Problem Set 4: Matching the U.S. income distribution by GMM

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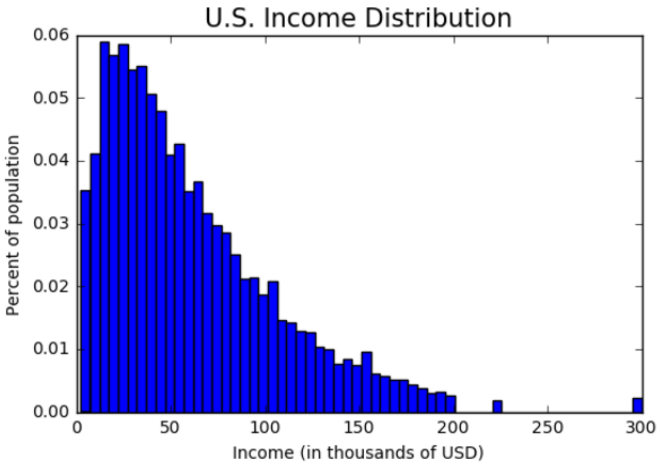
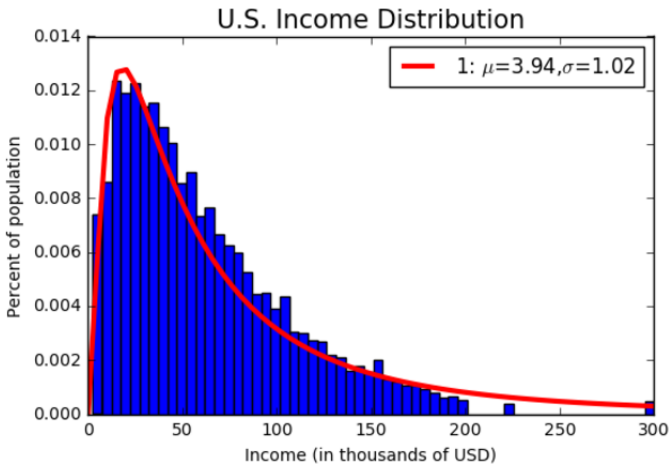
(a)

We plot the histogram implied by the moments in the .txt file:



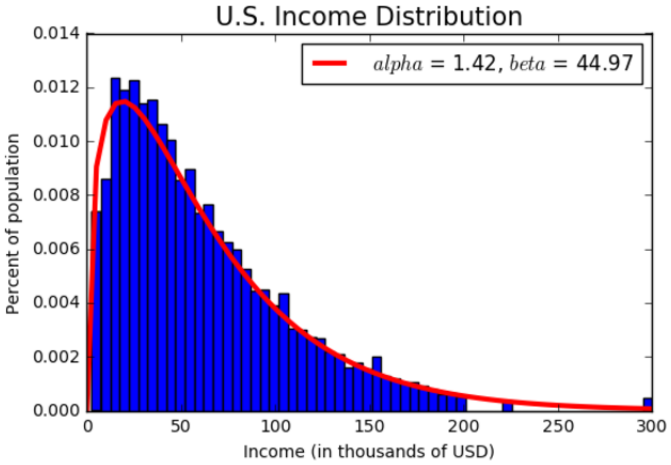
(b)

The GMM estimates for the Lognormal distribution are $\hat{\mu} \approx 3.94$ and $\hat{\sigma} \approx 1.02$ and we obtain the following histogram:



(c)

The GMM estimates for the Gamma distribution are $\hat{\alpha} \approx 1.42$ and $\hat{\beta} \approx 44.97$ and we obtain the following histogram:



(d)

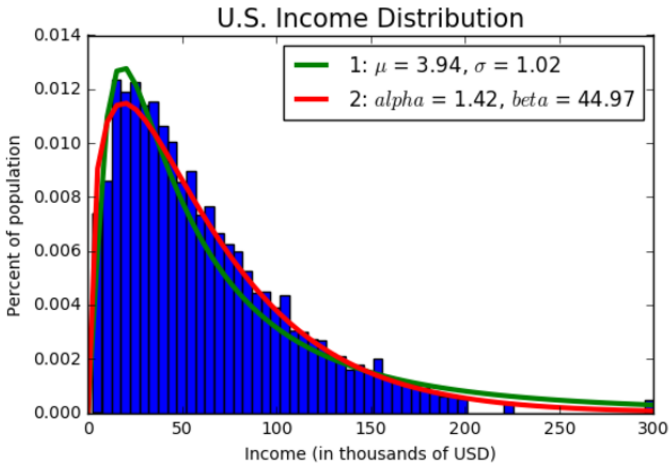
We plot both distribution over the US income histogram. To compare these distributions we calculate the criterion functions using both parameter vectors

and compare the two values. We obtain

$$e\left(x|\hat{\theta}_{LN}\right)^T We\left(x|\hat{\theta}_{LN}\right) \approx 0.03$$

$$e\left(x|\hat{\theta}_{GA}\right)^T We\left(x|\hat{\theta}_{GA}\right) \approx 0.007$$

Hence we conclude the Gamma distribution fits the data best.



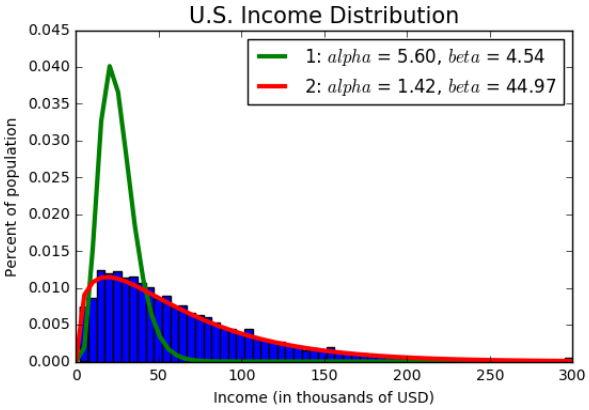
(e)

We repeat the estimation of the Gamma distribution from part (c) with the two-step estimator for the optimal weighting matrix $\hat{\mathbf{W}}_{twostep}$. We find $\hat{\alpha}_{twostep} \approx 5.60$ and $\hat{\beta}_{twostep} \approx 4.54$. We can see the estimates change considerably compared to the previous one. To compare the goodness of fit we can calculate the criterion function using both estimates:

$$e\left(x|\hat{\theta}_{GA}\right)^T We\left(x|\hat{\theta}_{GA}\right) \approx 0.007$$

$$e\left(x|\hat{\theta}_{twostep}\right)^T We\left(x|\hat{\theta}_{twostep}\right) \approx 6.4 \cdot 10^{-12}$$

These functions would imply the two step estimator has a better fit. However, we can also plot both distributions and clearly see the two step estimator does not have the best fit.


U.S. Income Distribution
1: $alpha$ = 5.60, $beta$ = 4.54
2: $alpha$ = 1.42, $beta$ = 44.97
Percent of population
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0
50
100
150
200
250
300
Income (in thousands of USD)