Linz, Theoretische Physik, WS 2025

Due date: Oct. 13, 2025

Polynomial interpolation:

The following 2 functions should be interpolated with a polynomials $p(x)$ of degree n on the interval $[-1, 1]$:

$$f(x) = \sin(\pi x)$$

$$g(x) = \frac{1}{1 + (3x)^2}$$

For this purpose, you can use the Lagrange polynomials from the lecture,

$$L_k(x) = \prod_{j(\neq k)}^{n} \frac{x - x_j}{x_k - x_j}$$

3. Choose an equidistant grid:
$$x_j = \frac{2j}{n} - 1 \quad j = 0, \ldots, n$$

    Try degree $n = 14$, and possibly a few other degrees.

    (a) Plot $L_k(x)$ as function of x for a few k's.

    (b) Compare the 2 functions above with their respective interpolating polynomial $p(x)$.

4. Repeat your calculations with a different grid:
$$x_j = \cos\left(\frac{\pi(j + 1/2)}{n + 1}\right), \quad j = 0, \ldots, n$$

    Try degree $n = 14$, and possibly a few other degrees.

    (a) Plot $L_k(x)$ as function of x for a few k's.

    (b) Compare the 2 functions above with their respective interpolating polynomial $p(x)$.