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Gearbox Design by means of Genetic Algorithm and CAD/CAE Methodologies

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ABSTRACT

The paper discusses a gearbox design method based on an optimization algorithm coupled to a full integrated tool to draw 3D virtual models, in order to verify both functionality and design. The aim of this activity is to explain how the state of the art of the gear design may be implemented through an optimization software for the geometrical parameters selection of helical gears of a manual transmission, starting from torque and speed time-histories, the required set of gear ratios and the material properties. This approach can be useful in order to use either the experimental acquisitions or the simulation results to verify or design all of the single gear pairs that compose a gearbox. Genetic algorithm methods are applied to solve the optimization problems of gears design, due to their capabilities of managing objective functions discontinuous, non-differentiable, stochastic, or highly non-linear. The final design tool, implemented in Matlab® environment, is based on the calculation of load capacity of helical gears, including the computation of tooth bending strength, of surface durability (pitting) and the estimation of service life under variable load, as suggested by International Standards. An automated macro procedure for Solidworks® interacts with the Matlab® environment to get the dimensional parameters of each gear and produces the models of each gear and their assembly.

INTRODUCTION

The design of gearboxes composed of several gear pairs sharing the same interaxis, i.e. the distance between the primary and the secondary shafts, can be handled by means of methods based on genetic algorithms coupled to full integrated tools to draw 3D virtual models, in order to verify both functionality and design.

The aim of this paper is to describe the optimization approach defined for evaluating the geometrical parameters related to the helical gears of a manual transmission, starting from torque and speed time-histories (representative of the typical operating conditions), the required set of gear ratios and the material properties. This approach can be useful in order to use either the experimental acquisitions or the simulation results to verify or design all of the single gear pairs that compose a gearbox.

ISO 6336 provides a coherent system of procedures for the calculation of the load capacity utilizing various influence factors of cylindrical involute gears with external and internal teeth. The formulas in ISO 6336 are intended to establish a uniformly acceptable method for calculating pitting resistance and bending strength capacity of cylindrical gears with straight or helical involute teeth. The procedures are based on testing and theoretical studies; the results of rating calculations made by following this method are in good agreement with previously accepted gear calculation methods ([1],2,[3]) for normal working pressure angles up to 25° and reference helix angles up to 25°. The influence factors presented in ISO 6336 are derived from results of research activities and field service and can be determined by various methods. In the international standard three methods characterized by an increasing accuracy and complexity are presented. The method B factors are derived with sufficient accuracy for most applications. ISO 6336 is primarily intended for verifying the load capacity of gears for which essential calculation data are available by way of part drawings, or in similar form. Unfortunately, the data available at the primary design stage is usually restricted. It is therefore necessary, at this stage, to make use of approximations or empirical values for some factors.

The design of several gear pairs sharing the same interaxis (the distance between the primary and the secondary shafts)

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and characterized by a given set of gear ratios, can be formalized in a certain number of minimization problems, each bounded (e.g. minimum and maximum helix angle), with both linear and non linear constrains, involving the contemporaneous interaxis, face widths and stress minimization. The genetic algorithm method, which is based on natural selection, is particularly fitted for that task because it can be applied to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly non-linear. By utilizing this approach it is possible to verify or design all of the single gear pairs that compose a gearbox using either the experimental acquisitions or the simulation results.

Genetic algorithm approaches were successfully used by Gologlu [15] and Yokota [16] minimizing the gear volume or weight, by Rao [18] to optimize epicyclic gear trains. Abersek [17] defined procedures to design and manufacture gears. Dantan [19] focused the problem of tolerance synthesis and production cost.

The aim of the CAD procedures and algorithms concerning gear design problems is mostly focused on gear pairs of extreme geometric complexity, like in Kaftanoglu [20], Jehng [21], Su [22] and Tsai [23]. No attempt regarding parametric and automatic gearbox assembly design by means of CAD system were found in the technical literature.

The approach of this paper is to combine an efficient genetic algorithm with the reliable design procedures of ISO6336 and a 3D parametric CAD software in a single effective tool, in order to optimize a design configuration and provide a complete 3D gearbox assembly advanced design.

The final design tool, implemented in Matlab® environment, is based on the calculation of load capacity of helical gears, including the computation of tooth bending strength, of surface durability (pitting) and the estimation of service life under variable load, as suggested by International Standards.

Starting from the geometrical fundamental parameters, an automated macro procedure for Solidworks® interacting with the Matlab® environment is presented in the following part, in order to get the dimensional parameters of each gear and to create the corresponding models and their assembly.

GEOMETRICAL PARAMETERS FOR HELICAL GEARS PROFILE

The main geometrical parameters for a helical gears profile is presented in Figure 1. For a cylindrical involute gears with external teeth, the parametric equation is defined by means of progressive rotations, in order to take into account the polar involute formula (profile I of Figure 1b)

$$\begin{cases} \mathrm{r} = \dfrac{\mathrm{rb}}{\cos\theta} \\ \varphi = \tan\theta - \theta \end{cases} , \tag{1}$$

the involute profile crossing the pitch point (profile II of Figure 1b, angle ψ), the involute profile placed symmetrically with respect to a tooth space (profile III of Figure 1b, angle σ) and the involute profile shifted for long and short-addendum teeth (profile IV of Figure 1b).

The overall rotation results:

$$\zeta = \psi - \sigma - \frac{360}{2\pi}\left(\frac{x\,\mathrm{m_n}\tan\alpha_t}{\mathrm{R}}\right) \tag{2}$$

