Starting point

~~**Building on work by ...**~~

~~[illegible]~~

- Keane, M. P. and Wolpin, K. I. (1994). The Solution and Estimation of Discrete Choice Dynamic Programming Models by Simulation and Interpolation: Monte Carlo Evidence. ~~*The Review of Economics and Statistics*, 76(4):648–672~~ REStat
- Standard model for...
- Useful b/c simple enough

Baseline ~~Basic~~ Model

under risk

Ingredients

- Objectives
- Constraints
  - Institutions
  - Information → This where Ambig. enters

⇒ Optimal Decision → How does this Δ?

~~Basic~~ Notation

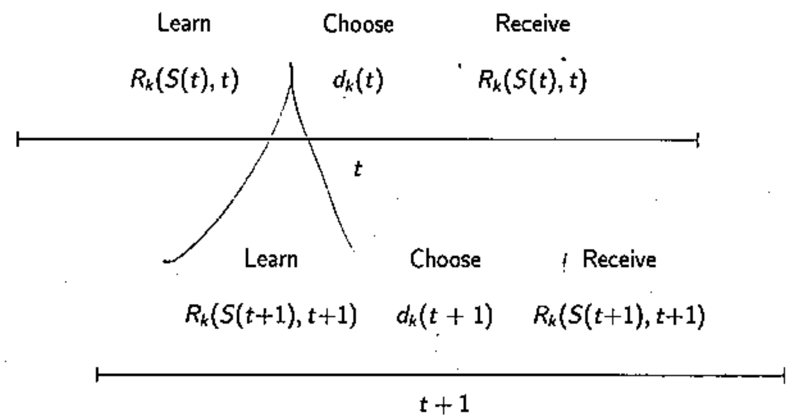
$k = 1, \dots, K$	alternative
$t = 1, \dots, T$	time
$d_k(t)$	indicator for alternative k at t
$R_k(t)$	rewards for alternative k at t
$S(t)$	state spate at t
δ	discount factor

Alternatives

- Occupation A
- Occupation B
- School
- Home

for a total of $T = 40$ periods

Structure of Information



Agents' Objective under Risk

$$\max_{\{d_k(t)\}_{k\in K}} E\left[\sum_{\tau=t}^{T} \delta^{\tau-t} \sum_{k\in K} R_k(\tau) d_k(\tau) \middle| S(t)\right]$$

Agents act as to maximize the expected value of their discounted lifetime reward.

Optimal Decisions under Risk

$$V(S(t), t) = \max_{k\in K}\{V_k(S(t), t)\},$$

where

$$V_k(S(t), t) = \begin{cases} R_k(S(t), t) + \delta E\left[V(S(t+1), t+1)\right] & \text{if } t \leq T-1 \\ R_k(S(T), T) & \text{if } t = T \end{cases}$$

Agent Characteristics

$x_{1,t}$ Years of Experience in Occupation A at time t

$x_{2,t}$ Years of Experience in Occupation B at time t

s_t Years of Schooling at time t

Reward Functions

$$R_1(t) = \exp\{\alpha_{10} + \alpha_{11}s_t + \alpha_{12}x_{1,t} - \alpha_{13}x_{1,t}^2 + \alpha_{14}x_{2,t} - \alpha_{15}x_{2,t}^2 + \epsilon_{1,t}\}$$

$$R_2(t) = \exp\{\alpha_{20} + \alpha_{21}s_t + \alpha_{22}x_{1,t} - \alpha_{23}x_{1,t}^2 + \alpha_{24}x_{2,t} - \alpha_{25}x_{2,t}^2 + \epsilon_{2,t}\}$$

$$R_3(t) = \beta_0 - \beta_1 \mathrm{I}\,[s_t \geq 13] - \beta_2(1 - d_3(t-1)) + \epsilon_{3,t}$$

