of fragmentation, $F_{q\bar{q}'}$ refers to the shower partons initiated by a hard parton. In the latter case of inclusive production, $F_{q\bar{q}'}$ refers to the q and $\bar{q}'$ that are produced by the collision and are to recombine in forming the detected meson. The equations for the two cases are similar because the physics of recombination is the same. In either case the major task is in the determination of the distribution $F_{q\bar{q}'}$.

We now focus on the fragmentation problem and regard Eq. (1) as the basis of the recombination model for fragmentation. The LHS is the FF, known from the parametrization that fits the data. The RHS has the RF that is known from previous studies of the recombination model [12,13] and will be specified in the next section. Thus, it is possible to determine the properties of $F_{q\bar{q}'}$ from Eq. (1). To facilitate that determination we shall assume that $F_{q\bar{q}'}$ is factorizable except for kinematic constraints, i.e., in schematic form we write it as

$$F_{q\bar{q}'}^{(i)}(x_1, x_2) = S_i^q(x_1) S_i^{\bar{q}'}(x_2), \tag{2}$$

where $S_i^q(x_1)$ denotes the distribution of shower parton q with momentum fraction x_1 in a shower initiated by a hard parton i. The exact form with proper kinematic constraints will be described in detail in the next section. Here we remark on the general implications of Eqs. (1) and (2).

The important point to emphasize is that we are introducing the notion of shower partons and their momentum distributions $S_i^j(x_1)$. The significance of the SPD is not to be found in problems that involve only the collisions of leptons and hadrons, for which the fragmentation of partons is known to be an adequate approach, and the recombination of shower partons merely reproduces what is already known. The knowledge about the SPD becomes crucial when the shower partons recombine with other partons that are not in the jet but are in the ambient environment. We shall illustrate this important point later.

It should be recognized that the SPD that we shall determine through the use of Eqs. (1) and (2) depends on the specific form of $R(x_1, x_2, x)$, which in turn depends on the wave function of the meson produced. It would be inconsistent to use our S_i^j given below in conjunction with some approximation of the RF that differs significantly from our R. The recombination of two shower partons must recover the FF from which the SPD's are obtained.

Finally, we remark that S_i^j should in principle depend on Q^2 at which the $D(x, Q^2)$ is used for its determination, since Q^2 evolution affects both. It is outside the scope of this paper to treat the Q^2 dependence of S_i^j. Our aim here is to show how S_i^j can be determined phenomenologically, and how it can be applied, when Q^2 is fixed. The same method can be used to determine S_i^j at other values of Q^2. In practice, the Q^2 dependence of S_i^j is not as important as the inclusion of the role of the shower partons in the first place at any reasonably approximate Q^2 in heavy-ion collisions where hard partons are produced in a range of transverse momentum.

III. SHOWER PARTON DISTRIBUTIONS

In order to solve Eqs. (1) and (2) for S_i^j, we first point out that there are various $D(x)$ functions corresponding to vari-

ous fragmentation processes. We shall select five of them, from which we can determine five SPD's. Three of them form a closed set that involves no strange quarks or mesons. Let us start with those three. Consider the light quarks u, d, $\bar{u}$, $\bar{d}$, and gluon g. They can all fragment into pions. To reduce them to three essential FF's, we consider the three basic types D_V^π, D_S^π, and D_G^π, that correspond to valence, sea, and gluon fragmentation, respectively. If the fragmenting quark has the same flavor as that of a valence quark in π, then the valence part of the fragmentation is described by D_V^π, e.g., $u_v \rightarrow \pi^+$, $d_v \rightarrow \pi^-$, $\bar{u}_v \rightarrow \pi^-$, the subscript v referring to the valence component. All other cases of quark fragmentation are described by D_S^π, e.g., $u \rightarrow \pi^-$, $d \rightarrow \pi^+$, $\bar{u} \rightarrow \pi^+$. If the initiating parton is a gluon, then we have D_G^π for any state of π. Those FF's are given by Ref. [9] in parametric form. We shall use them even though they are older than the more recent ones [10,11,14], which do not give the D_V^π and D_S^π explicitly. Our emphasis here is not on accuracy, but on the feasibility of extracting the SPD's from the FF's of the type discussed above. We shall determine S_i^j from the parametrization of Ref. [9] with Q^2 fixed at 100 GeV^2 and demonstrate that the use of shower partons is important in heavy-ion collisions.

