Risk and Ambiguity in Educational Choices

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Ambiguity in Dynamic Models of Educational Choices

- Plausible
  - better description of agent decision problem
- Meaningful
  - reinterpretation of economic phenomenon
  - reevaluation of policy interventions
- Tractable

Starting point ...

Keane, M. P. and Wolpin, K. I. (1994a). The Solution and Estimation of Discrete Choice Dynamic Programming Models by Simulation and Interpolation: Monte Carlo Evidence. *The Review of Economics and Statistics*, 76(4):648–672.

Transparency, Recomputability, and Extensibility

https://github.com/robustToolbox/package

- Documentation
  - Source Codes
  - Test Suite
- Teaching Material

Basic Model under Risk

Ingredients

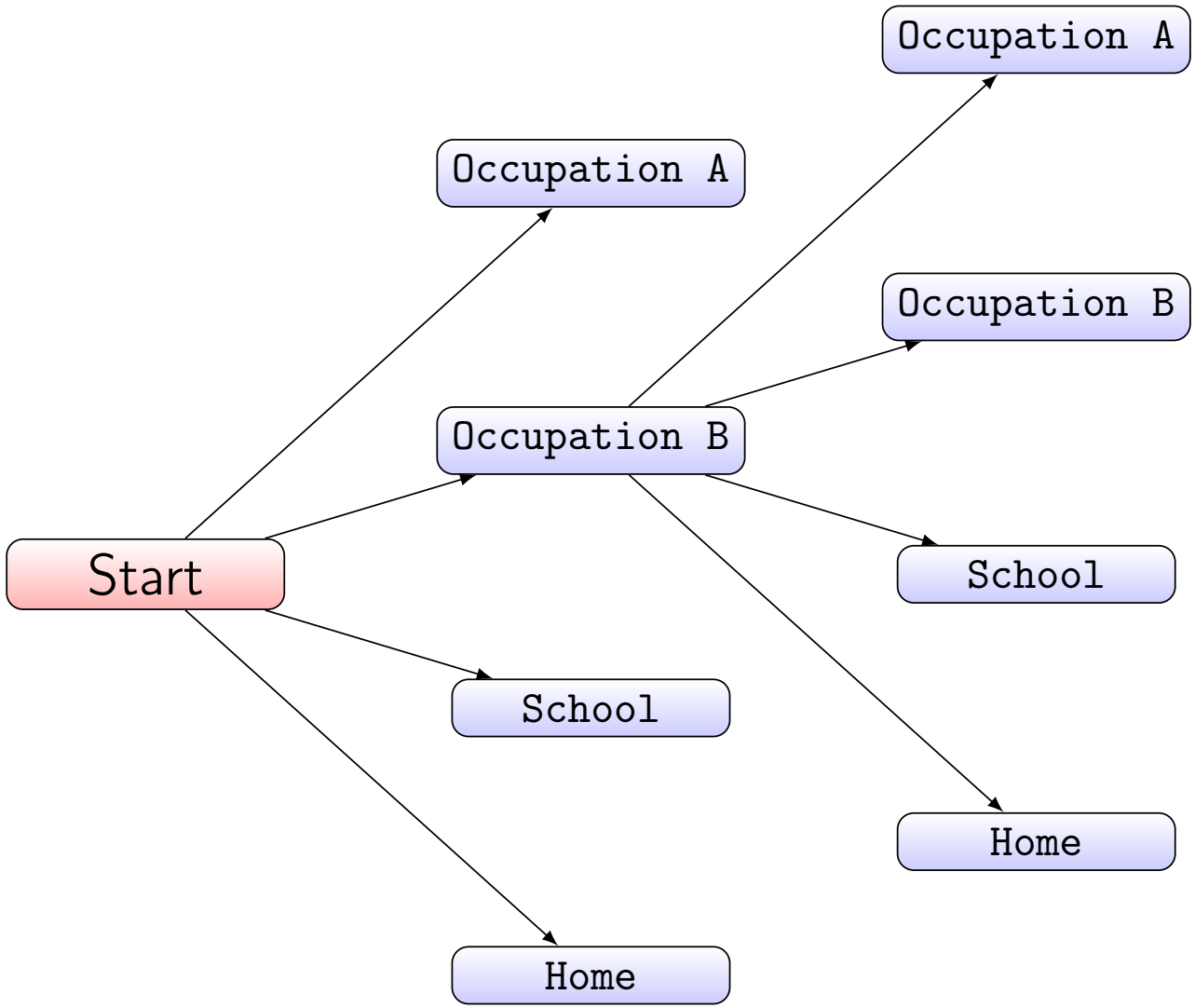
- Objectives
- Constraints
  - Institutions
  - Information

