Integrated structural and control system design (Co-Design) for robust flutter performance

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ABSTRACT

This article presents an integrated approach to achieve system-optimal wing designs for robust flutter performance using co-design, a design methodology that accounts directly for the synergistic coupling between physical and control system design. Direct Transcription method is used to find the optimal system. The final results show the gain due to co-design methods. In the first paragraph a short introduction introduces flutter problem.

The second and third sections describe the importance of co-design methods and some tools useful for the system analysis. Direct Transcription method is explicated in the fourth part, whereas the application to the wing case of study is illustrated in the fifth paragraph. The analysis results are discussed in the sixth section before the conclusion.

I. Introduction

Aeroelasticity is a classic problem in aerospace engineering, it should take into account the interaction between several disciplines: aerodynamics, structures and control systems. Central to the entire discipline of aeroelasticity is the concept of flutter, "a structural dynamical instability, which occurs in a solid elastic structure interacting with a flow of gas or fluid and consists of violent vibrations of the structure with rapidly increasing amplitudes"[1]. The objective of this study is to optimize the whole system so as to achieve the best possible system performance.

II. State of the art

In this paragraph the common approach used today in systems engineering is described.

Many engineered systems operate using control systems. The design of these systems is usually split into several sequential steps: mechanical system design, control system design, and real-time software development. In current practice the system is typically designed in this sequence, where part of the system design is fixed at each stage, reducing design flexibility and opportunity for improvement. In fact all these stages of the design process are coupled (fig.1). In other words, to achieve the best possible system performance, an integrated design approach that considers all aspects of system design simultaneously is required.

III. Principles of optimal design - System design

In this section some tools for the analysis of multidisciplinary design optimization methods are described.

As explained in [2], a system can be defined as something that comprises elements interacting with each other, that function together. Therefore a system can be partitioned into its elements, and its overall function is achieved through properly tracking how the elements function together. According to the principle of decomposition, it is possible to break the problem into simpler problem pieces (subproblems linked through common design variables and analysis interactions), solve each subproblem separately, and coordinate the subproblem solution so that you obtain the solution to original problem. Methods for decomposition-based design optimization, are called with the term multidisciplinary design optimization (MDO). For a given partition a coordination strategy must be derived. During the system design we should consider:

- interactions: subsystems may share some variables that they must jointly determine; a subsystem may have variables whose output values are inputs to another subsystem (fig.1);
- partitioning methods: hierarchical decomposition involves partitioning the system problem into a master problem and subproblems. In a nonhierarchical strategy the problem is partitioned into subproblems. The master problem is an optimal design problem in the variables that couple the subproblems (linking variables). Each subproblem is an optimal design problem with its own set of variables (local variables);
- coordination strategies: Single-level formulations correspond to a single optimization problem. Multilevel formulations utilize distributed optimization: a separate optimization problem is defined for each subproblem;
- coupling: measures of coupling.

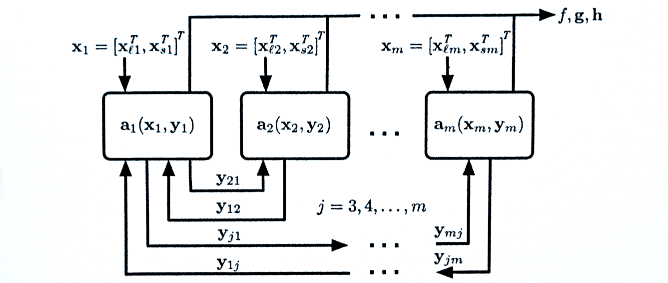

Fig. 1. Input and output relationships for a system of analysis functions.

The importance and the complexity of MDO can be appreciated through the example by Lockheed Martin shown in figure 2 [3]: a high number of analysis with the multidisciplinary optimization could leads to a high computational coast and time.

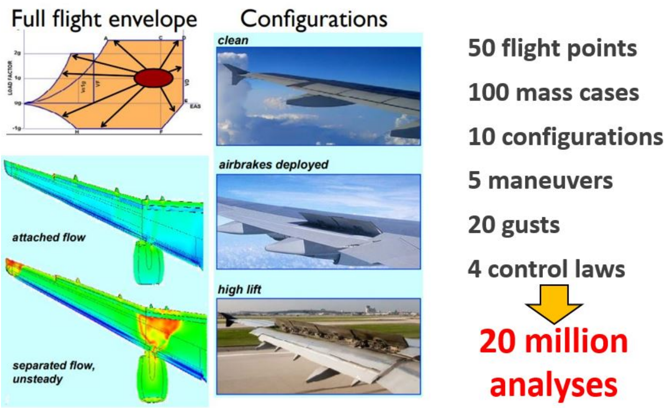

Fig. 2. What next generation MDO will be computationally demanding.

