Robust Decision over the Life-Cycle*

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Introduction

Welcome to *Risk and Ambiguity in Educational Choices.* In my research agenda, I investigate the potential additional insights gained by introducing ambiguity in dynamic economic models. Currently, I work on **dynamic discrete choice models**.In these models, agents make repeated decisions over multiple periods. I study educational choices; starting at age eighteen agents each year decide whether to increase their education or work in the labor market.

These **structural economic models** make explicit the agents' objective and the informational and institutional constraints under which they operate.

They allow to assess the relative importance of competing economic mechanisms that guide agents decisions and conduct ex ante evaluation of alternative policy proposals. Agents are **forward-looking** and thus take the future consequences of their immediate actions into account. Their problem is **dynamic** as current investment into education, increases the rewards of future labor market participation. The agents operate in an **uncertain** economic environment, at least parts of their future payoffs are not known at the time of their decisions. For example think of it as labor market luck.

The *existing work on dynamic models of schooling* offers insights on the reasons for observed heterogeneity in educational attainment. The literature investigates (1) heterogeneity in returns to education, decomposing returns into benefits and costs of education, (2) selectively binding credit constraints, (3) heterogeneity in preferences (risk aversion, time preferences), and (4) the role of uncertainty.

I hope to *contribute to the literature* in the following way. The existing literature on these models studied **decisions under risk**. That is, agents know the exact distribution of the future random components. When face with a decision, they simply integrate out the random component and pursue the option with the higher expected value. I extend this literature by focusing on **decisions under ambiguity**.I instill a fear of model misspecification into the agents. Agents are not entirely sure about the distribution of the random components, but they try to make **robust decisions**. Decisions that work well under a variety of alternative forecasting models. They do not only consider one distribution, but a whole set. A make their decision by maximizing their worst-case outcome under the whole set of so called **admissible distributions**.

Why is this potentially productive extension? In previous work with Prof. Heckman, we studied educational choices under risk in a standard version of the model. There we identified so called *psychic cost* as a major factor in explaining educational enrollment patterns in the NLSY dataset. Another way to put it, these *psychic cost* could be interpreted as effort costs. This datasets tracks educational choices of a cohort of individuals over time. Final educational attainment was simply too low compared to the returns in the data. However, pointing towards *psychic costs* is very unsatisfactory as they remain as an essentially unexplained residual. As it turns out there is a **modeling trade-off**. If I fit a model where agents make decisions under risk, while in fact the economic environment is ambiguous, then this misspecification error shows up as *psychic costs*. That is why I am now exploring ambiguity as a more interpretable economic mechanism leading to the observed patterns in the data.
Summing up, my goal for the next hour is the following: I hope to convince you that acknowledging ambiguity in dynamic models educational choice is (1) plausible, (2) meaningful, and (3) tractable.

Starting point ...

And this is how I intent to do it. The whole existing literature studying educational choices under risk was started and still relies on the key components outlined and developed in this paper. From a computational perspective solving these models boils down to a solving a finite horizon dynamic programming problem under risk. It involves several numerical challenges to make estimation of these models feasible. In particular function approximation and numerical integration.

This basic model was of course extended to account for alternative mechanisms to account for the heterogeneity in educational attainment. However, all that remaining within the **paradigm of decisions under risk**.

I am now branching off from this literature and study educational choices under ambiguity. All the basic ideas and challenges are part of this baseline model. However, a lot of the bells and whistles are absent and allow for a clear focus on this new mechanism. Of course, the final goal is clear. Have add all the mechanisms studied in the existing literature.

The **final goal** is clear, of course. Fit a general model of educational choices under ambiguity to the NLSY that then allows to asses the relative importance of competing economic mechanisms proposed in the existing risk-based literature and conduct an ex ante evaluation of alternative policy proposals.

Just as a word of caution, this is very much work in progress. In fact, it is the first time I am presenting this work. My only goal for the talk is to illustrate the challenges involved in introducing ambiguity in these models and show you my first steps in doing so. At the end, I hope we agree, that it is project worth continued efforts.

Basic Model under Risk

Ingredients

These **ingredients** determine how we think about an agent's optimal decisions. In particular we will study how to think about *optimal decision* in environment that are characterized by different assumptions about the information available to agent's information, i.e. study agent decisions under risk and ambiguity.

Let's now establish some basic notation to describe these ingredients in a more formal way.

Notation

More precisely, we will have a set of four alternatives: (1) Occupation A, (2) Occupation B, (3) School, and (4) Home. The time of agents in our model is set to 40 years, the discount factor is set to 0.95.

- When discussion rewards, preview observed and unobserved components.

Decision Tree

In period 40, there are around 13,000 different nodes.

Timing of Events

Agent Characteristics

Agents are characterized by occupation-specific human capital, some learning by doing. As it turns out, at least some tasks will be similar in both occupations, so skills are at least partly transferable.

State Space

Representation reflects fact of serially independence, initial conditions $x_{10}, x_{20} = 0, s_t = 10$. There is no depreciation of human capital.

Agents' Objective under Risk

Agents act as to maximize the expected value of their discounted lifetime reward.

Calibration

Now, we will specialize the model even further. We will add functional form and distributional assumptions, and settle on a particular parametrization. The last figure shows the effect of schooling on wages for the two occupations. In *Occupation A*, starting wages are higher but the returns to schooling are lower compared to *Occupation B*. As agents accumulate more and more schooling at the beginning of their life-cycle, they are drawn towards *Occupation B*.

Occupation A

The earnings equations are motivated by Mincer (1958, 1974). Log earnings are linear in years of schooling, and linear and quadratic in years of labor market experience. The residual captures labor market luck. The rate of return to schooling is the same for all schooling levels.

