# Portfolio Replication

Non-Negative LASSO

Business, Economics and Financial Data

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LASSO and Elastic Net



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- Logistic Regression



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## INTRODUCTION

### WHAT IS PORTFOLIO REPLICATION?

### WHICH APPROACHES?

- Active
- Passive

WHY LASSO?

# SHRINKAGE METHODS

## LASSO REGRESSION

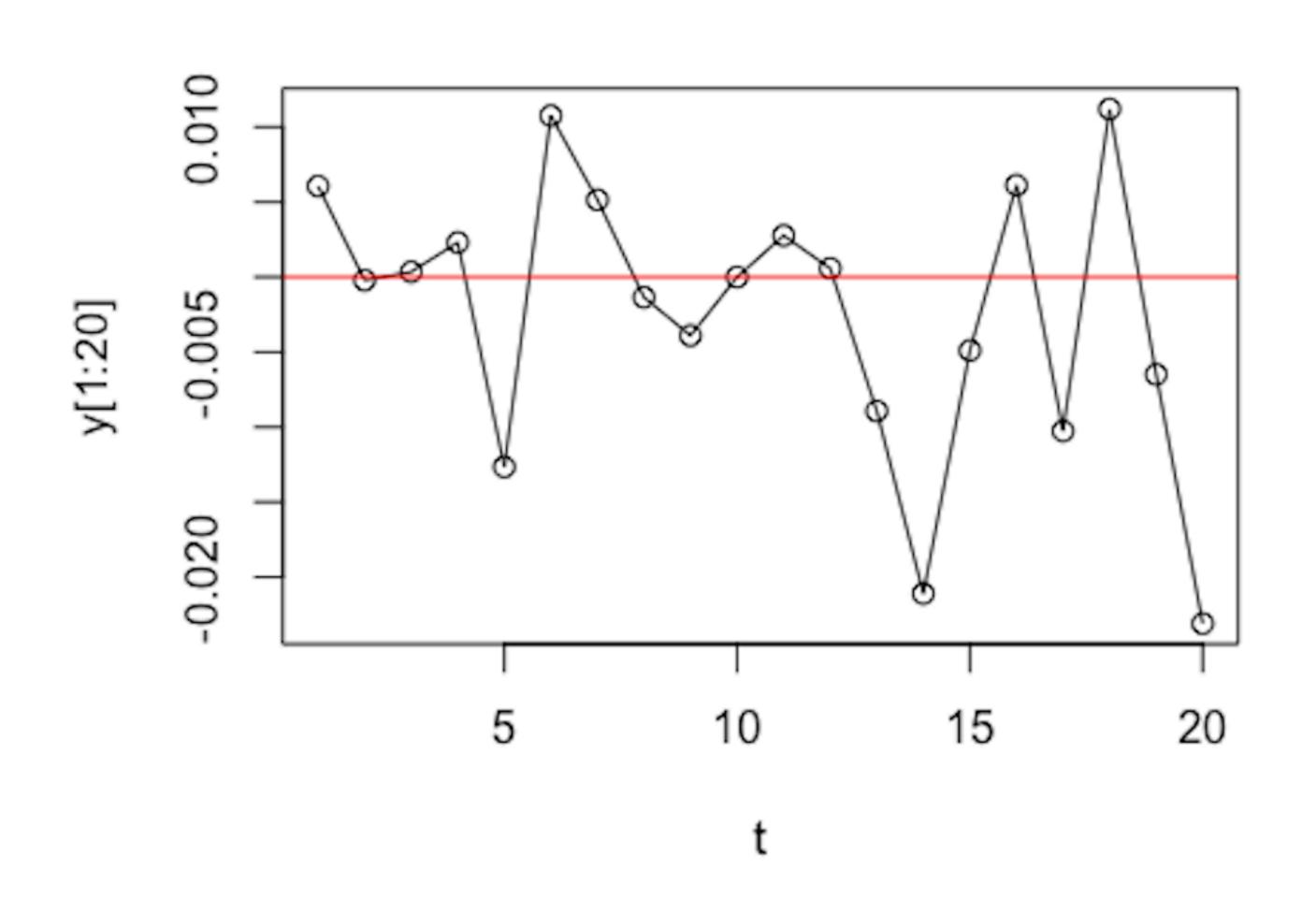
$$\min_{\boldsymbol{\beta}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\| + \lambda \sum_{j=1}^{p} |\beta_j|$$

## ELASTIC NET REGRESSION

$$\min_{\boldsymbol{\beta}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\| + \lambda \sum_{j=1}^{p} (\alpha \beta_j^2 + (1 - \alpha)|\beta_j|)$$