IV. Direct transcription

In this section, a particular method for multidisciplinary design optimization is described. This method is called direct transcription (DT) and it has been used to solve the problem illustrated in the following paragraphs. For a more in-depth explanation of the method, see reference [4]. We assume the system to be designed can be modeled using a system of continuous differential equations:

$$\dot{\xi} = f_d(\xi(t), u(t), t) \quad (1)$$
$$0 = f_a(\xi(t), u(t), t) \quad (2)$$
$$0 = f_b(\xi(t_F), u(t_F), t_F) \quad (3)$$

Equation (1) defines the time derivatives of a system model, where $\xi(t)$ (length n_s) is the vector of state variables, u(t) (length n_c) is the control input, and $f_d(\cdot)$ (length n_s) is the time derivative function. Equation (2) is the algebraic or path constraint. Equation (3) is an eventual boundary condition enforced at the simulation end time t_F. The open-loop optimal control design problem can be written as

$$\min_{\xi(t),u(t),t_F} J = \phi(\xi(t), u(t), t_F) \quad (4)$$

subjects to Eqs. (1)-(3), where J is the response of a cost function. The cost function take into account the impact of both physical system and control system design changes on system performance. Feedback control can be accommodated by replacing u(t) with control design variables x_c. It is possible to reformulate the problem in this form:

$$\min_{x_c} J = \phi(\Xi, x_c) \quad (5)$$

where Ξ is a discretized representation of the state variables (the i-th row of Ξ is $\xi_i = \xi(t_i)$, i = 1,...,n_t, and n_t is the number of time steps). As showed in figure 3, the simulation is partitioned into n_T time segments using multiple shooting technique (MS): the state at the interfaces between time segments (Y) is controlled directly by the optimization algorithm, and state continuity is enforced at convergence by defect constraints $\zeta(\Xi, Y)$:

$$\begin{aligned} &\min_{x_c} J = \phi(\Xi, x_c) \\ &s.t.\ \zeta_i(\Xi, Y) = 0,\ i = 1, ..., n_{T-1} \end{aligned} \quad (6)$$

Defect constraints require the state at the end of time segment j to match the j-th row of Y. In figure 3 the size of the

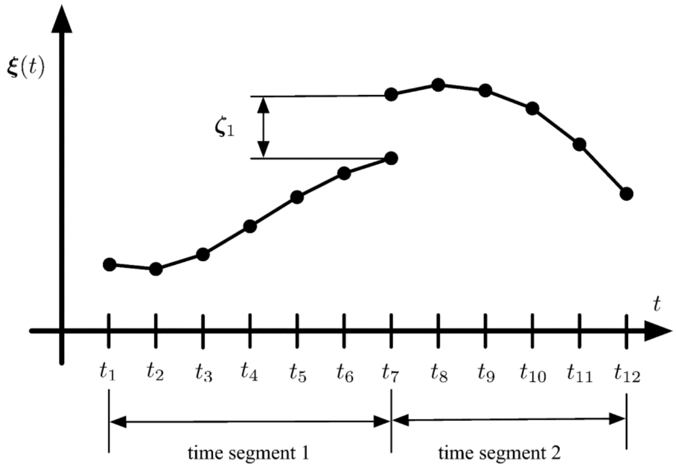

Fig. 3. Direct Transcription - Multiple shooting.

gap at the interface is the value of the corresponding defect constraint ζ_1, and this gap is driven to zero by the optimization algorithm. In the limit, as $n_T \to n_t - 1$, the simulation for each time segment collapses to a single difference equation. The set of defect constraints replace simulation completely and Y expands into the full state matrix Ξ. This is Direct Transcription (DT); the infinite-dimensional optimal control problem is transcribed directly to a finite-dimension nonlinear programming problem:

$$\begin{aligned} &\min_{\Xi, x_c, t_F} J = \phi(\Xi, x_c, t_F) \\ &s.t.\ \zeta_i(\Xi, x_c, t_F) = 0 \\ &\hat{f_{ai}}(\Xi, x_c, t_F) = 0 \\ &where\ i = 2, 3, ..., n_t - 1, n_t \end{aligned} \quad (7)$$

