Report: Sixth Assignment (optional assignment)

Homography

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1 Introduction

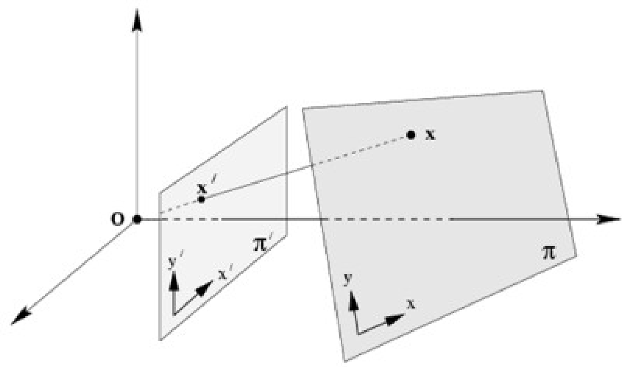
1.1 Homography

The Homography transformation, also called projective transformation, is a popular geo-referencing technique used worldwide. It is a linear transformation of homogeneous vector represented by a non-singular matrix and it is based on quite complex geometric and mathematic concepts known as *homogeneous coordinate* and *projective planes.* The familiar Cartesian plane is composed by a set of points which have a one-to-one correlation to pairs of real numbers. The *projective planes* instead is a superset of that real plane where for each point we also consider all possible (infinite) straight lines towards space. In this scenario every 2D point can be projected in any other plane in the space. Based on these concepts, it is defined the **homography between 2 planes** which, simply speaking, means that given 4 points(the least possible number possible in order to estimate the matrix H) in a plane, there always exists a relationship that transforms them into the corresponding 4 points in another plane. Initially the homography has not been used for map geo-referencing but for other purposes, such as to rectify a perspective image, for example to generate a *plan* view of a building from a *perspective* photo. The planar homography relates the transformation between two planes (up to a scale factor):

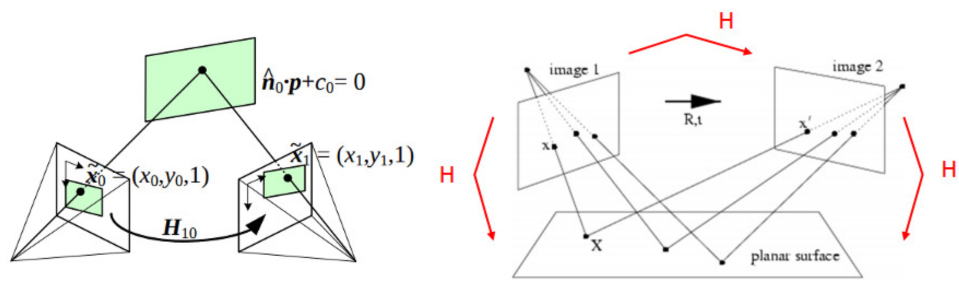
$$s \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = H \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

The homography matrix is a 3×3 matrix but with 8 degrees of freedom, as it is estimated up to a scale. The following examples show different kinds of transformation but all relate a transformation between two planes.

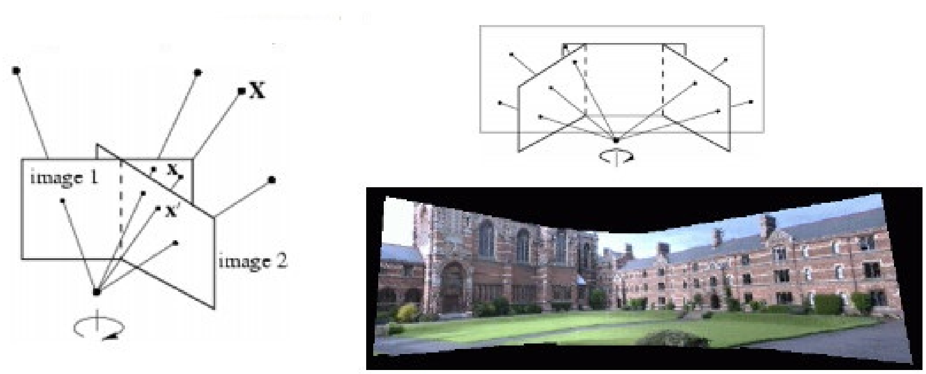
- A planar surface and the image plane



- A planar surface viewed by two camera positions



- A rotating camera around its axis of projection, equivalent to consider that the points are on a plane at infinity

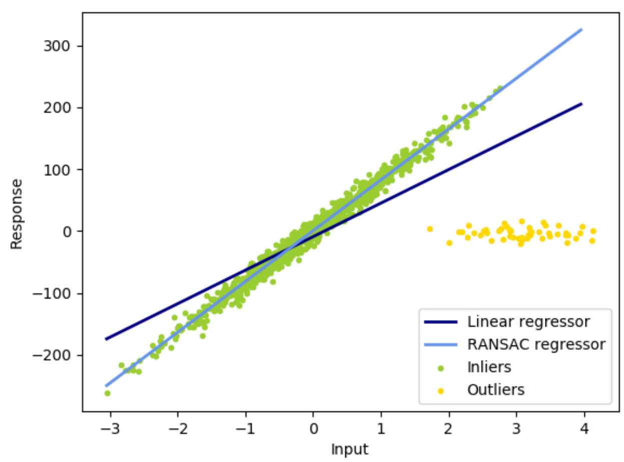


How the homography transformation can be useful?

