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Tutorial 3

Exercise 14

Random Walk

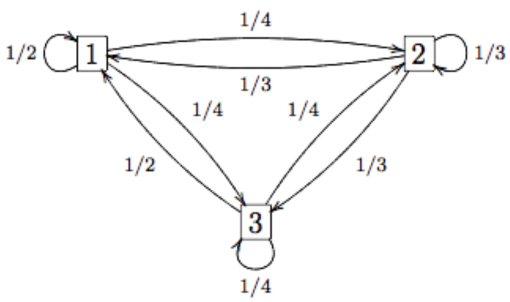
a.) Implement the algorithm of Week 5, use it to generate a path of the random walk for $p = 1/2$ and $N = 8$, and plot your generated path.

b.) Let now $n = 8$ be fixed. Generate 10^5 realizations of the random variable X_n using $p = 1/2$. Convince yourself that the distribution of X_n can be described with a binomial distribution (see Week 5) by comparing the histogram obtained from the simulated values with the true pmf (probability mass function) obtained from the binomial distribution.

Exercise 15

Three-state Markov chain

Consider a Markov chain given by the transition diagram



Assume that $X_0 = 1$ and let

$$\tau = \inf\{n > 0 | X_n \neq 1\}$$

be the time of the first jump away from state 1.

a.) Determine the transition matrix P.

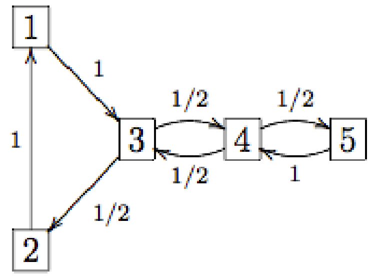
b.) Find $\mathbb{P}(\tau = 1)$, $\mathbb{P}(\tau = 2)$ and $\mathbb{P}(\tau = 3)$. What is the name of the distribution of τ?

Remark: This is a written exercise. Please send me your solution as a pdf.

Exercise 16

Five-state Markov chain

Consider the Markov chain with initial value $X_0 = 1$ and state space $S = \{1, 2, 3, 4, 5\}$ given by the transition diagram



a.) Find the transition matrix P and a proper initial vector α.

b.) Implement the algorithm of Week 5, use it to generate a path of the Markov chain for $N = 20$, and plot your generated path.

Exercise 17

Five-state Markov chain

Consider again the Markov chain of the previous exercise with initial value $X_0 = i \in \{1, \dots, 5\}$.

a.) Implement a function `MC5stateProbability(m,j,i,transMat)` that computes $p_{ij}^m = \mathbb{P}(X_m = j | X_0 = i)$ with transition matrix `P=transMat` and compute the probability $\mathbb{P}(X_4 = 3 | X_0 = 3)$.

b.) Argue that there may exist a stationary distribution of the above Markov chain by considering P^l, $l = 10, 100, 1000$. Compute the stationary distribution (by hand).

Remark: The last part is a written exercise. Please send me your solution as a pdf.

Exercise 18

Homogeneous Poisson process I

a.) Implement an algorithm to generate a path of the homogeneous Poisson process $(N_t)_{t\in[0,T]}$ with rate λ.

b.) Plot a path for $\lambda = 3$ and $T = 10$.

Exercise 19

Homogeneous Poisson process II

Continuation of Exercise 18. Choose again $\lambda = 3$.

c.) Write an algorithm to obtain $M = 10^4$ realizations of the random variable N_2. Compare the empirical pmf of the simulations with the true pmf of N_2.

d.) Write an algorithm to obtain $M = 10^4$ realizations of the jumping time T_4. Compare the empirical pdf (probability density function) of the simulations with the true pdf of T_4.

Exercise 20

Non-homogeneous Poisson process

a.) Implement an algorithm to generate a path of the non-homogeneous Poisson process $(N_t)_{t\in[0,T]}$ with rate function $\lambda(t) = 5\sin^2(t/2)$.

b.) Plot a path for $T = 30$.

c.) Produce $M = 10^4$ realizations of the random variable N_5. Compare the empirical pmf of the simulations with the true pmf of N_5.