4. Exercise for Computational Physics I

Linz, Theoretische Physik, WS 2025

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7. Finding real roots numerically:
   From the lecture you know two methods, bisection and Newton-Raphson, to calculating the roots $\bar{x}$ of a function $f(x)$, i.e. solving $f(x) = 0$. Both methods iteratively produce a series $x_n$ that should converged to the exact root $\bar{x}$.

   Use and compare both methods for the following functions:

   (a) (1 point) Determine the root(s) of $f(x) = x^2 - 1$.

   (b) (1 point) Determine the root(s) of $f(x) = (x - 1)^2$.

   (c) (1 point) Determine the root(s) of $f(x) = (x - 1)^3$.

   For all cases plot the error $|x_n - \bar{x}|$ as function of iteration step $n$.

8. Finding complex roots numerically:
   The equation $z^3 - 1 = 0$ has the three roots $\bar{z} = 1, e^{2\pi i/3}, e^{4\pi i/3}$, which we want to calculate numerically. Our bisection method only works in one dimension, but $\mathbb{C}$ is effectively two-dimensional. Therefore we use the Newton-Raphson method, leading to the following complex iteration

$$z_{n+1} = z_n - \frac{z_n^3 - 1}{3z_n^2}$$

   (a) (1 point) Calculate the 3 complex roots starting with appropriate initial values $z_0$.

   (b) (1 point) Study systematically how the initial value $z_0$ influences which of the 3 roots are found, i.e. what's the shape of the three "basins of convergence"?
   Proceed as follows: fill an area, e.g. the square $[-1, 1] \times [-i, i]$, with a grid of initial values $z_0$ and see which root the iteration converges to (if at all). For visualization, you can plot the results by coloring the initial point $z_0$ depending on which root it finds. If you use a dense grid of initial values $z_0$, a strange shape appears...