EXERCISE - STATISTICS FOR AI
Summer Semester 2025 (Mag. Thomas Forstner)

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56. A sample space Ω_1 is defined as $\Omega_1 = \{1, 2, 3\}$, with $P(\{1\}) = 1/2$, $P(\{2\}) = 1/6$, and $P(\{3\}) = 1/3$.
    - What is $P(\{1, 2\})$? 0.667 What is $P(\{1, 2, 3\})$? 1
    - List all events A such that $P(A) = 1/2$. {1} and {2,3}

57. Another sample space Ω_2 is defined as $\Omega_2 = \{1, 2, 3\}$, with $P(\{1\}) = 1/2$ and $P(\{1, 2\}) = 2/3$. What must $P(\{2\})$ be? 0.167

58. Three events are defined as: $S = \{x \mid 0 < x < 12\}$, $M = \{x \mid 1 < x < 9\}$, $N = \{x \mid 0 < x < 5\}$. Define:
    - $M \cup N$: $\{x \mid 0 < x < 9\}$
    - $M \cap N$: $\{x \mid 1 < x < 5\}$

59. A basketball team has 5 players. All these players leave the team's locker room at the start of the game.

    How many different orders are possible for leaving the team's locker room? 120

60. A student has to answer at least 6 out of 12 questions in an exam correctly.
    a) How many possibilities does she have to pass the exam? 2510
    b) How many possibilities are there, if she still has to answer at least 6 out of 12 questions in total correctly and also, the first 5 questions correctly? 127
    c) How many possibilities are there, if she still has to answer at least 6 out of 12 questions in total correctly and also, at least 4 of the first 5 questions correctly? 727

61. One bag contains 5 white balls and 2 black balls, and a second bag contains 4 white balls and 6 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black? 0.571

62. A ship carries some colored signal flags (2 yellow, 2 red, 2 blue, 2 green, 2 white, 2 black) for the purpose of signaling to other ships. Exactly two raised flags make a signal. The order of the flags does not play a role, so that the signals can be understood from any direction. How many different signals can be signaled by this ship? 21

63. Three cards are drawn in succession, without replacement, from an ordinary deck of playing cards (definition see lecture slides 5).

    Find the probability that the event $A1 \cap A2 \cap A3$ occurs, where A1 is the event that the first card is a red ("hearts or diamonds") ace, A2 is the event that the second card is a 10 or a jack, and A3 is the event that the third card is greater than 3 but less than 8. 0.193%

64. From the numbers 1 to 9, two are randomly selected without replacement. What is the probability that both numbers are even, when you know that their sum is even? 37.5%

65. A bicycle lock has a four-digit combination, with each digit ranging from 0 to 9. Unfortunately, the owner has forgotten the right combination for the lock.

    What is the **probability** that the lock can be opened on the first attempt when he …

    a) no longer remembers any of the digits of the combination? 0.01%
    b) remembers that there is exactly one "6" among these digits? 0.034%
    c) remembers that the "6" is the first digit and that there is exactly one "6"? 0.137%

66. A club with 20 members has 3 members with the first name "John". Now, a random delegation of exactly 5 persons is selected.

    a) How likely is it that exactly one "John" is in this delegation? 46.053%
    b) How likely is it that no "John" is in this delegation? 39.912%
    c) How likely is it that at least one "John" is in this delegation? 60.088%

67. A small company has 12 female employees and 10 male employees. An employee organization consisting of 5 women and 3 men should be selected. The 2 male sales representatives working in the field should not be selected into the employee organization, but the female employee from the financial accounting department should be selected in any case.

    How many ways are there to form an employee organization? 18480

68. Someone has found a phone, and this person knows that this type of phone has a 5-digit passcode that contains the numbers 0 through 9 in a specific order.

    a) How many different passcodes are possible? 100000
    b) What if there are 5 smudges over 5 digits on the screen and these smudges indicate the numbers used in this 5-digit passcode. How many different passcodes are possible now? 120

69. We consider only two biological sexes (boy, girl) and assume that the probability of having a girl or a boy is the same and that the order of birth is relevant.

    a) How many equally likely biological sex combinations are possible, in the case of a family with two children? 4
    b) Calculate the probability that at least one of the two children is a girl? 75%
    c) Calculate the probability that the two children have the same biological sex? 50%

70. The probability that a regularly scheduled train departs on time is P(D) = 0.75. The probability that it arrives on time is P(A) = 0.85, and the probability that it departs and arrives on time is $P(D \cap A) = 0.65$. Find the probability that a train …

    - arrives on time, given that it departed on time. 86.667%
    - departed on time, given that it has arrived on time. 76.47%

71. A survey of students found that 75% of all students are regularly eating in the cafeteria, and that 35% of all students want longer opening hours of the cafeteria. 30% of all students are regularly eating in the cafeteria and also want longer opening hours.

    How likely is it that a student who wants longer opening hours of the cafeteria eats in the cafeteria regularly? 85.714%

72. A survey of employees found that 60% of all employees regularly use the company gym, and that 40% of all employees want more gym equipment. 25% of all employees regularly use the company gym and also want more gym equipment.

    What is the probability that an employee who does not use the company gym regularly wants more gym equipment? 37.5%

73. The octal system uses exactly 8 digits: 0, 1, 2, 3, 4, 5, 6, and 7. Each octal number contains only these digits.

    How many 5-digit octal numbers are there that start with the digit 4? 4096

74. The hexadecimal system uses exactly the 16 “digits” 0, 1, 2, ... , 9, A, B, C, D, E, F. Each hexadecimal number only contains these 16 digits.

    How many 6-digit hexadecimal numbers are there that begin with the digit A and contain neither the digit 4 nor the digit 2? 537824

75. Consider passing a lecture exam and the corresponding exercise exam as related events. The probability that a student will pass the lecture exam is 60%, and the probability that a student will pass the exercise exam, knowing that he has already passed the lecture exam, is 70%.

    What is the joint probability that a student passes both exams? 42%

76. At the end of the semester, 40% of the students of a specific study program did not pass physics, 30% did not pass biology, and 20% did not pass both.

    A student is randomly selected. What is the probability that this student did not pass physics or biology? 50%

Please keep the formal guidelines for submitting the homework assignments in mind to avoid losing points unnecessarily.