Currents: an Interview

Giovanni Canarecci

Department of Mathematics and Statistics
University of Helsinki

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Who do I study?

I study **Currents** = linear functionals from the space of differential forms $\mathcal{D}^k(M)$ to the real line.

$$S : \mathcal{D}^k(M) \to \mathbb{R}$$
$$\varphi \mapsto S(\varphi)$$

where a differential form φ is a map

$$\varphi : \mathcal{T}(M) \to \mathbb{R}$$

How do I study them?

We consider only those currents to whom we can associate a good set!

Observation

Any good[1] set S can be seen as a current S:

$$S : \varphi \in \mathcal{D}^k(M) \longrightarrow \int_S \langle \vec{S}(x), \varphi \rangle \mu(x) d\mathcal{H}^k \in \mathbb{R}$$

And we call these currents "Rectifiable Currents".

[1] oriented $k-$dim rectifiable

Where do I study them?

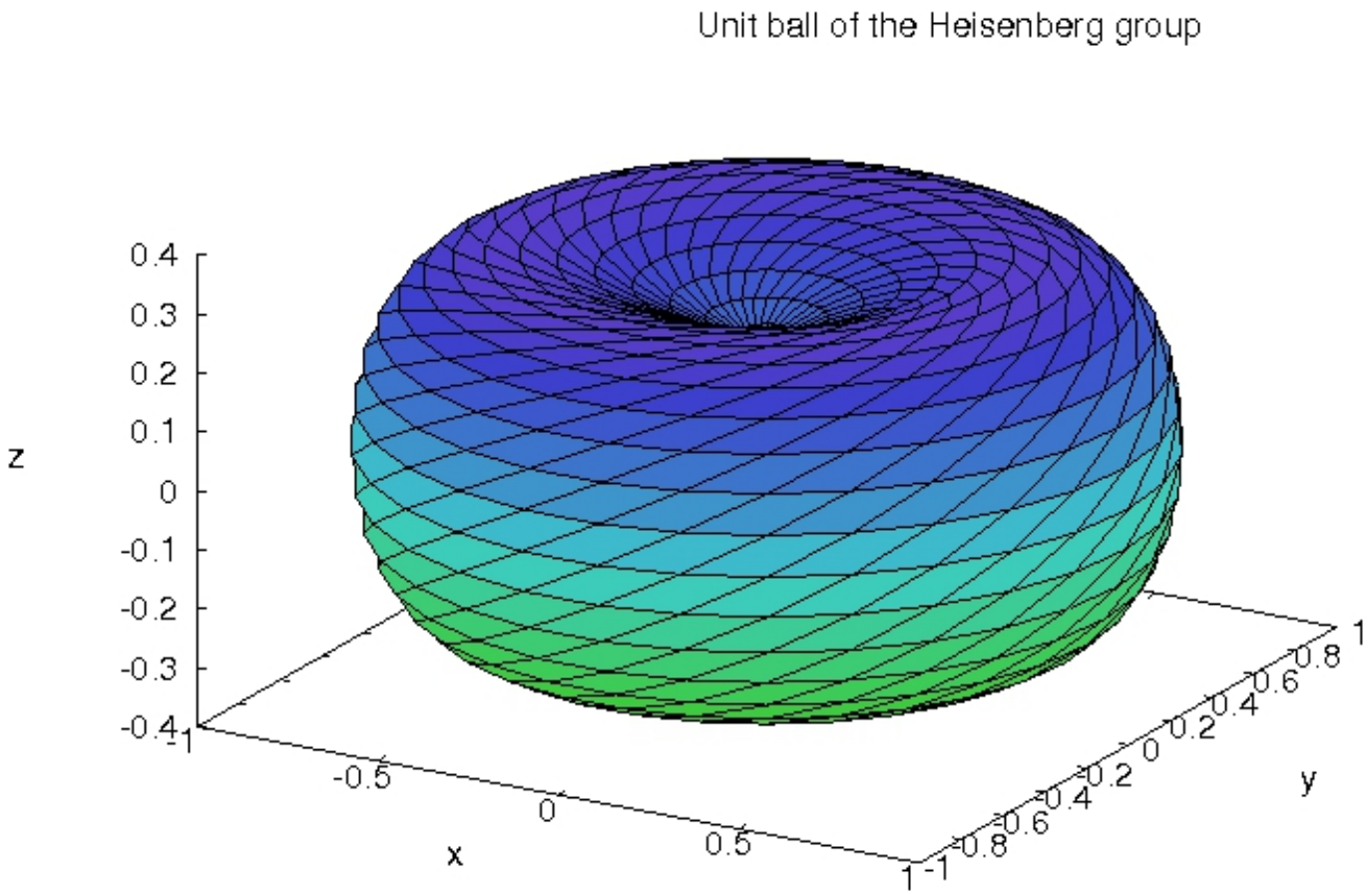
In the Heisenberg Group:

$$\mathbb{H}^1 := (\mathbb{R}^3, *)$$

that is $\mathbb{R}^3$ with a strange product:

$$(x_1, y_1, t_1) * (x_2, y_2, t_2) = (x_1 + x_2, y_1 + y_2, t_1 + t_2 - 2(x_1 y_2 - x_2 y_1)) .$$

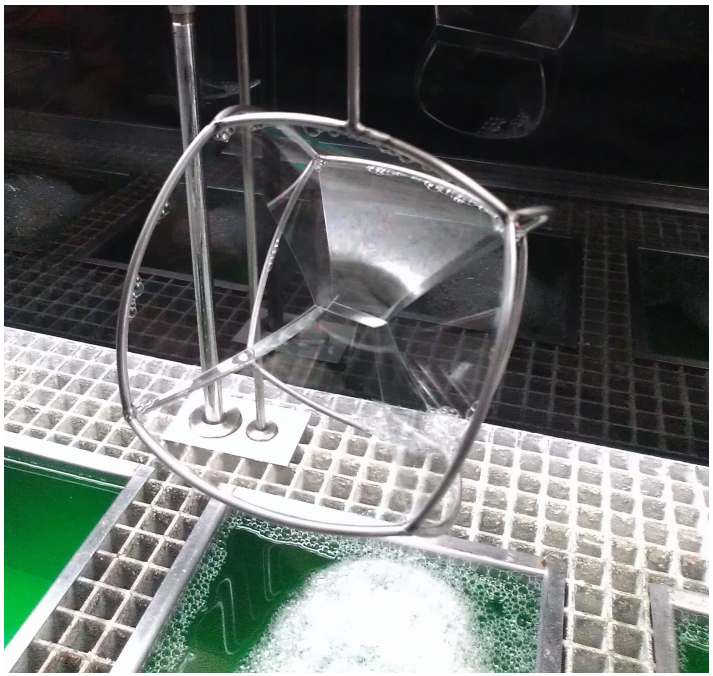
Clear? ... maybe a picture would help!

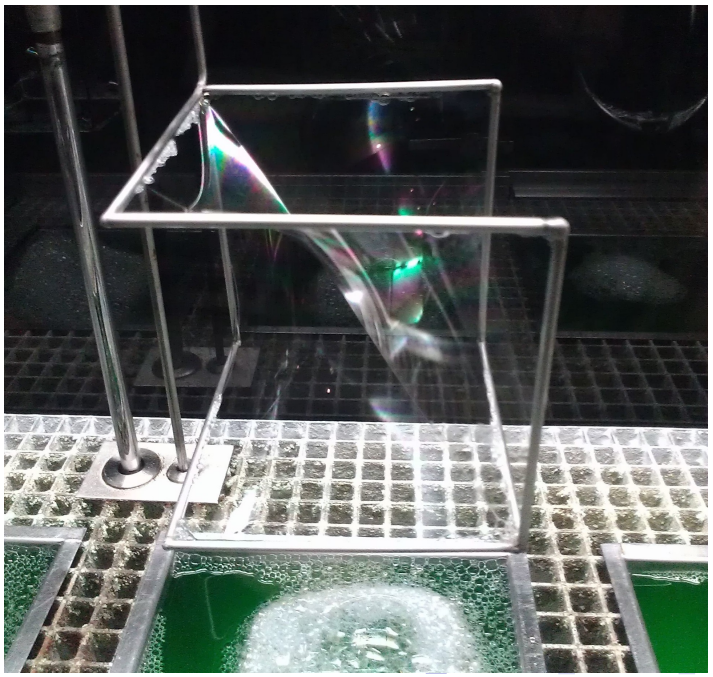


What do I study with them? The Plateau Problem!

Definition (Plateau Problem)

Given a **boundary** with some kind of regularity, the Plateau Problem consists in finding the **minimal surface** that fits that boundary.





Why?

A complete answer for this question is currently object of research.

Kiitos paljon!

Thank you!

Grazie mille!