For the SPD's we use the notation K_{NS} and L for valence and sea-quark distributions, respectively, in a shower initiated by a quark or antiquark, and G for any light quark distribution in a gluon-initiated shower. That is, for example, $K_{NS} = S_u^{u_v}$, $L = S_u^d$, $G = S_g^u$. It should be recognized that L also describes the sea quarks of the same flavor, such as $S_u^{u_{sea}}$, so that the overall distribution of shower quark that has the same flavor as the initiating quark (e.g., $u \rightarrow u$) is given by

$$K = K_{NS} + L. \tag{3}$$

It is evident from the above discussion that there is a closed relationship that is independent of other unknowns. It follows from Eq. (1) when restricted to sea-quark fragmentation:

$$x D_S^\pi(x) = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} L(x_1) L\left(\frac{x_2}{1-x_1}\right) R_\pi(x_1, x_2, x). \tag{4}$$

The sea-SPD $L(z)$ can be determined from this equation alone. In Eq. (4) we have exhibited the argument of the second L function that reflects the momentum constraint, i.e., if one shower parton has momentum fraction x_1, then the momentum fraction of the other recombining shower parton cannot exceed $1 - x_1$, and can only be a fraction of the balance $x_2/(1-x_1)$. Symmetrization of x_1 and x_2 is automatic by virtue of the invariance of $R_\pi(x_1, x_2, x)$ under the exchange of x_1 and x_2.

After $L(z)$ is determined from Eq. (4), we next can obtain K_{NS} from

$$x D_V^\pi(x) = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \{K_{NS}(x_1), L(x_2)\} R_\pi(x_1, x_2, x), \tag{5}$$

where the curly brackets define the symmetrization of the leading parton momentum

$$\{K_{NS}(x_1),L(x_2)\} \equiv \frac{1}{2}\left[K_{NS}(x_1)L\left(\frac{x_2}{1-x_1}\right) \right.$$
$$\left. + K_{NS}\left(\frac{x_1}{1-x_2}\right)L(x_2) \right]. \qquad (6)$$

Finally, we have the closed equation for the gluon-initiated shower

$$xD_G^\pi(x) = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}G(x_1)G\left(\frac{x_2}{1-x_1}\right)R_\pi(x_1,x_2,x). \qquad (7)$$

In this nonstrange sector we have three SPD's (L, K_{NS}, and G) to be determined from the three phenomenological FF's (D_S^π, D_V^π, and D_G^π).

In extending the consideration to the strange sector, we must make use of L and G determined in the above set and two FF's D_S^K and D_G^K that describe the fragmentation of a nonstrange quark and gluon, respectively, to a kaon. That is, we have

$$xD_S^K(x) = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\{L(x_1),L_s(x_2)\}R_K(x_1,x_2,x), \qquad (8)$$

$$xD_G^K(x) = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\{G(x_1),G_s(x_2)\}R_K(x_1,x_2,x), \qquad (9)$$

where L_s and G_s are two additional SPD's specifying the strange quark distributions in showers initiated by nonstrange and gluon partons, respectively. R_K is the RF for kaon.

To complete the description of the integral equations, we now specify the RF's. They depend on the square of the wave functions of the mesons, π and K, whose structures in momentum space have been quantified in the valon model [12,13]. Unlike the case of the proton, whose structure is well studied by deep inelastic scattering so that the valon distribution can be obtained from the parton distribution functions [15], the RF for the pion relies on the parton distribution of the pion probed by the Drell-Yan process [16]. The derivation of the RF's for both π and K is given in Ref. [13]; they are

$$R_\pi(x_1,x_2,x) = \frac{x_1x_2}{x^2}\delta\left(\frac{x_1}{x}+\frac{x_2}{x}-1\right), \qquad (10)$$

$$R_K(x_1,x_2,x) = 12\left(\frac{x_1}{x}\right)^2\left(\frac{x_2}{x}\right)^3\delta\left(\frac{x_1}{x}+\frac{x_2}{x}-1\right). \qquad (11)$$

The δ functions guarantee the momentum conservation of the recombining quarks and antiquarks, which are dressed and become the valons of the produced hadrons.