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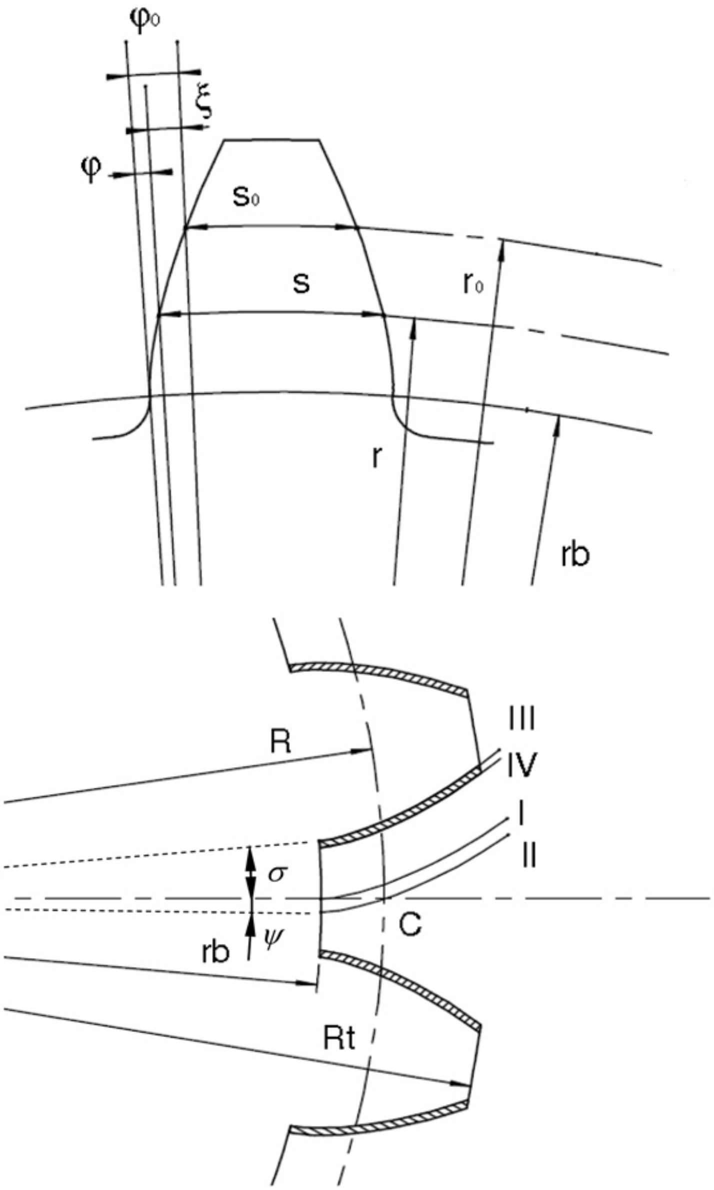

Figure 1. Involute profile parameters (1a, above) and progressive rotations (1b, below).

In Figure 2 an example of two helical gears is shown. The functional model is obtained through the modification of parametric 3D parts driven by an automated macro procedure developed in Solidworks® with API (Application Programming Interface) software [10]. Appling this procedure for each gear pairs, this tool provides a complete 3D gearbox assembly.

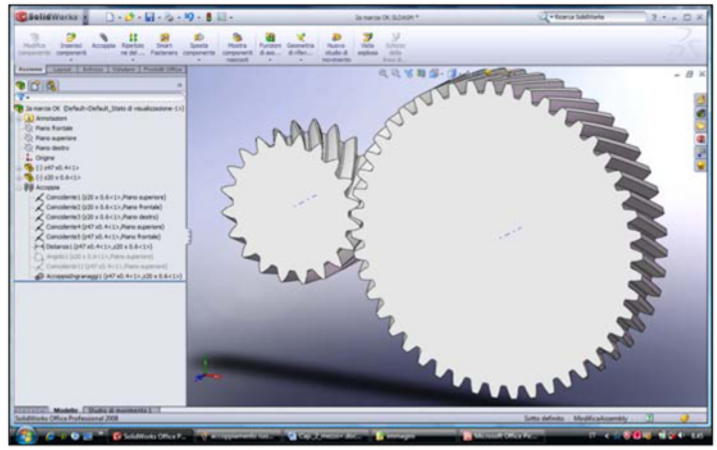

Figure 2. Example of two helical gears obtained through parametric models.

CALCULATION OF LOAD CAPACITY OF HELICAL GEARS

Some of the formulas used for the optimization algorithm implementation, in particular the ones relating to the gears stress minimization, are extracted from ISO 6331: "Calculation of load capacity of spur and helical gears" [1]. This International Standard propose a very accurate gears design and verification process; it provides a coherent system of procedures for the calculation of the load capacity of cylindrical involute gears with external and internal teeth. These procedures are based on testing and analytical studies and the results of these formulas are in good agreement with previously accepted gear calculation methods.

A series of influence factors have to be evaluated, these numeric coefficients can be determined by three different method (called Method A, B and C) depending on the quantity of data available at the design stage. Method A requires all gear and loading data, while Method C uses simplifying assumption in order to obtain average values which can be used for common fields of application.

ISO 6336 is primarily intended for verifying the load capacity of gears for which essential calculation data are available by way of detail drawings, or in similar form. Unfortunately, the data available at the primary design stage is usually restricted. It is therefore necessary, at this stage, to make use of approximations or empirical values for some factors.

The formulas in ISO 6336 are intended to establish a uniformly acceptable method for calculating the pitting resistance and bending strength capacity of cylindrical gears with straight or helical involute teeth.

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ISO 6336-2: CALCULATION OF SURFACE DURABILITY (PITTING)

The second part of ISO 6336 [2] was used to calculate the surface durability of the gears.

The contact stress σ_H shall be less than the permissible contact stress σ_{HP}, for both the wheel (subscript 1) and the pinion (subscript 2).

$$\sigma_{H1} = Z_B \, \sigma_{H0} \sqrt{K_A \, K_V \, K_{H\beta} \, K_{H\alpha}} \tag{3}$$

$$\sigma_{H2} = Z_D \, \sigma_{H0} \sqrt{K_A \, K_V \, K_{H\beta} \, K_{H\alpha}} \tag{4}$$

where the common factor σ_{H0} is

$$\sigma_{H0} = \sqrt{\frac{F_t}{d_1 \, b} \frac{\tau + 1}{\tau}} \, Z_H \, Z_E \, Z_\varepsilon \, Z_\beta \tag{5}$$

F_t is the nominal tangential load; b the facewidth, d_1 is the reference diameter of the pinion; $\tau = z_2/z_1$ is the gear ratio, positive for the external gears, negative for internal gears.

The permissible contact stress is calculated from

$$\sigma_{HP} = \frac{\sigma_{H,lim} \, Z_{NT}}{S_{H,min}} Z_L \, Z_R \, Z_V \, Z_W \, Z_X \tag{6}$$

where $\sigma_{H,lim}$ is the allowable stress of the standard reference test gears, S_{Hmin} is the minimum required safety factor with regard to Hertzian pressure (see the Appendix for a complete definition of the factors).

The values of the influence parameters are calculable using the equations and tables proposed in [2].

In particular some of these parameters, the three relating to the influence of the lubricant film: z_L, z_R, z_V, the work hardening factors z_W and the size factor z_X, assume different values for static and fatigue analysis. All these factors are equal to 1 in the static case.

The life factor z_{NT} also can assume different values depending on the requested life (cycles number) of the component. Figure 3 shows the fatigue characteristic of a case-hardened steel extensively used for the gear manufacturing in industrial, automotive and aerospace applications.

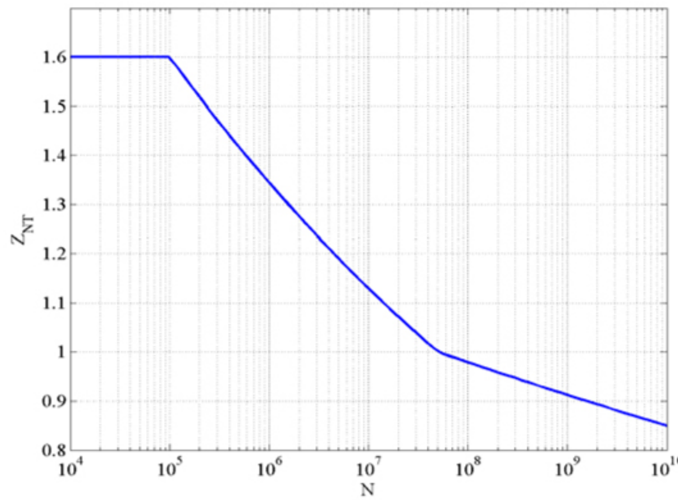

Figure 3. Life factor versus the number of cycles for a case-hardened steel (no pitting is permitted).