# DATASET ANALYSIS

## DATASET ANALYSIS



### What are we working on?

- Sp500 index

• Daily log returns 
$$\log(y_{t+1}) - \log(y_t) = \log\left(\frac{y_{t+1}}{y_t}\right)$$

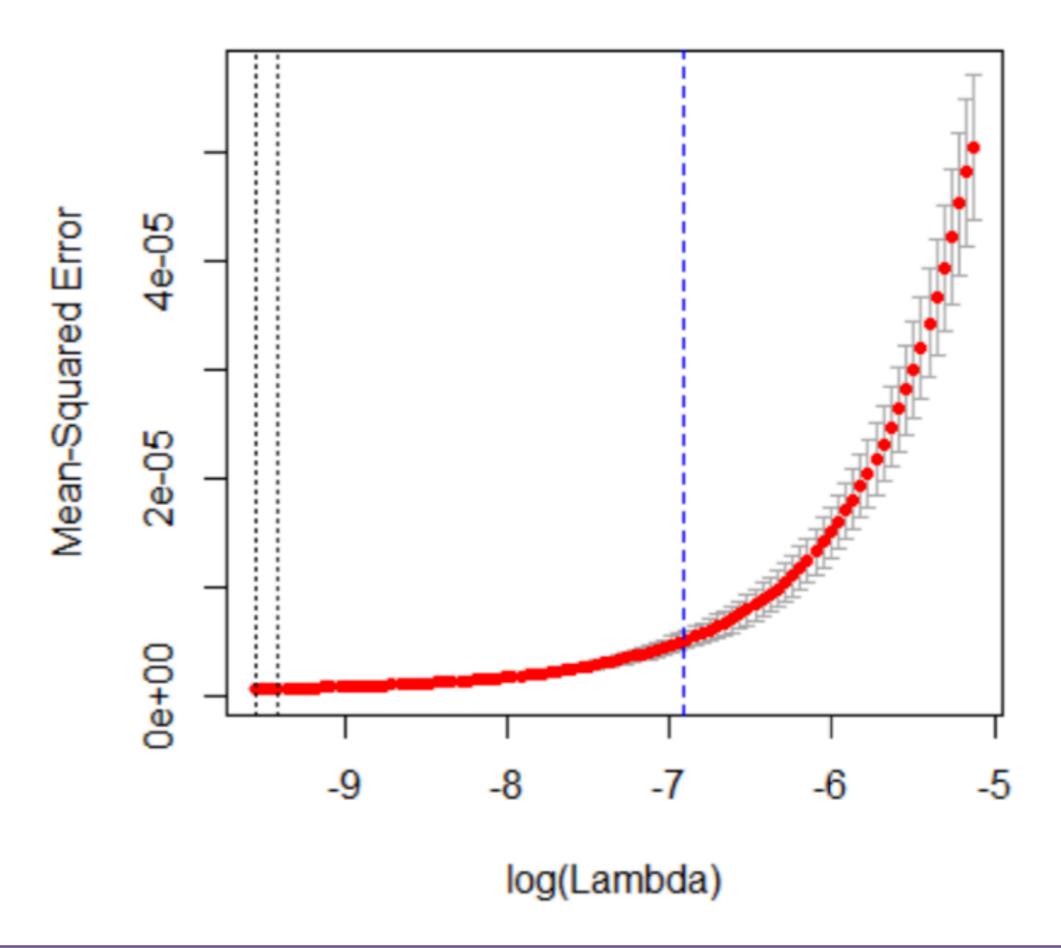
### **Problems**

• How to predict  $y_{t+1}$ ?

