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Tutorial 2

Hint: You can use the R-function runif to generate realisations from $U(0,1)$.

Exercise 7

Inverse Transform Method - continuous case

a.) Construct an algorithm for generating M realisations of a Weibull random variable, which has cumulative distribution function (cdf)

$$F(x) = 1 - \exp(-\alpha x^{\beta}), \qquad 0 < x < \infty, \quad \alpha, \beta > 0.$$

b.) Use your algorithm to simulate $M = 1000$ realisations of a Weibull random variable with parameters $\alpha = 2$ and $\beta = 4$, and compare the empirical cdf (ecdf) of the realisations with the true cdf.

c.) Use your algorithm to simulate $M = 1000$ realisations of an $Exp(\lambda)$-distributed random variable for $\lambda = 0.2$. Compare the ecdf of the realisations with the true cdf.

Exercise 8

Inverse Transform Method - continuous case

If $X_1, \ldots, X_n$ are independent random variables with $X_i \sim Exp(\lambda)$, then

$$\sum_{i=1}^{n} X_i \sim \Gamma(n, \lambda^{-1}),$$

where Γ denotes the Gamma distribution.

a.) Implement an algorithm that generates M realisations of a $\Gamma(n, \lambda^{-1})$-distributed random variable.

b.) Generate $M = 1000$ realisations with your algorithm and compare the ecdf with the true cdf.

c.) Compare also the empirical density with the true pdf.

Exercise 9

Inverse Transform Method - discrete case

Let X be a random variable defined by

$$X = \max\left\{ n : \sum_{i=1}^{n} \log(U_i) > -3 \right\},$$

where U_i are independent and identically distributed (iid) and $U_i \sim U(0,1)$ for $i = 1, \ldots, n$.

a.) Write a function `simx(M)` for simulating a vector consisting of M realisations of the random variable X.

b.) Estimate $\mathbb{E}[X], \mathrm{Var}(X)$ and $\mathbb{P}(X = i)$ based on $M = 10^4$ realisations of X.

c.) Convince yourself that the generated variates come from a Poisson distribution with parameter $\lambda = 3$, and that all estimated values agree with the true ones.

Exercise 10

Rejection Sampler

Consider a distribution having probability density function (pdf)

$$f(x) = \frac{e^x}{e-1}, \qquad 0 \leq x \leq 1.$$

a.) Implement the rejection sampler algorithm to generate M realisations of that distribution.

b.) Use your algorithm to generate $M = 10^5$ realisations. Use them to estimate the mean and the variance, and compare the obtained results with the theoretical ones.

Exercise 11

Acceptance-Rejection method

Construct an algorithm based on the acceptance-rejection method for generating 10000 realisations from a distribution having pdf

$$f(x) = 20x(1-x)^3, \quad 0 < x < 1.$$

Hint: Choose Y (having pdf g) to be $U(0,1)$-distributed.

a.) Compute the acceptance probability by simulation, and compare it with the true value.

b.) Compare the empirical density with the true pdf f.

Exercise 12

Box-Muller algorithm

a.) Implement the Box-Muller algorithm to generate $N = 2 \cdot n$ realisations of the standard normal distribution, and consider $n = 10^5$.

b.) Compare the empirical density of the $N = 2 \cdot n = 2 \cdot 10^5$ generated values with the true pdf of the standard normal distribution.

c.) Perform a Kolmogorov-Smirnov test to check for normality by using the R function `ks.test`.

Exercise 13

Polar algorithm

a.) Implement the polar algorithm to generate realisations of the standard normal distribution.

b.) Compare the empirical density of the generated values with the true pdf of the standard normal distribution.

c.) Perform a Kolmogorov-Smirnov test to check for normality by using the R function `ks.test`.

d.) Calculate the number of generated realisations from the standard normal distribution obtained from the $2 \cdot n = 2 \cdot 10^5$ realisations of $U(0,1)$.