Problem Set #3

MACS 40200, Dr. Evans
Due Tuesday, Jan. 24 at 12:00pm

1. **MLE estimation of simple macroeconomic model (10 points).** You can observe time series data in an economy for the following variables: $(c_t, k_t, w_t, r_t$. Data on (c_t, k_t, w_t, r_t) can be loaded from the file `MacroSeries.txt`. The first line of this file contains variable labels. These data have 100 periods, which are quarterly (25 years). Suppose you think that the data are generated by a process similar to the Brock and Mirman (1972). A simplified set of characterizing equations of the Brock and Mirman model are the following.

$$(c_t)^{-1} - \beta E\left[r_{t+1}(c_{t+1})^{-1}\right] = 0 \tag{1}$$

$$c_t + k_{t+1} - w_t - r_t k_t = 0 \tag{2}$$

$$w_t - (1-\alpha)e^{z_t}\left(k_t\right)^{\alpha} = 0 \tag{3}$$

$$r_t - \alpha e^{z_t}\left(k_t\right)^{\alpha-1} = 0 \tag{4}$$

$$\begin{aligned} z_t &= \rho z_{t-1} + (1-\rho)\mu + \varepsilon_t \\ &\text{where} \quad \varepsilon_t \sim N(0,\sigma^2) \end{aligned} \tag{5}$$

The variable c_t is aggregate consumption in period t, k_{t+1} is total household savings and investment in period t for which they receive a return in the next period (this model assumes full depreciation of capital). The wage per unit of labor in period t is w_t and the interest rate or rate of return on investment is r_t. Total factor productivity is z_t, which follows an AR(1) process given in (5). The rest of the symbols in the equations are parameters that must be estimated $(\alpha, \beta, \rho, \mu, \sigma)$. The constraints on these parameters are the following.

$$\alpha, \beta \in (0,1), \quad \mu, \sigma > 0, \quad \rho \in (-1,1)$$

Assume that the first observation in the data file variables is $t = 1$. Let k_1 be the first observation in the data file for the variable k_t. Assume that $z_0 = \mu$ so that $z_1 = \mu$. Assume that the discount factor is known to be $\beta = 0.99$.

   (a) (4 points) Use the data (w_t, k_t) and equations (3) and (5) to estimate the four parameters $(\alpha, \rho, \mu, \sigma)$ by maximum likelihood. Given a guess for the parameters $(\alpha, \rho, \mu, \sigma)$, you can use the two variables from the data (w_t, k_t) and (3) to back out a series for z_t. You can then use equation (5) to compute the probability of each $z_t \sim N\Big(\rho z_{t-1} + (1-\rho)\mu, \sigma^2\Big)$. The maximum likelihood estimate $(\hat{\alpha}, \hat{\rho}, \hat{\mu}, \hat{\sigma})$ maximizes the likelihood function of that normal distribution of z_t's. Report your estimates and the inverse hessian variance-covariance matrix of your estimates.

(b) (4 points) Now we will estimate the parameters another way. Use the data (r_t, k_t) and equations (4) and (5) to estimate the four parameters $(\alpha, \rho, \mu, \sigma)$ by maximum likelihood. Given a guess for the parameters $(\alpha, \rho, \mu, \sigma)$, you can use the two variables from the data (r_t, k_t) and (4) to back out a series for z_t. You can then use equation (5) to compute the probability of each $z_t \sim N\Big(\rho z_{t-1} + (1-\rho)\mu, \sigma^2\Big)$. The maximum likelihood estimate $(\hat{\alpha}, \hat{\rho}, \hat{\mu}, \hat{\sigma})$ maximizes the likelihood function of that normal distribution of z_t's. Report your estimates and the inverse hessian variance-covariance matrix of your estimates.

(c) (2 points) According to your estimates from part (a), if investment/savings today is $k_t =?$ and the productivity shock today $z_{t-1} = \hat{\mu}$, what is the probability that the interest rate this period will be greater than ?. That is, solve for $Pr(r_t >?|\hat{\theta}, k_t, c_t, z_{t-1})$. [HINT: Use equation (4) to solve for the $z_t = z^*$ such that $r_t =?$. Then use (5) to solve for the probability that $z_t > z^*$.]

References

Brock, William A. and Leonard J. Mirman, "Optimal economic growth and uncertainty: The discounted case," *Journal of Economic Theory*, June 1972, *4* (3), 479–513.