Occupation B

Wages and Experience

Schooling is set to ten years, which is the initial conditions all agents start out with. All agents start out with ten years. Let us briefly pause and compare the two types of occupations. ... Experience is set to their mean value in the sample, average education is 12 years. What do I know about the distribution of schooling. For how many is the 16 relevant? Occupation B is more skill intensive in the sense that schooling has a higher return and own experience has a higher return. Schooling an experience one provide general skill that is useful in both occupations, while occupation two only provides experience useful in Occupation B

Wages and Schooling

Interaction between schooling and occupational choice. Schooling has a positive consumption value. occupation two has a lower mean wage at $t = 0$. Experience is fixed at the mean values in the application.

School

consumption Value and adjustment costs

Shocks

idiosyncratic time-varying shocks

Choices over Time

All agents start out identically, different choices over the life cycle are the cumulative effects of different shocks. Initially, 60% increase their schooling but the share of agents in school in each period declines sharply. The share working in Occupation A starts to increase from 30% and peaks out at about 60% around period 15. Then declines back to 50%. Occupation B increases continuously, initially only 2% work in Occupation B but this share increases to about 40%. Around 5% stay at home each period.

Ambiguity

Modeling Ambiguity

Set of Admissible Beliefs

Agents center their beliefs around a **baseline model** $\mathcal{N}_0$ and center their beliefs for the shock distribution and consider **local perturbations** around it. θ governs the size of the ambiguity set, if $\theta = 0$ then we are in the **special case** where the decision maker is acting under risk. The set is the **same for all agents** and remains constant over time, i.e. the uncertainty cannot be reduced over time. However, even though the set is the same for all agents, the relevant worst-case distribution still differs between agents.

In effect, the decision makers gains nothing from by having future actions depend explicitly on past realizations of uncertainty. This leads to a separability that is crucial for establishing the robust counterpart of the Bellman recursion (Iyengar, 2005). Applicability treats each state as very different, so the applicability might depend on the time horizon (e.g. seconds in financial data or years in occupational choice).

- This is going to be different than just having a different value for the intercept.
- Concepts from decision theory: Consequentialism, Dynamic Consistency.

Exploring Set of Admissible Beliefs

A key economic assumption is rectangularity. It is best interpreted in an adversarial setting, where the decision maker chooses its policy and an adversary observes it. The adversary then observes it and and selects the distribution that minimizes the rewards. It is a form of an independence assumption. The choice of distribution in a particular state does not restrict the choices of the adversary in the future.

- This is going to be different than just having a different value for the intercept.
- There is no belief heterogeneity.

Exploring Expected Total Values

Why not just call them value functions? T - 1. Here ambiguity does not matter, late in life-cycle. But I choose this setup as it allows me to have a clear cut comparison between the cases. The disturbances are set to zero in the $T-1$

We are looking at an agent at the end of his career in our model. It is in second to last period. He has worked most of his life. Accumulated worked for 9 years in Occupation A and 20 years in Occupation B, just one year of additional schooling.

Preferences under Ambiguity

While there is agreement that maximization of expected reward is the right approach for decisions under risk, there is no such consensus for decision under ambiguity. Axiomatic decision theory can clarify the tradeoffs Stoye (2012).

Minimax loss is the best known alternative to a the Bayes Approach. It evaluates decision rules by imputing a worst-case scenario as opposed to aggregating loss with respect to a prior.

Agents compute the expected utility with respect to each admissible probability measure and act as to maximize the expected value of their discounted lifetime reward under the worst-case scenario (Maccheroni et al., 2006; Gilboa and Schmeidler, 1989; Hansen and Sargent, 2007).

Hansen and Sargent (2001) distinguish between model and payoff uncertainty, ECB Error modeling

The next figure shows the share of individuals in school over time. Overall, investment in schooling declines as ambiguity increases. Embracing ambiguity can thus provide a more interpretable explanation for low enrollment rates of income-maximizing agents than the presence of large psychic costs investigated in Eisenhauer et al. (2015).

The next figure shows the increase in average schooling (in percentage terms) for a $500 tuition subsidy for different levels of ambiguity. The impact of the policy decreases as the level of ambiguity increases. In the case of ambiguous payoffs, agents do not account for the full value of the subsidy. They only adjust their worst-case evaluation.

The next figure documents the share of agents that end up in each of the two occupations in the last period for different levels of ambiguity. Agents reduce their schooling investments as ambiguity increases and thus less and less end up working in *Occupation B*.

Understanding Economic Mechanism

Slide on Economic Mechanism

Assessing Model Misspecification

Now I am breaking away from the common setup, where the **econometrican and the agent share the same model**. Instead, I illustrate how our conclusions are potentially flawed when we the econometrican fit a model of educational choices under risk to data generated that recorded agent decisions and outcomes under ambiguity.

Slide on Model Misspecification

Conclusion

So, I hope I made the case that incorporating ambiguity in dynamic discrete choice models is potentially fruitful undertaking. It is a plausible, meaningful, and tractable extension to the standard model under risk.

The **final goal** is clear. Fit a more general model of educational choices under ambiguity to the NLSY that then allows to asses the relative importance of competing economic mechanisms and conduct an ex ante evaluation of alternative policy proposals.

However, there are a couple of papers that I want to exist to make this a sound extension to the standard model.

- I want to work with a graduate students to provide an upgraded recomputation of the results in the original paper.
- I am working with Stefan Wild from Argonne National Labs to meet the numerical challenges of the standard model with state of the art approaches.
- Then we need to assess the performance of all the numerical components for models of ambiguity.

These improvements to the standard model provide a sound foundation to tackle the additional challenges posed by studying ambiguity.

Alright, that is it. Thank you very much for your attention. i very much look forward to any further questions an comments from your side.

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