# PARSIMONY ACCURACY

#### Lasso accuracy given lambda

160 139 103 82 63 43 26 17 4 0



#### How to choose lambda?

Compromise between number of coefficients and prediction error

# PARAMETERS ESTIMATION

### Moving window approach

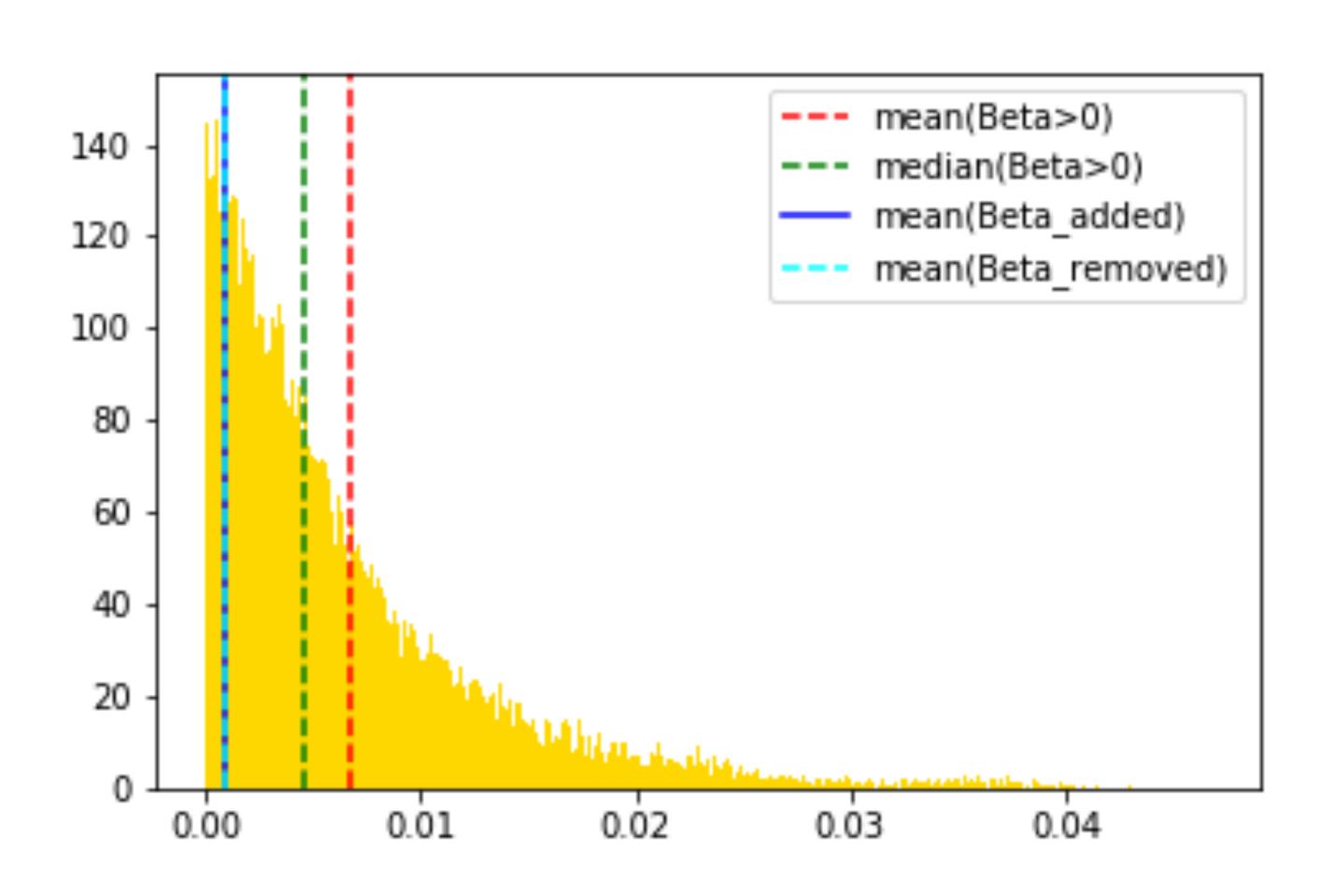
 Are beta coefficients time invariant?

### Beta-parameters interpretation

$$\log(y_{t+1}) - \log(y_t) = \log\left(\frac{y_{t+1}}{y_t}\right)$$

Sp500	APPLE	MICROSOFT	AMAZON.COM	EXXON.MOBIL
0.006063	-0.007184	0.007720	0.011116	0.014049
-0.000212	0.006318	-0.018014	0.009726	-0.003270
0.000348	-0.012855	-0.006453	-0.002267	-0.009776
0.002304	-0.006692	0.014252	-0.008389	0.007589
-0.012656	0.005221	-0.029853	-0.016941	-0.019793
0.010760	0.019702	0.022613	0.016639	0.005767
0.005153	0.019879	0.027021	-0.004209	-0.003436
-0.001348	-0.005596	0.003530	-0.000177	0.001618
-0.003903	-0.024807	-0.013921	0.009580	0.002221

