Portfolio Replication

Non-Negative LASSO

Business, Economics and Financial Data

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WHICH APPROACHES?

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WHY LASSO?

SHRINKAGE METHODS

LASSO REGRESSION

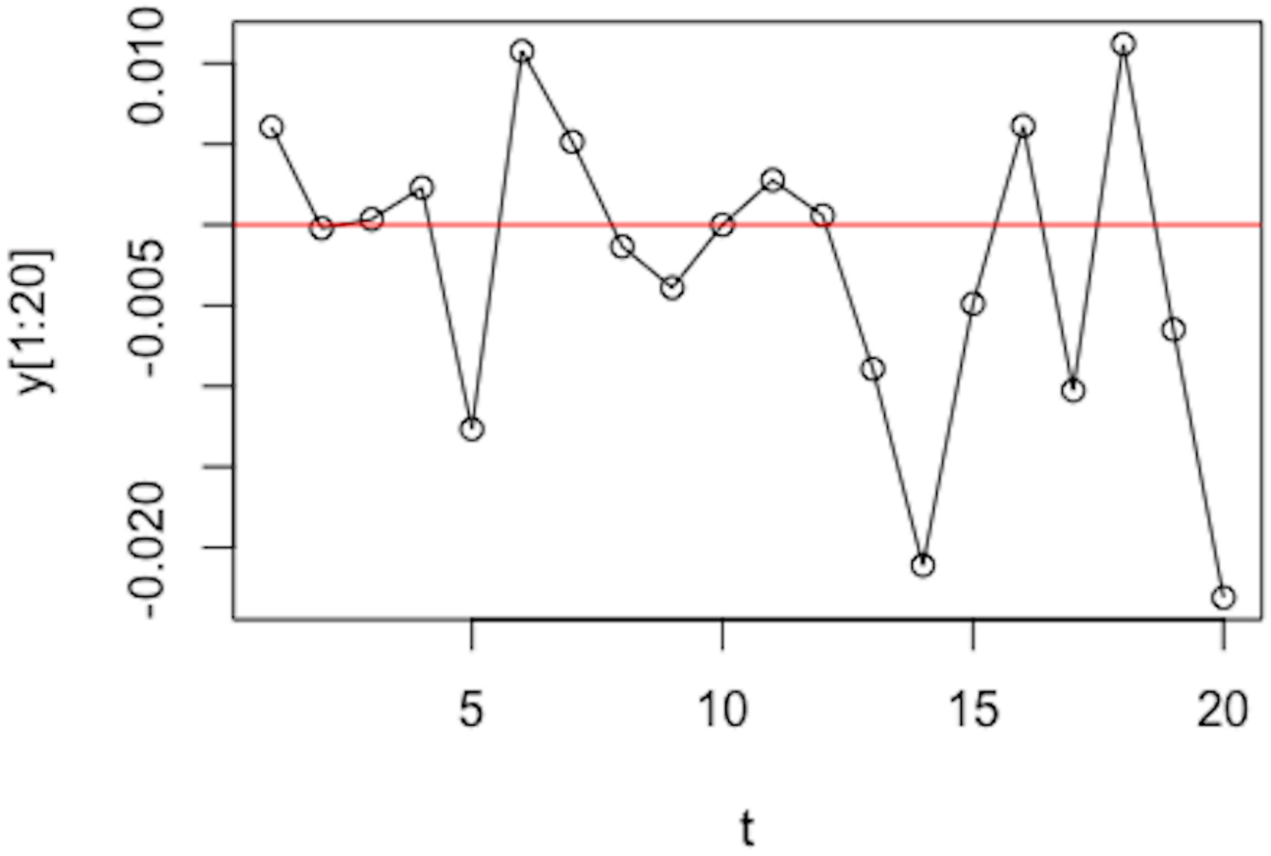
$$\min_{\boldsymbol{\beta}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\| + \lambda \sum_{j=1}^{p} |\beta_j|$$

ELASTIC NET REGRESSION

$$\min_{\boldsymbol{\beta}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\| + \lambda \sum_{j=1}^{p} (\alpha\beta_j^2 + (1-\alpha)|\beta_j|)$$

DATASET ANALYSIS

DATASET ANALYSIS



What are we working on?

