Problem Set #4

MACS 40200, Dr. Evans

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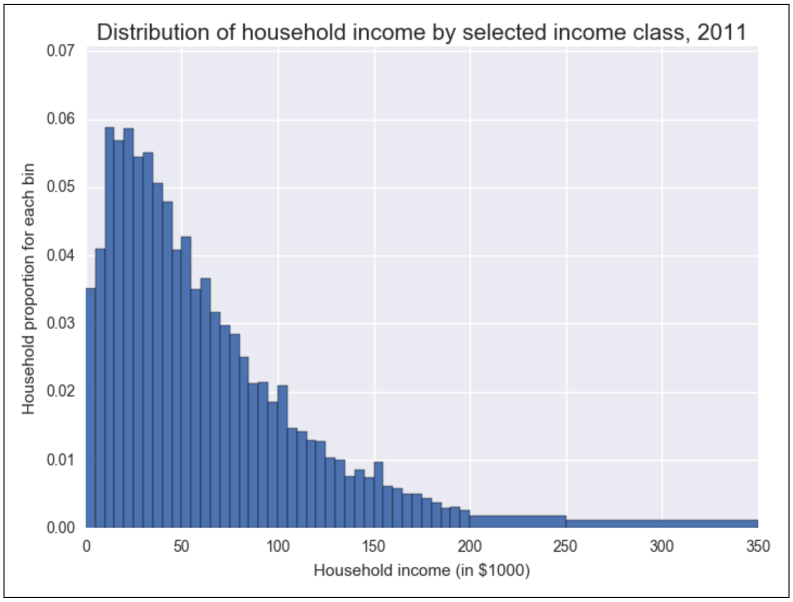
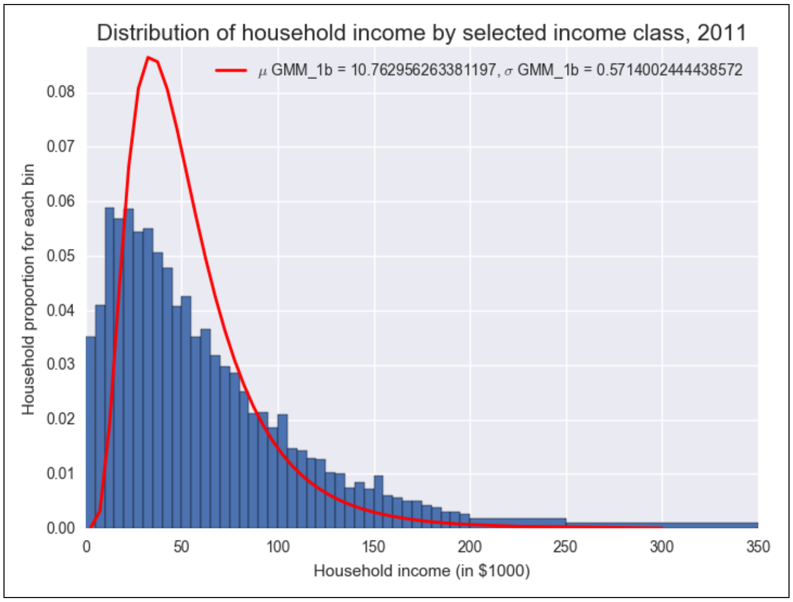
1. **Matching the U.S. income distribution by GMM.**

   (a) Plot the histogram implied by the moments in the tab-delimited text file `usincmoms.txt`.

   (b) Using GMM, fit the lognormal LN(x; μ, σ) distribution defined in the MLE notebook to the distribution of household income data using the moments from the data file. Report your estimated values for $\hat{\mu}$ and $\hat{\sigma}$, as well as the value of the minimized criterion function. Plot the histogram from part (a) overlayed with a line representing the implied histogram from your estimated lognormal (LN) distribution.

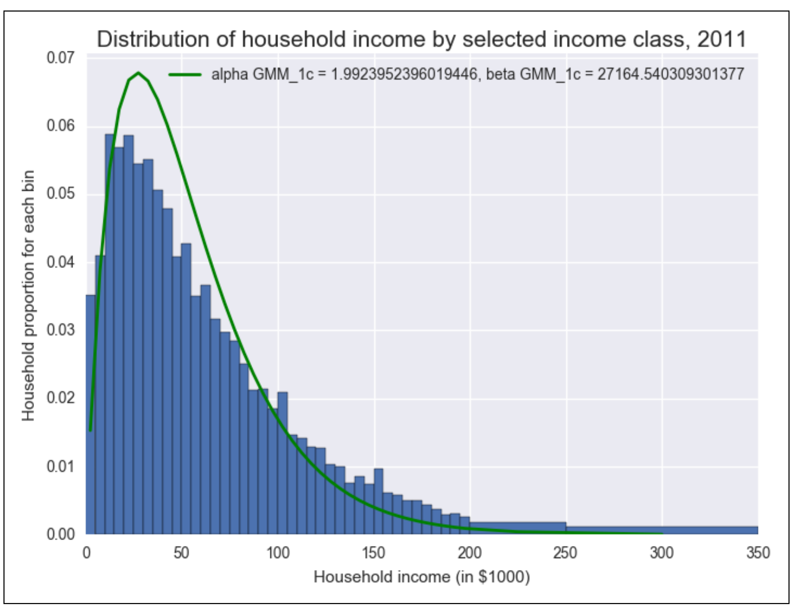


GMM crietron function value: 13.41130601

GMM estimate for μ, with $\mathbf{W} = \mathbf{I}$: 10.76295626

GMM estimate for σ, with $\mathbf{W} = \mathbf{I}$: 0.57140024

(c) Using GMM, fit the gamma GA(x; α, β) distribution defined in the MLE notebook to the distribution of household income data using the moments from the data file. Report your estimated values for $\hat{\alpha}$ and $\hat{\beta}$, as well as the value of the minimized criterion function. Plot the histogram from part (a) overlayed with a line representing the implied histogram from your estimated gamma (GA) distribution.