- Camera pose estimation from coplanar points for augmented reality with marker for instance
- Perspective removal / correction
- Panorama stitching

1.2 RANSAC

RANdom SAmple Consensus, or RANSAC is an iterative and statistic method to estimate parameters of a mathematical model from a set of observed data that contains outliers, these are to be accorded no influence on the values of the estimates. Therefore, it also can be interpreted as an outlier detection method. It is a non-deterministic algorithm in the sense that it produces a reasonable result only with a certain probability, this probability increases as much as many iterations are allowed. One of the usages of this method is to determine the points in the space that project onto an image into a set of landmarks with known locations. A basic assumption is that the data consists of *inliers*, data whose distribution can be explained by some set of model parameters, though may be subject to noise, and *outliers* which are data that do not fit the model. The outliers can come, for example, from extreme values of the noise or from erroneous measurements or incorrect hypotheses about the interpretation of data. RANSAC also assumes that, given a (usually small) set of inliers, there exists a procedure which can estimate the parameters of a model that optimally explains or fits this data. A simple example of a RANSAC method application is shown in the following figure:



2 Problem Description

This assignment consists on three exercises mainly focused on better understanding the homography and its effects in different possible applications and problems solving; a paragraph is dedicated for each different problem' s and exercise' s description. In each exercise it is requested to discard the interpolation, to vary the number of points, to add the normalization of the points considered and finally to use a RANSAC procedure to compute a more robust homography matrix H.

We have changed the organization of the scripts provided to us: a script called *Lab*06 has been added, this one asks to the user which one of the three scripts he wants to run, how many points he wants to consider, then to select them, and finally to choose which of the two possible algorithms, Homography or RANSAC, he wants to execute.

2.1 Correcting the perspective distortion of an image

In the first exercise it is requested to correct the perspective distortion of an image.
Regarding the results coming from the direct mapping and the inverse mapping methods it can be noted from the Figure 2 that in the first case there are lots of black lines in the output image since the algorithm tries to find a correspondence between each pixel in the input image and the corresponding one in the output image, so as much as the image is distorted as many black lines we will have in the output image because the algorithm does not find a valid correspondence between some pixels. The inverse mapping has better results as expected since normally it uses the bilinear interpolation in order to compute the pixel' s value also depending on the values of its neighbors pixels and so the output image will have less black lines as shown in Figure 3; the effects of discarding the bilinear interpolation are discussed in the following.