A defect constraint is defined for each time step, and path constraints are discretized ($\hat{f_{ai}}(\cdot)$). The differential equations from the original optimal control problem are discretized and converted to a system of algebraic equations. Using the implicit trapezoidal method, we can write:

$$\xi_i = \xi_{i-1} + \frac{h_i}{2}(f_d(\xi_{i-1}, x_c, t_{i-1}) + f_d(\xi_i, x_c, t_i)) \quad (8)$$

Therefore if the implicit trapezoidal method is used to convert state equations to algebraic equations, the defect constraint functions become

$$\zeta_i(\Xi, x_c) = \xi_i - \xi_{i-1} - \frac{h_i}{2}(f_d(\xi_{i-1}, x_c, t_{i-1}) + f_d(\xi_i, x_c, t_i)) \quad (9)$$

At every optimization iteration, Ξ is specified completely, so each of the n_{t-1} defect equations and path constraints are independent, enabling fine-grained-parallel computing. The ability to manage nonlinear inequality constraints is a primary motivation for extending DT to co-design.

An interesting aspect of Direct Transcription is shown in figure 4: a nested method can move only in one subspace

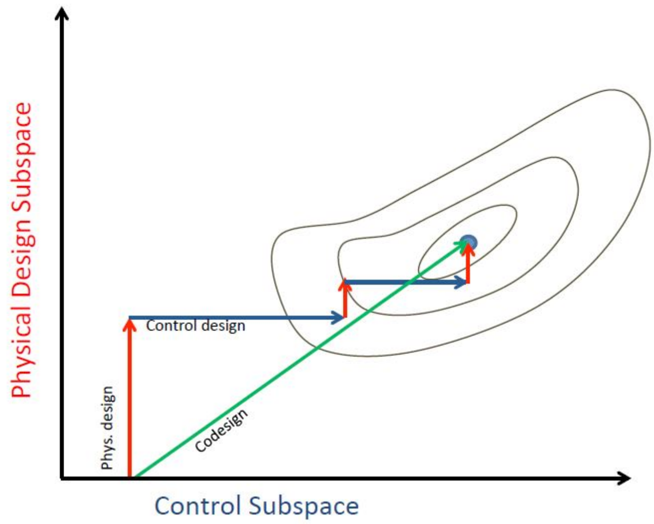

Fig. 4. Conceptual solution trajectories through design and state subspaces.

at a time, whereas Direct Transcription can move in both simultaneously, tracing more direct path to the solution. The results of two cases of study by Allison [4,5,6] are set out below:

	Sequential	Nested	Simultaneous
AEP (kW·h)	2996.9	3231.5	3231.5
% *AEP* Improvement	–	8.03	8.03

Fig. 5. Co-design example: horizontal axis wind turbine.

	Sequential	Simultaneous
J_{ramp}	52.4	55.8
J_{rough}	2.09	1.57
$J = 0.01 J_{\text{ramp}} + J_{\text{rough}}$	2.63	2.12

Fig. 6. Co-design example: Active suspension.

As shown in figure 5 and figure 6, the simultaneous optimization leads to an increase of the annualized energy production (AEP) in the first example (about horizontal axis wind turbines) and to the best objective function J in the second example (about car active suspensions).

V. Mathematical Modelling of a Simple Aeroelastic System with a Control Surface

In this section the mathematical model of the problem is described. At first the methods used for aerodynamic modelling are introduced; then the wing mathematical model is analyzed [7].

The quasi-steady assumption is not sufficiently accurate for flutter and gust response calculations and a more advanced unsteady aerodynamic analysis must be used in order to predict accurately the dependency of aerodynamic forces and moments on the frequency content of dynamic motions. In order to understand the effect of aerofoil heave and/or pitch motions on the aerodynamic loads and moments generated, the result of instantaneous changes in the angle of incidence and harmonic motion of the aerofoil need to be considered. The key tools to analyse these effects are Wagners and Theodorsens functions respectively. Wagners function can be used to consider the case of general motion (in the time domain), whereas the related Theodorsens function is an important component in predicting the onset of flutter (in the frequency domain) and in the analysis of the response to continuous turbulence.

As can be seen in figure 7 [7], the model is composed of a uniform rigid rectangular wing with pitch θ and flap κ degrees of freedom introduced via two springs at the root. A full span rigid control surface is included. It has an infinite stiffness attachment to the wing but can be moved to any angle β that is demanded.