$\Rightarrow$ Optimal Decision

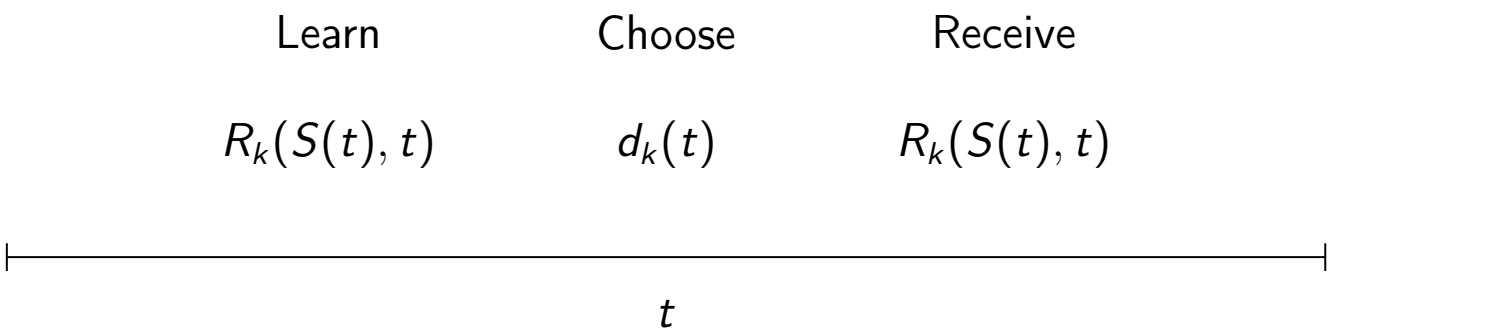
Notation

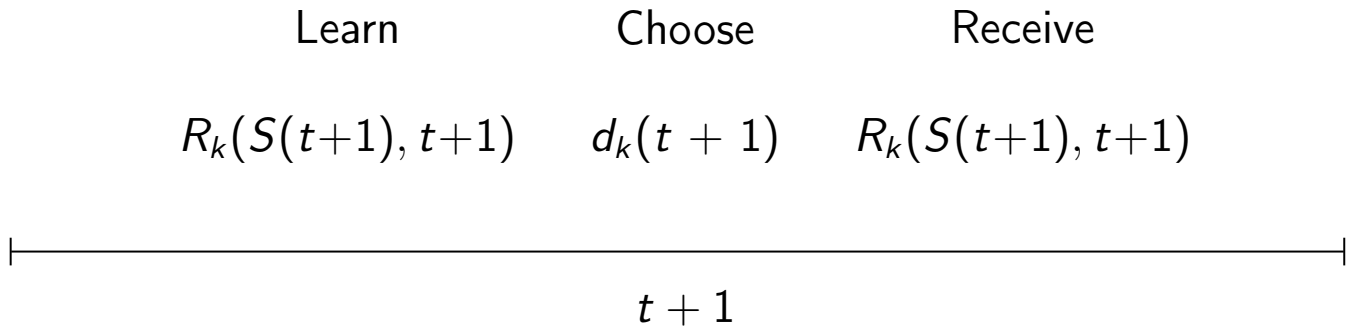
$k = 1, \ldots, K$	Alternative
$t = 1, \ldots, T$	Time
$S(t)$	State Space at Time t
$R_k(S(t), t)$	Rewards for Alternative k at Time t
$d_k(t)$	Indicator for Alternative k at Time t
δ	Discount Factor

Decision Tree



Timing of Events





Agents' Objective under Risk

$$V(S(t),t) = \max_{\{d_k(t)\}_{k\in K}} E\left[\sum_{\tau=t}^{T} \delta^{\tau-t} \sum_{k\in K} R_k(\tau) d_k(\tau) \middle| S(t)\right]$$

Bellman Equations

$$V(S(t), t) = \max_{k \in K} \{ V_k(S(t), t) \},$$

where for all but the final period:

$$V_k(S(t), t) = R_k(S(t), t) + \delta E \left[V(S(t+1), t+1) \mid S(t), d_k(t) = 1 \right]$$

Calibrated Example

Agent Characteristics

$x_{1,t}$ Experience in Occupation A at Time t

$x_{2,t}$ Experience in Occupation B at Time t

s_t Years of Schooling at Time t

Occupation A

$$R_1(t) = \exp\{\alpha_{10} + \alpha_{11} s_t + \alpha_{12} x_{1,t} - \alpha_{13} x_{1,t}^2 + \alpha_{14} x_{2,t} - \alpha_{15} x_{2,t}^2 + \epsilon_{1,t}\}$$

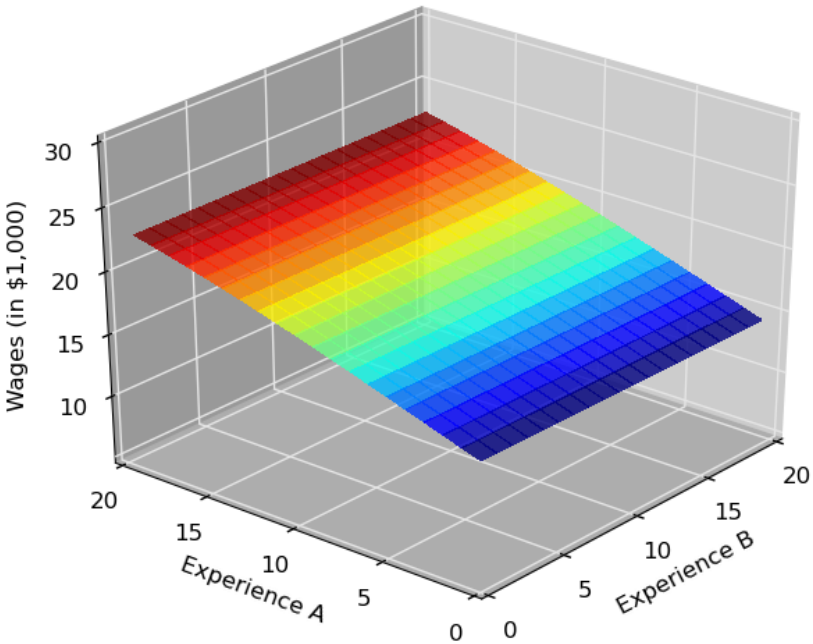
Parameters	α_{10}	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}
Values	9.21	0.04	0.033	0.0005	0.00	0.00