$$R_4(t) = \gamma_0 + \epsilon_{4,t}$$

State Space

$$x_{1,t+1} = x_{1,t} + d_1(t)$$

$$x_{2,t+1} = x_{2,t} + d_2(t)$$

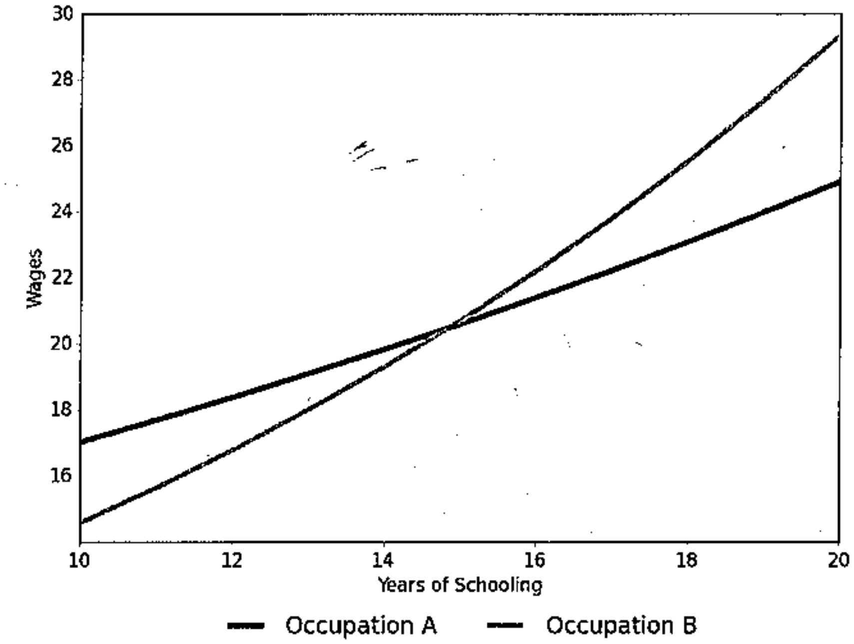
$$s_{t+1} = s_t + d_3(t)$$

$$f(\epsilon_{t+1} \mid S(t), d_k(t)) = f(\epsilon_{t+1} \mid \bar{S}(t), d_k(t))$$

Calibrated Example

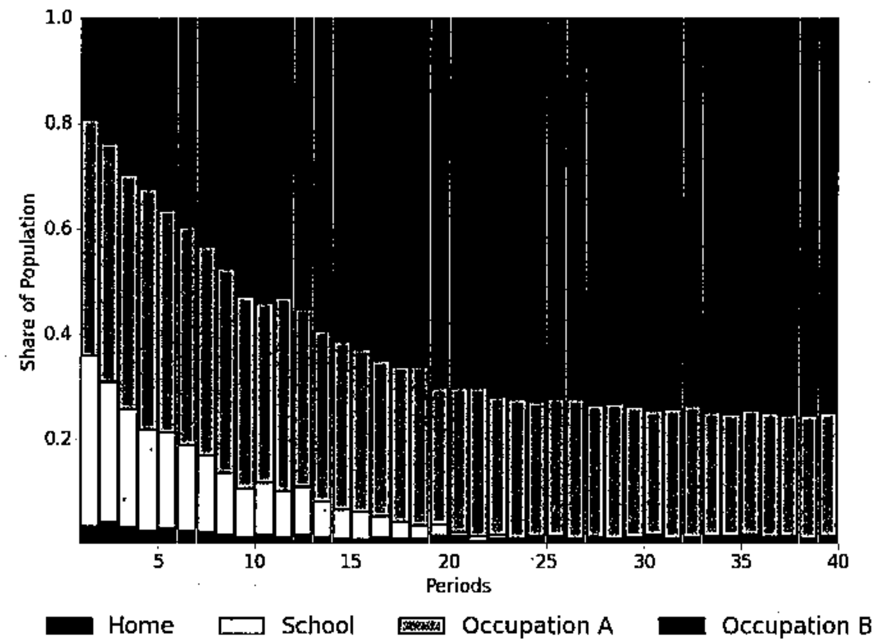
↳ Appendix

Wages and Schooling



The last figure shows the effect of schooling on wages for the two occupations. In *Occupation A*, starting wages are higher but the returns to schooling are lower compared to *Occupation B*. As agents accumulate more and more schooling at the beginning of their life-cycle, they are drawn towards *Occupation B*.

Choice Patterns



Model under Knightian Uncertainty

~~Embracing~~ Ambiguity

- Modeling Ambiguity
  - Set of Admissible Beliefs
  - Preferences under Ambiguity
- Economic Implications
  - Investment in Schooling
  - Occupational Choices
  - Effectiveness of Policies

Modeling Ambiguity

Set of Admissible Beliefs

$$\mathbb{N} = \{\mathcal{N} \in \mathrm{Q} : D_{KL}(\mathcal{N}_0 \mid \mathcal{N}) \leq \theta\},$$

where D_{KL} denotes the Kullback-Leibler divergence between the baseline distribution $\mathcal{N}_0$ of ϵ and an alternative $\mathcal{N}$. The set $\mathbb{N}$ contains all admissible distributions.