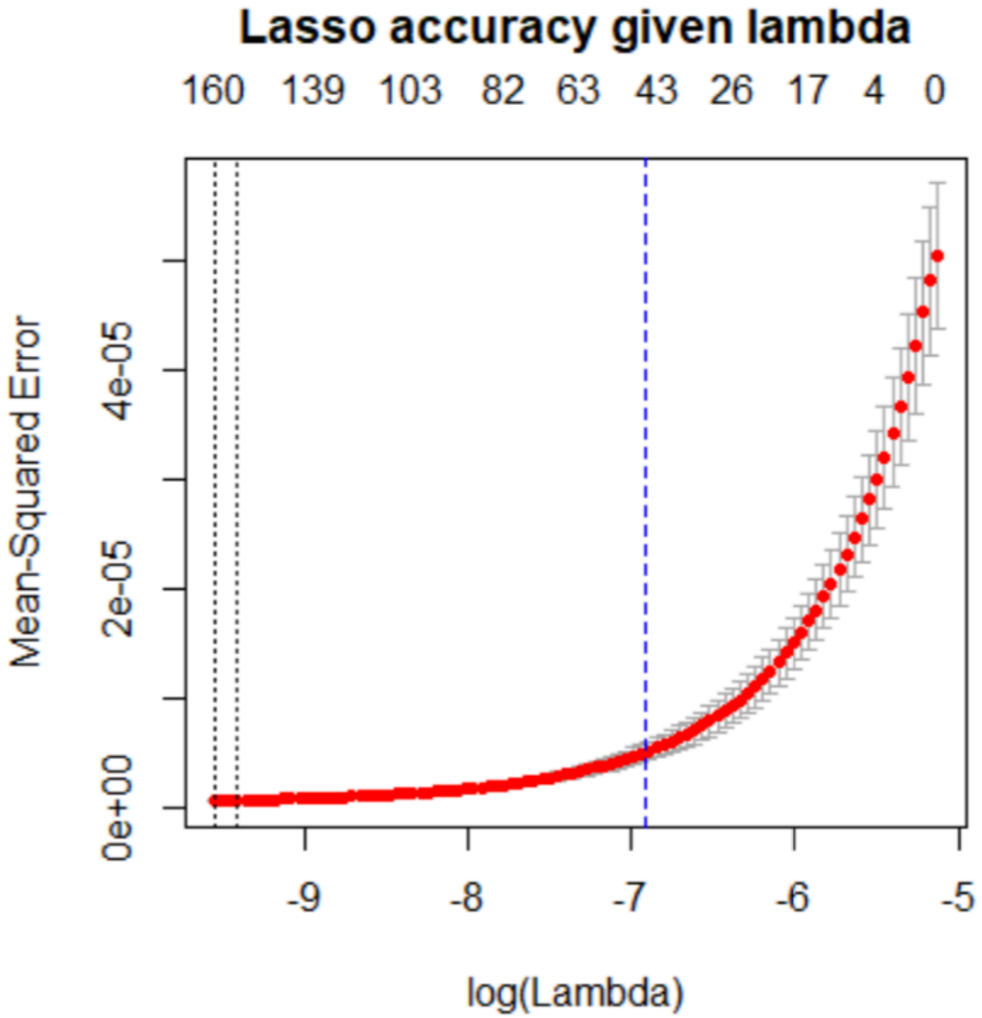
- Sp500 index
- Daily log returns

$$\log(y_{t+1}) - \log(y_t) = \log\left(\frac{y_{t+1}}{y_t}\right)$$

Problems

- How to predict y_{t+1}?

PARSIMONY ⟷ ACCURACY



How to choose lambda?