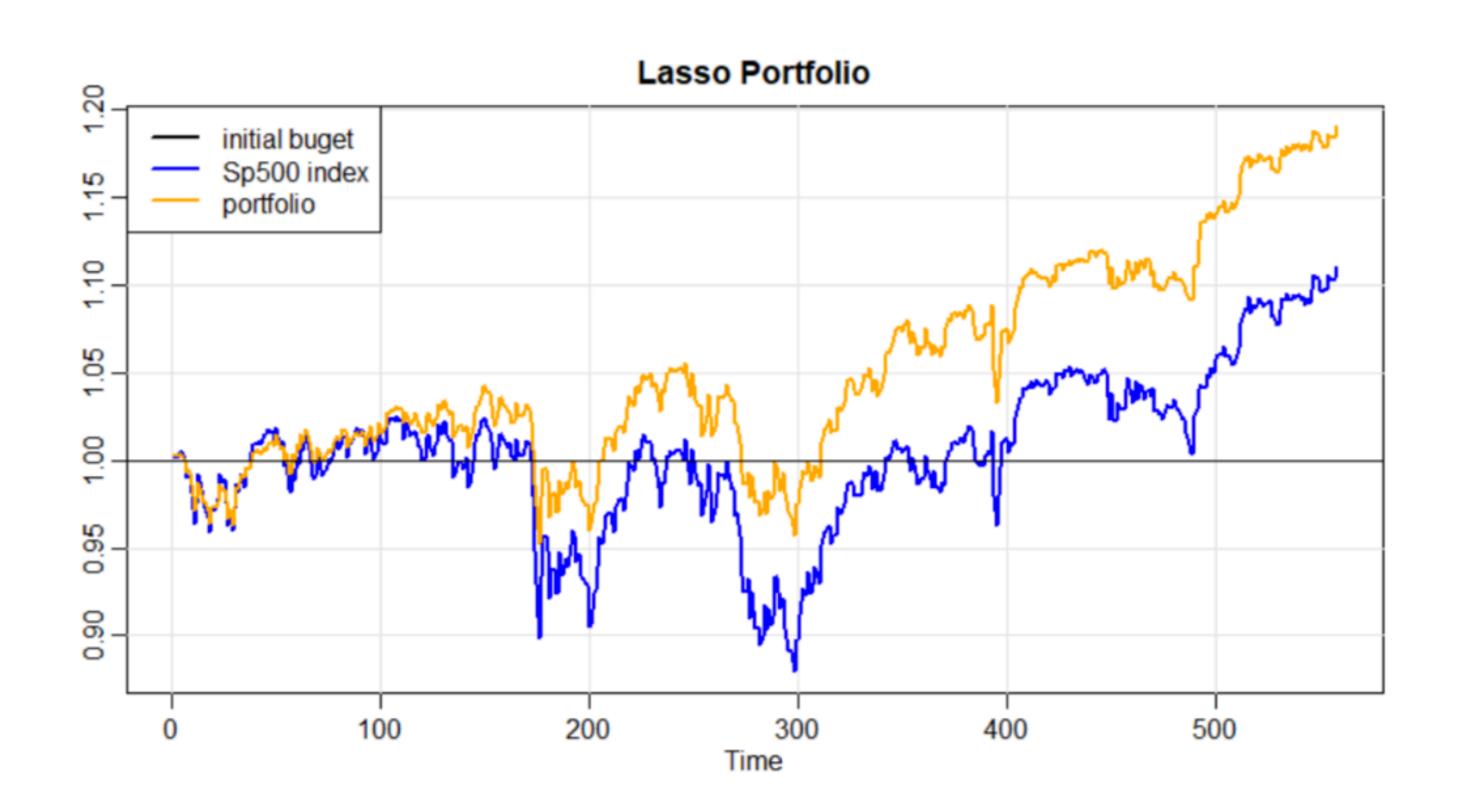
## BETA TURNOVER



- Low turnover (4%)
- Added and removed variables have low weights (12%)

