

*Presentation and Discussion of*  
**“Volatility Managed Portfolio”**  
A. Moreira, T. Muir, JF 2017

Giovanni Aiello

Rice University

April 20, 2022

# Overview

# What do they do

- They construct **Volatility Managed Portfolio**: the weights of the portfolio are determined by  $1/\sigma_t^2$ 
  - Take more risk when volatility is low and viceversa

# What do they do

- They construct **Volatility Managed Portfolio**: the weights of the portfolio are determined by  $1/\sigma_t^2$ 
  - Take more risk when volatility is low and viceversa
- From the simple optimal portfolio choice

$$w_t = \frac{1}{\gamma} \frac{\mu_t}{\sigma_t^2}$$

# What do they do

- They construct **Volatility Managed Portfolio**: the weights of the portfolio are determined by  $1/\sigma_t^2$ 
  - Take more risk when volatility is low and viceversa
- From the simple optimal portfolio choice

$$w_t = \frac{1}{\gamma} \frac{\mu_t}{\sigma_t^2}$$

- They use the weak relationship between volatility and returns

# What do they find

## Volatility managed portfolios

- Generate large alpha on original factors
- Take less risk in recession when  $\sigma$  is high
- Sell after market crashes (1929, 1987, 2008)

# Volatility Managed Portfolio Empirically

- Factors:
  - **FF3**: Excess Mkt Return (Mkt), size (SMB), value (HML); **FF5**: profitability (RMW), investment (CMA), **Momentum** (Mom)
  - **HXZ**: Investment (IA), ROE
  - **Lustig et al 2011** Currency return (Carry or FX)
- **Daily** and Monthly data for each factor
- Sample: 1926-2015 (Mkt, SMB, HML, Momentum), Post 1960 for the rest
- All numbers annualized



# Managed Volatility Factors

- Let  $f_{t+1}$  be an excess return, construct

$$f_{t+1}^{\sigma} = \frac{c}{\sigma_t^2(f)} \times f_{t+1}$$

- $\sigma_t(f)$  previous month realized volatility (daily data)
  - choose  $c$  so  $f^{\sigma}$  has same unconditional volatility as  $f$
- Regression:

$$f_{t+1}^{\sigma} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$$

# Volatility Managed Factors: alphas

	(1) Mkt $\sigma$	(2) SMB $\sigma$	(3) HML $\sigma$	(4) Mom $\sigma$	(5) RMW $\sigma$	(6) CMA $\sigma$	(7) MVE $\sigma$	(8) FX $\sigma$	(9) ROE $\sigma$	(10) IA $\sigma$
MktRF	0.61 (0.05)									
SMB		0.62 (0.08)								
HML			0.57 (0.07)							
Mom				0.47 (0.07)						
RMW					0.62 (0.08)					
CMA						0.68 (0.05)				
MVE							0.58 (0.03)			
Carry								0.71 (0.08)		
ROE									0.63 (0.07)	
IA										0.68 (0.05)
$\alpha$	4.86 (1.56)	-0.58 (0.91)	1.97 (1.02)	12.51 (1.71)	2.44 (0.83)	0.38 (0.67)	4.12 (0.77)	2.78 (1.49)	5.48 (0.97)	1.55 (0.67)
N	1,065	1,065	1,065	1,060	621	621	1,060	360	575	575
R2	0.37	0.38	0.32	0.22	0.38	0.46	0.33	0.51	0.40	0.47
rmse	51.39	30.44	34.92	50.37	20.16	17.55	25.34	21.78	23.69	16.58

The appraisal ratio

$$\frac{\alpha}{\sigma_{\epsilon}}$$

gives us a measure of the extent to which we are expanding the slope of the MVE frontier

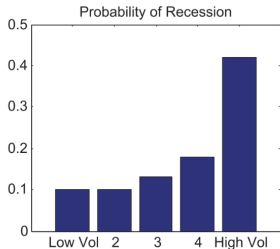
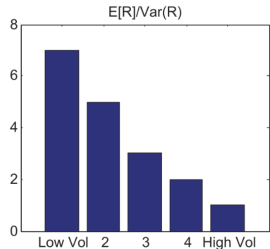
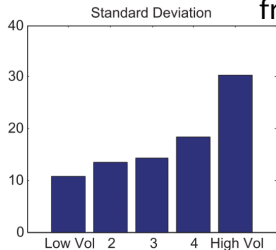
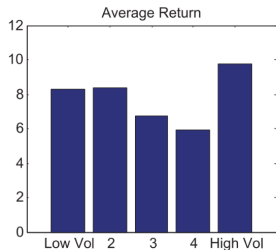
- MKT (0.33), HML (0.20), MOM (0.88), Profitability (0.41), Carry (0.44), ROE (0.80), Investment (0.32)