Compromise between number of coefficients and prediction error

PARAMETERS ESTIMATION

Moving window approach

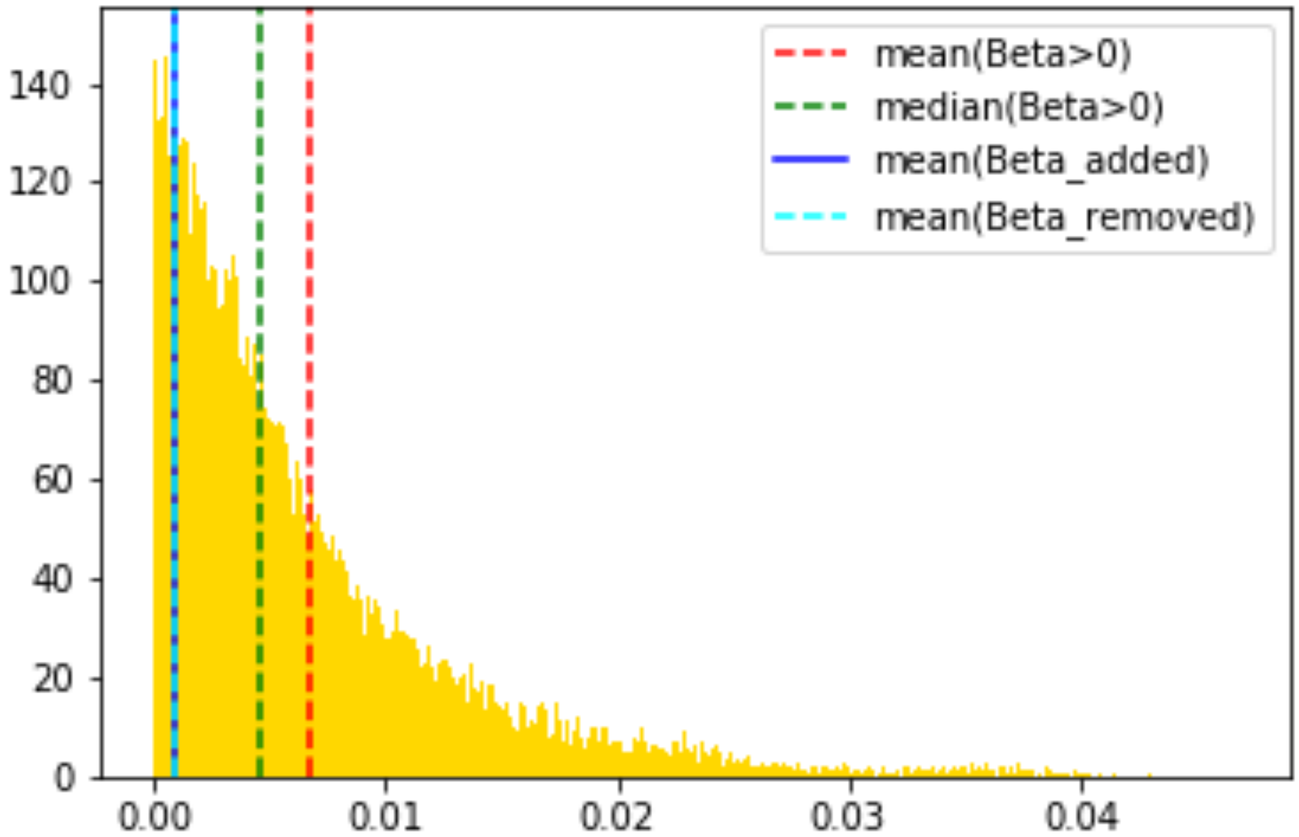
- Are beta coefficients time invariant?

Beta-parameters interpretation

$$\log(y_{t+1}) - \log(y_t) = \log\left(\frac{y_{t+1}}{y_t}\right)$$

Sp500	APPLE	MICROSOFT	AMAZON.COM	EXXON.MOBIL
0.006063	-0.007184	0.007720	0.011116	0.014049
-0.000212	0.006318	-0.018014	0.009726	-0.003270
0.000348	-0.012855	-0.006453	-0.002267	-0.009776
0.002304	-0.006692	0.014252	-0.008389	0.007589
-0.012656	0.005221	-0.029853	-0.016941	-0.019793
0.010760	0.019702	0.022613	0.016639	0.005767
0.005153	0.019879	0.027021	-0.004209	-0.003436
-0.001348	-0.005596	0.003530	-0.000177	0.001618
-0.003903	-0.024807	-0.013921	0.009580	0.002221