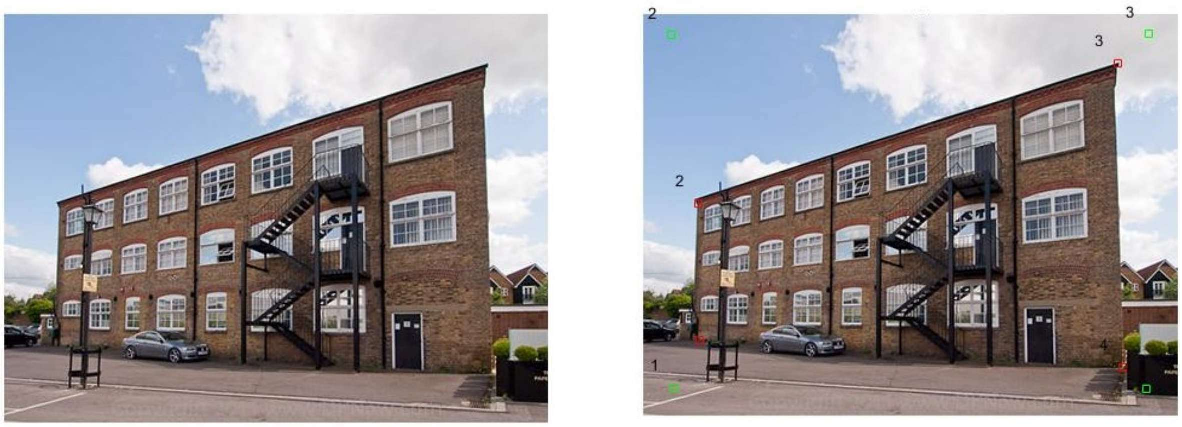


Figure 1: The original and image and the selection points step

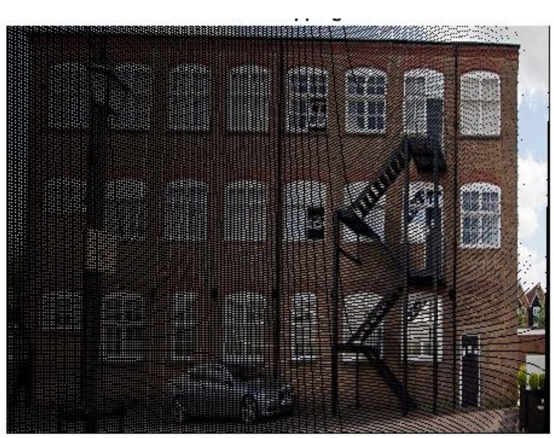


Figure 2: The direct mapping

Figure 3: The inverse mapping without an with bilinear interpolation

The comparison and the evaluation of the results deriving from the perspective distortion of the input image with the bilinear interpolation and the ones deriving without the bilinear interpolation are as expected that the first ones are clearly better, in particular it can be noted that the edges in the resulting image are less smoothed and so the objects are better defined in the output image thanks to the bilinear interpolation.
After that it is requested to change the number of the points considered, also in this case the previous consideration about the usage or not of the bilinear interpolation are valid. In this case some important considerations are that as many points the user decides to consider as much the edges of the resulting image without the interpolation appear irregular, moreover if the points are randomly selected in the input image the effects of the correction of the perspective are not clearly interpretable, so it is recommended to select them in a reasonable way in order to obtain a good result.
The normalization of the points considered before computing the homography was the next step requested, for doing that the function called *normalise2dpts* is used and from the results deriving from this operation any consistent difference can be note since the input images are characterized by the same brightness and so the normalization has not appreciable results in this case.
The last step of the first exercise was to incorporate a robust estimate of H through a RANSAC procedure and through a code given in the text and stored in two function called *ransacH* and *testH*. The results of applying a RANSAC procedure in order to have a more robust matrix H is as much relevant as many points are considered: if the user is considering few points probably the matrix H will not change using the RANSAC procedure but as soon as the user select a bigger amount of points this matrix will change its values also depending on the threshold that is used and that is the third parameter of the function *ransacH*, in fact as much as the threshold is high as many points will be considered as inliers and less as outliers. These evaluations can be seen in Figure in the appendix.

2.2 Estimating the homography between a pair of images

In the second exercise the aim was to estimate the homography matrix H between a pair of images, again manually selecting the corresponding points in the images, and so to try to modify the first input image in order to make it equal to the second input one.
From the point of view of the efficiency of the homography method and the normalization of the points' vector the same considerations of the previous exercise are valid, in fact the homogeneous matrix H does not change if we apply or not the normalization, while it is changing if the user consider more and more points and then he will apply the RANSAC procedure in fact as many points (that will be considered as inliers) will be considered in the RANSAC procedure as much the matrix H will be robust. The effects of these exercise are shown in Figure 6:

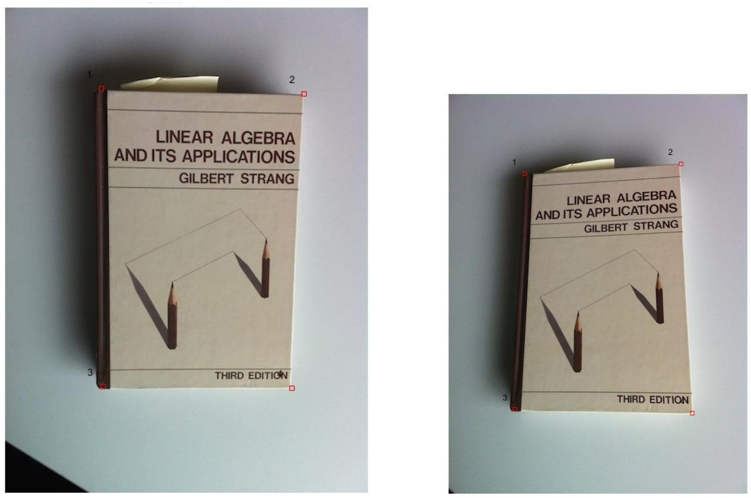


Figure 4: The selection points step

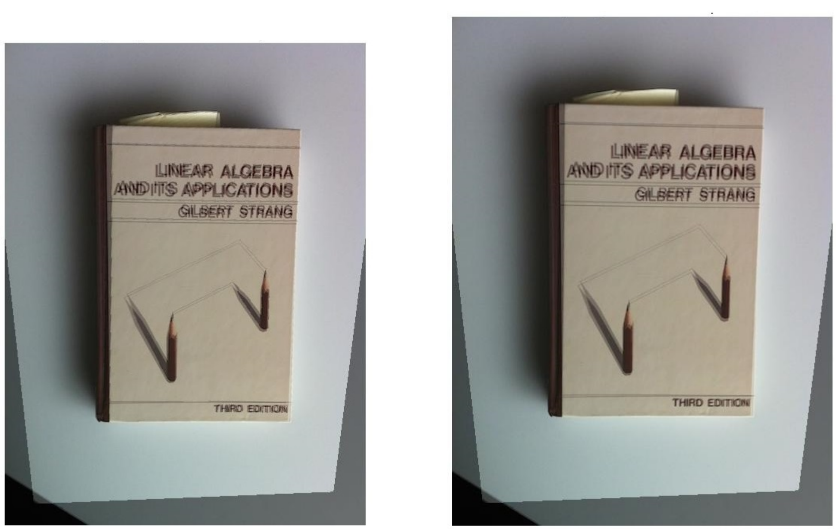


Figure 5: The average image without and with bilinear interpolation

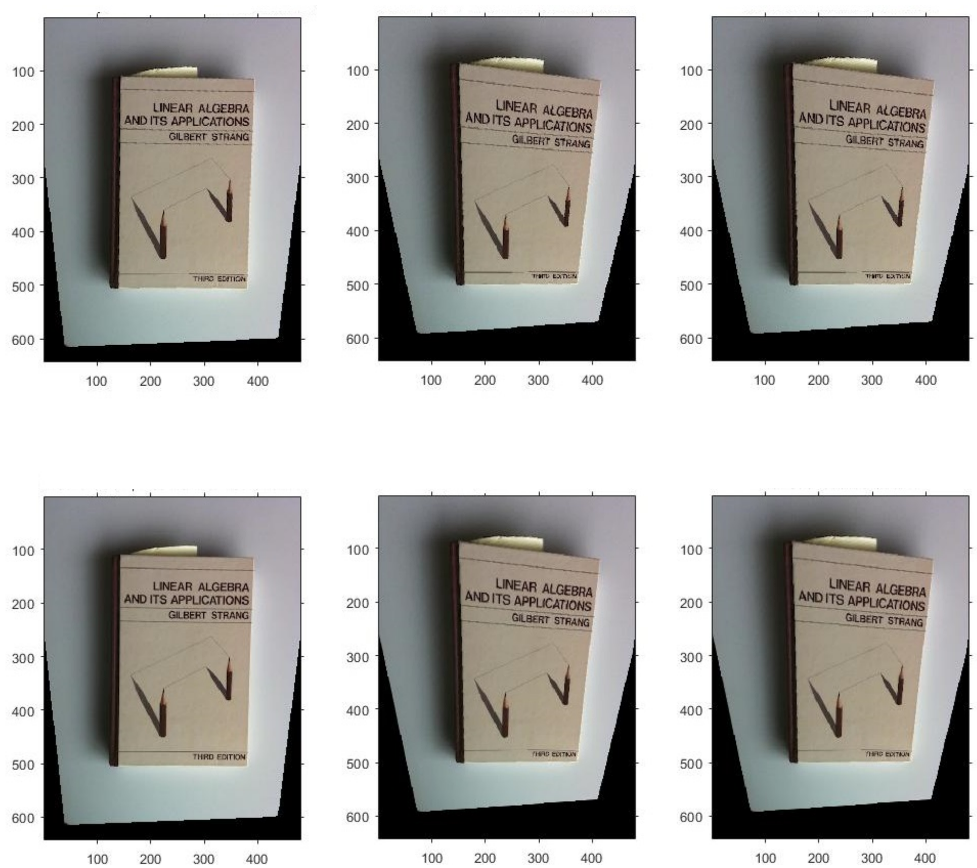


Figure 6: Methods comparison, we have: Homography not normalized, normalized and Ransac. The difference is that in the first row the image do not have the bilinear interpolation while it present in the second.

2.3 Building a mosaic from a sequence of images

The last exercise consists on building images mosaic from a sequence of input images acquired by purely rotating the camera and on exploiting homographies.
With this code we compute a *collage* image built from a set of images, in this case unlike previous exercises we need to have 8 correspondence points, in fact the images are not $2D$ and so the homography matrix has 8 degrees of freedom. The correspondence is done taking one reference image (the one in the middle) and selecting the correspondence points between this and all the other images. The effects of this algorithm are really appreciable and are shown in Figure 7.

Figure 7: Building a mosaic from a sequence of images

3 GitHub Repository

https://github.com/federicotomat/Computer-Vision

References

[1] Richard Szeliski, *Computer Vision: Algorithms and Applications*, (University of Washington, 2010).

[2] Francesca Odone & Fabio Solari, *Computer Vision Course: Image Foundamental*, (University of Study of Genoa, 2019).

A Appendix