The factors z_H, z_B, z_D account for the influence of tooth flank curvature on contact stress. The combination of sufficiently high helix and pressure angles and a non symmetric profile shift, for the pinion and for the wheel, can significantly reduce the contact stress (Figure 4).

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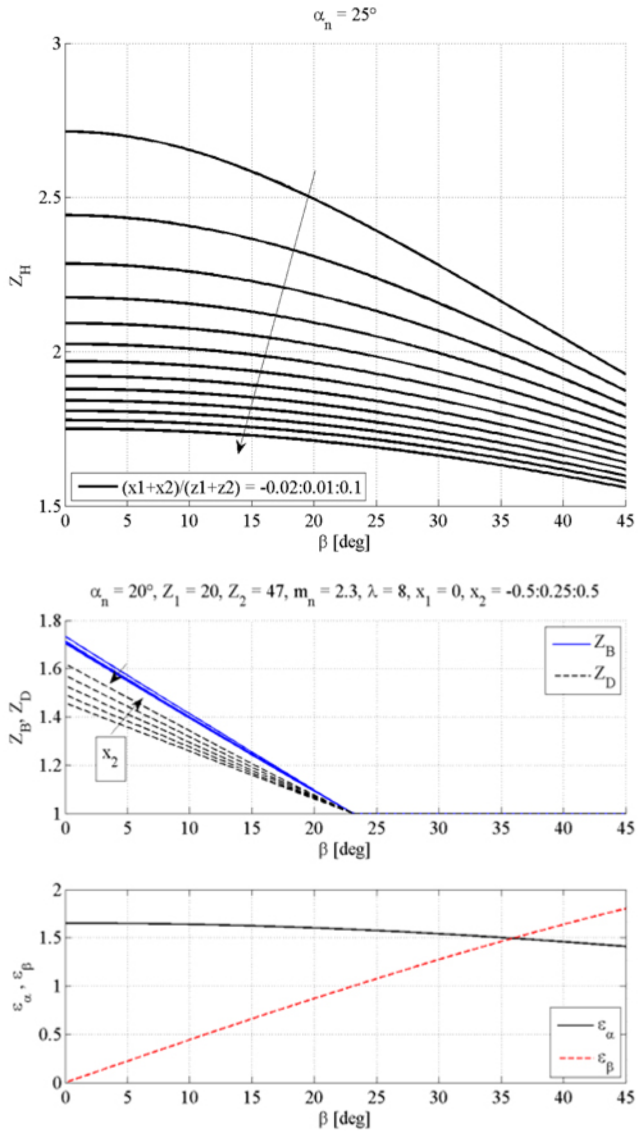

Figure 4. Influence factors for the evaluation of the tooth flank curvature effect on the contact stress.

Finally the factors z_E, z_ε, z_β account respectively for the elastic properties of the material (eventually different for pinion and wheel), for the influence of transverse contact and overlap ratios, and for the influence of the helix angle on surface load capacity.

ISO 6336-3: CALCULATION OF TOOTH BENDING STRENGTH

The third part of ISO 6336 [3] was used to calculate the bending strength of the gears.

The actual tooth root stress σ_F shall be less than the permissible tooth root bending stress σ_{FP}, for both the wheel and the pinion.

$$\sigma_F = \sigma_{F,0} K_A K_V K_{F\beta} K_{F\alpha} \tag{7}$$

where

$$\sigma_{F,0} = \frac{F_t}{b\, m_n} y_F\, y_S\, y_\beta\, y_B\, y_{DT} \tag{8}$$

The permissible contact stress is calculated from

$$\sigma_{FP} = \frac{\sigma_{F,lim}\, y_{ST}\, y_{NT}}{S_{Fmin}} y_{\delta\, rel\, T}\, y_{R\, rel\, T}\, y_X \tag{9}$$

where $\sigma_{F,lim}$ is the allowable stress (bending) of the standard reference test gears and S_{Fmin} is the minimum required safety factor for tooth root strength.

Also in the case of bending some of the influence factors, the last three terms in equation (9), are equal to one in the static case. These are the parameters relating to the notch sensitivity of the material $y_{\delta\ rel\ T}$, to the surface condition (roughness) $y_{R\ rel\ T}$, and to the size of the gear y_X.

In perfect analogy with the pitting analysis, the life factor y_{NT} represent the fatigue behavior of the material. It is defined as the ratio between the higher tooth root stress for a limited number of cycles and the allowable stress at 3 millions of cycles. Let consider as an example a case-hardened wrought steel, the value assumed by this factor is 2.5 for static stress and 1 for reference stress calculation. The stress correction factor y_{ST} is equal to 2.

With reference to eq. (6) the y_F is the form factor, a function of the normal chordal dimension of the teeth and the moment arm relevant to the load application. It is possible to reduce the value of this parameter, and consequently also the actual tooth root stress, by increasing the virtual number of teeth of the helical gear.

The stress correction factor y_S is used to convert the nominal tooth root stress in an equivalent local tooth root stress, taking into account the complex stress system at the tooth critical section. The helix angle factor y_β reduce the actual root tooth stress accordingly to the increase of both the helix angle and the face width (Figure 5). The rim thickness factor y_B and the deep tooth factor y_{DT} can be in general considered unitary.

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All the mathematical procedures for the single factor calculation are implemented separately in the form of function in Matlab environment.