Since the recombination process involves the quarks and antiquarks, one may question the fate of the gluons. This problem has been treated in the formulation of the recombination model [12], where gluons are converted to quark-antiquark pairs in the sea before hadronization. That is, the sea is saturated by the conversion to carry all the momentum, save the valence parton. Such a procedure has been shown to give the correct normalization of the inclusive cross section

TABLE I. Parameters in Eq. (12) for $Q^2 = 100$ GeV^2.

	N	α	β	γ
D_S^π	2.7236	−0.734	3.384	−5.471
D_V^π	0.2898	−1.040	1.608	−0.111
D_G^π	0.7345	−1.112	2.547	−0.541
D_S^K	0.2106	−1.005	2.548	−0.620
D_G^K	0.0768	−1.481	2.489	−0.778

of hadronic collisions [12]. In the present problem of parton fragmentation we implement the recombination process in the same framework, although gluon conversion is done only implicitly. What is explicit is that the gluon degree of freedom is not included in the list of shower partons. It means that in the equations for D_V^π, D_S^π, and D_G^π (and likewise in the strange sector) only K_{NS}, L, and G appear; they are the SPD's of quarks and antiquarks that are to recombine. Those quarks and antiquarks must include the converted sea, since they are responsible for reproducing the FF's through Eqs. (4), (5), and (7) without gluons. Thus the shower partons whose momentum distributions we calculate are defined by those equations that have no gluon component for recombination, and would not be the same as what one would conceptually get (if it were possible) in a pQCD calculation that inevitably has both quarks and gluons.

It should be noted that our procedure of converting gluons to $q\bar{q}$ pairs is essentially the same as what is done in Ref. [7], whose branching processes terminate at the threshold of the nonperturbative regime. In that approach, nearby quarks and antiquarks that are the products of the conversion from different gluons form color-singlet clusters of various invariant masses that subsequently decay (or fragment as in strings) sequentially through resonances to the lowest lying hadron states [17]. A similar but not identical approach is taken in Ref. [18], where gluons are not directly converted to $q\bar{q}$ pairs, but are either absorbed or annihilated by $g+g\rightarrow q+\bar{q}$ Born-diagram processes.

IV. RESULTS

We now proceed to solve the integral equations for the five FF's, which are known from Ref. [9]. Those equations relate them to the five unknown SPD's: K_{NS}, L, G, L_s, and G_s. If those equations were algebraic, we obviously could solve them for the unknowns. Being integral equations, they can nevertheless be "solved" by a fitting procedure that should not be regarded as being unsatisfactory for lack of mathematical rigor, since the FF's themselves are obtained by fitting the experimental data in some similar manner. Indeed, the FF's in the next-to-leading order are given in parametrized forms [9]

$$D_k^h(x) = Nx^\alpha(1-x)^\beta(1+x)^\gamma, \qquad (12)$$

where the parameters for $Q^2 = 100$ GeV^2 are given in Table I for $k=S,V,G$, and $h=\pi,K$.

All five SPD's are denoted collectively by S_i^j with $i = q,\bar{q},g$ and $j=q,s,\bar{q},\bar{s}$, where q can be either u or d. If in i
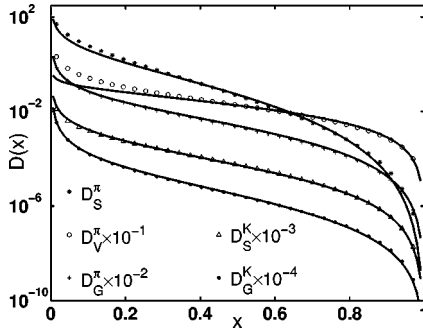
FIG. 1. Fragmentation functions, as parametrized in Ref. [9], are shown in symbols, while those calculated in the recombination model are shown by the solid lines. All curves are for $Q^2 = 100$ GeV^2.

the initiating hard parton is an s quark, it is treated as q. That is not the case if s is in the produced shower. Our parametrization of S_i^j has the form

$$S_i^j(z) = Az^a(1-z)^b(1+cz^d), \qquad (13)$$

where the dependences of the parameters A, a, etc., on i and j are not exhibited explicitly, just as in Eq. (12). Substituting Eqs. (10) and (11) into Eqs. (4)–(9), we can determine the parameters one equation at a time, i.e., L from D_S^π, G from D_G^π, and then K_{NS} from D_V^π and so on. In most cases it can be shown that the $x \rightarrow 1$ limit requires $b = \beta$. The final results of the fits are shown in Fig. 1 with the corresponding parameters given in Table II.