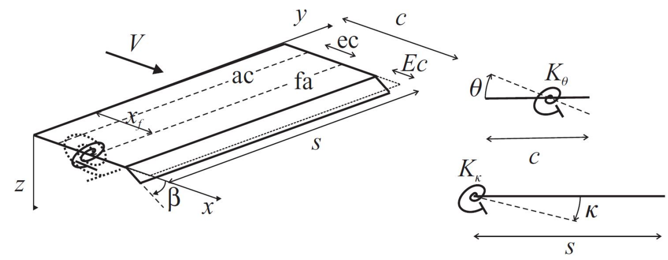

Fig. 7. Binary flutter system with a control surface.

Evaluation of Lagranges equations across the entire semi-span of the wing, adding in the prescribed motion of the control surface and the effect of gusts and turbulence, gives the expression for the open loop system:

$$
\begin{aligned}
&\begin{bmatrix} I_\kappa & I_{\kappa\theta} \\ I_{\kappa\theta} & I_\theta \end{bmatrix} \begin{bmatrix} \ddot{\kappa} \\ \ddot{\theta} \end{bmatrix} + \rho V \begin{bmatrix} \frac{cs^3 a_w}{6} & 0 \\ -\frac{ec^2 s^2 a_w}{4} & -\frac{c^3 s}{8} M_{\dot{\theta}} \end{bmatrix} \begin{bmatrix} \dot{\kappa} \\ \dot{\theta} \end{bmatrix} \\
&\quad + (\rho V^2 \begin{bmatrix} 0 & \frac{cs^2 a_w}{4} \\ 0 & -\frac{ec^2 s a_w}{2} \end{bmatrix} + \begin{bmatrix} K_\kappa & 0 \\ 0 & K_\theta \end{bmatrix}) \begin{bmatrix} \kappa \\ \theta \end{bmatrix} \\
&= \rho v^2 cs \begin{bmatrix} -\frac{s a_c}{4} \\ \frac{c b_c}{2} \end{bmatrix} \beta + \rho V cs \begin{bmatrix} -\frac{s}{4} \\ \frac{c}{4} \end{bmatrix} w_g = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \beta + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} w_g
\end{aligned}
\tag{10}
$$

where:

$$\begin{cases} I_{ij} : \ inertia \\ \rho : \ air \ density \\ V : \ air \ speed \\ c : \ chord \\ s : \ semi \ span \\ a_w : \ lift \ curve \ slope \ (2\pi) \\ e : eccentricity \ between \ flexural \ axis \ and \ aero \ centre \\ M_{\dot{\theta}} : \ unsteady \ aero \ damping \ term \\ K_i : stiffness \\ a_c, \ b_c : \ functions \ of \ a_w \ and \ of \ the \ fraction \ of \ chord \\ \qquad made \ up \ by \ control \ surface \end{cases}$$

In particular this is the vector of the design variables:

$$x_d = [s, c, m, f_f, f_\theta, perc\_x_{cm}, perc\_x_f]$$

where s is the semi span, c is the chord, m is the unit mass per area of wing, k_{freq} and θ_{freq} are the flapping and pitch frequencies, x_{cm} and x_f are the position of center of mass and flexural axis from nose (normalized with the chord). The problem parameters are:

$$\begin{cases} \rho = 1.225 \frac{kg}{m^3} \\ V = 100 m/s \\ a_w = 2\pi \\ M_{\dot{\theta}} = -1.2 \end{cases}$$

The other functions are:

$$\begin{cases} e = \frac{xf}{c} - 0.25 \\ K_i = (f_i \pi 2)^2 I_{ii} \ where \ i = k, \theta \\ a_c = \frac{a_w}{\pi}(cos^{-1}(1 - 2EE) + 2\sqrt{EE(1 - EE)}) \\ b_c = -\frac{a_w}{\pi}(1 - EE)\sqrt{EE(1 - EE)}) \end{cases}$$

where EE is the fraction of chord made up by control surface (EE=0.1)

We can rewrite the equations in compact form:

$$A\ddot{q} + \rho V B \dot{q} + (\rho V^2 C + E) q = g\beta + h w_g \tag{11}$$

where A, B, C, E are the structural inertia, aerodynamic damping, aerodynamic stiffness and structural stiffness matrices respectively, β is the control surface angle and w_g is gust term. This expression for the open loop system with control and gust input can be reformulated into the first-order state space form, such that:

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -A^{-1}(\rho V^2 C + E) & -A^{-1}(\rho V B) \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ A^{-1} g \end{bmatrix} \beta + \begin{bmatrix} 0 \\ A^{-1} h \end{bmatrix} w_g \tag{12}$$

The equivalent expression is:

$$\dot{\xi} = A_s \xi + B_s u + h_s w_g \tag{13}$$

Thus the vector of the state variables is $\xi = [k, \theta, \dot{k}, \dot{\theta}]$, the control variable is the control surface angle $u = \beta$. The system is excited by a gust. Two types of gust inputs are used to test the wing: "1-cosine" input and random turbulence input.