# Volatility timing works due to weak risk-return trade-off (market)

They show in a continuous theoretical framework

$$\alpha = -\text{cov}\left(\frac{\mu_t}{\sigma_t^2}, \sigma_t^2\right) \frac{c}{E[\sigma_t^2]}$$

- The mean-variance trade-off weakens in periods of high volatility.
- A standard mean-variance investor should take more (less) risk when volatility is low (high)



# Multiple Factors

- Some investors invest in multiple factors beyond the market
- Extend our approach to the (static) MVE portfolio
  - For given set of factors construct in sample MVE:  $f_{t+1}^* = b^* F_{t+1}$
  - Volatility time the MVE portfolio:  $f_{t+1}^{\sigma,*} = \frac{c}{\sigma_t^2(f^*)} f_{t+1}^*$

# MVE Portfolios

	(1) Mkt	(2) FF3	(3) FF3 Mom	(4) FF5	(5) FF5 Mom	(6) HXZ	(7) HXZ Mom
Alpha ( $\alpha$ )	4.86 (1.56)	4.99 (1.00)	4.04 (0.57)	1.34 (0.32)	2.01 (0.39)	2.32 (0.38)	2.51 (0.44)
Observations	1,065	1,065	1,060	621	621	575	575
R-squared	0.37	0.22	0.25	0.42	0.40	0.46	0.43
rmse	51.39	34.50	20.27	8.28	9.11	8.80	9.55
Original Sharpe	0.42	0.52	0.98	1.19	1.34	1.57	1.57
Vol Managed Sharpe	0.51	0.69	1.09	1.20	1.42	1.69	1.73
Appraisal Ratio	0.33	0.50	0.69	0.56	0.77	0.91	0.91

- A positive MVE alpha implies that our volatility-managed strategy increases Sharpe ratios relative to the best buy-and-hold Sharpe ratio achieved by someone with access to the multiple factors

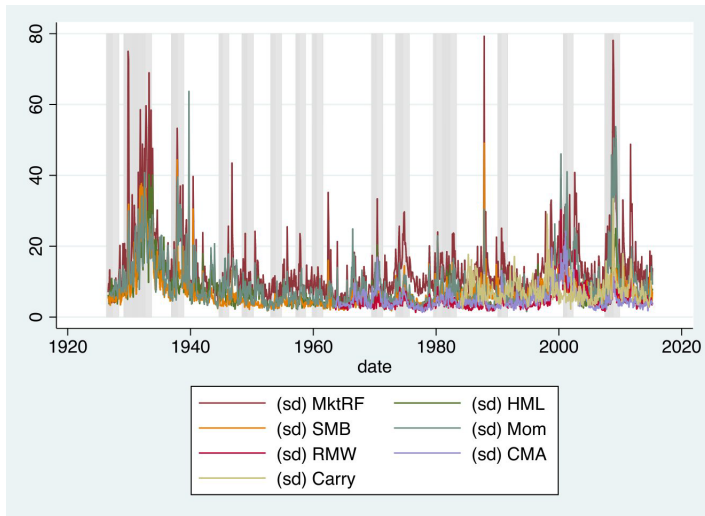
# Robustness/ Additional Empirical Results

# Take Less Risk in Recession

	(1) Mkt $\sigma$	(2) HML $\sigma$	(3) Mom $\sigma$	(4) RMW $\sigma$	(5) CMA $\sigma$	(6) FX $\sigma$	(7) ROE $\sigma$	(8) IA $\sigma$
MktRF	0.83 (0.08)							
MktRF $\times 1_{rec}$	-0.51 (0.10)							
HML		0.73 (0.06)						
HML $\times 1_{rec}$		-0.43 (0.11)						
Mom			0.74 (0.06)					
Mom $\times 1_{rec}$			-0.53 (0.08)					
RMW				0.63 (0.10)				
RMW $\times 1_{rec}$				-0.08 (0.13)				
CMA					0.77 (0.06)			
CMA $\times 1_{rec}$					-0.41 (0.07)			
Carry						0.73 (0.09)		
Carry $\times 1_{rec}$						-0.26 (0.15)		
ROE							0.74 (0.08)	
ROE $\times 1_{rec}$							-0.42 (0.11)	
IA								0.77 (0.07)
IA $\times 1_{rec}$								-0.39 (0.08)
Observations	1,065	1,065	1,060	621	621	362	575	575
R-squared	0.43	0.37	0.29	0.38	0.49	0.51	0.43	0.49