## PASSIVE APPROACH

## LASSO REGRESSION



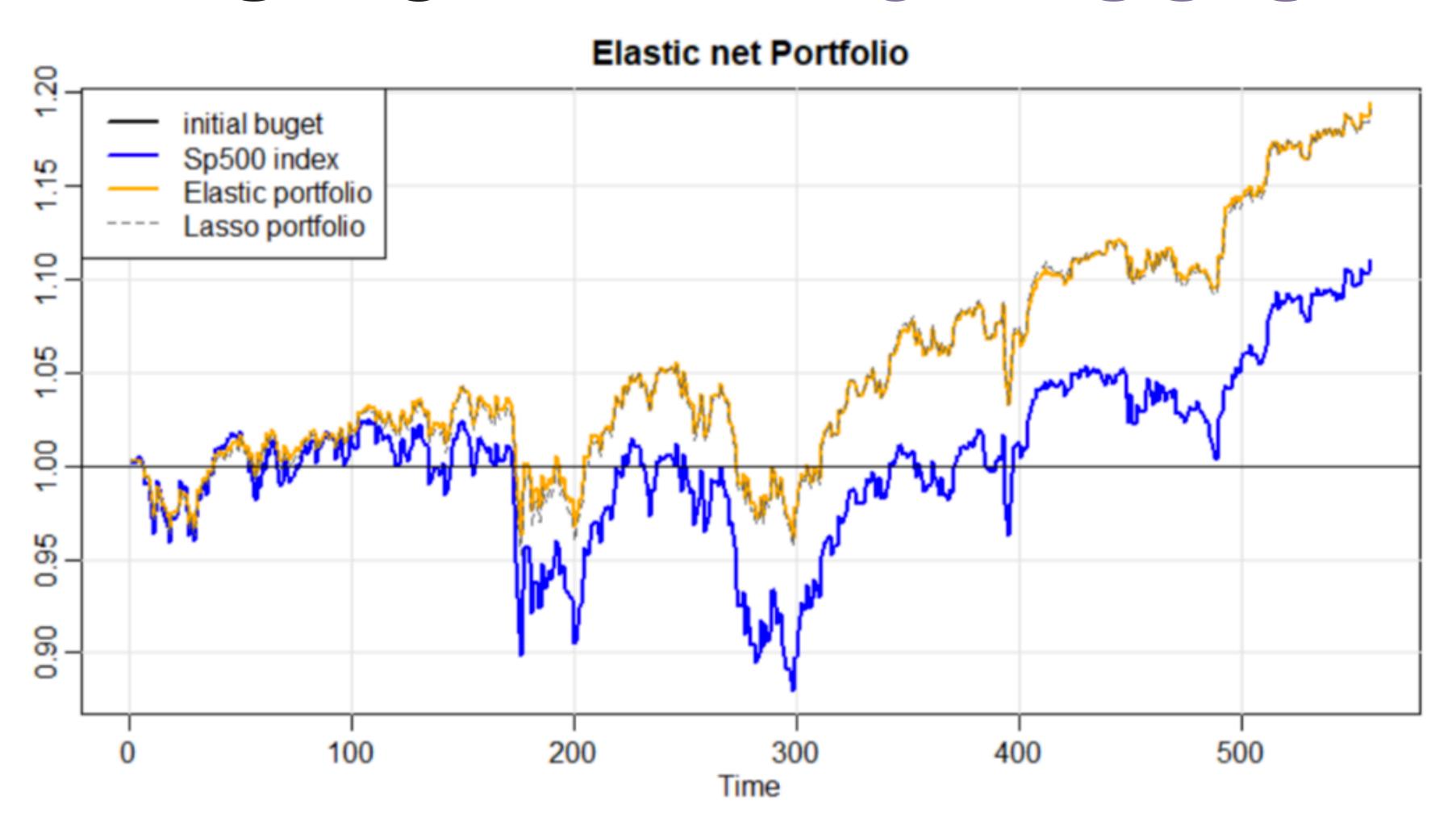
### PROs:

- Low Fees
- Growing Index 
   Positive Returns

### CONs:

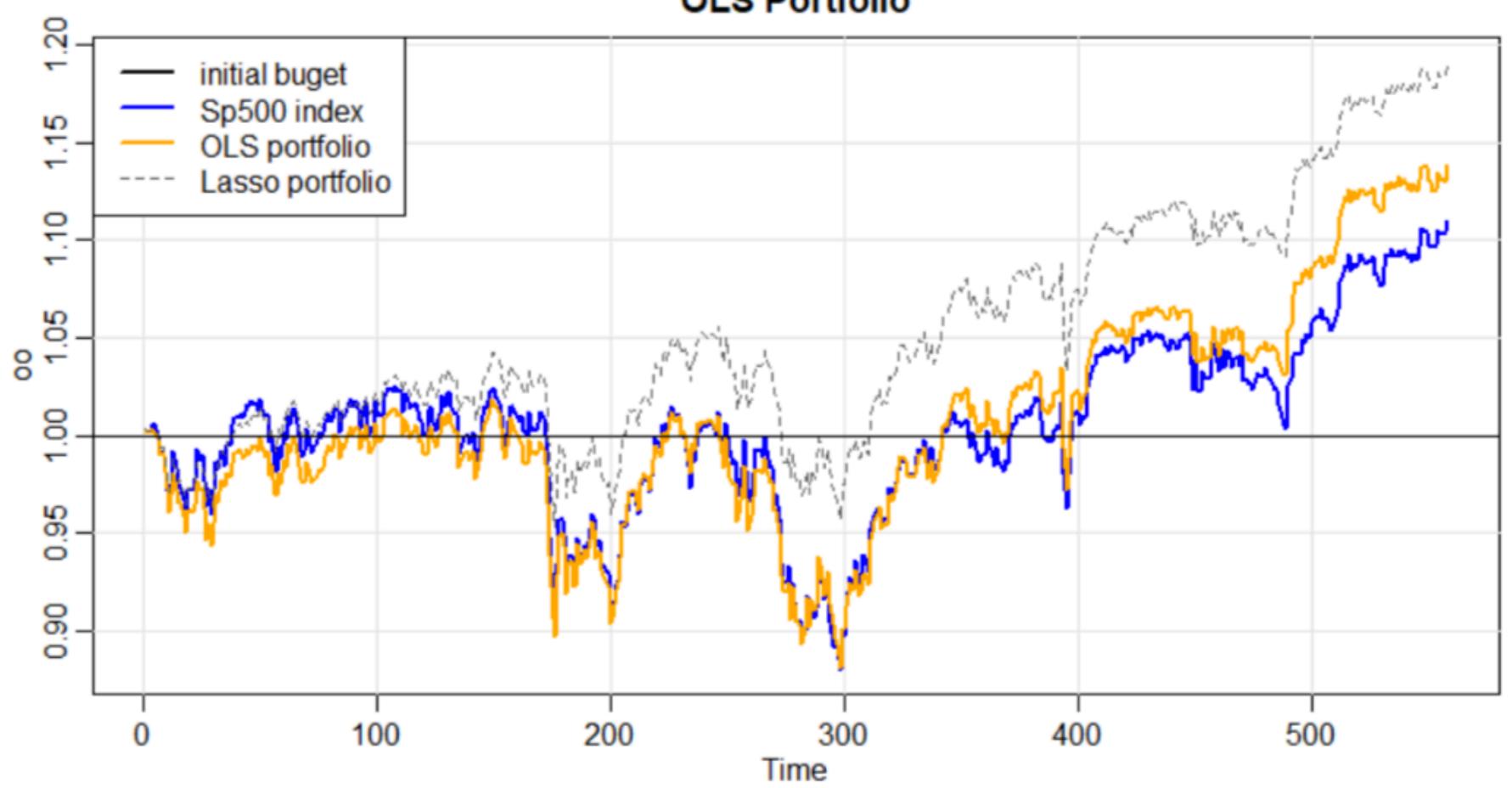
- How often update?
- Descending Index 
   Losses