Occupation B

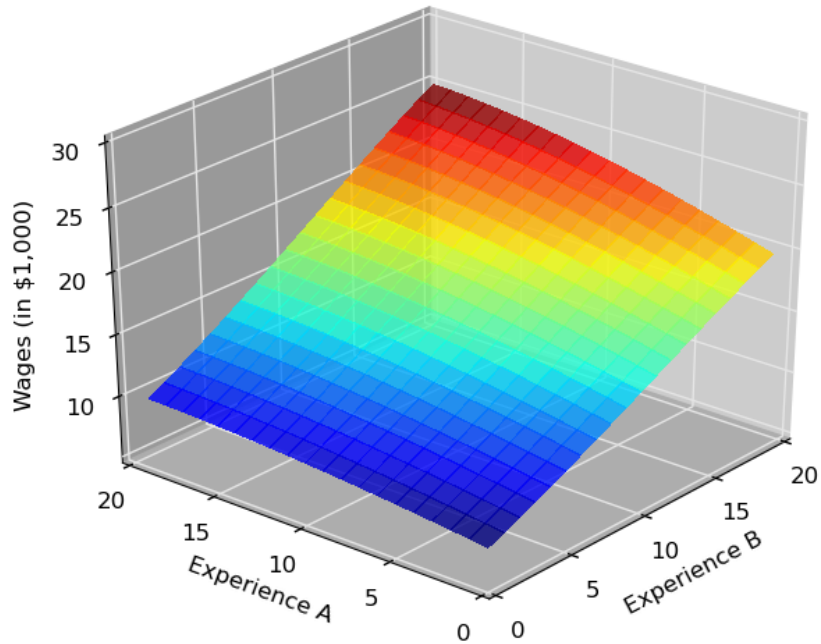
$$R_2(t) = \exp\{\alpha_{20} + \alpha_{21} s_t + \alpha_{22} x_{1,t} - \alpha_{23} x_{1,t}^2 + \alpha_{24} x_{2,t} - \alpha_{25} x_{2,t}^2 + \epsilon_{2,t}\}$$

Parameters	α_{20}	α_{21}	α_{22}	α_{23}	α_{24}	α_{25}
Values	8.20	0.08	0.022	0.0005	0.067	0.001

Wages and Experience

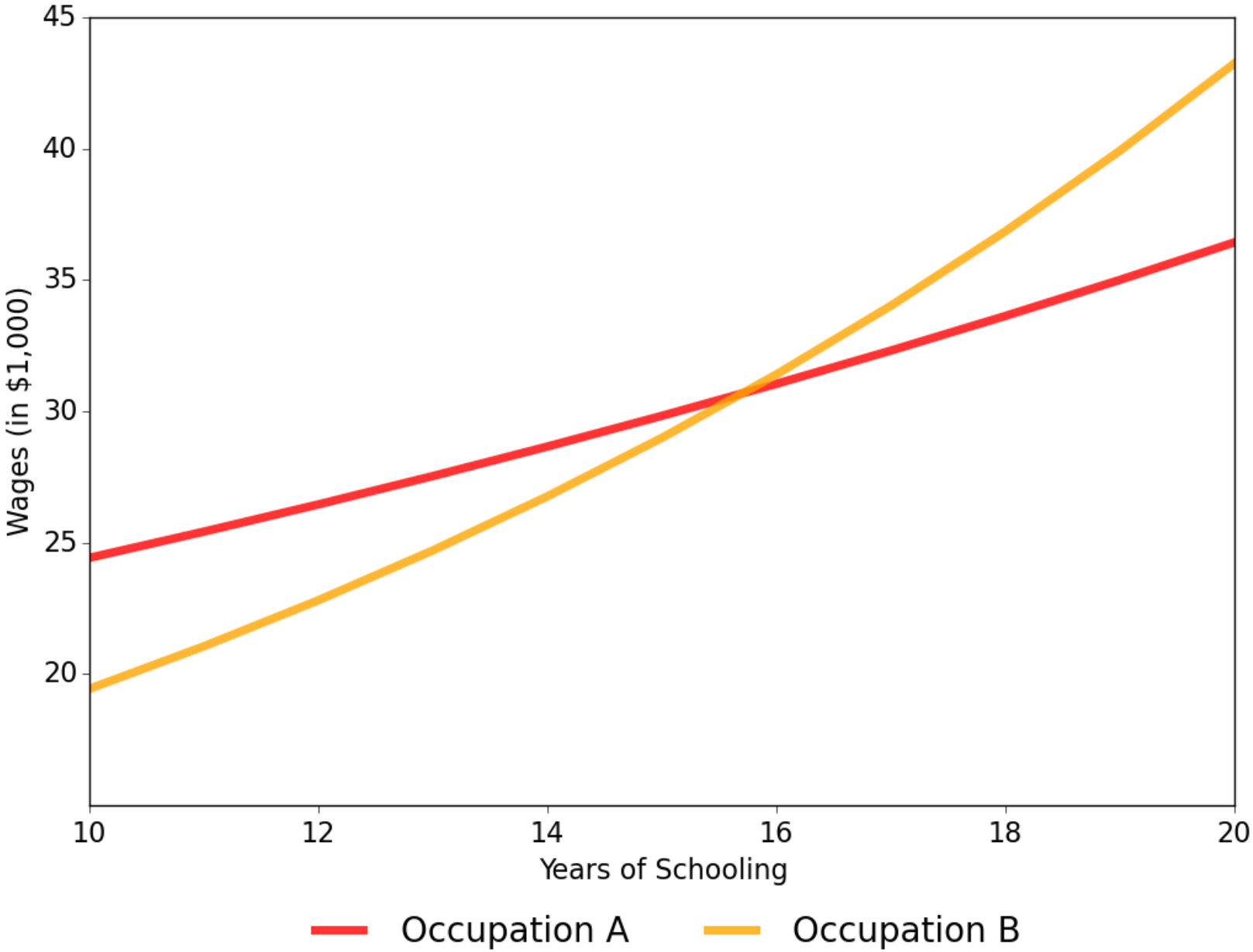


Occupation A



Occupation B

Wages and Schooling



School

$$R_3(t) = \underbrace{(\beta_0 - \beta_1(1 - d_3(t-1)))}_{\text{Psychic Costs}} - \beta_2 \mathrm{I}\left[s_t \geq 13\right] + \epsilon_{3,t}$$