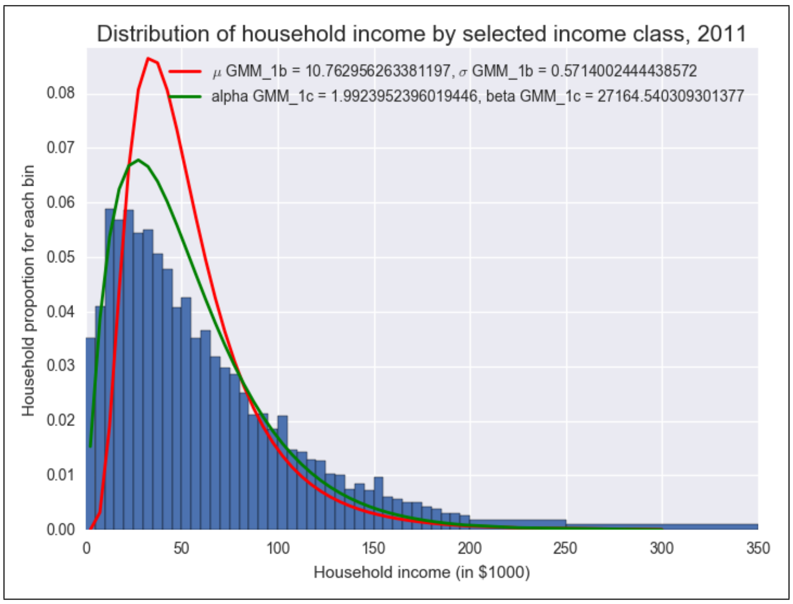


GMM crietron function value: 6.70630716

GMM estimate for α, with $\mathbf{W} = \mathbf{I}$: 1.9923952396

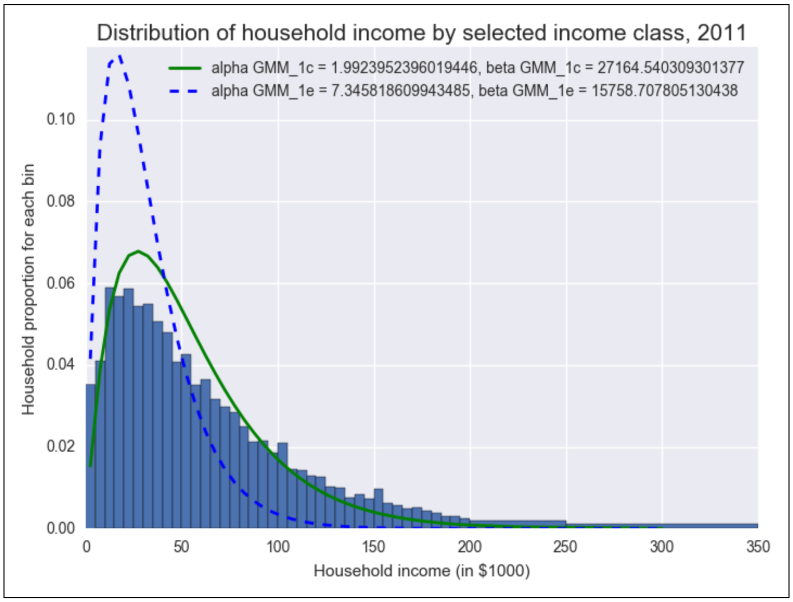
GMM estimate for β, with $\mathbf{W} = \mathbf{I}$: 27164.5403093

(d) Plot the histogram from part (a) overlayed with the line representing the implied histogram from your estimated lognormal (LN) distribution from part (b) and the line representing the implied histogram from your estimated gamma (GA) distribution from part (c). What is the most precise way to tell which distribution fits the data the best? Which estimated distribution–LN or GA–fits the data best?



As illustrated by the plot above, the GA distribution fits the data with greater precision than the LN distribution. In addition, using the weight matrix $\mathbf{W} = \mathbf{I}$, we can compare the criterion function values. This comparison also suggests that the GA distribution fits the data better.

(e) Repeat your estimation of the GA distribution from part (c), but use the two-step estimator for the optimal weighting matrix. Do your estimates for α and β change much? How can you compare the goodness of fit of this estimated distribution versus the goodness of fit of the estimated distribution in part (c)?



GMM crietron function value: 7.46064292e-09
GMM estimate for α, with $\hat{\mathbf{W}}_{twostep}$: 7.34581860994
GMM estimate for β, with $\hat{\mathbf{W}}_{twostep}$: 15758.7078051

The GMM estimates for α and β changed significantly. With the same initial input for parameters, the esitmate for α changed from 1.99 to 7.35, and the estimate for β from 27164.54 to 15758.71. The different weight matrices make it not ideal for me to simply compare the criterion function values. The plot suggests that the first GMM estimates with the weighting matrix $\mathbf{W} = \mathbf{I}$ better fits the distribution to the data than the second estimates with the two-step weighting matrix, $\hat{\mathbf{W}}_{twostep}$.