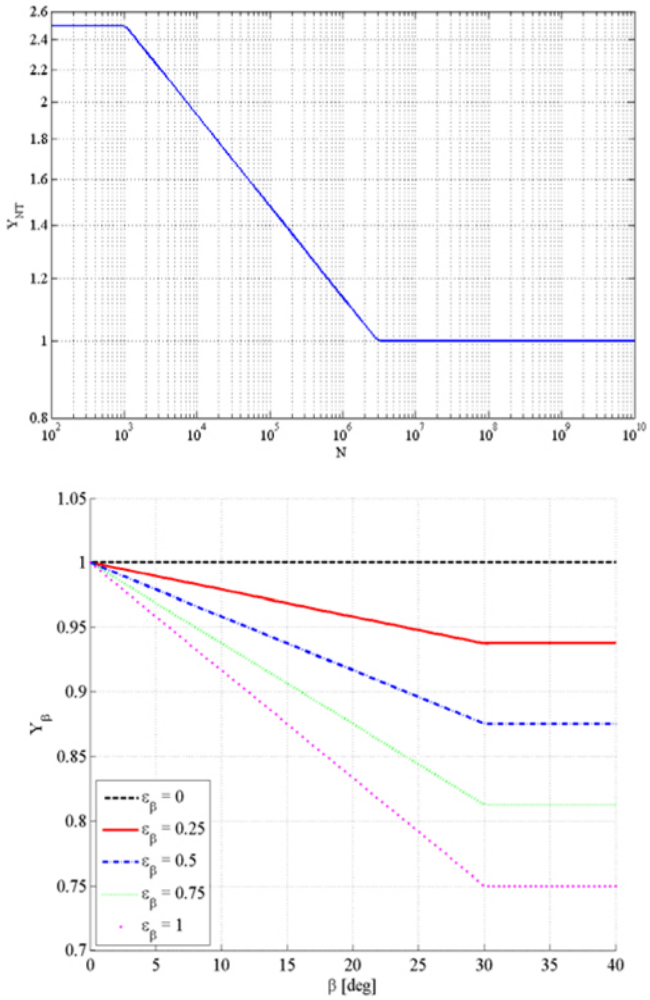

Figure 5. Life factor versus the load cycles numbers (above) and helix angle factor vs helix angle for different overlap ratios (below).

GENERAL INFLUENCE FACTOR

Taking into account eqs.(3), (4) and (7), the *load application factor* K_A quantifies the load increase, with respect to the nominal value, due to external sources (driving and driven machines characteristics, inertias and stiffness of the transmission architecture). It can be calculated starting from the load spectra or using empirical guidance values [5], depending on the input data available during the design stage.

The *internal dynamic factor* K_V was computed using Method B [1] in order to consider the increase of the internal dynamic tooth loading due to resonant forced vibrations. A torsional resonance frequency arise from the elasticity of the meshing gears teeth. The definition of a resonance ratio N_R as the ratio between the running speed and the resonance speed (see eq. (10)) can be used to divide the entire running speed range into three sectors (Figure 6):

$$N_R = \frac{\pi}{30000}\sqrt{\frac{m_{eq}}{k_m}}\, n_1 z_1 \tag{10}$$

where m_{eq} [kg/mm] is the equivalent mass of the gear pair per unit face width, referred to the base radius; k_m [N/mm/μm] is the mesh stiffness per unit face width; n_1 [rpm] is the running speed. The three sectors are subcritical, main resonance, intermediate and supercritical range. Different equations are proposed for each speed range.

Figure 6 shows the internal dynamic factor and resonance ratio versus the input speed of the transmission at full load. A part from the first two gear pairs, the risk of resonance is possible for all the others inside the working speed range of the engine. Hence, it is possible to conclude that the simplest Method C that is based on the main simplifying hypothesis of subcritical running speed range, should not be adopted for the gearbox design and verification.

The parameters $K_{F\alpha}$ and $K_{H\alpha}$, the *transverse load distribution factor* for bending and pitting respectively, are related to the effect of the non uniform distribution of transverse load between several contacting gear teeth pairs. Since it is not so easy to parameterize the deflection under load of the gears, and in particular the base pitch deviations (f_{pb}) of pinion and wheel, we impose a value of f_{pb} only dependant on the accuracy grade [6], e.g. 35 μm for an accuracy grade of 7. The value of both these parameters increases with the increase of the total contact ratio (see Figure 7).

The *face load factors*, for pitting $K_{H\beta}$ and bending $K_{F\beta}$, are set to one. The effect of non-uniform load distribution over the gear face width, due to misalignments and deformations of the whole elastic system is so neglected; the calculation of these parameters would require a complete knowledge of the gearbox architecture also for the application of the simplest method C (in particular the position of the pinion in relation to the mechanical power path through the transmission have to be known).

The factor K_V was calculated using Method-B [1] at full load and varying the pinion speed from 0 to 7000 rpm.

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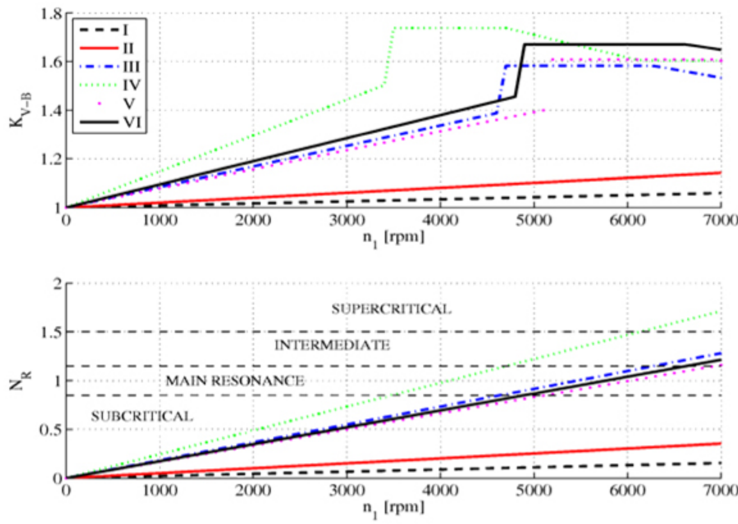

Figure 6. Dynamic factor (above) and resonance ratio (below) for the six gear pair of a 400 Nm gearbox.

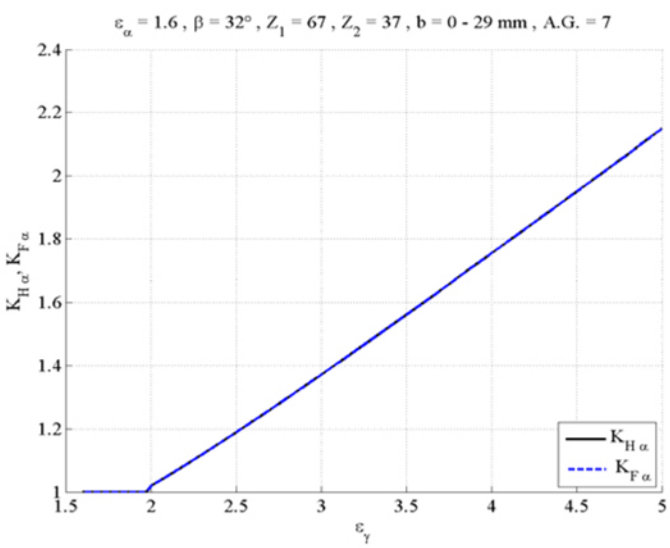

Figure 7. Transverse load factors as a function of the total contact ratio. The variation of the contact ratio is obtained by varying the face width of the gears, the other parameters are relating to the 6th gear of the gearbox.