Parameters	β_0	β_1	β_2
Values	5,000	15,000	5,000

Home

$$R_4(t) = \gamma_0 + \epsilon_{4,t}$$

Parameter	γ_0
Value	14,500

State Space

- at time t

$$S(t) = \{s_t, x_{1,t}, x_{2,t}, d_3(t-1), \epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}, \epsilon_{4,t}\}$$

- laws of motion

$$x_{j,t+1} = x_{j,t} + d_j(t) \qquad \forall \quad j \in \{1, 2\}$$

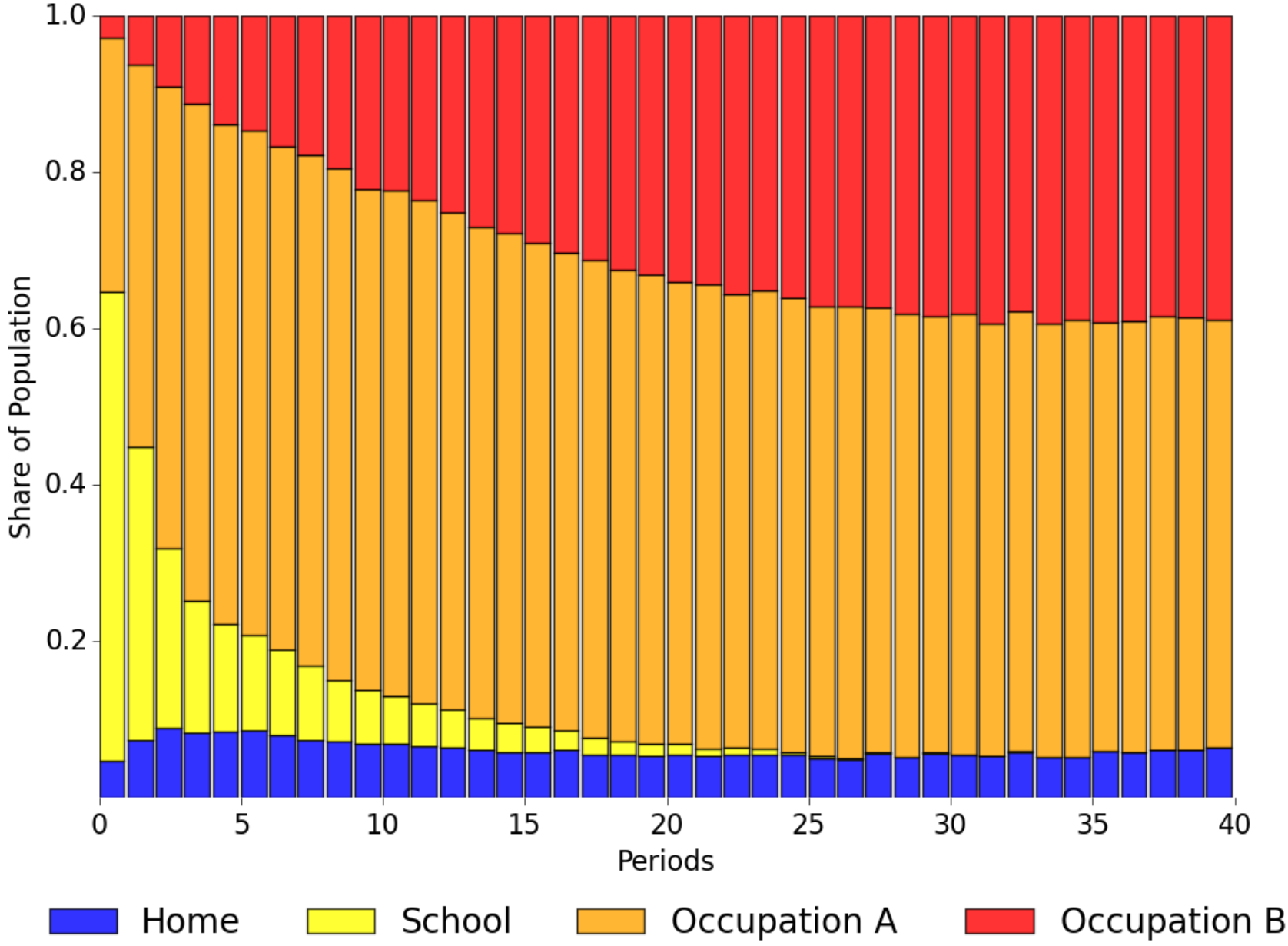
$$s_{t+1} = s_t + d_3(t)$$

$$f(\epsilon_{t+1} \mid S(t), d_k(t)) = f(\epsilon_{t+1} \mid \bar{S}(t), d_k(t))$$

Shocks

$$\begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{pmatrix} \sim \mathcal{N}_0 \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 16 \times 10^{-4} & 0.00 & 0.00 & 0.00 \\ 0.00 & 25 \times 10^{-2} & 0.00 & 0.00 \\ 0.00 & 0.00 & 36 \times 10^{6} & 0.00 \\ 0.00 & 0.00 & 0.00 & 36 \times 10^{6} \end{pmatrix} \right]$$

Choices over Time



Basic Model under Ambiguity

- Modeling Ambiguity
- Understanding Economic Mechanism
- Assessing Model Misspecification

Modeling Ambiguity

To fix ideas, let us study the decision problem of *Agent Blue* in the second to last period:

- 9 Years of Experience in Occupation A
- 20 Years of Experience in Occupation B
- 1 Year of Additional Schooling