# Take Less Risk in Recession



# Transaction Costs

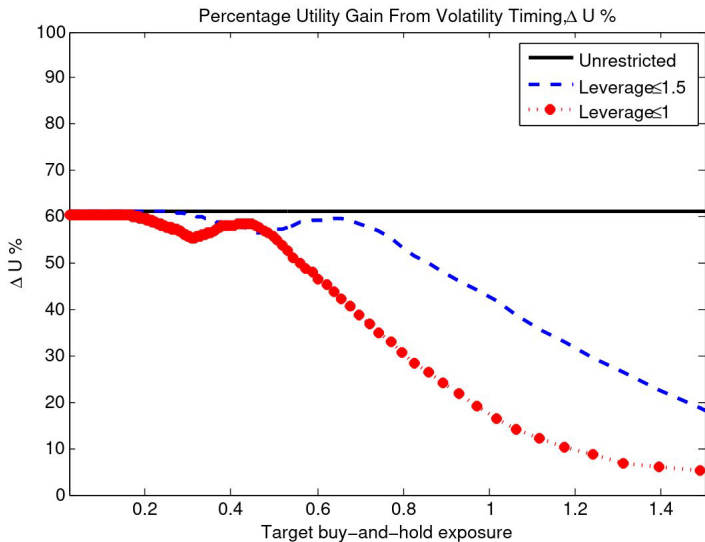
- Results for the market portfolio
- Transaction cost from Frazzini, Israel and Moskowitz (2015)

$w$	Description	$ \Delta w $	$E[R]$	$\alpha$	$\alpha$ After Trading Costs			
					1bps	10bps	14bps	Break Even
$\frac{1}{RV_t^2}$	Realized Variance	0.73	9.47%	4.86%	4.77%	3.98%	3.63%	56bps
$\frac{1}{RV_t}$	Realized Vol	0.38	9.84%	3.85%	3.80%	3.39%	3.21%	84bps
$\frac{1}{E_t[RV_{t+1}^2]}$	Expected Variance	0.37	9.47%	3.30%	3.26%	2.86%	2.68%	74bps
$\min\left(\frac{c}{RV_t^2}, 1\right)$	No Leverage	0.16	5.61%	2.12%	2.10%	1.93%	1.85%	110bps
$\min\left(\frac{c}{RV_t^2}, 1.5\right)$	50% Leverage	0.16	7.18%	3.10%	3.08%	2.91%	2.83%	161bps

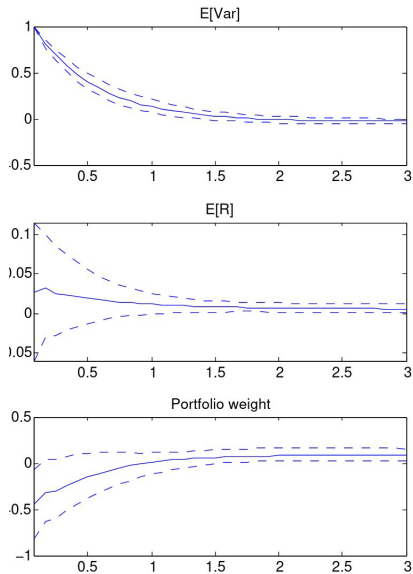
# Works with Leverage Constraint

Volatility Timing and Leverage								
Panel A: Weights and Performance for Alternative Volatility Managed Portfolios								
$w_t$	Description	$\alpha$	Sharpe	Appraisal	Distribution of Weights $w$			
					P50	P75	P90	P99
$\frac{1}{RV_t^2}$	Realized Variance	4.86 (1.56)	0.52	0.34	0.93	1.59	2.64	6.39
$\frac{1}{RV_t}$	Realized Volatility	3.30 (1.02)	0.53	0.33	1.23	1.61	2.08	3.36
$\frac{1}{E_t[RV_{t+1}^2]}$	Expected Variance	3.85 (1.36)	0.51	0.30	1.11	1.71	2.38	4.58
$\min\left(\frac{c}{RV_t^2}, 1\right)$	No Leverage	2.12 (0.71)	0.52	0.30	0.93	1	1	1
$\min\left(\frac{c}{RV_t^2}, 1.5\right)$	50% Leverage	3.10 (0.98)	0.53	0.33	0.93	1.5	1.5	1.5
Panel B: Embedded Leverage Using Options: 1986-2012								
	Buy and hold	Vol Timing	Vol Timing With Embedded Leverage					
			Calls	Calls + puts				
Sharpe Ratio	0.39	0.59	0.54	0.60				
$\alpha$	—	4.03	5.90	6.67				
$s.e.(\alpha)$	—	(1.81)	(3.01)	(2.86)				
$\beta$	—	0.53	0.59	0.59				
Appraisal Ratio	—	0.44	0.39	0.46				

# Works with Leverage Constraint



# The dynamics



- Substantial implication: higher return with lower risks
- Cederburg et al (2020) show they do not outperform their unmanaged counterparts out of sample
- Barroso and Detzel (2020) show that they don't survive transaction costs