It is evident from Fig. 1 that all the fits are very good, except in the low x region of $D_V^\pi(x)$. In the latter case we are constrained by the condition

The integral is a sum of two beta functions.

$$\int \frac{dz}{z} K_{NS}(z) = 1 \qquad (14)$$

that is imposed by the requirement that there can be only one valence quark in the shower partons. However, the fit for $x > 0.4$ is excellent, and that is the important region for the determination of $K_{NS}(z)$. In application to $u \rightarrow \pi^+$, say, the u quark in the shower must have both valence and sea quarks so the shower distribution for the u quark is always the sum $K(z) = K_{NS}(z) + L(z)$. Since $L(z)$ is large at small z, and is accurately determined, the net result for $K(z)$ should be quite satisfactory.

TABLE II. Parameters in Eq. (13).

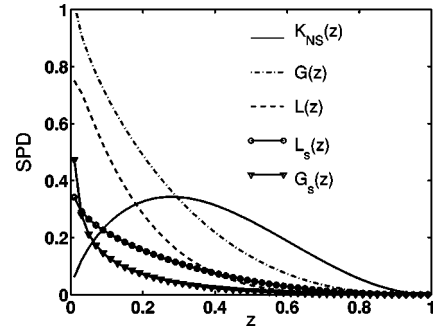|	A	a	b	c	d
K_{NS}	0.333	0.45	2.1	5.0	0.5
L	1.881	0.133	3.384	−0.991	0.31
G	0.811	−0.056	2.547	−0.176	1.2
L_s	0.118	−0.138	2.3	0.90	0.1
G_s	0.069	−0.425	2.489	−0.5	1.1

FIG. 2. Shower parton distributions determined in the recombination model, corresponding to the parametrization given in Table II.

It is remarkable how well the FF's in Fig. 1 are reproduced in the recombination model. The corresponding SPD's that make possible the good fit are shown in Fig. 2. They have very reasonable properties, namely: (a) valence quark is harder, (b) sea quarks are softer, (c) gluon jet has a higher density of shower partons, and (d) the density of produced s quarks is lower than that of the light quarks.

It is appropriate at this point to relate our approach to those of Marchesini-Webber [7] and Geiger [8], which are serious attempts to incorporate the QCD dynamics in their description of the branching and collision processes. The former is done in the momentum space only, whereas the latter is formulated in space time as well as in momentum space. The parton cascade model of Geiger is a very ambitious program that treats a large variety of processes ranging from e^+e^- annihilation [18] to deep inelastic scattering [19] to hadronic and nuclear collisions [20]. The evolution of partons is tracked by use of relativistic transport equations with gain and loss terms. Cluster formation takes into account the invariant distance between near-neighbor partons. Cluster decay makes use of the Hagedorn spectrum and the particle data table. Because of the complexity of the problems, both QCD models are implemented by Monte Carlo codes. The predictive power of the models is exhibited as numerical outputs that cannot easily be adapted for comparison with our results on the SPD's. Our approach makes no attempt to treat the QCD dynamics; however, the SPD's obtained are guaranteed to reproduce the FF's on the one hand, and are conveniently parametrized for use in other context that goes beyond fragmentation, as we shall show in the next section. From the way the color-singlet clusters are treated in the QCD models, it is clear that our shower partons do not correspond to the partons of those models at the end of their evolution processes, except in the special case when the cluster consists of only one particle. In our approach, the nonperturbative part of how the shower partons dress themselves and recombine to form hadrons with the proper momentum-fraction distributions is contained in the RF's. Such shower partons that are ready to hadronize are sufficiently far from other shower partons as to be independent from them. In general, they cannot be identified with the q and $\bar{q}$ that form the color-singlet clusters in the QCD models, but are more closely related to the constituents of the final hadrons, as in the case of quarkonium formation [21]. The distribution of
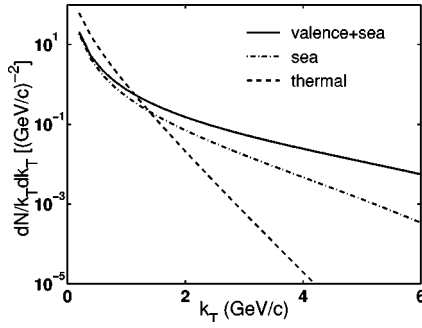
FIG. 3. Parton distributions in transverse momentum k_T for valence+sea quark (solid line), sea quark (dashed-dotted line), and thermal partons (dashed line).
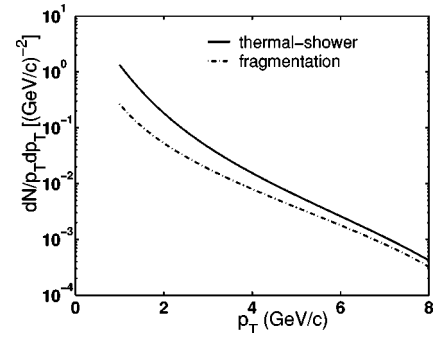