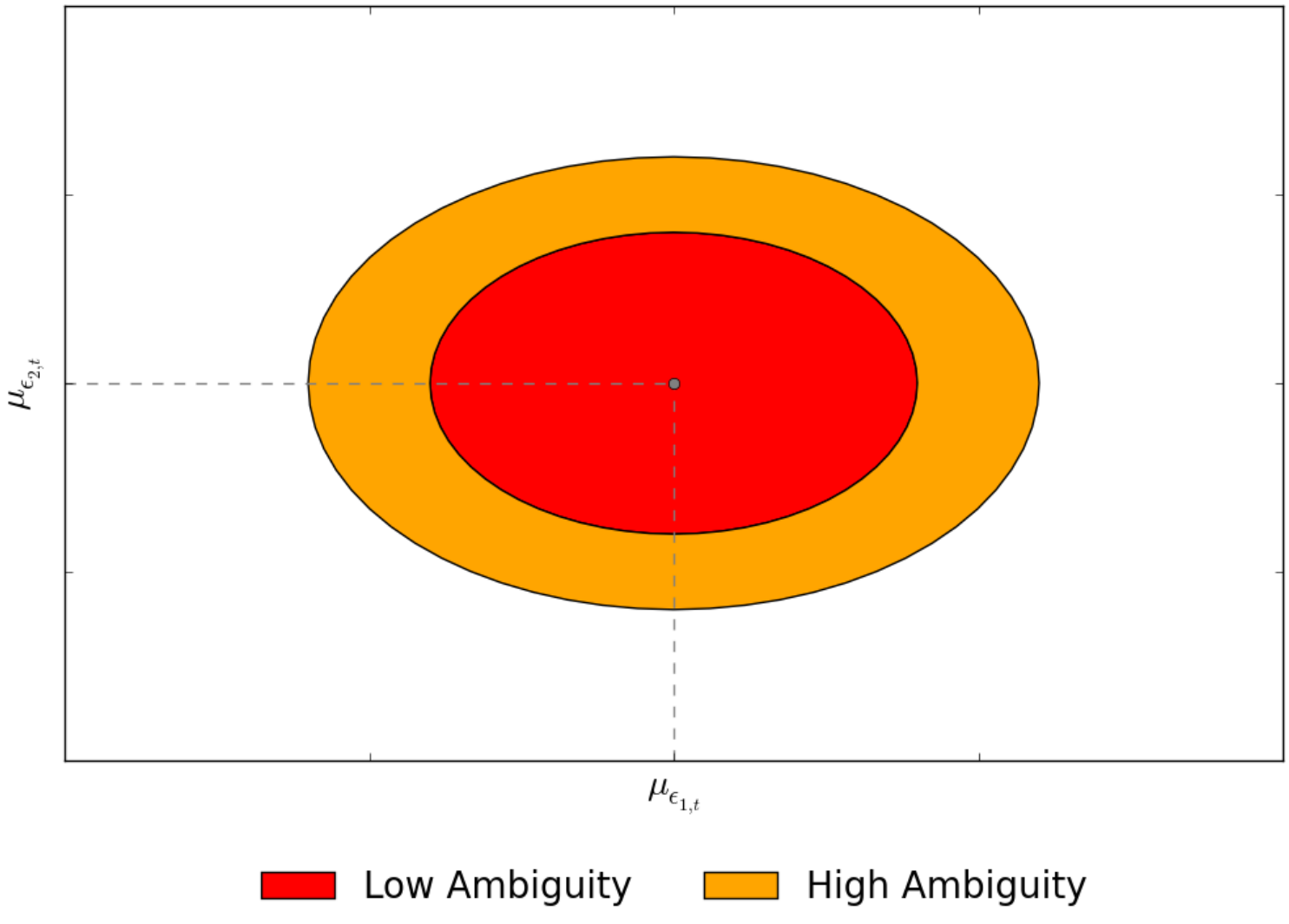
Set of Admissible Beliefs

$$\mathbb{N} = \{\mathcal{N} \in \mathrm{Q} : D_{KL}(\mathcal{N}_0 \mid \mathcal{N}) \leq \theta\}$$