Add reference

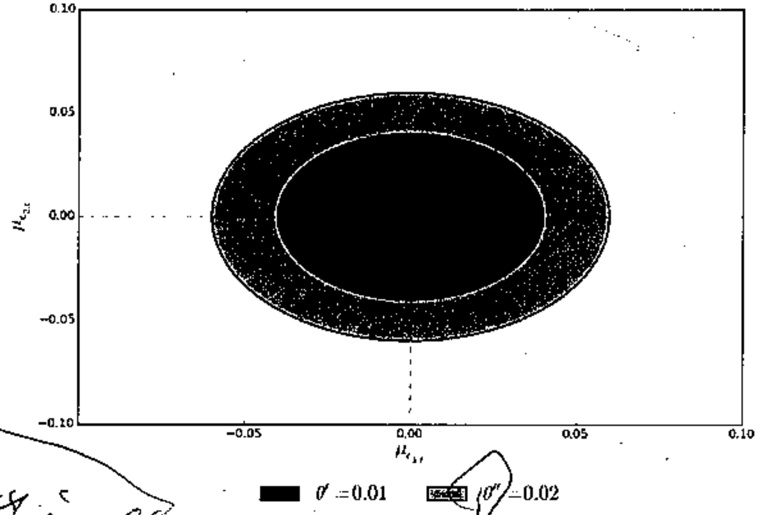
For now:

- ambig. only about

~~We start by introducing ambiguity about~~ future wages. Agents are ambiguous about the means $(\mu_{\epsilon_{1,t}}, \mu_{\epsilon_{2,t}})$ of the normal distribution from which the future random components are drawn.

- Everything normal
- Agents are ambig ...

Exploring Set of Admissible Beliefs



Technically:
- Θ measures degree of ambig.:
- For now: explore quantitative meaning
- Eventually: estimate Θ in simulation

Agents' Objective under Ambiguity

$$\max_{\{d_k(t)\}_{k\in K}} \left\{ \min_{\mathcal{N}\in\mathbb{N}} E_{\mathcal{N}} \left[\sum_{\tau=t}^{T} \delta^{\tau-t} \sum_{k\in K} R_k(\tau) d_k(\tau) \middle| S(t) \right] \right\}$$

Agents compute the expected utility with respect to each admissible probability measure and act as to maximize the expected value of their discounted lifetime reward under the worst-case scenario (Maccheroni et al., 2006; Gilboa and Schmeidler, 1989; Hansen and Sargent, 2007).

Key Assumption: *Rectangularity*

Rectangularity is a form of an independence assumption. The choice of a particular distribution $\mathcal{N}$ in a state-action pair at time t does not limit the choice of distributions in the future. ~~This leads to a~~ separability that is crucial for establishing the robust counterpart of the Bellman recursion (Iyengar, 2005).

~~Economic Implications~~

Quant. effects of varying Θ relative to parameter K/W 1994

Θ̂ matches with Θ = 0

The following slides show selected economic implications as I vary the level of ambiguity in the economy. All other structural parameters remain unchanged.

I document changes to:

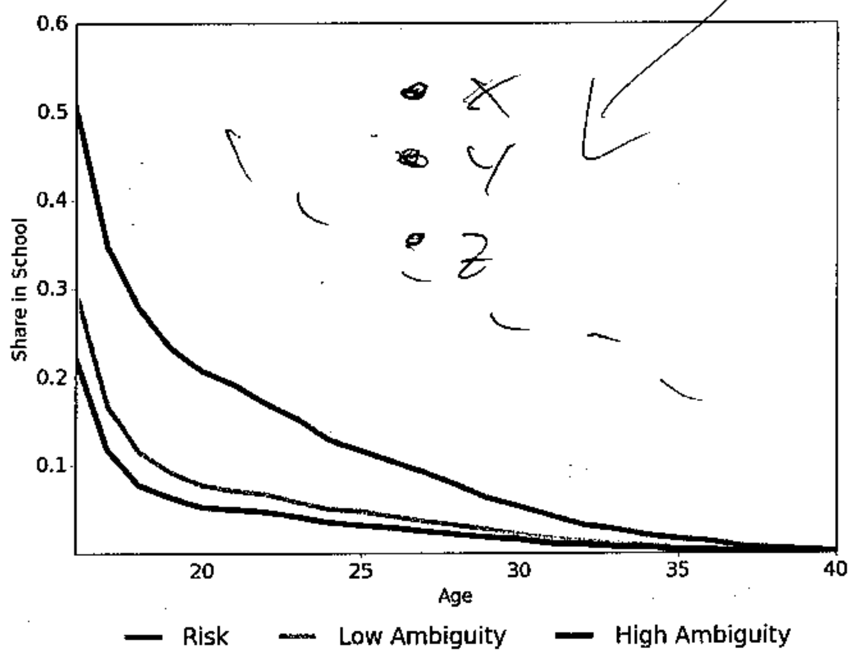
- Investment in Schooling
- Occupational Choices
- Effectiveness of Policies

The next figure shows the share of individuals in school over time. Overall, investment in schooling declines as ambiguity increases. Embracing ambiguity can thus provide a more interpretable explanation for low enrollment rates of income-maximizing agents than the presence of large psychic costs investigated in Eisenhauer ~~et al.~~ (~~2015~~).