FIG. 4. Distributions of π^+ in p_T arising from thermal-shower recombination (solid line) and shower-shower recombination, i.e., fragmentation (dashed-dotted line).

those constituents in a hard-parton shower cannot be displayed in the QCD models, but are determined by us by solving Eqs. (4) and (9).

V. APPLICATIONS

As we have stated in the introduction, the purpose of determining the SPD's is for their application to problems where the FF's are insufficient to describe the physics involved. We consider in this section two such problems as illustrations of the usefulness of the SPD's. The first is when a hard parton is produced in the environment of thermal partons, as in heavy-ion collisions. The second is the determination of two-pion distribution in a jet.

Let us suppose that a u quark is produced at $k_T = 10$ GeV/c in a background of thermal partons whose invariant k_T distribution is

$$T(k_T) = k_T \frac{dN}{dk_T} = Ck_T e^{-k_T/T}. \quad (15)$$

Let the parameters C and T be chosen to correspond to a typical situation in Au+Au collisions at $\sqrt{s} = 200$ GeV [6]

$$C = 23.2 \text{ GeV}^{-1}, \quad T = 0.317 \text{ GeV}. \quad (16)$$

The high-k_T u quark generates a shower of partons with various flavors. Consider specifically u and $\bar{d}$ in that shower. The valence quark distribution is given by $K_{NS}(x_1)$, while the $\bar{d}$ sea-quark distribution (including the ones converted from the gluons) is given by $L(x_1)$. In Fig. 3 we plot $dN/k_T dk_T$ for (a) u quark (valence and sea) in solid line, (b) $\bar{d}$ sea antiquark in dashed-dotted line, and (c) $\bar{d}$ thermal antiquark in dashed line. They correspond to $k_T^{-2} \times$ (invariant distributions $K = K_{NS}+L$, L, and T, respectively), in which $K(x_1)$ and $L(x_1)$ are evaluated at $k_T = x_1 k_T^{max}$, with $k_T^{max} = 10$ GeV/c. Note that the thermal distribution is higher than the shower parton distributions for $k_T < 1$ GeV/c. That makes a crucial difference in the recombination of those partons. Such a thermal distribution is absent in pp collisions, whose soft partons are at least two orders of magnitude lower. In e^+e^- annihilation there are, of course, no soft partons at all.

We now calculate the production of π^+ from the assemblage of u and $\bar{d}$ partons. The thermal-shower (TS) recombination gives rise to

$$\frac{dN_{\pi^+}^{TS}}{p_T dp_T} = \frac{1}{p_T^3} \int_0^{p_T} dk_T K(k_T/k_T^{max}) T(p_T - k_T), \quad (17)$$

where Eq. (10) has been used in an equation such as Eq. (1) for xdN_π/dx, but expressed for $dN_\pi/p_T dp_T$. Using Eqs. (3) and (15) and the parametrizations given in Table II, the integral in Eq. (17) can readily be evaluated. The result is shown by the solid line in Fig. 4. It is to be compared with the p_T distribution from the fragmentation of the u quark to π^+, which is

$$\frac{dN_{\pi^+}^{SS}}{p_T dp_T} = \frac{1}{p_T^3} \int_0^{p_T} dk_T \left\{ K\left(\frac{k_T}{k_T^{max}}\right), L\left(\frac{p_T - k_T}{k_T^{max} - k_T}\right) \right\}, \quad (18)$$