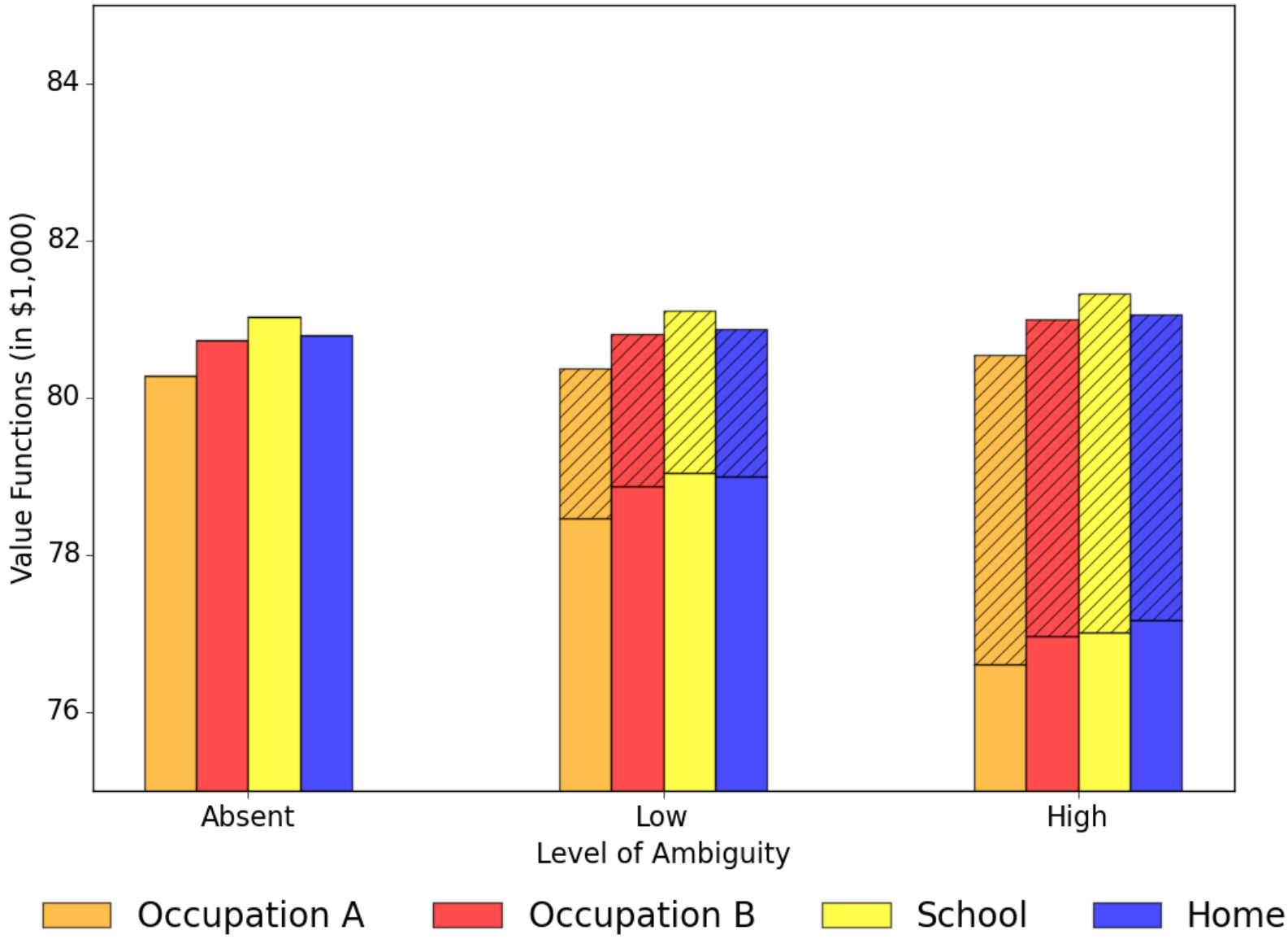
Distribution of Labor Market Shocks

$$\begin{pmatrix} \epsilon_{\mathbf{1},t} \\ \\ \epsilon_{\mathbf{2},t} \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mu_{\epsilon_{\mathbf{1}},t} \\ \\ \mu_{\epsilon_{\mathbf{2}},t} \end{pmatrix}, \begin{pmatrix} 16 \times 10^{-4} & 0.00 \\ \\ 0.00 & 25 \times 10^{-2} \end{pmatrix}\right]$$

Exploring Set of Admissible Beliefs



Exploring Admissible Value Functions



Agents' Objective under Ambiguity

$$V^*(S(t),t) = \max_{\{d_k(t)\}_{k\in K}} \left\{ \min_{\mathcal{N}\in\mathbb{N}} E_{\mathcal{N}} \left[\sum_{\tau=t}^{T} \delta^{\tau-t} \sum_{k\in K} R_k(\tau) d_k(\tau) \middle| S(t) \right] \right\}$$

See: Epstein and Schneider (2003), Hansen and Sargent (2007)

Bellman Equations

$$V^*(S(t), t) = \max_{k \in K} \{ V_k^*(S(t), t) \},$$

where for all but the final period:

$$V_k^*(S(t), t) = R_k(S(t), t) + \delta \min_{\mathcal{N} \in \mathbb{N}} E_{\mathcal{N}} \left[V^*(S(t+1), t+1) \mid \cdot \right]$$

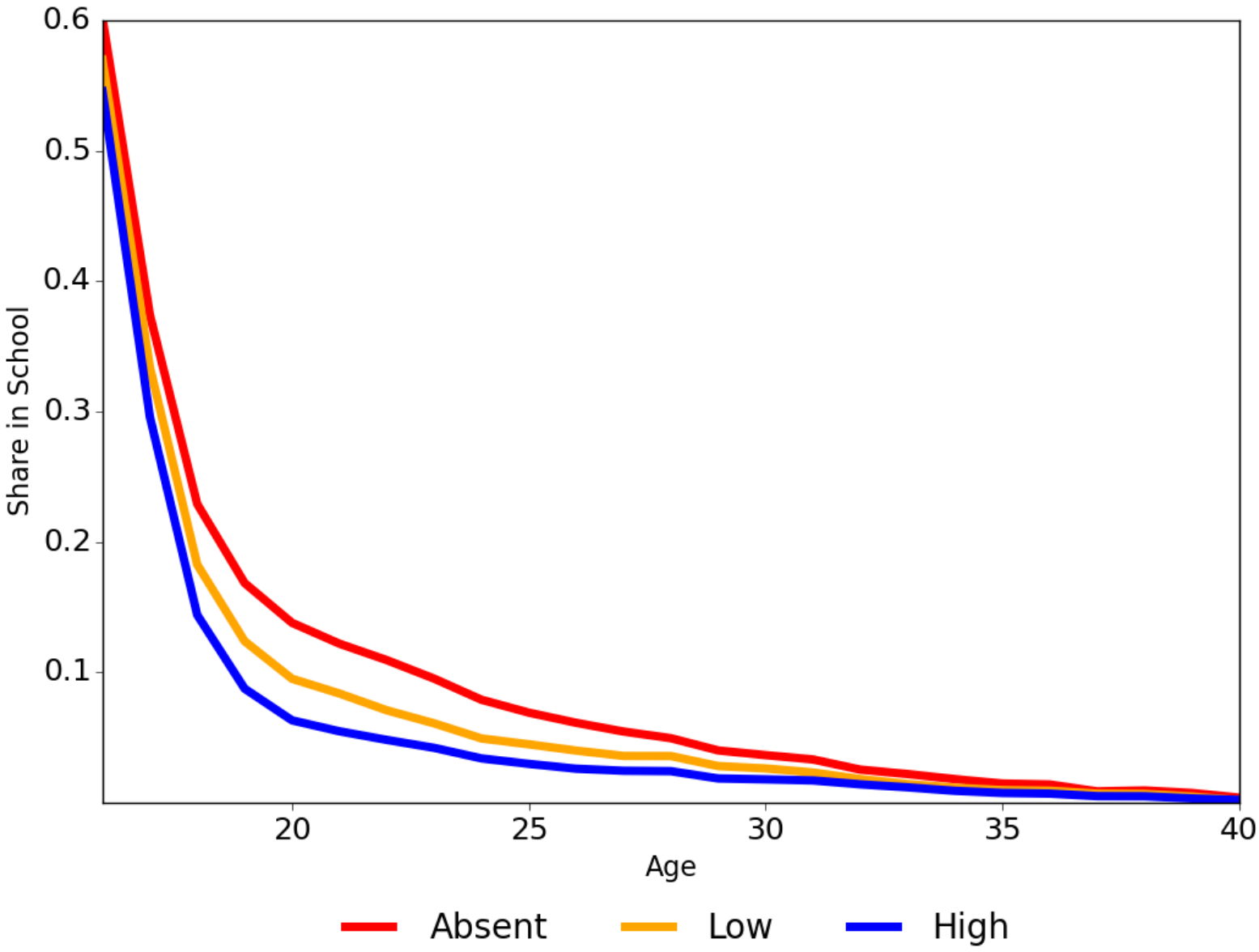
See: Iyengar (2005)