## ELASTIC NET REGRESSION

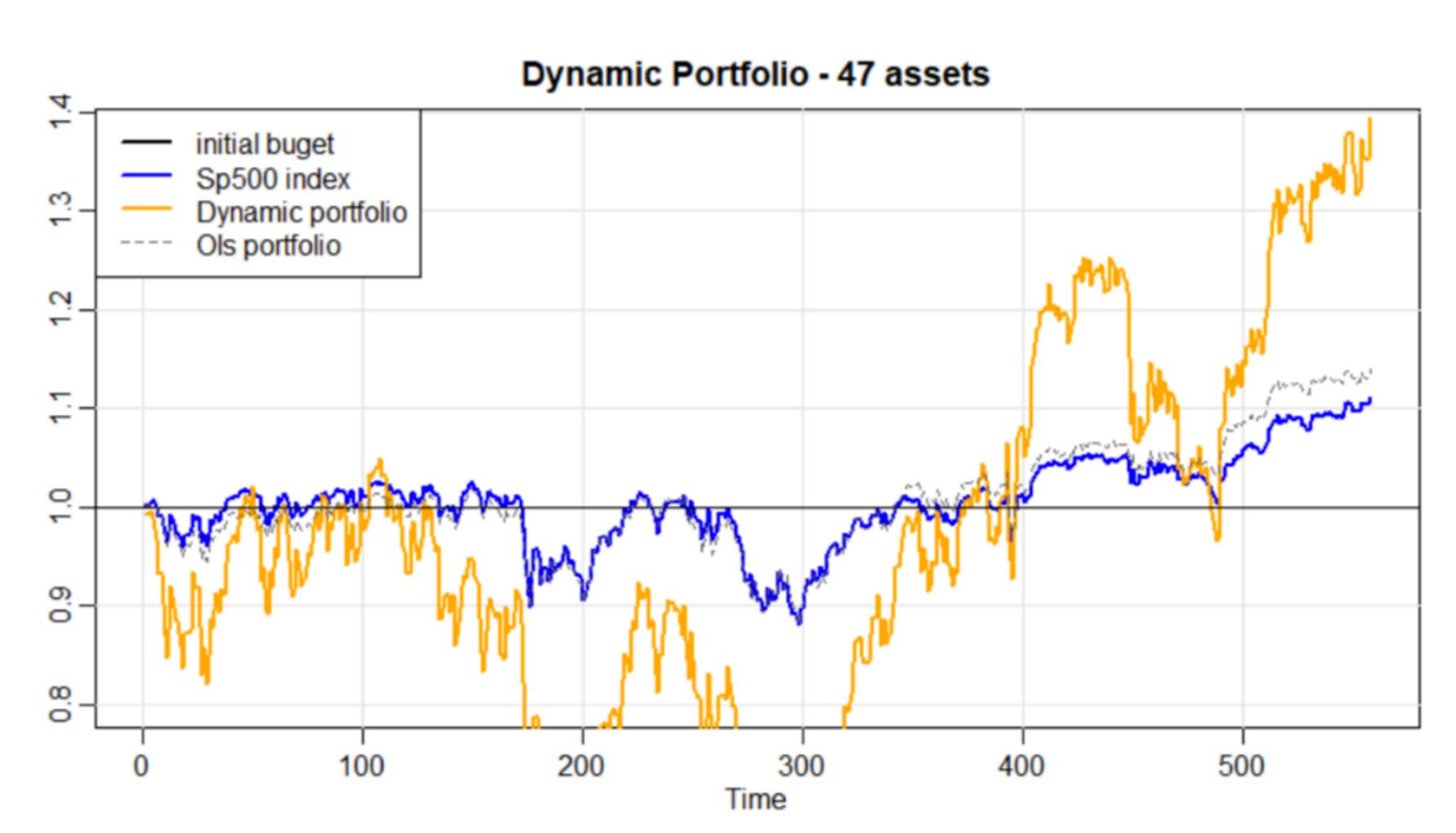


## OLS REGRESSION





## DYNAMIC REGRESSION



# ACTIVE APPROACH

### Shifted windows approach

