

Stochastic differential equations

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1. Write a program to integrate the following stochastic differential equation:

$$dx = -xdt + \sqrt{2}dW$$

Use Euler integrator (i.e. just replace dt with Δt and dW with $\sqrt{\Delta t}R$ where R is a Gaussian number with zero average and unitary variance; choose a short timestep $\Delta t=0.001$). To generate Gaussian numbers, you can copy the `gasdev` routine from `simplemd`. Use any programming language you like (suggested: C, FORTRAN, python).

2. Using this program, compute ten trajectories with same initial condition ($x = 10$) and different seeds for the random number generator. At fixed value of time t , compute the average over the trajectories of the value of x and its standard deviation. How do these two quantities depend on t ?
3. Analyze the trajectories and compute their time increment, defined as $g(t) = x(t + \tau) - x(t)$. How does the increment depend on τ ? How does it depend on x ?