EXERCISE - STATISTICS FOR AI
Summer Semester 2025 (Mag. Thomas Forstner)

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77. At the end of the semester, 35% of the students in a study program fail mathematics, 25% fail chemistry, and 20% fail both mathematics and chemistry. Now, a student is chosen at random.

    What is the probability that this student failed mathematics, given that he already failed chemistry? 80%

78. The Hotel Statistic Inn has a fire alarm system which gives an alarm with a probability of 99.4% in the event of a fire outbreak. Occasionally, the system gives false alarms. According to the night porter, this happens five times a year (corresponds to a probability of 5/365). The probability of a fire breaking out on a certain night is 0.02%.

    Someone spends a night at the Statistic Inn and hears the fire alarm. What is the probability of a fire now? 1.431%

79. A TV network wants to produce a new TV show, “CSI Statistics”. A pilot episode was produced and shown to some people. Of the viewers of this pilot episode, 65% were over 20 years old. 60% of the viewers over 20 years and 50% of the rest liked the pilot episode.

    A viewer of the pilot episode “CSI Statistics”, who liked the show, is randomly selected. What are the chances that he is over 20 years old? 69.027%

80. Joe is randomly selected for a drug test. On average, 8% of all tested people are drug users. The drug test used correctly identifies drug users 96% of the time and correctly identifies non-drug users 93% of the time.

    Assuming that Joe has tested positive for drugs, calculate the probability that Joe really has consumed drugs. 54.391%

81. A spam filter works by having users train the filter. It looks for patterns in mails marked as spam by the user. For example, it may have learned that the word “free” appears in 30% of the mails marked as spam. Assuming 4% of non-spam mails include the word “free” and 45% of all mails received by the user are spam.

    Calculate the probability that a mail is spam, if it contains the word “free”. 85.987%

82. A pregnancy test should be positive with a probability of 95%, if the woman is pregnant (“test indicates pregnancy”). If the woman is not pregnant, the test should be negative with a probability of 96% (“test does not indicate pregnancy”). 25% of the women who take this pregnancy test are actually pregnant.

    What is the probability that the woman is pregnant, if the test is negative? 1.706%

83. A new screening test for the early detection of a specific metabolic disease in infants has been developed. This screening test does not detect the present metabolic disease in 0.03% of the cases. In 0.1% of the cases, it mistakenly detects the metabolic disease. Furthermore, it's known that on average, 100 infants out of 1,1 million infants have this metabolic disease.

What is the probability that an infant, which was diagnosed as sick, has this metabolic disease? 8.331%

84. A box contains 3 red balls and 5 blue balls. Balls are randomly drawn without replacement until a red ball is drawn. The random variable X describes the number of draws needed for a red ball (including the draw of the red ball).

State - in tabular form - the probability distribution function in the box below.

Extrations	Probability
1	0.375
2	0.267
3	0.178
4	0.107
5	0.053
6	0.017

85. Someone wants to send a Christmas card by mail to 10 friends. 2 of these friends live in Vienna. Unfortunately, the person only has only 4 stamps, so he decides to randomly select 4 out of the 10 friends and only these 4 selected friends will get a Christmas card. Let X be the number of friends who live in Vienna among the 4 selected friends.

Calculate the expected value and the variance of X. 0.8 ; 0.427

86. A continuous random variable X has the density:

$$f(x) = \begin{cases} (1 - \frac{x}{10}) \cdot k & 0 \le x \le 10 \\ 0 & else \end{cases}$$

a) Calculate the constant k so that the probability density function is valid. **k =** 0.2

b) State the cumulative distribution function. x/5 - x^2 / 100 for 0<= x <= 10

c) Calculate: P(X > 8) and P(|X - 5| ≤ 3).). 0.04 ; 0.60

d) Calculate the expected value and the variance of X. 3.333 ; 5.556

e) Below which value are 30% of the distribution mass? 1.633

87. Suppose that the error in the reaction temperature, in degree Celsius, for a controlled laboratory experiment is a continuous random variable X having the probability density function below:

$$f(x)=\begin{cases}\dfrac{x^2}{3} & -1<x<2\\ 0 & else\end{cases}$$

a) Calculate: $P(X < 0.5)$ and $P(0.5 < X < 1.1)$. 0.125 ; 0.134

b) Calculate the expected value and the variance of X. 1.25 ; 0.638

88. The joint density function for the random variables X and is defined as:

$$f_{xy}(x,y)=\begin{cases}2 & 0<x<y<1\\ 0 & else\end{cases}$$

a) Calculate the marginal density function $f_x(x)$ and $f_y(y)$. 2-2x ; 2y

b) Calculate the probability: $P(1/4 < X < 1/2 \mid Y = 3/4)$. 0.333

89. The joint density function for the random variables X and Y is defined as:

$$f_{xy}(x,y)=\begin{cases}4x^2y+2y^5 & 0\le x\le 1,\ 0\le y\le 1\\ 0 & else\end{cases}$$

a) Calculate the expected value of $f_x(x)$, $f_y(y)$, and $f_{xy}(xy)$**.** 0.667 ; 0.730 ; 0.476

b) Calculate the covariance of these random variables. -0.011

c) Calculate the correlation coefficient of these random variables. -0.183

Hint: The idea of the Bravais-Pearson correlation coefficient “COV(X,Y)/(SD(X)*SD(Y))” can also be applied to random variables.

90. The random variable X is defined as the sum of the numbers on 3 cards, which are drawn simultaneously from a deck of 5 cards numbered from 1 to 5.

State - in tabular form - the probability distribution function in the box below.

Sum	Frequency	Prob
6	1	0.1
7	1	0.1
8	2	0.2
9	2	0.2
10	2	0.2
11	1	0.1
12	1	0.1

Please keep the formal guidelines for submitting the homework assignments in mind to avoid losing points unnecessarily.