

EXERCISE - STATISTICS FOR AI
Summer Semester 2025 (Mag. Thomas Forstner)

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108. The weight of a chocolate bar (in grams) is distributed normally with a mean of $\mu = 50$ and a variance of $\sigma^2 = 4$.
- a) How likely is it that such a chocolate bar weighs less than 52 g. 0.8413
 - b) How likely is it that such a chocolate bar weighs more than 52 g. 0.1587
 - c) How likely is it that such a chocolate bar weighs exactly 52 g. 0
109. The lifetime of a certain type of smartphone battery is normally distributed with a mean of $\mu = 500$ charge cycles and a variance of $\sigma^2 = 25$ charge cycles².
- a) Calculate the probability that a randomly selected battery will last for less than 490 charge cycles. 0.023
 - b) Calculate the probability that a randomly selected battery will last for more than 490 charge cycles. 0.977
 - c) Calculate the probability that a randomly selected battery will last between 490 and 510 charge cycles. 0.954
110. A small gas station monitors the daily fuel demand (in liters) for its customers. The daily fuel demand is modeled as a normally distributed random variable X . This random variable is defined as $X \sim N(1000, 10000)$.
- How much fuel should the gas station stock to ensure a 99% chance that the fuel will be sufficient for tomorrow, assuming the gas station has run out of fuel today? 1232.63 l
111. The weight of a certain species of elephants is normally distributed. The standard deviation of the weight of this species is known to be 300 kg. It is also known that 50% of these elephants weigh less than 5000 kg.
- a) Calculate the expected value of the weight. 5000 kg
 - b) Which weight is exceeded with a probability of 97.5%? 4412 kg
 - c) Calculate the limits of an interval, which covers the central 95% of the distribution. [4412, 5588] kg
- Hint: The "central 95%" of a distribution are distributed symmetrical around the expected value.
112. A smartphone manufacturer claims that the battery life, measured as the number of charge cycles without significant degradation, is normally distributed with a mean of $\mu = 500$ charge cycles. 5% of all batteries degrade after 296 charge cycles or fewer.
- a) What is the standard deviation of this normal distribution? 124.012
 - b) Calculate the percentage of batteries that last for 700 charge cycles or more. 5.341%

113. It is known, that the points on the verbal part of a Standard Achievement Test (SAT) are normally distributed with a mean of 466 and a variance of 6360. It is also known, that the points on the math part of this Standard Achievement Test (SAT) are also normally distributed with a mean of 520 and a variance of 5730. Now two students are selected at random. What is the probability that the first student's math score is higher than the second student's verbal score? 68.833%
114. On average, 5% of all passengers who reserved seats do not show up for their booked flight. The airline Econair knows this and reserves 230 flight tickets for 220 available seats. What are the chances that all passengers who actually come to a flight get a seat?
- It can be assumed that the passengers make their decisions independently of each other.
- Calculate the exact probability. 0.718
 - Calculate an appropriate asymptotic solution using the Poisson distribution. 0.711
 - Calculate an appropriate asymptotic solution using the normal distribution. 0.727
115. A ferry operator knows that the weight of vehicles, transported with the ferry is normally distributed with $\mu = 1500$ kg and $\sigma^2 = 22500$ kg². The ferry can carry 40 vehicles.
- Calculate the expected value and variance for the total weight of the vehicles on a fully loaded ferry. 60000 kg ; 900000 kg²
 - Calculate the probability that the total weight of the vehicles on a fully loaded ferry is greater than 62000 kg. 0.0175
116. The weight of coffee bags is normally distributed with $\mu = 246$ g. 10% of these bags weigh less than 237 g. Calculate the probability that ...
- one bag weighs more than 255 g. 0.10
 - under 10 bags at least 9 bags weigh less than 255 g. 0.736
117. Due to random errors in the manufacturing process, the actual weight of a chocolate bar often differs slightly from the target weight. This difference can be described by a normal distribution. The weight of a randomly selected 100-gram chocolate bar is normally distributed with a mean of 102 grams and a standard deviation of 2 grams. Chocolate is also sold in 300-gram packs, which are normally distributed with a mean of 305 grams and a standard deviation of 4 grams.
- Somebody has bought three 100-gram chocolate bars. Calculate the expected value and variance of the summed-up weight of these three 100-gram chocolate bars.
306 g ; 12 g²
 - Selecting chocolate bars at random, what is the probability that the sum of three 100-gram chocolate bars exceeds the weight of one 300-gram pack?
0.575

118. A bakery states that the biscuits it produces have an average weight of 50 grams. A statistics student who is suspicious buys $n = 9$ biscuits and weighs all the biscuits. He receives the following biscuit weights in grams:

49 46 49 48 49 50 49 51 50

- Calculate an unbiased and consistent estimator for the true mean and the true variance based on the given sample data. 49 g ; 2 g^2
 - Calculate a 90% confidence interval for the true mean weight of the biscuits based on the given sample data. [48.123, 49.878] g
119. Someone owns a sawmill. He gets 1943 logs delivered. He wants to estimate the average circumference of the logs, with a reliability of $1 - \alpha = 95\%$. The circumference of the logs is normally distributed. He now draws a random sample out of the 1943 delivered logs and measures the circumference x of each log in millimeters:
- 1111 1150 1200 1435 1130 1050 1302 1451
- Calculate a 95% confidence interval for the true mean circumference of the logs based on the given sample data. [1102.266, 1354.984] mm
 - The sawmill owner now wants to determine a confidence interval for the true mean circumference of the logs, which is 80 mm wide and still has the same confidence of 95%. Calculate the approximately needed sample size for this confidence interval. 55
120. On behalf of the owner of a winery, the true average bottled quantity of wine, which is bottled in 750 ml wine bottles, should be estimated based on a 99% confidence interval. The filling quantity X is regarded as normally distributed. Ten bottles are randomly selected, and the filling quantity of these bottles is checked.
- Quantity of wine [ml] of the sample bottles:
- 743 759 748 745 746 751 748 762 759 751
- Calculate a 99% confidence interval based on the sample data. [744.421, 757.989] ml
 - Which confidence level has to be chosen, so that with a sample of 50 bottles a confidence interval with a length of at most 1 milliliter is achieved? An approximate solution is sufficient. 0.41
121. A political candidate finds that in a random sample of 600 people, 240 people support her party. Calculate a 95% confidence interval for the support she has. [0.361, 0.439]
122. A survey of a sample of $n = 1700$ people reveals a share of 30% party X voters. From this, it is deduced that the unknown proportion of party X voters in the population is covered by a confidence interval from 29% to 31%. What is the approximate probability that the true proportion is covered by this confidence interval? 0.632

Please keep the formal guidelines for submitting the homework assignments in mind to avoid losing points unnecessarily.