The constraints are described in the following lines. First the wing area has to be constant:

$$S = c * s = 15 m^2 \tag{14}$$

Then the eccentricity between flexural axis and aerodynamic center shall be between 0.1 and 1:

$$0.1 < e < 1 \tag{15}$$

Finally the maximum vertical displacement is imposed to be less than 1m:

$$-1 \ m < z_{max} < 1 \ m \tag{16}$$

The plant design variable bounds $\underline{x_d} \leq x_d \leq \overline{x_d}$ used here are:

$$\underline{x_d} = [4, \ 1, \ 70, \ 3, \ 8, \ 0.1, \ 0.1]$$
$$\overline{x_d} = [10, \ 3, \ 130, \ 7, \ 12, \ 0.9, \ 0.9]$$

The system objective function incorporates handling, comfort, and control cost using a Lagrange term:

$$J = \int_0^{t_F} (r_1 z^2 + r_2 \ddot{z}^2 + r_3 u^2) dt \tag{17}$$

where the weights $r_1 = 10^3$, $r_2 = 10^{-2}$, $r_3 = 10^2$ ensure each component of the objective is approximately the same magnitude. As described above, this design problem incorporates two different inputs: "1-cosine" gust and random turbulence. The overall objective function is the summation of the two profiles:

$$J = J_{gust} + J_{turb} \tag{18}$$

where J_{gust} and J_{turb} are the values of the above integral for the gust and turbulence inputs, respectively.

VI. Results

In this paragraph the final analysis results are discussed (time response and objective functions values) and sequential and simultaneous optimization are compared.

The time responses are set out in the next page. First the sequential optimization, than the simultaneous optimization are shown. For both sequential and simultaneous optimization, the pitching and flapping angles (θ, κ) and the control input are illustrated. On the left we can find the "1-cosine" gust input, whereas on the right there is the random turbulence input. If we see the time responses, there are not many differences between the two types of optimization. However, in order to understand the importance of co-design, we need to analyze the objective function because it takes into account all the components (handling, comfort and control cost). The objective function value is equal to 16.48 for the sequential optimization, whereas it is equal to 4.67 for the simultaneous optimization. Therefore there is a gain of 28.33 % in the case of co-design. The final result is the optimal design given by the plant design vector:

$$x_d = [6.85, \ 2.18, \ 122, \ 3.277, \ 9.318, \ 0.5, \ 0.35]$$

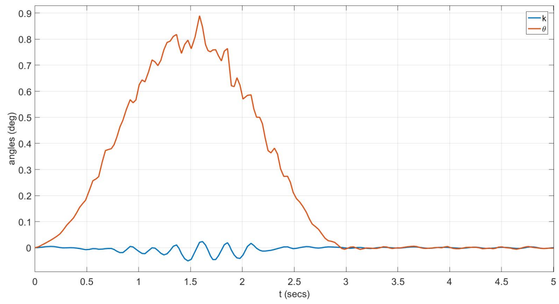

Fig. 8. Flap and Pitch Angles - "1-cosine" input - sequential.

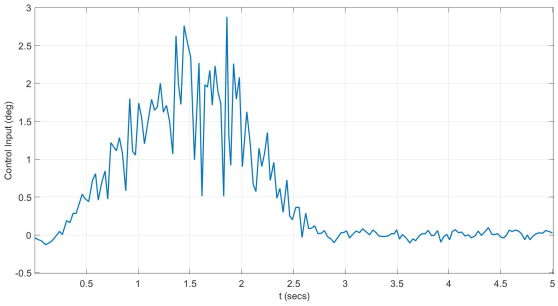

Fig. 9. Control input - "1-cosine" input - sequential.