CUMULATIVE LOAD SPECTRUM FROM TIME HISTORIES

The input data for the design stage of a mechanical component should be the most accurate description of the expected load acting on the component itself. The data normally available, coming from experimental testing or the simulation of dynamic models, are in the form of time histories. In the gears design field, instead, is more practical to manage the cumulative load spectra as input data for damage calculation. In the following the algorithm developed to convert the torque (T) and speed (n) time histories to the cumulative torque spectrum is reported. This procedure does not need a constant sampling frequency, so it is adapted also for variable-step simulation results.

The duration of the time step (from sample i to i+1) is obviously

$$\Delta t_i = t_{i+1} - t_i \ [s] \tag{11}$$

The average values of speed and torque over the time interval Δt_i are

$$\overline{n}_i = \frac{n_{i+1} + n_i}{2} \qquad \overline{T_i} = \frac{(T_{i+1} + T_i)}{2} \tag{12}$$

The number of cycles during the time step Δt_i at the medium torque T_i is

$$N_i = \frac{\overline{n_i}}{60} \Delta t_i \tag{13}$$

Then we need to divide the range of measured (or calculated) load into bins or classes. M is the total number of classes (e.g. M=64), m is the identifier of the class, $T_{UP,m}$ and $T_{LO,m}$ are the upper and the lower bound of the load level n, T_{MAX} is the maximum torque.

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$$T_{UP,m} = \frac{T_{MAX}}{M} m \qquad T_{LO,m} = \frac{T_{MAX}}{M}(m-1) \tag{14}$$

The execution of a loop, with the following conditionally executed statements, allows to identify the number of cycles for each class $N_{C,m}$:

$$\text{if} \quad T_{LO,m} < \overline{T_i} \le T_{UP,m} \quad \text{then} \qquad N_{C,m} = N_{C,m} + N_i \tag{15}$$

The passage from the load spectrum to the cumulative can be easily obtained by summing, class by class, the previous number of cycles starting from the higher level:

$$N_{C,m,cum} = N_{C,m+1} + N_{C,m} \tag{16}$$

In this way, the gearbox design procedure, could be applied starting from the time histories of speed and torque obtained either from simulations or from experimental tests. It is worth noting that, in case of direct measurements or accurate dynamic simulations of the mechanical system both the application factor K_A and the dynamic factor K_V previously introduced have to be set to 1.

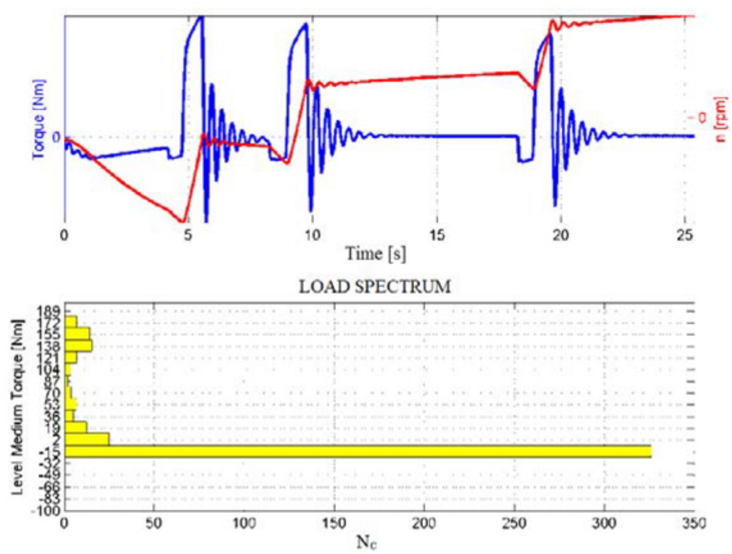

Figure 8. Comparison between time history (above) and load spectrum representation (below).

SAFETY FACTORS CALCULATION PROCEDURE

In order to evaluate the safety factors for both bending and pitting, that will be used in the minimization algorithm, starting from the load spectrum, it is necessary to compute the allowable and the equivalent loads. Using eq. (3), (4), (5), (6) for pitting and eq. (7), (8), (9) for bending and imposing unitary safety factors and life factors, it is possible to calculate the allowable forces for both the cases:

$$F_{tF\lim 1,2} = \frac{B_{F1,2}}{A_{F1,2}} b\,\sigma_{Flim} \quad ,$$

$$F_{tH\lim 1,2} = \frac{\tau}{\tau+1}\left(\frac{B_H\,\sigma_{Hlim}}{A_H}\right)^2 b\,d_1 \cdot \begin{cases} 1/{z_B}^2 \\ 1/{z_D}^2 \end{cases} \tag{17}$$

where

$$A_F = \frac{1}{m_n} y_F\, y_S\, y_\beta\, y_B y_{DT}\left(K_A\, K_V\, K_{F\beta}\, K_{F\alpha}\right) \quad , \quad B_F = y_{ST}\, y_{\delta\,rel\,T}\, y_{R\,rel\,T} \tag{18}$$

and

$$A_H = z_H\, z_E\, z_\varepsilon\, z_\beta \sqrt{K_A\, K_V\, K_{H\beta}\, K_{H\alpha}} \quad , \quad B_H = z_L\, z_R\, z_V\, z_W\, z_X \tag{19}$$

Using the well known Palgrem-Miner accumulation method [5] together with the life factors equations, the equivalent tangential forces are:

$$F_{tF\,eq\,1,2} = \left(\frac{\sum_{i=1}^{N\_dam} {F_{ti}}^{p_F} \cdot N_i}{\sum_{i=1}^{N\_tot} N_i}\right)^{1/p_F} \quad ,$$

$$F_{tH\,eq\,1,2} = \left(\frac{\sum_{i=1}^{N\_dam} {F_{ti}}^{p_H} \cdot N_i}{\sum_{i=1}^{N\_tot} N_i}\right)^{1/p_H} \tag{20}$$

where the summations at the denominators account for all the load cycles, and the one at the numerator just for the cycles that involve damage, F_{ti} is the force for bin i of the load spectrum ($F_{ti} = T_i/r$), p_F and p_H are the slopes of the Woehler-damage lines (suggested values for case hardened steel are p_F =8.7 and p_H =6.6).

Finally the safety factors, two for bending and two for pitting (subscripts 1 and 2 stand for pinion and wheel), can be expressed as a function of the required duration of the gears through the life factors y_{NY} and z_{NT}:

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$$S_{F1,2} = \frac{B_{F1,2}\,\sigma_{F,lim}\,y_{NT}}{A_{F1,2}\dfrac{F_{t\,F\,eq\,1,2}}{b}} \quad , \quad S_{H1,2} = \frac{B_H\,\sigma_{H,lim}\,z_{NT}}{A_H\sqrt{\dfrac{F_{t\,H\,eq}}{d_1 \cdot b}\dfrac{\tau+1}{\tau}}} \cdot \begin{cases} 1/z_B \\ 1/z_D \end{cases} \tag{21}$$

One of the target of the genetic algorithms can be to find the best parameter combinations in order to maximize all of the former safety factors.

