

Due date: Oct. 27, 2025

5. Show that the derivative of a function $f(x)$ can be calculated with the help of Fouriertransformation as follows:

$$f'(x) = \text{FT}^{-1}[i q F(q)]$$

where FT^{-1} is the inverse Fouriertransformation.

6. *Numerical Differentiation:*

Calculate the first derivative $f'(x)$ of the Gaussian function $f(x) = e^{-x^2/(2\sigma^2)}$.

- (a) Calculate $f'(x)$ using the forward or backward difference. Try different step sizes h and compare with the analytical result. Confirm that the error of the numerical differentiation scales as shown in the lecture.
- (b) Calculate $f'(x)$ using the central difference. Try again different step sizes h and compare with the analytical result. Confirm again that the error of the numerical differentiation scales as shown in the lecture.
- (c) Use your program from problem 4 to calculate $f'(x)$ via Fourier transformation – alternatively you can use the FFT algorithm for DFT via a library of your choice (the best known is the “FFTW” library). How does the result depend on step size and interval size? Compare the results for $f'(x)$ with those obtained in (a) and (b) (plot the error).