- Predictions are quite inaccurate
- Also time series approach not working

#### PROs:

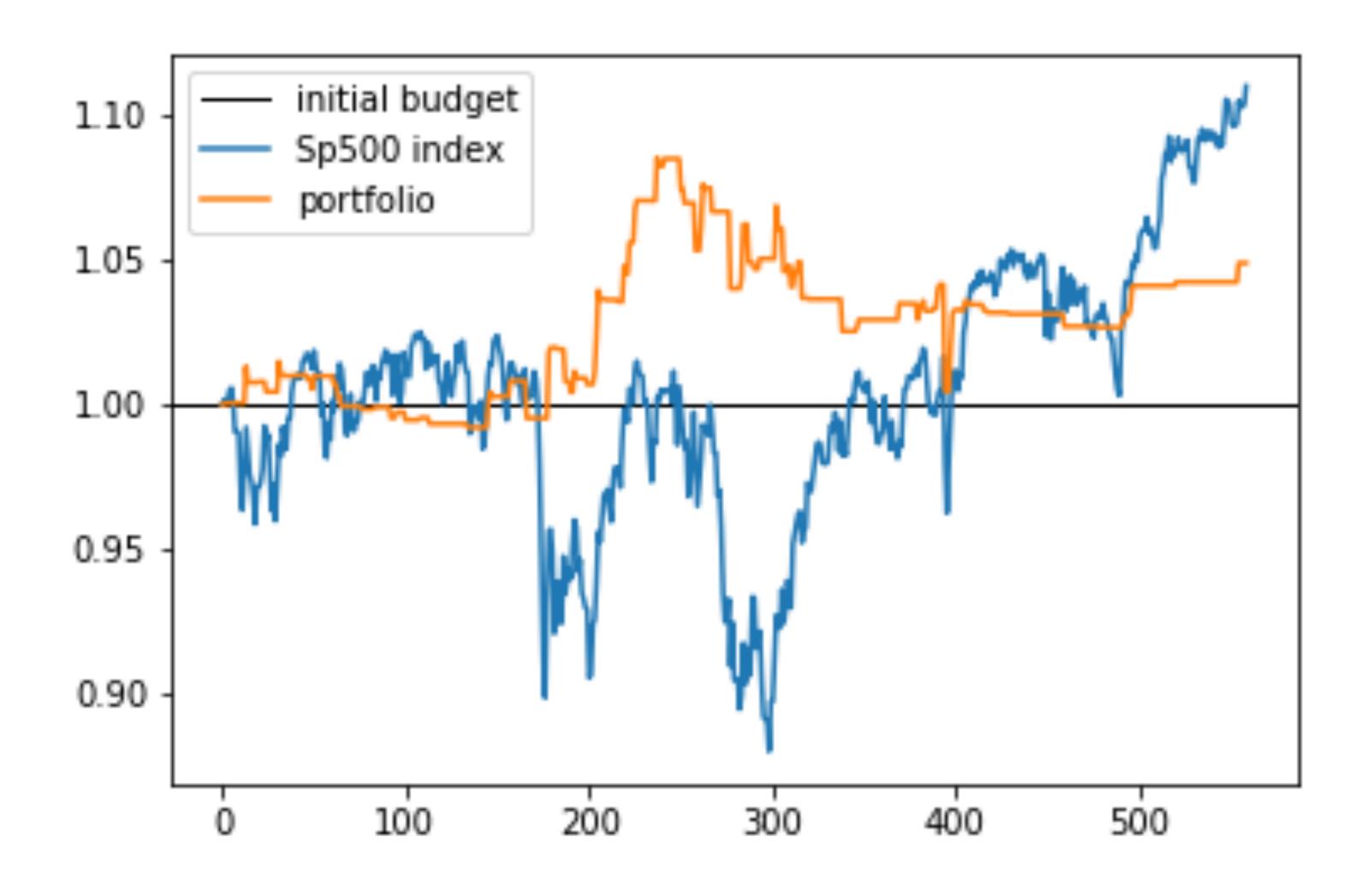
- Financial Strategy
- Possible high revenue