GEARBOX DESIGN PROCESS AS A MINIMIZATION PROBLEM

OPTIMIZATION DESIGN AND DEFINITION OF PARAMETERS CONSTRAINTS

The aim of this activity has been the optimization of the geometrical parameters of profile shifted helical gears of a manual transmission gearbox by means of the formulation and the solution through the genetic algorithm of a bounded minimization problem.

First of all it is necessary to highlight the different and sometimes opposite requirements of a manual transmission gearbox:

- Functional objectives: the main focus of a gearbox is obviously to introduce different gear ratios between the engine and the wheels. For a given vehicle and engine type, the gear ratios are generally imposed by performance and fuel consumption requirements; consequently a quite tight tolerance is generally allowed on this set of parameters.
- Structural objectives: moreover the gearbox has to be able to transfer the power from the engine to the driving wheels, with a certain mission profile, for a given amount of kilometers without showing significant damage due to Hertzian contacts or root tooth bending.
- Layout constraints: other constraints are imposed by the mechanical architecture of the gearbox, in particular the number and the spatial disposition of the shafts. The necessity to design several gear pairs sharing the same interaxis introduce therefore other rigid constraints.

Also the space occupancy of the gearbox, in particular for the front wheel drive vehicles with transverse powertrain disposition, both in the radial and in the axial directions is an important parameter to control and minimize during the design phase.

The gearbox design process therefore can be seen as a set of minimization problems, each of them bounded with both linear and non linear constraints on the input parameter. The ideal global minimization algorithm should fully satisfy the single problems, or, more reasonably, it should find a trade-off between the various possible combinations of the input data.

The fixed parameters, the ones that cannot be modified by the algorithm, are: the load mission profile of the gearbox (e.g. the torque and speed time histories); the material properties, i.e. the allowable reference stress for tooth bending and pitting ($\sigma_{F\ lim}$ and $\sigma_{H\ lim}$), the Young and Poisson modulus (E,ν); the dimensional tolerances used during the gears manufacturing (Accuracy Grade); the surface roughness and the lubricant properties, i.e. the viscosity.

Instead, the input variables of the minimization algorithm are the following: the number of teeth of the gears ($z_{1,i}$, $z_{2,i}$), the normal moduli ($m_{n,i}$), the normal pressure angles ($\alpha_{n,i}$), the helix angles ($\beta_{n,i}$), the profile shifts ($x_{1,i}$, $x_{2,i}$), and the face width (b_i). The subscript $_i$ identify the specific gear pair of the manual gearbox.

The constancy of the interaxis (a_x) between several profile shifted helical gear pairs can be treated as a non linear constraint between the input variables, that the algorithm has to guarantee during all of the iterations. Equations (22) and (23) show the non linear relationship between the center distance a_x and the input variables for a single gear pair. It results:

$$a_x = \frac{z_1 + z_2}{2}\frac{\cos\alpha_t}{\cos\beta\cos\alpha_{wt}} \tag{22}$$

where

$$\mathrm{inv}\alpha_{wt} = 2\tan\alpha_n \frac{x_1 + x_2}{z_1 + z_2} + \mathrm{inv}\alpha_t \tag{23}$$

The non linear constraint of interaxis equality between the gears can be expressed as:

$$\sum_{i=1}^{ng-1}\left(a_{x,i} - a_{x,i+1}\right)^2 = 0 \tag{24}$$

where *ng* are the number of gear pair considered. This function is satisfied only if $a_{x,i} = a_{x,i+1}$ for every i.

It is important to define a flexible fitness function $f_{fitness}$, i.e. the function to be minimized, with opportune weights in order to favor some of the design criterions more than others and to force the convergence towards a particular solution. It has been proposed the following fitness function:

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$$f_{fitness} = \sum_{i=1}^{ng}\left(S_{F1,i} + S_{F2,i} + S_{H1,i} + S_{H2,i}\right) + \ldots$$
$$k_a \sum_{i=1}^{ng} a_{x,i} + k_b \sum_{i=1}^{ng} b_i + k_u \sqrt{\sum_{i=1}^{ng}\left(u_i - \tau_i\right)^2} \quad (25)$$

where the first summation accounts for the stress optimization (both root tooth bending and pitting are considered, see eq.(21)), the second and the third terms regard respectively the interaxis and the face widths minimization due to the necessity to save space in the radial and the axial directions, the fourth term is the root mean square of the gear ratios errors, been u_i the actual and τ_i the required value for the i-th gear. Parameters k_a, k_b, k_u allow to adimensionalize the single terms of eq.(25) and to set the relative weight of the single requirements.

APPLICATION EXAMPLE

A simple application of the genetic algorithm approach, applied to the gearbox design, is reported in the following. By taking into account only a single gear pair, the module minimization problem can be formulated rearranging equation (8) and introducing the applied torque T, the number of teeth z and the parameter $\lambda = b/m_n$ (a typical design constraint). Therefore the normal module results:

$$m_n = \sqrt[3]{\frac{2T\cos\beta\, y_F\, y_S\, y_\beta\, y_B y_{DT}}{\sigma_{F,0}\, z\, \lambda}} \quad (26)$$

It is evident that an improvement of material properties or a reduced load acting on the gear have a relevant positive effect on the optimization process.

The genetic algorithm allows to reach a global minimum of the fitness function in comparison with other approaches (i.e. the generalized simplex method or the recursive quadratic programming) that give local minima dependent on the selected starting point.

In Figure 9 the population evolution of the fitness function value during the optimization process is shown for the case of the module minimization and the corresponding final physical parameters values are reported on the bottom. The final results could be obtained in about 50 iterations and the mean fitness function is not monotonically decreasing. This behavior is due to the fact that the algorithm covers and mixes the local optima of each iteration evaluated on the entire previously defined domain. It is worth noting that the physical parameters are bounded and the sensitivity of the tunable design variables is function of the optimization result. In the specific case the input torque and the material properties are fixed by application and manufacturing constraints, whilst the number of teeth, the helix angle and the λ parameter may be optimized in a bounded range. It is evident from Figure 10 that the sensitivity for a given static root tooth stress is higher for z and β and the minimum of the objective function is obtained by increasing all the three parameters.

<figures 9, 10 here>

In this way most of the main objectives of the gearbox design phase, as previously discussed, have been mathematically expressed in a unique fitness function that can be minimized using genetic algorithm.

GEARBOX FUNCTIONAL MODELING

Figure 11a shows the design workflow implemented into a macro command who defines the parametric CAD sketches, functions and assembly part positioning. The result of the GA optimization are saved into a text file for reference and the same data are passed to the CAD modeler by OLE call of a macro. The operations inside the macro are organized with the single parts modeling first (gears and shafts), followed by the positioning of the shafts into the assembly model by means of axis coincidence or distance from principal planes. The gears are loaded and positioned concentric to the shaft with assigned distances (Figure 11b) and the gear constraint is applied so to completely define the assembly kinematics.

