Lecturer: Dr. Amira Meddah
Email: amira.meddah@jku.at

Tutorial 1

Exercise 1

Linear congruential generator

Consider the *multiplicative congruential generator* with modulus $m = 31$, multiplier $a = 7$ and seed $x_0 = 19$.

a. Compute (by hand) the resulting sequence of pseudo-random numbers.

b. What is the period of the sequence?

c. Why does it not have maximum period $m - 1 = 30$?

d. Find a value for a such that the generator has maximum period.

Exercise 2

Linear congruential generator

a. Implement the *linear congruential generator*, i.e., write a function `lcg(a,c,m,x0)`, which returns a sequence $(x_1, \ldots, x_p)$ of p (period) integer numbers. Recall: a is the multiplier, c the increment, m the modulus and x_0 the seed.

b. Run the function `lcg(a,c,m,x0)`, using the values from Exercise 1, i.e., $a = 7$, $c = 0$, $m = 31$ and $x_0 = 19$.

Exercise 3

Linear congruential generator

Consider the *linear congruential generator* from Exercise 2, in particular

```
(A) lcg(17,0,2^13-1,1)
(B) lcg(29,0,2^13-1,1)
(C) lcg(197,0,2^13-1,1)
```

a. What is the period of each generated sequence?

b. Plot all pairs of consecutive numbers and consider the auto-correlation for each of the three cases.

c. What is the best choice of multiplier a between (A), (B) and (C)?

Exercise 4

Kolmogorov-Smirnov test

a. Write a function `Fks(x)` to compute the Kolmogorov cumulative distribution function, i.e. $\mathrm{Fks}(x) = \mathbb{P}(K \leq x)$, where K is the Kolmogorov random variable. Use `Vectorize(Fks)(x)` to apply the function to a vector x.

   Hint: Define $\mathrm{Fks}(0) = 0$ *and truncate the series of the Kolmogorov distribution.*

b. Define a function `Kalpha(alpha)` having a number $\alpha \in (0, 1)$ as input and returning a value K_α such that

$$\mathbb{P}(K \leq K_\alpha) = 1 - \alpha.$$

   Hint: Use `uniroot` *or* `optim`.

c. Write a function `Dno(A)` returning the Kolmogorov distance for a vector A.

d. Write a function `pval(A)` returning the p-value for the Kolmogorov-Smirnov test.

e. Write a function `ourkstest(A,alpha)` performing a Kolmogorov-Smirnov test at a α confidence level, using a dataset A. In particular, the function should return both the p-value and the Kolmogorov distance for A.

f. Run the following code

```
set.seed(11)
Asim<-runif(50)
```

   and perform Kolmogorov Smirnov tests with $\alpha = 0.01$ and $\alpha = 0.05$, using `ourkstest`. Formulate conclusions about the tests. Compare your results with those coming from the R-function `ks.test`.

Exercise 5

Quadratic congruential generator

Implement the *quadratic congruential generator*, i.e., write a function `qcg(d,a,c,m,x0)`, which returns a sequence $(x_1, \ldots, x_p)$ of p (period) integer numbers.

Exercise 6

Consider `qcg(d,a,c,m,x0)` from Exercise 5.

a. We fix the parameters $c = 1, x_0 = 1$ and $m = 65536$. Choose d and a such that the generator has full period p, and run `qcg(d,a,1,65536,1)` with the chosen parameter d and a.

b. Perform a test for uniformity on the corresponding generated sequence $(u_1, \ldots, u_p)$.

c. Plot 1000 pairs of corresponding consecutive numbers u_{i-1} and u_i.