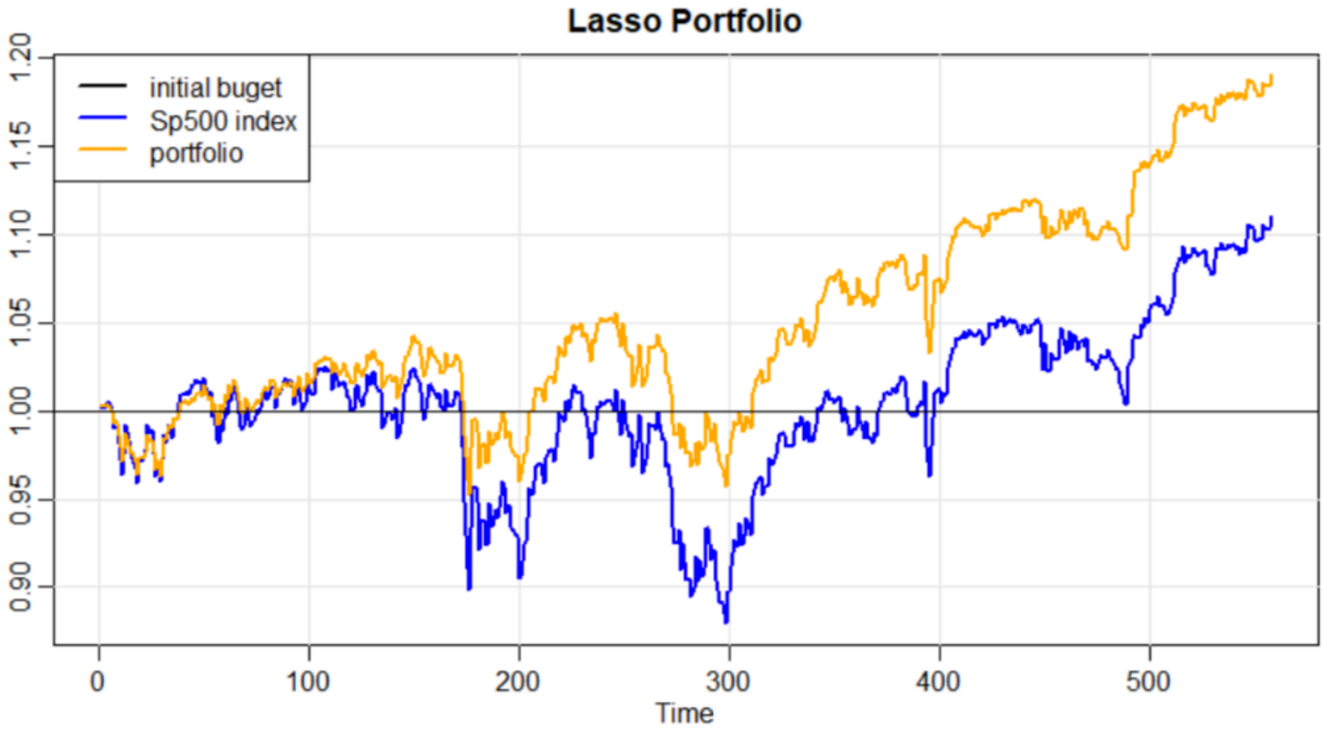
BETA TURNOVER



- **Low turnover (4%)**
- **Added and removed variables have low weights (12%)**

PASSIVE APPROACH

LASSO REGRESSION



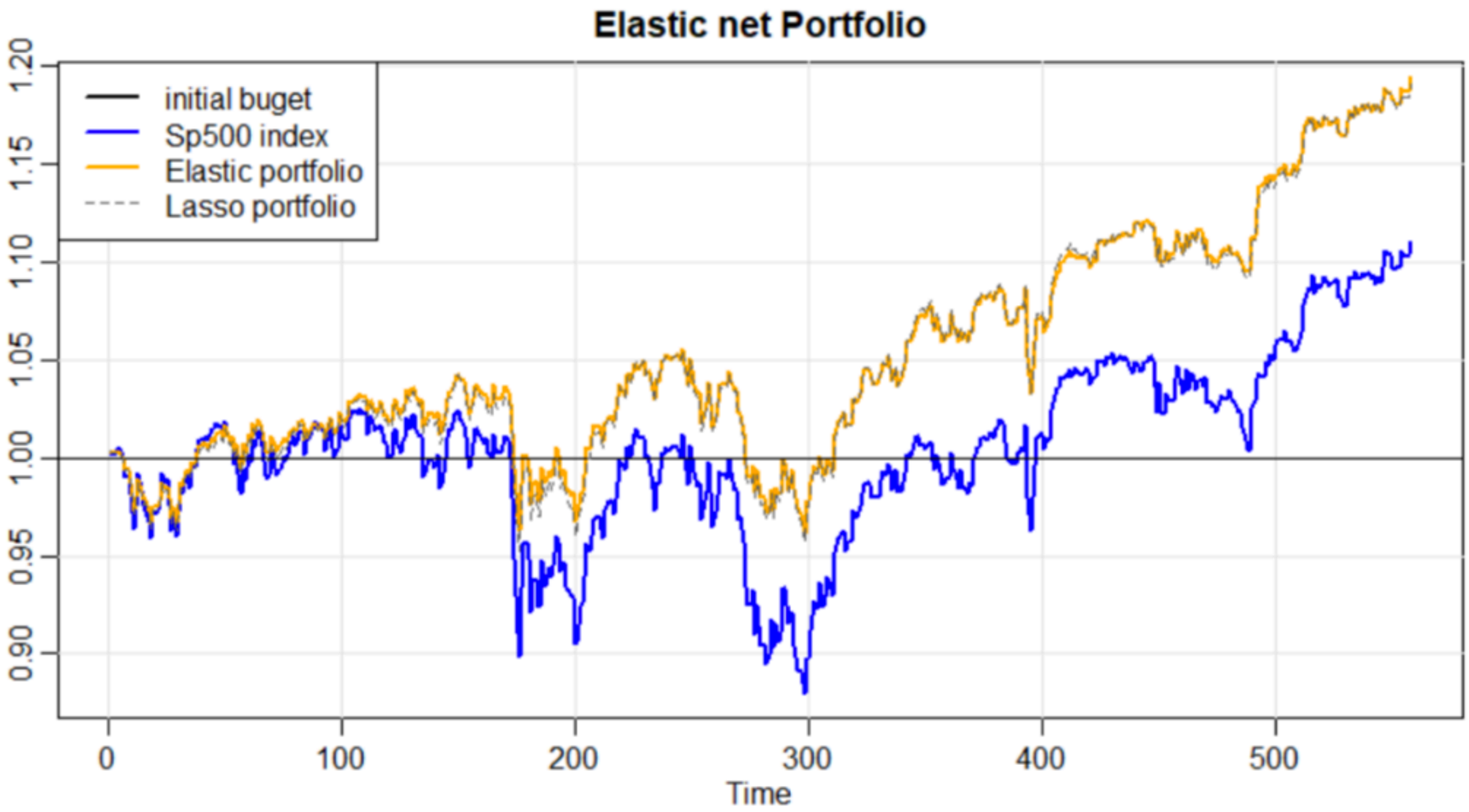
PROs:

- Low Fees
- Growing Index → Positive Returns

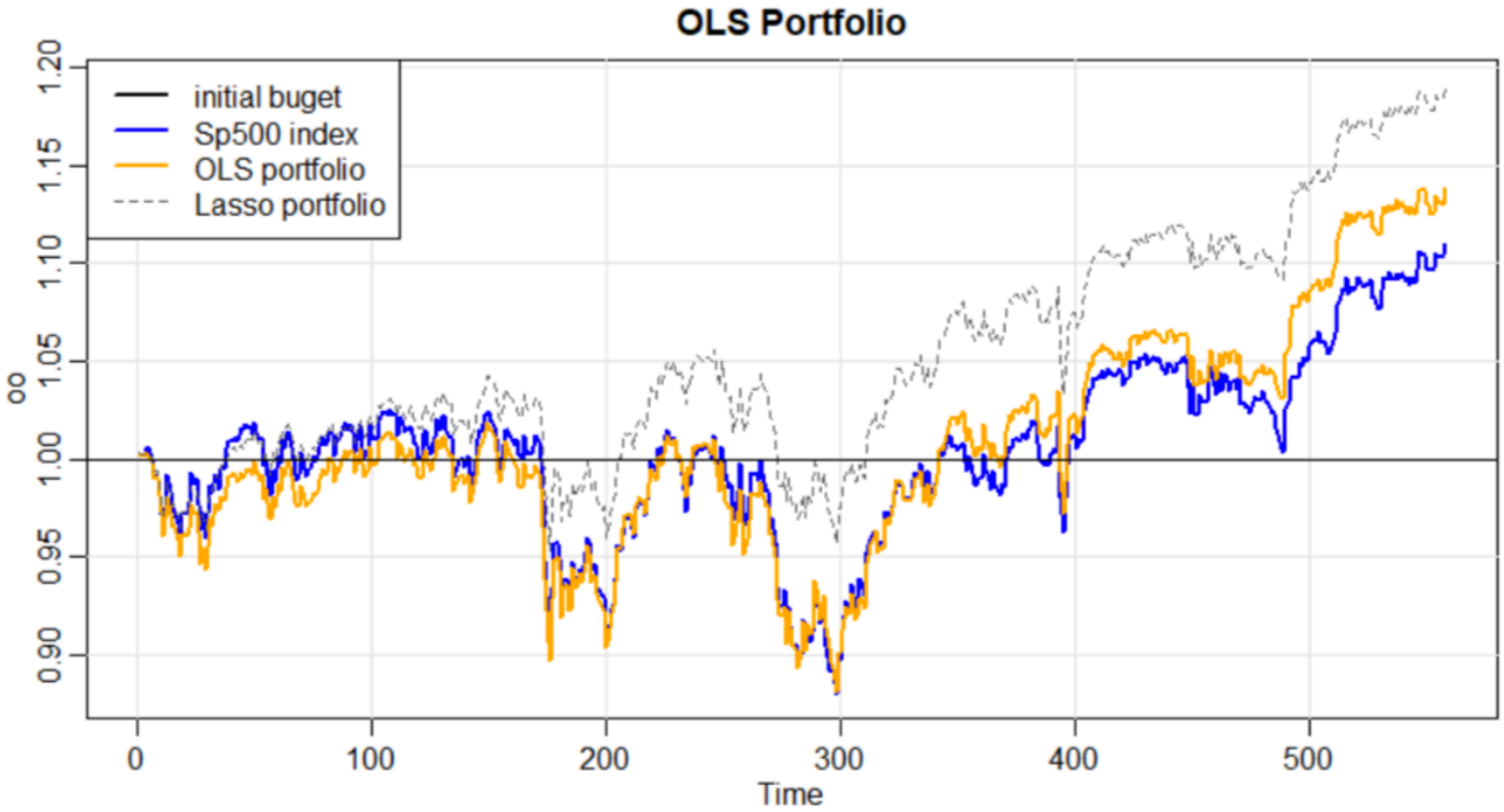
CONs:

- How often update?
- Descending Index → Losses

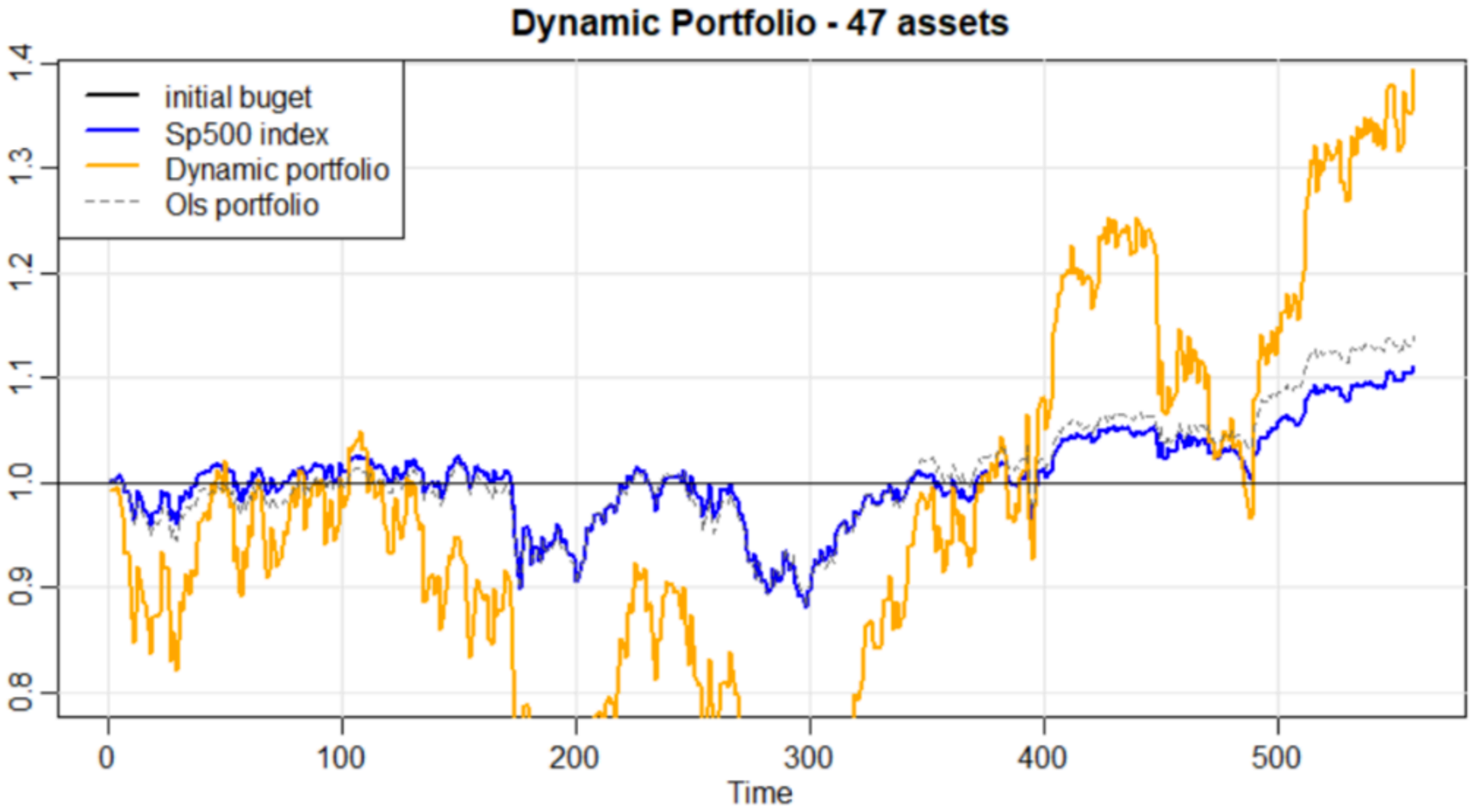
ELASTIC NET REGRESSION



OLS REGRESSION



DYNAMIC REGRESSION



ACTIVE APPROACH

ONE STEP AHEAD PREDICTION

Shifted windows approach

- Predictions are quite inaccurate
- Also time series approach not working

PROs:

- Financial Strategy
- Possible high revenue

CONs:

- High fees
- Possible huge losses even with growing index