Since this is just retracing the path in which we obtained K and L from the D function in the first place, Eq. (18) can more directly be identified with

$$\frac{dN_{\pi^+}^{frag}}{p_T dp_T} = (p_T k_T^{max})^{-1} \left[D_V^\pi\left(\frac{p_T}{k_T^{max}}\right) + D_S^\pi\left(\frac{p_T}{k_T^{max}}\right) \right]. \quad (19)$$

The result is shown by the dashed-dotted line in Fig. 4. Evidently, the contribution from the thermal-shower recombination is much more important than that from fragmentation in the range of p_T shown. Despite the fact that $T(k_T)$ is lower than $L(k_T)$ for $k_T > 1.5$ GeV/c, its dominance at $k_T < 1.5$ GeV/c is enough to result in the TS recombination to dominate over the SS recombination for all $p_T < 8$ GeV/c. This example demonstrates the necessity of knowing the SPD's in a jet, since $K(x_1)$ is used in Eq. (17). If SS recombination is the only important contribution, as in pp collisions, then fragmentation as in Eq. (19) is all that is needed, and the search for SPD's plays no crucial role. In realistic problems the hard-parton momentum k_T^{max} has to be integrated over the weight of the jet cross section. However, for our illustrative purpose here, that is beside the point.

Our next example is the study of the dihadron distribution in a jet. We need only carry out the investigation here for a jet in vacuum, since the replacement of a shower parton by a thermal parton for a jet in a medium is trivial, having seen how that is done in the replacement of Eq. (18) by Eq. (17) in the case of the single-particle distribution. Consider the joint distribution of two π^+ in a jet initiated by a hard u

quark. As we shall work in the momentum fraction variables, the value of the momentum of the initiating u quark is irrelevant, except that it should be high. Let X_1 and X_2 denote the momentum fractions of the two π^+, and x_i denotes that of the ith parton, $i = 1, \ldots, 4$. Then, since only one u quark can be valence, the other three quarks being in the sea, we have one K, three L, and two R functions. Combinatorial complications arise when we impose the condition that $\Sigma_i x_i < 1$ for $i = 1, 2, 3, 4$. There are two methods to keep the accounting of the different orderings of the four x_i.

Method 1.

Let one ordering be

$$
SPD(x_1 x_2 x_3 x_4) = K(x_1) L\left(\frac{x_2}{1 - x_1}\right) L\left(\frac{x_3}{1 - x_1 - x_2}\right)
$$
$$
\times L\left(\frac{x_4}{1 - x_1 - x_2 - x_3}\right). \tag{20}
$$

There are 4! ways to rearrange the four x_i in all orders. However, they are to be convoluted with $R_\pi(x_1, x_2, X_1)$, which is symmetric in $x_1 \leftrightarrow x_2$, and similarly with $R_\pi(x_3, x_4, X_2)$. Thus there are $4!/2!2!$ independent terms. Since K can appear at any one of the four positions in Eq. (20), we have altogether 24 terms. Thus we have

$$
X_1 X_2 \frac{dN_{\pi^+ \pi^+}}{dX_1 dX_2} = \int \left(\prod_{i=1}^{4} \frac{dx_i}{x_i}\right) \left[\frac{1}{24} \sum_P SPD(x_1 x_2 x_3 x_4)\right]
$$
$$
\times R_\pi(x_1, x_2, X_1) R_\pi(x_3, x_4, X_2), \tag{21}
$$

where Σ_P symbolizes the permutation of all x_i and summing over all four positions of K, but eliminating redundant terms that are symmetric under the interchanges of $x_1 \leftrightarrow x_2$ and $x_3 \leftrightarrow x_4$.

Method 2.

Let us fix the ordering in Eq. (20) but permute the contributing x_i to X_1 and X_2. There are six arrangements of x_i and x_j in $R_\pi(x_i, x_j, X_1) R_\pi(x_{i'}, x_{j'}, X_2)$, while counting in $x_{i'}$ and $x_{j'}$ is unnecessary. Let us denote the summation over them by Σ_Q. Thus we have

$$
X_1 X_2 \frac{dN_{\pi^+ \pi^+}}{dX_1 dX_2} = \int \left(\prod_{i=1}^{4} \frac{dx_i}{x_i}\right) \left[\frac{1}{4} \sum_K SPD(x_1 x_2 x_3 x_4)\right]
$$
$$
\times \left[\frac{1}{6} \sum_Q R_\pi(x_i, x_j, X_1) R_\pi(x_{i'}, x_{j'}, X_2)\right] \tag{22}
$$

where Σ_K denotes summing over the four positions of K. Equation (22) is equivalent to Eq. (21).