### CONs:

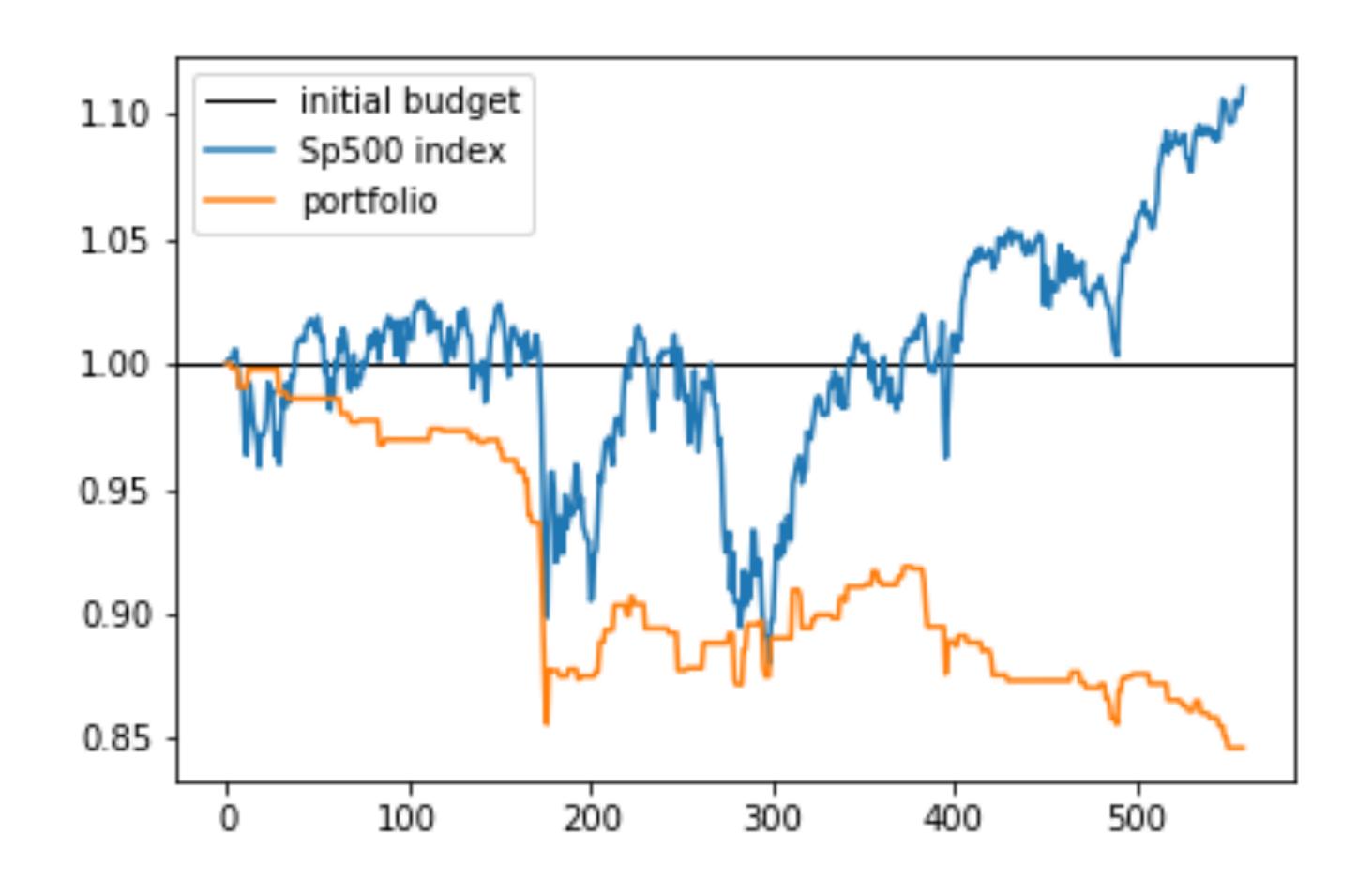
- High fees
- Possible huge losses even with growing index

Sp500	APPLE	MICROSOFT	AMAZON.COM	EXXON.MOBIL
0.006063	-0.007184	0.007720	0.011116	0.014049
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## LASSO REGRESSION



## LOGISTIC REGRESSION



# CONCLUSIONS

- Portfolio replication works quite well
- Passive approach performs better than active
- We need a better prediction model to exploit active approach