Sp500	APPLE	MICROSOFT	AMAZON.COM	EXXON.MOBIL
0.006063	-0.007184	0.007720	0.011116	0.014049
-0.000212	0.006318	-0.018014	0.009726	-0.003270
0.000348	-0.012855	-0.006453	-0.002267	-0.009776
0.002304	-0.006692	0.014252	-0.008389	0.007589
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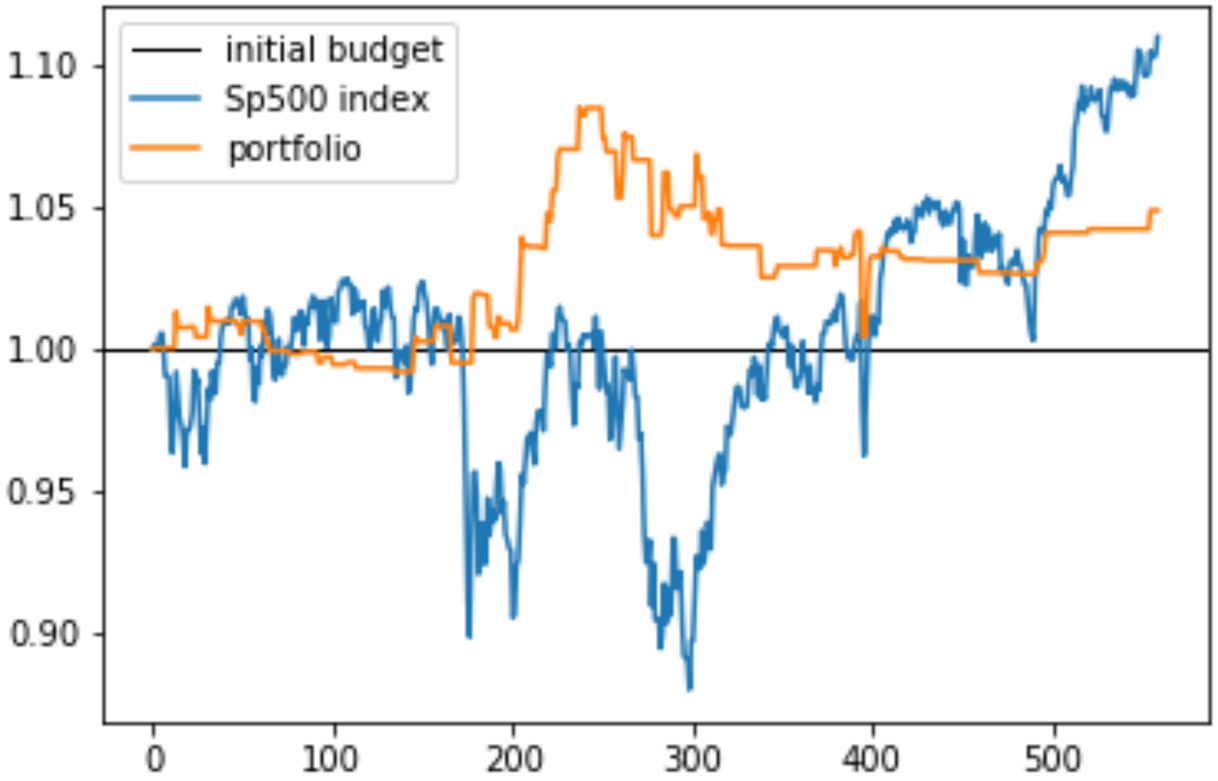
-0.003903	-0.024807	-0.013921	0.009580	0.002221