The modeling of each gear is made by cutting the gear space from a solid cylinder of the outside diameter. The involute is computed, inserted into a sketch, mirrored and closed by outside and root arcs as indicated in eqs. (1) and (2). The sketch is cut-extruded following an helix curve and these operations are repeated with a circular pattern to cut all the other spaces, so to obtain the teeth.

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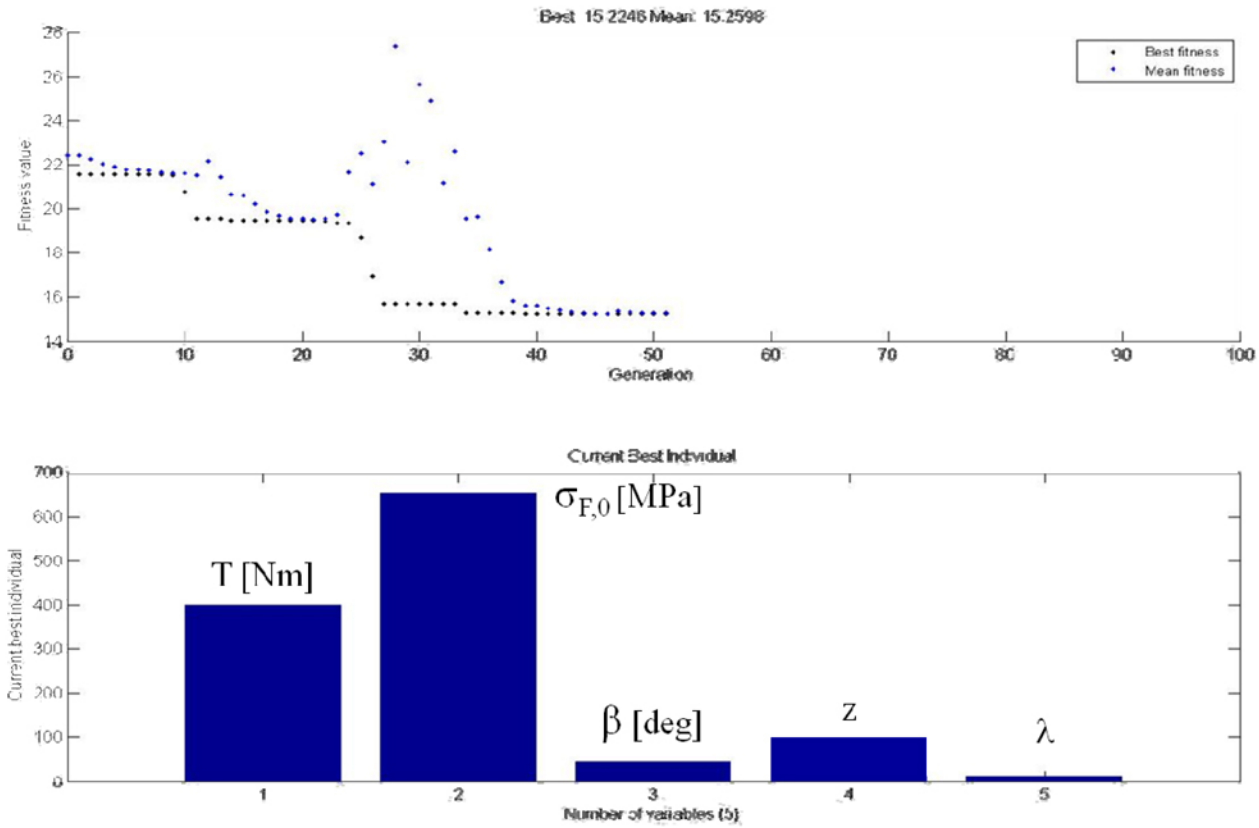

Figure 9. Fitness function values during the genetic optimization process (above) and final physical parameters values (below).

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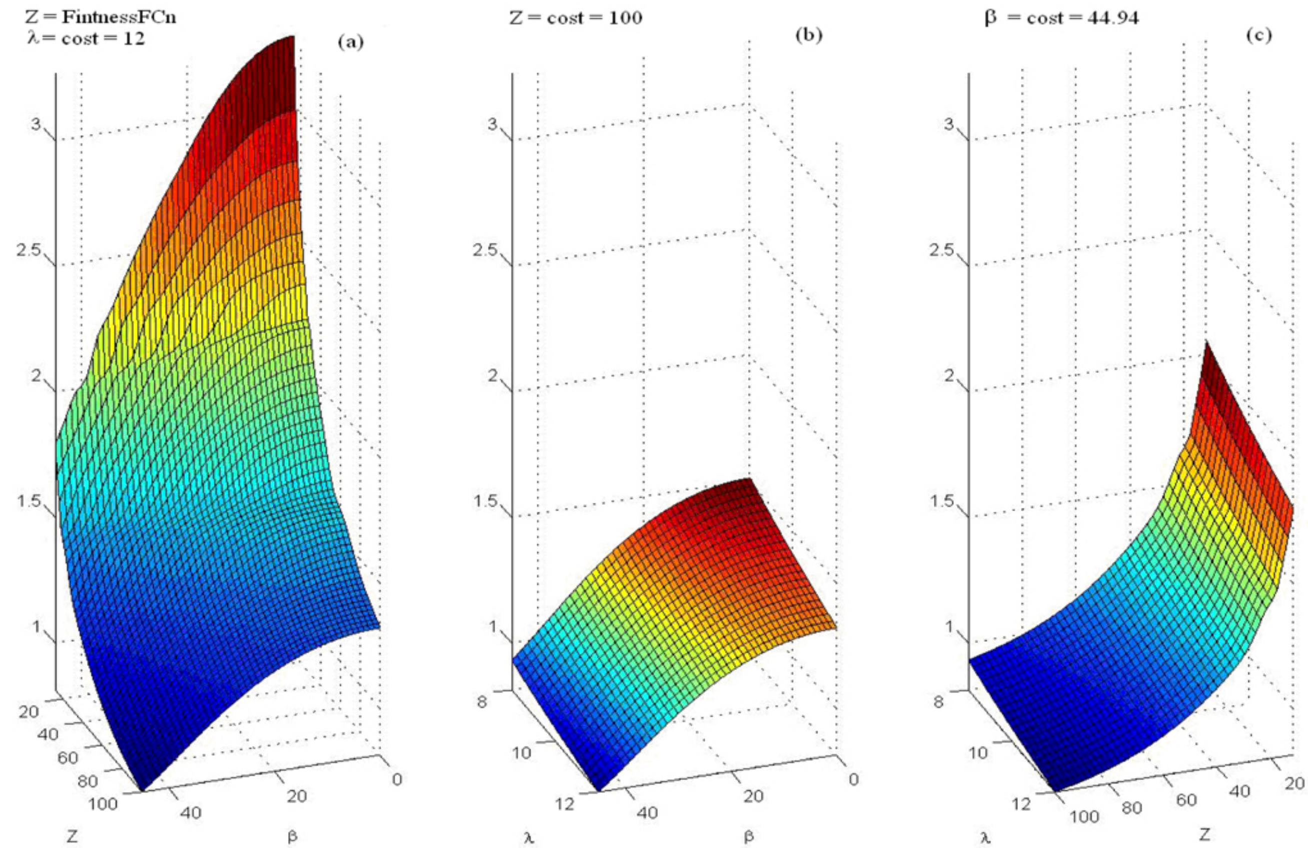

Figure 10. Sensitivity analysis on the normal module minimization for a given static root tooth stress.