Heckman/Mosso (2015)

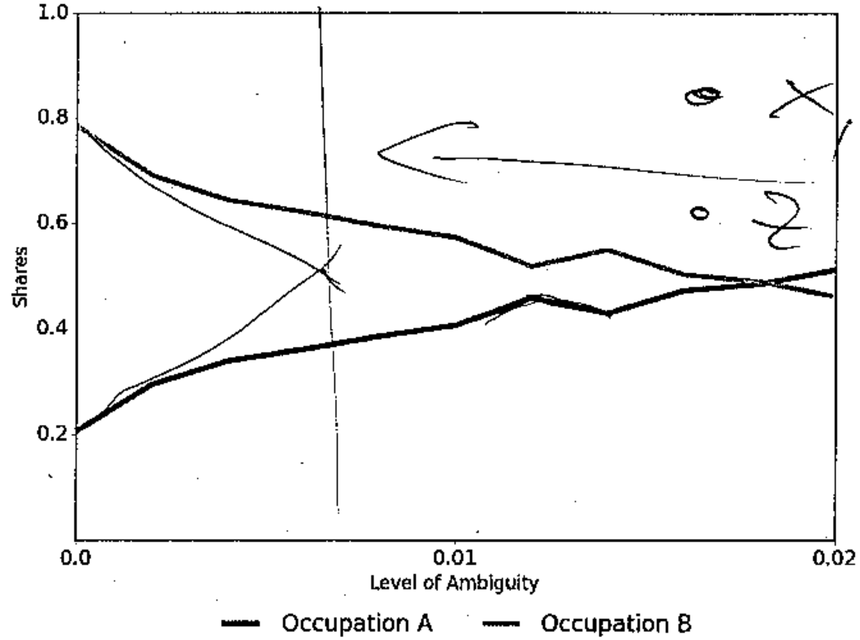
Why?

Changing Investment in Schooling



The next figure documents the share of agents that end up in each of the two occupations in the last period for different levels of ambiguity. Agents reduce their schooling investments as ambiguity increases and thus less and less end up working in *Occupation B*.

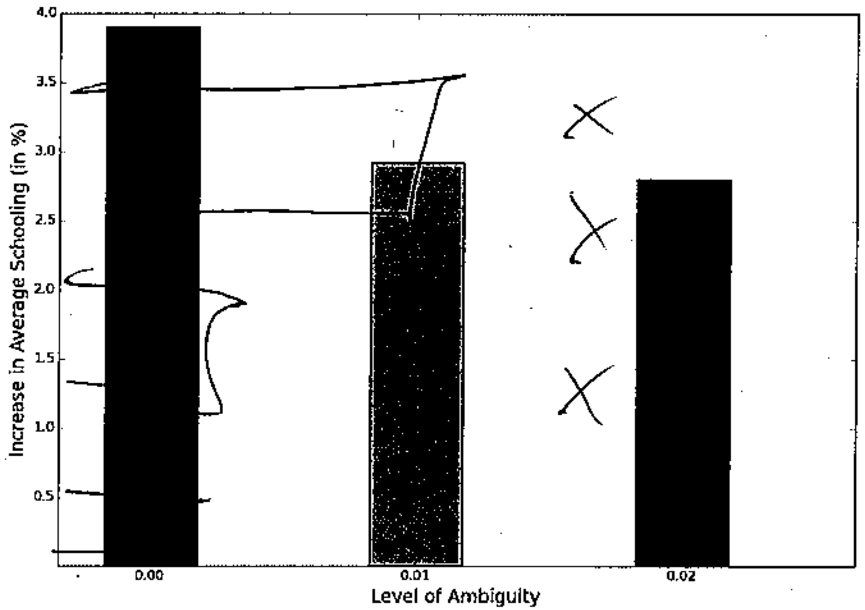
~~Changing~~ Occupational Choices in the last period



The next figure shows the increase in average schooling (in percentage terms) for a $500 tuition subsidy for different levels of ambiguity. The impact of the policy decreases as the level of ambiguity increases. In the case of ambiguous payoffs, agents do not account for the full value of the subsidy. They only adjust their worst-case evaluation.

? → only put in if economically story

Changing Effectiveness of Tuition Policy



... to be continued