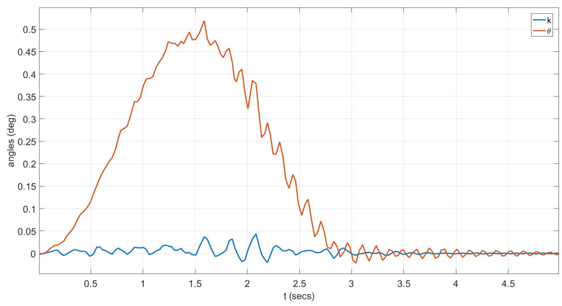

Fig. 10. Flap and Pitch Angles - "1-cosine" input - DT.

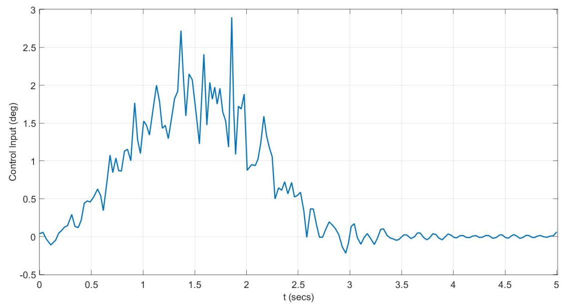

Fig. 11. Control input - "1-cosine" input - DT.

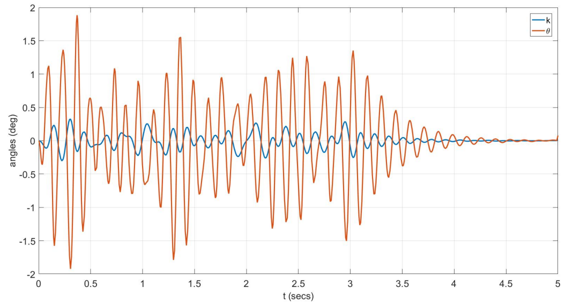

Fig. 12. Flap and Pitch Angles - random turbulence input - sequential.

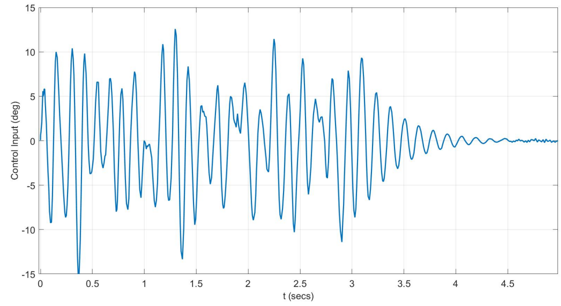

Fig. 13. Control input - random turbulence input - sequential.

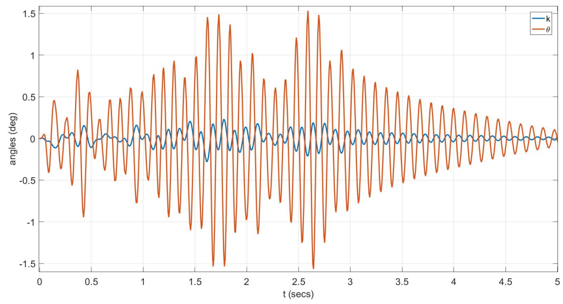

Fig. 14. Flap and Pitch Angles - random turbulence input - DT.

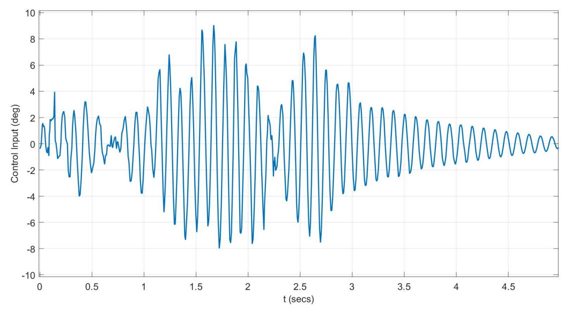

Fig. 15. Control input - random turbulence input - DT.

VII. Conclusions

This article presented an approach for optimizing wing design using a co-design method for robust flutter performance to achieve system optimal solutions. Solution of this problem provides significant insight via exploration of design alternatives that are overlooked when using conventional sequential design. Balanced co-design is essential for producing more meaningful solutions to multidisciplinary optimization problems for dynamic systems. This paper first presented MDO, then introduced DT method. This method is used in the following of the article to solve the wing case of study. The co-design approach produced system-optimal designs that resulted in an objective function improvement of more than 28% compared to sequential design optimization.

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