MACS 40200 Problem Set #3

Alejandro Parraguez

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MLE Estimation of simple macroeconomic model

The set of characterizing equations of the Brock and Mirman model are:

$$
\begin{aligned}
(c_t)^{-1} - \beta E\left[r_{t+1}\left(c_{t+1}\right)^{-1}\right] &= 0 \qquad (1)\\
c_t + k_{t+1} - w_t - r_t k_t &= 0 \qquad (2)\\
w_t - (1-\alpha)\, e^{z_t}\left(k_t\right)^{\alpha} &= 0 \qquad (3)\\
r_t - \alpha e^{z_t}\left(k_t\right)^{\alpha-1} &= 0 \qquad (4)\\
z_t = \rho z_{t-1} + (1-\rho)\,\mu + \varepsilon_t & \qquad\qquad (5)
\end{aligned}
$$

$$\text{where } \varepsilon_t \sim N\left(0, \sigma^2\right)$$

(a)

From equations (3)

$$\ln\left(w_t\right) - \ln\left(1-\alpha\right) - \alpha \ln\left(k_t\right) = z_t$$

and from (5) we know

$$z_t \sim N\left(\rho z_{t-1} + (1-\rho)\,\mu, \sigma^2\right)$$

Thus combining these two equations we get can use MLE to estimate the parameters. We find:

$$
\begin{aligned}
\hat{\alpha} &\approx 0.467245\\
\hat{\rho} &\approx 0.716920\\
\hat{\mu} &\approx 9.454959\\
\hat{\sigma} &\approx 0.092454
\end{aligned}
$$

To calculate the estimates the only mehtod that converged was the SLSQP method which does not contain hess_inv in results.

(b)

From equations (4)

$$\ln\left(r_t\right) - \ln\alpha - (\alpha-1)\ln\left(k_t\right) = z_t$$

and from (5) we know

$$z_t \sim N\left(\rho z_{t-1} + (1-\rho)\mu, \sigma^2\right)$$

Thus combining these two equations we get can use MLE to estimate the parameters. We find similar values as before:

$$\begin{aligned}
\hat{\alpha} &\approx 0.462484 \\
\hat{\rho} &\approx 0.716940 \\
\hat{\mu} &\approx 9.282153 \\
\hat{\sigma} &\approx 0.092454
\end{aligned}$$

(c)

Using equation (4) and the estimates from (a) we can write:

$$\ln(r_t) - \ln(0.47) - (0.46 - 1)\ln(k_t) = z_t$$

If $k_t = 7,500,000$ and $r_t = 1$ then

$$\begin{aligned}
z^* &= -\ln(0.47) + 0.54\ln(7,500,000) \\
z^* &\approx 9.3
\end{aligned}$$

Furthermore, from equation (5)

$$z_t \sim N\left(\rho z_{t-1} + (1-\rho)\mu, \sigma^2\right)$$

Using the estimates from (a) and $z_{t-1} = 10$

$$\begin{aligned}
z_t &\sim N\left(0.72 \cdot 10 + (1-0.72) \cdot 9.45, (0.09)^2\right) \\
z_t &\sim N(9.846, 0.0081)
\end{aligned}$$

Thus

$$\begin{aligned}
\mathrm{P}\left(r_t > 1 | \hat{\theta}, k_t, z_{t-1}\right) &= \mathrm{P}\left(z_t > z^* | \hat{\theta}, k_t, z_{t-1}\right) \\
\mathrm{P}\left(r_t > 1 | \hat{\theta}, k_t, z_{t-1}\right) &= 1 - \mathrm{P}\left(z_t \leq 9.3 | \hat{\theta}, k_t, z_{t-1}\right) \\
\mathrm{P}\left(r_t > 1 | \hat{\theta}, k_t, z_{t-1}\right) &\approx 0
\end{aligned}$$