It should be noted that not all terms in these equations can be expressed in the form factorizable FF's. One example that can is
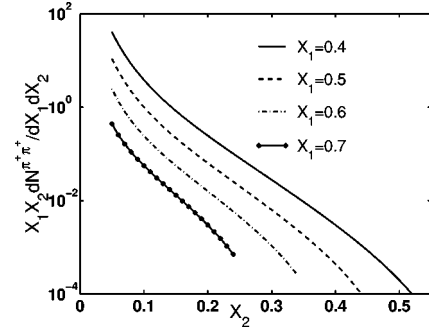

FIG. 5. Two π^+ correlated distribution in a u-quark initiated jet.

$$
\int \left(\prod_{i=1}^{4} \frac{dx_i}{x_i}\right) \frac{1}{2}\left[K(x_1) L\left(\frac{x_2}{1 - x_1}\right)\right.
$$
$$
+ L(x_1) K\left(\frac{x_2}{1 - x_1}\right)\right] R_\pi(x_1, x_2, X_1)
$$
$$
\times L\left(\frac{x_3}{1 - x_1 - x_2}\right) L\left(\frac{x_4}{1 - x_1 - x_2 - x_3}\right) R_\pi(x_3, x_4, X_2)
$$
$$
= D_u^{\pi^+}(X_1) D_S^{\pi^+}[X_2/(1 - X_1)]. \tag{23}
$$

Because of the presence of terms that cannot be written in factorizable form, the two-particle distribution cannot be adequately represented by the FF's only.

Using the SPD's obtained in the previous section, we get the results shown in Fig. 5, which exhibits the X_2 distribution for four fixed values of X_1. This type of correlation in parton fragmentation has never been calculated before. Although the shapes of the X_2 distributions look similar in the log scale in Fig. 5, there is significant attenuation as $X_2 \to 1 - X_1$ for each value of X_1. Thus, the effective slope becomes steeper for larger X_1. Recent experiments at the relativistic heavy-ion collider have begun to measure the distribution of particles associated with triggers restricted to a small interval. The extension of our calculation here to such problems will need the input of jet cross sections for all hard partons in heavy-ion collisions and the participation of thermal partons in the recombination. Here we only demonstrate the utility of the SPD's in the study of dihadron correlation.

VI. CONCLUSION

We have described the fragmentation process in the framework of recombination. The shower parton distributions obtained are shown to be useful in problems where the knowledge of the fragmentation functions alone is not sufficient to provide answers to questions concerning the interaction between a jet and its surrounding medium or between particles within a jet. Such questions arise mainly in nuclear collisions at high energies.

In our view the basic hadronization process is recombination, even for fragmentation in vacuum. Since the recombination process can only be formulated in the framework of a model, the shower parton distributions obtained are indeed model dependent. That is a price that must be paid for the

study of hadrons produced at intermediate p_T where the interaction between soft and semihard partons cannot be ignored, and where perturbative QCD is not reliable. Once recombination is adopted for treating hadronization in that p_T range, the extension to higher p_T can remain in the recombination framework, since the fragmentation process is recovered by the recombination of two shower partons. For hadron production in heavy-ion collisions at superhigh energies, such as the large hadron collider, then the high density of hard partons produced will require the consideration of recombination of hard partons from overlapping jets. Thus it is sensible to remain in the recombination mode for all p_T.

We have shown in this paper how the SPD's can be determined from the FF's. Although we have determined the SPD's at only one value of Q^2 for the FF's, it is clear that the same procedure can be followed for other values of Q^2. The formal description of how the Q^2 dependences of the FF's can be transferred to the Q^2 dependences of the SPD's is a problem that is worth dedicated attention. While the numerical accuracy of the SPD's obtained here can still be improved, especially at lower Q^2, for the purpose of phenomenological applications the availability of the parametrizations given in Table II is far more important than not taking into account at all the shower partons and their interactions with the medium in the environment. The Q^2 evolution of the SPD's may have to undergo a long process of investigatory evolution of its own just as what has happened to the FF's. That can proceed in parallel to the rich phenomenology that can now be pursued in the application of the role of shower partons to heavy-ion collisions.

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