Stability score Algorithm Let be C the count matrix with $N \times M$ dimension where N is the gene number and M is cell number.

$$C = \begin{bmatrix} c_{11} & c_{12} & \dots \\ \vdots & \ddots & \\ c_{N1} & & c_{NM} \end{bmatrix}$$

Then we define C^p_q as a matrix derived by matrix C in which q columns (randomly selected in permutation p) are removed. The function $RemoveCell(p)$ takes as input a specific permutation p and returns a list L^p containing all the removed cells in p(observe that the size of L^p is q). We denote $\mathbf{cl}^p$ the vector with length $M - q$ storing the output of the clustering algorithm in the permutation p. Hence, $\mathbf{cl}^p[i]$ identified the cluster in which the ith cell is inserted in permutation p. Moreover we use the notation **cl** for indicating the the output of the clustering algorithm obtained by all cells.

The relation symmetric matrix R^p with dimension $M \times M$ is defined as follows:

$$R^p = \begin{bmatrix} r^p_{1,1} & r^p_{1,2} & \dots \\ \vdots & \ddots & \\ r^p_{M,1} & & r^p_{M,M} \end{bmatrix}$$

where $r^p_{i,j}$ is:

$$r^p_{i,j} = \begin{cases} 1 \; if \; \mathbf{cl}^p[i] = \mathbf{cl}^p[j] \\ 0 \; otherwise \end{cases}$$

Similarly we defined R relation symmetric matrix obtained considering all cells. Then we introduce a function $RMV()$ that takes as input R and the list of removed cells in a permutation (i.e. l^p) and returns an new matrix R' in which the columns and the rows associated with cells in l^p are removed.

Function $length(j, p, k)$ counts the occurrences of k in the row j of the matrix $R + R^p$. It is formally defined as follows:

$$length(j, p, k) = \sum_i = 1^{M-|L^p|} 1_{R[i,j]} + R^p[i, j] = k$$

where 1_A is an indicator function returning 1 when condition A is satisfied.

Finally we define a permutation score $pscore_{j,p}$ as:

$$pscore_{j,p} = \frac{length(j, p, 2)}{length(j, p, 2) + length(j, p, 1)}$$

where p is a permutation and j a cell.

We define $tscore_{m,s}$ as follow

$$tscore_{m,s} = \frac{\mathit{1}}{P} \sum_{p \in P} \mathit{1}_{score_{m,p}>=s}$$

where P is the permutation number and s the threshold score.

1 Example

Be $\mathbf{cl} = \begin{Bmatrix}1 & 2 & 2 & 1 & 2 & 1\end{Bmatrix}$

Be $\mathbf{L} = \begin{Bmatrix}6 & 2 & 2 & 4\end{Bmatrix}$

Be $\mathbf{cl}^1 = \begin{Bmatrix}1 & 2 & 1 & 1 & 2\end{Bmatrix}$

Be $\mathbf{cl}^2 = \begin{Bmatrix}1 & 2 & 1 & 2 & 2\end{Bmatrix}$

Be $\mathbf{cl}^3 = \begin{Bmatrix}1 & 2 & 1 & 1 & 2\end{Bmatrix}$

Be $\mathbf{cl}^4 = \begin{Bmatrix}1 & 2 & 2 & 1 & 2\end{Bmatrix}$

$\forall\ p \in \{1, 2, 3, 4\},\ R_p$ is calculated

for istance hereafter I reported R and R^1

$$R = \begin{bmatrix}1 & 0 & 0 & 1 & 0 & 1\\ 0 & 1 & 1 & 0 & 1 & 0\\ 0 & 1 & 1 & 0 & 1 & 0\\ 1 & 0 & 0 & 1 & 0 & 1\\ 0 & 1 & 1 & 0 & 1 & 0\\ 1 & 0 & 0 & 1 & 0 & 1\end{bmatrix}$$

$$R^1 = \begin{bmatrix}1 & 0 & 1 & 1 & 0\\ 0 & 1 & 0 & 0 & 1\\ 1 & 0 & 1 & 1 & 0\\ 1 & 0 & 1 & 1 & 0\\ 0 & 1 & 0 & 0 & 1\end{bmatrix}$$

$\forall\ p\ R' + R^p\ \mathit{is\ calculated}$

for istance hereafter I reported R' + R^1

$$R' + R^1 = \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 0 & 2 & 1 & 0 & 2 \\ 1 & 1 & 2 & 1 & 1 \\ 2 & 0 & 1 & 2 & 0 \\ 0 & 2 & 1 & 0 & 2 \end{bmatrix}$$

$\forall\ p,\ pscore\ is\ evaluated\ with\ S = 0.6$ for istance hereafter I reported $pscore_1$

$$pscore_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

This means that in permutation 1 the third cell is unstable "jumping" from cluster number 1 to cluster number 2 in P. $\text{tscore}_{m,s}$ is evaluated then for each cell $tscore_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.25 \\ 0.25 \\ 0 \end{bmatrix}$

In this Example number of permutation is 4, for statistical relevance, an higher number of permutation is required.