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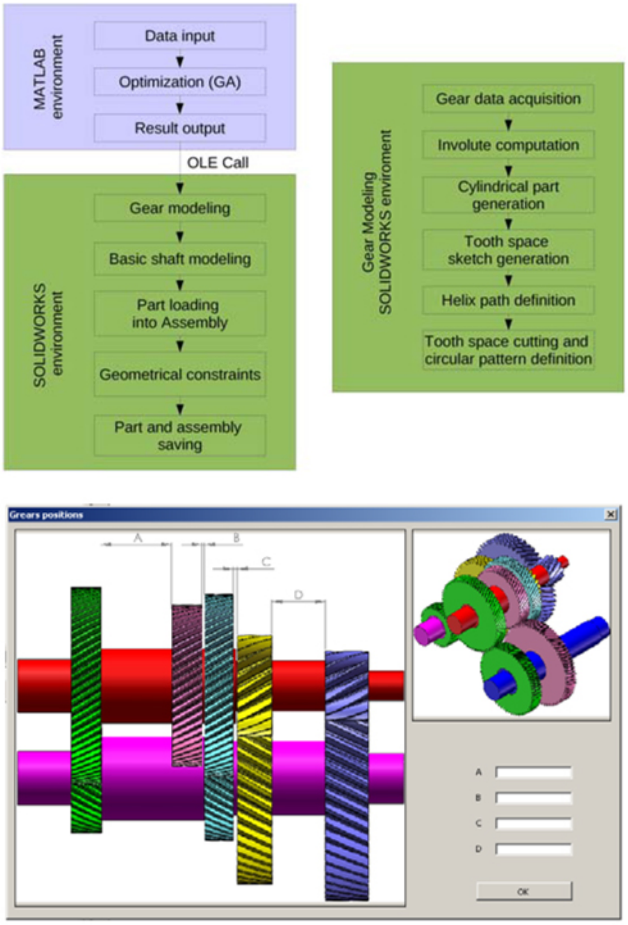

Figure 11. Macro workflow (11a, above), gear distance definition (11b, below).

The assembed gearbox is shown in Figure 12: the integrated CAD tool allows to obtain and verify the optimization design based on the genetic algorithm.

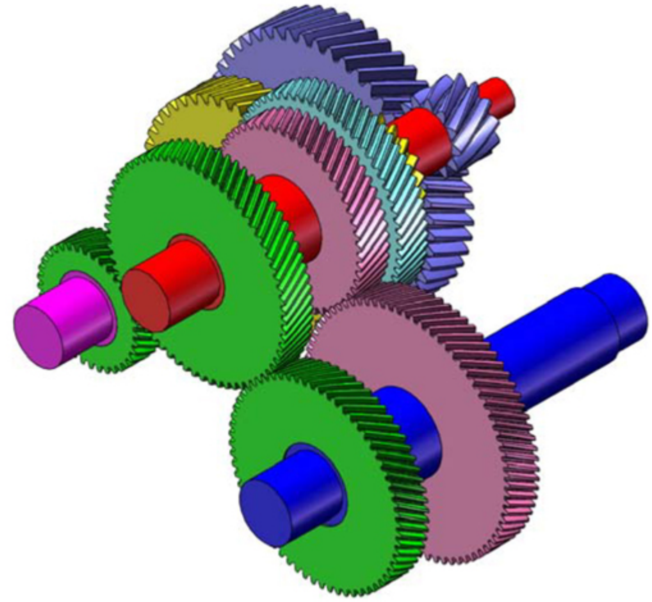

Figure 12. Assembled gearbox.

CONCLUSIONS

The proposed methodology allows the design of an automotive manual gearbox in which gear pairs share the same interaxis, with stress minimization on teeth and satisfying geometrical constraints (helix angle, contact ratios, etc.), by means of a genetic algorithm. The optimized results are passed to a CAD software and processed with macros in order to design the gear models and a complete assembly. This methodology can be extended to other geometrical gearbox configurations, so allowing the analysis of different scenarios and the advanced design of the part which can be further developed and customized (synchronizers, meshes, etc.). The volumes, weights and the costs of these different gearbox configurations can be compared so to obtain the most suitable and competitive solution.

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DEFINITIONS/ABBREVIATIONS

SHARED SYMBOLS

T
: torque

$\mathbf{F_t}$
: nominal tangential load

b
: facewidth

$\mathbf{m_n}$
: normal module

z
: number of teeth

$\boldsymbol{\alpha_n}$
: normal pressure angle

$\boldsymbol{\alpha_t}$
: frontal pressure angle

β
: helix angle

x
: profile shift

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τ
gear ratio (z_2/z_1)

d_1
reference diameter of pinion

R
reference radius

SYMBOLS USED FOR BENDING

$\sigma_{F,lim}$
nominal bending stress limit value

S_{Fmin}
minimum required safety factor

Y_F
form factor

Y_S
stress correction factor

Y_β
helix angle factor

Y_B
rim thickness factor

Y_{DT}
deep tooth factor

K_A
load application factor

K_V
dynamic (or speed) factor

$K_{F\beta}$
face load distribution factor

$K_{F\alpha}$
transverse load distribution factor

Y_{NT}
life factor

Y_{ST}
stress correction factor

$Y_{\delta\ rel\ T}$
relative notch sensitivity factor

$Y_{R\ rel\ T}$
relative surface factor

Y_X
size factor

SYMBOLS USED FOR PITTING

$\sigma_{H,lim}$
nominal contact stress limit value

$S_{H\ min}$
minimum required safety factor

K_A
load application factor

K_V
dynamic (or speed) factor

$K_{H\beta}$
face load distribution factor

$K_{H\alpha}$
transverse load distribution factor

Z_H
zone factor

Z_E
elasticity factor

$Z\varepsilon$
contact ratio factor

Z_β
helix angle factor

Z_L
lubricant factor

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Z_V
velocity factor

Z_R
roughness factor

Z_W
work hardening factor

Z_X
size factor

Z_{NT}
life factor

The Engineering Meetings Board has approved this paper for publication. It has successfully completed SAE's peer review process under the supervision of the session organizer. This process requires a minimum of three (3) reviews by industry experts.

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