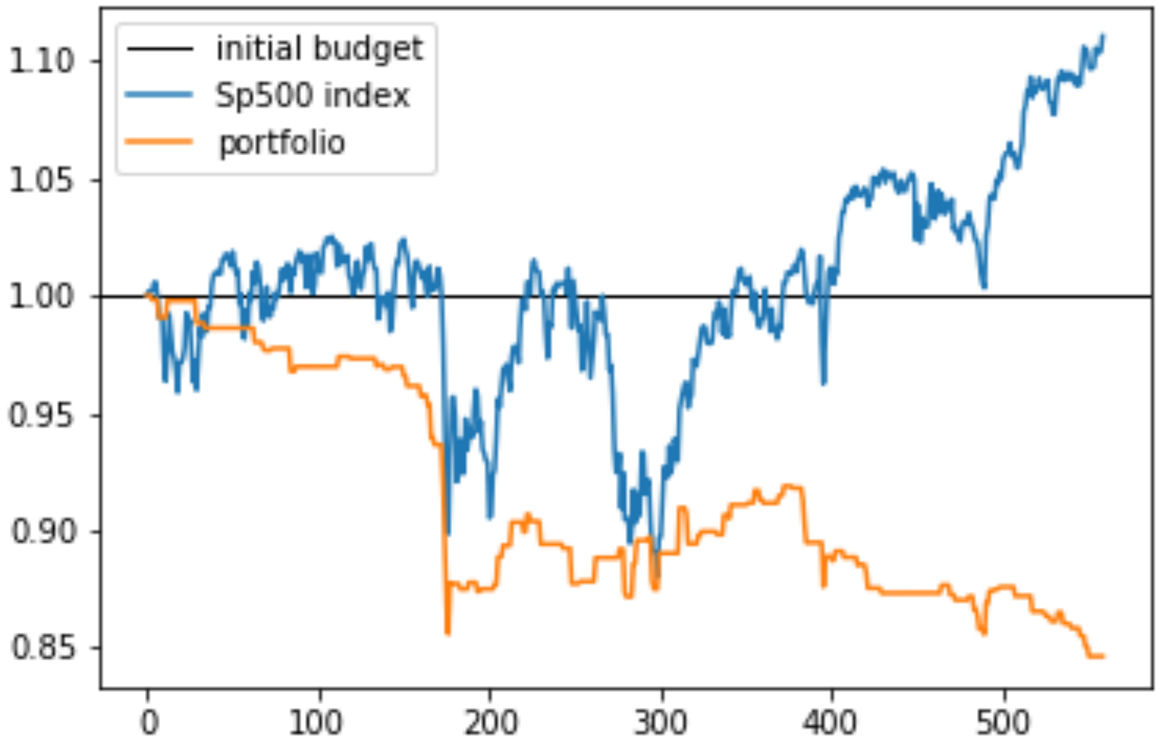
LASSO REGRESSION

	y	pred_y	real_gain
650	-0.008873	-0.000961	-0.007511
651	0.003337	-0.002482	0.006421
652	-0.002874	0.000829	-0.000308
653	0.013136	-0.001296	0.014291
654	0.001401	0.001277	0.003233
...	...	...	...
745	0.005451	0.001041	0.006650
746	-0.003586	-0.000771	-0.001410
747	0.002663	-0.001345	0.003020
748	-0.004392	-0.000360	-0.000268
749	0.005432	-0.001891	0.005809



LOGISTIC REGRESSION

	y	y_0/1	pred_y	real_gain
250	-0.000257	0.0	0.0	-0.000958
251	-0.002141	0.0	0.0	-0.001480
252	-0.002002	0.0	1.0	-0.001816
253	0.001302	1.0	0.0	-0.000422
254	-0.001141	0.0	0.0	-0.000995
...	...	...	...	...
803	0.005237	1.0	0.0	0.004295
804	-0.004120	0.0	0.0	-0.003826
805	-0.001775	0.0	0.0	-0.002612
806	-0.001309	0.0	1.0	-0.000020
807	0.003734	1.0	0.0	0.004221



CONCLUSIONS

- **Portfolio replication works quite well**
- **Passive approach performs better than active**
- **We need a better prediction model to exploit active approach**