# Problem Set #5 GMM and SMM

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## Estimating the Brock and Mirman (1972) model by GMM

We estimate $\alpha, \beta, \rho$ and $\mu$ by GMM using the moment conditions

$$\begin{aligned} E\left[z_{t+1} - \rho z_t - (1-\rho)\mu\right] &= 0 \qquad (1)\\ E\left[(z_{t+1} - \rho z_t - (1-\rho)\mu)\cdot z_t\right] &= 0 \qquad (2)\\ E\left[\beta\alpha e^{z_{t+1}} k_{t+1}^{\alpha-1}\frac{c_t}{c_{t+1}} - 1\right] &= 0 \qquad (3)\\ E\left[\left(\beta\alpha e^{z_{t+1}} k_{t+1}^{\alpha-1}\frac{c_t}{c_{t+1}} - 1\right)\cdot w_t\right] &= 0 \qquad (4) \end{aligned}$$

From the capital demand equation we know that:

$$\log r_t - \log\alpha + (1-\alpha)\log k_t = z_t$$

The empirical analogues of the moment conditions $(1)-(4)$ are

$$\begin{aligned} &\frac{1}{T}\cdot\textstyle\sum_{t=0}^{T-1}\left[z_{t+1} - \rho z_t - (1-\rho)\mu\right] \qquad (5)\\ &\frac{1}{T}\cdot\textstyle\sum_{t=0}^{T-1}\left[(z_{t+1} - \rho z_t - (1-\rho)\mu)\cdot z_t\right] \qquad (6)\\ &\frac{1}{T}\cdot\textstyle\sum_{t=0}^{T-1}\left[\beta\alpha e^{z_{t+1}} k_{t+1}^{\alpha-1}\frac{c_t}{c_{t+1}} - 1\right] \qquad (7)\\ &\frac{1}{T}\cdot\textstyle\sum_{t=0}^{T-1}\left[\left(\beta\alpha e^{z_{t+1}} k_{t+1}^{\alpha-1}\frac{c_t}{c_{t+1}} - 1\right)\cdot w_t\right] \qquad (8) \end{aligned}$$

The results we obtain are

$$\begin{aligned} \hat{\alpha} &\approx 0.29\\ \hat{\beta} &\approx 0.99\\ \hat{\rho} &\approx 0.81\\ \hat{\mu} &\approx 12.28 \end{aligned}$$

and the criterion function is

$$\epsilon^T\cdot I\cdot\epsilon = 0.003$$

# Estimating the Brock and Mirman (1972) model by SMM

The household decision has a known analytical solution:

$$k_{t+1} = \alpha\beta e^{z_t} k_t^{\alpha}$$

We assume $z_1 = \mu$ and $k_1 = \text{mean}\,(k_t)$ from the data. In order to do this we build a $(S, T, 6)$ matrix which will have the simulated data and calculate the model moments from there.

The moments we target are: $\text{mean}(c_t)$, $\text{mean}(k_t)$, $\text{var}(c_t)$, $\text{var}(k_t)$, $\text{corr}(c_t, k_{t,})$ and $\text{corr}(k_t, k_{t+1})$. The results we obtain are

$$\begin{aligned}
\hat{\alpha} &\approx 0.29 \\
\hat{\beta} &\approx 0.89 \\
\hat{\rho} &\approx 0.81 \\
\hat{\mu} &\approx 0.99 \\
\hat{\sigma} &\approx 0.500001
\end{aligned}$$

For the estimation we use the percentage deviation of moments as the error:

$$e\,(x|\theta) = \frac{\hat{m}\,(x|\theta) - m\,(x)}{m\,(x)}$$

The vector of errors at the optimum is:

$$\epsilon = \begin{bmatrix} -0.99 \\ -0.98 \\ -1 \\ -0.80 \\ -0.07 \\ 0.76 \end{bmatrix}$$

and the criterion function is

$$\epsilon^T \cdot I \cdot \epsilon = 3.65$$