Understanding Economic Mechanism

Quantifying Level of Ambiguity

Ambiguity	Lifetime Value	Relative Change	θ
Absent	\$405,258	—	0.0000
Low	\$395,129	-2.5%	0.0033
High	\$384,988	-5.0%	0.0142

Changing Schooling Investment

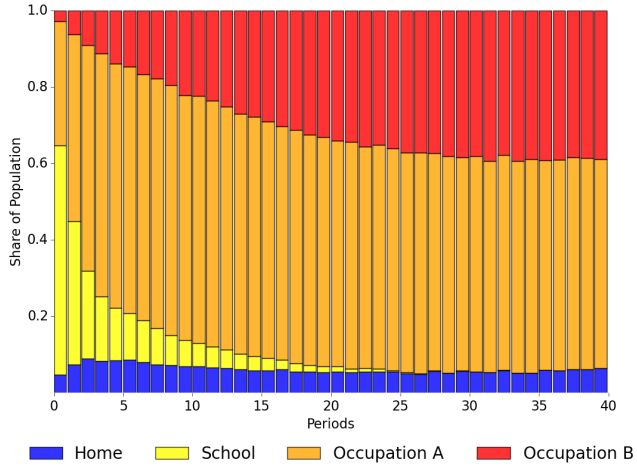


Changing Occupational Sorting

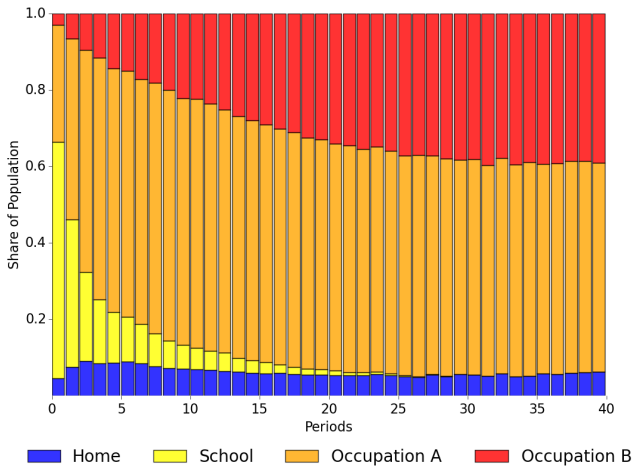
	Share in Occupation	
Ambiguity	A	B
Absent	55%	39%
Low	57%	35%
High	60%	32%

Assessing Model Misspecification

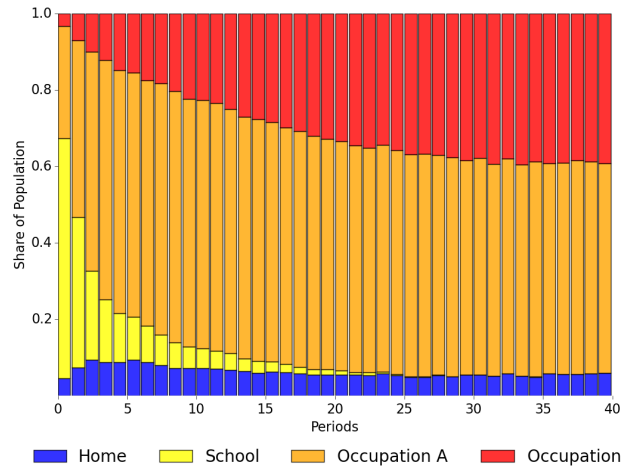
Ambiguity and Psychic Costs



Absent

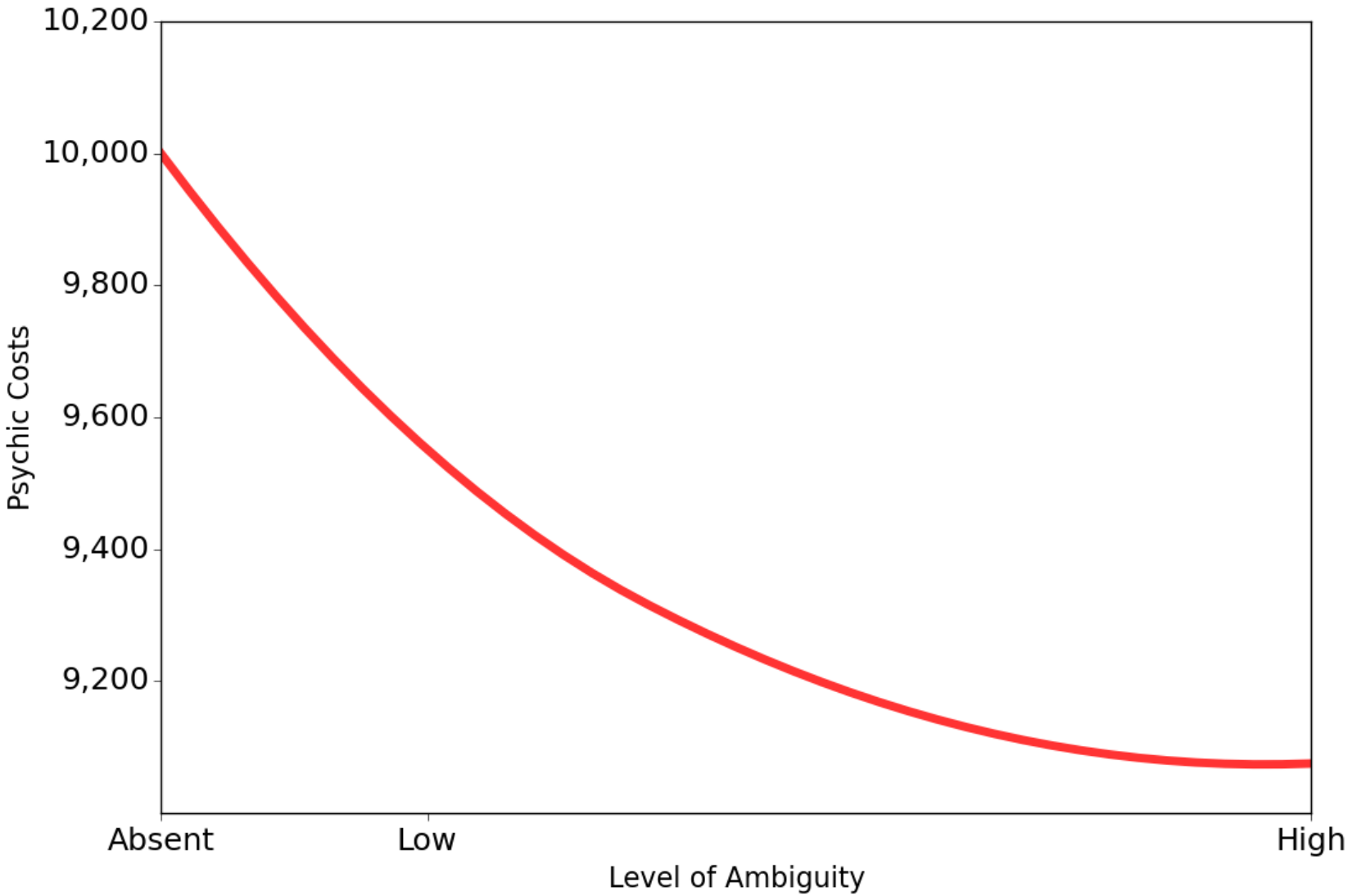


Low



High

Modeling Trade-off



Model Misspecification and Psychic Costs Estimates

	Psychic Costs		
Ambiguity	True	Estimate	Discrepancy
Absent	10,000	10,000	—
Low	9,550	10,000	450
High	9,075	10,000	925

Model Misspecification and Policy Assessment

	Average Schooling		
Ambiguity	True	Estimate	Discrepancy
Absent	1.18	1.18	—
Low	1.12	1.18	0.06
High	1.10	1.18	0.08

Conclusion

Ambiguity in Dynamic Models of Educational Choices

- Plausible
  - better description of agent decision problem
- Meaningful
  - reinterpretation of economic phenomenon
  - reevaluation of policy interventions
- Tractable

Appendix

Content

- Contact
- References

Contact

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Repository	https://github.com/peisenha

Project	http://www.policy-lab.org/structRobust

References

Eisenhauer, P., Heckman, J. J., and Mosso, S. (2015). Estimation of Dynamic Discrete Choice Models by Maximum Likelihood and Simulated Method of Moments. *International Economic Review*, 56(2):331–357.

Epstein, L. G. and Schneider, M. (2003). Recursive Multiple Priors. *Journal of Economic Theory*, 113(1):1–31.

Epstein, L. G. and Schneider, M. (2007). Learning Under Ambiguity. *Review of Economic Studies*, 74(4):1275–1303.

Epstein, L. G. and Schneider, M. (2010). Ambiguity and Asset Markets. *Annual Review of Financial Economics*, 2(1):315–346.

Gilboa, I. and Schmeidler, D. (1989). MaxMin Expected Utility With Non-Unique Priors. *Journal of Mathematical Economics*, 18(2):141–153.

Hansen, L. P. and Sargent, T. J. (2007). *Robustness*. Princeton University Press, Princeton, NJ.

Heckman, J. J., Lochner, L. J., and Todd, P. E. (2006). Earnings functions, rates of return and treatment effects: The mincer equation and beyond. In Hanushek, E. A. and Welch, F., editors, *Handbook of the Economics of Education*, volume 1, pages 307–485. North-Holland Publishing Company.

Iyengar, G. N. (2005). Robust Dynamic Programming. *Mathematics of Operations Research*, 30(2):257–280.

Keane, M. P. and Wolpin, K. I. (1994a). The Solution and Estimation of Discrete Choice Dynamic Programming Models by Simulation and Interpolation: Monte Carlo Evidence. *The Review of Economics and Statistics*, 76(4):648–672.

Keane, M. P. and Wolpin, K. I. (1994b). The Solution and Estimation of Discrete Choice Dynamic Programming Models by Simulation and Interpolation: Monte Carlo Evidence. *Federal Reserve Bank of Minneapolis*, No. 181.

Mincer, J. (1974). *Schooling, Experience and Earnings*. National Bureau of Economic Research, New York, NY.

Mincer, J. A. (1958). Investment in Human Capital and Personal Income Distribution. *Journal of Political Economy*, 66(4):281–302.

Navarro, S. (2011). Using Observed Choices to Infer Agent's Information: Reconsidering the Importance of Borrowing Constraints, Uncertainty and Preferences in College Attendence. *CIBC Working Paper Series*.

Stoye, J. (2012). New Perspective on Statistical Decisions Under Ambiguity. *Annual Review of Economics*, 4:257–282.