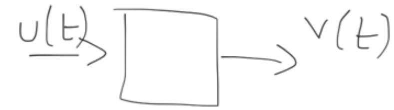


$$\ddot{v}(t) - 4v(t) = 2\dddot{u}(t) + \dot{u}(t) - u(t)$$

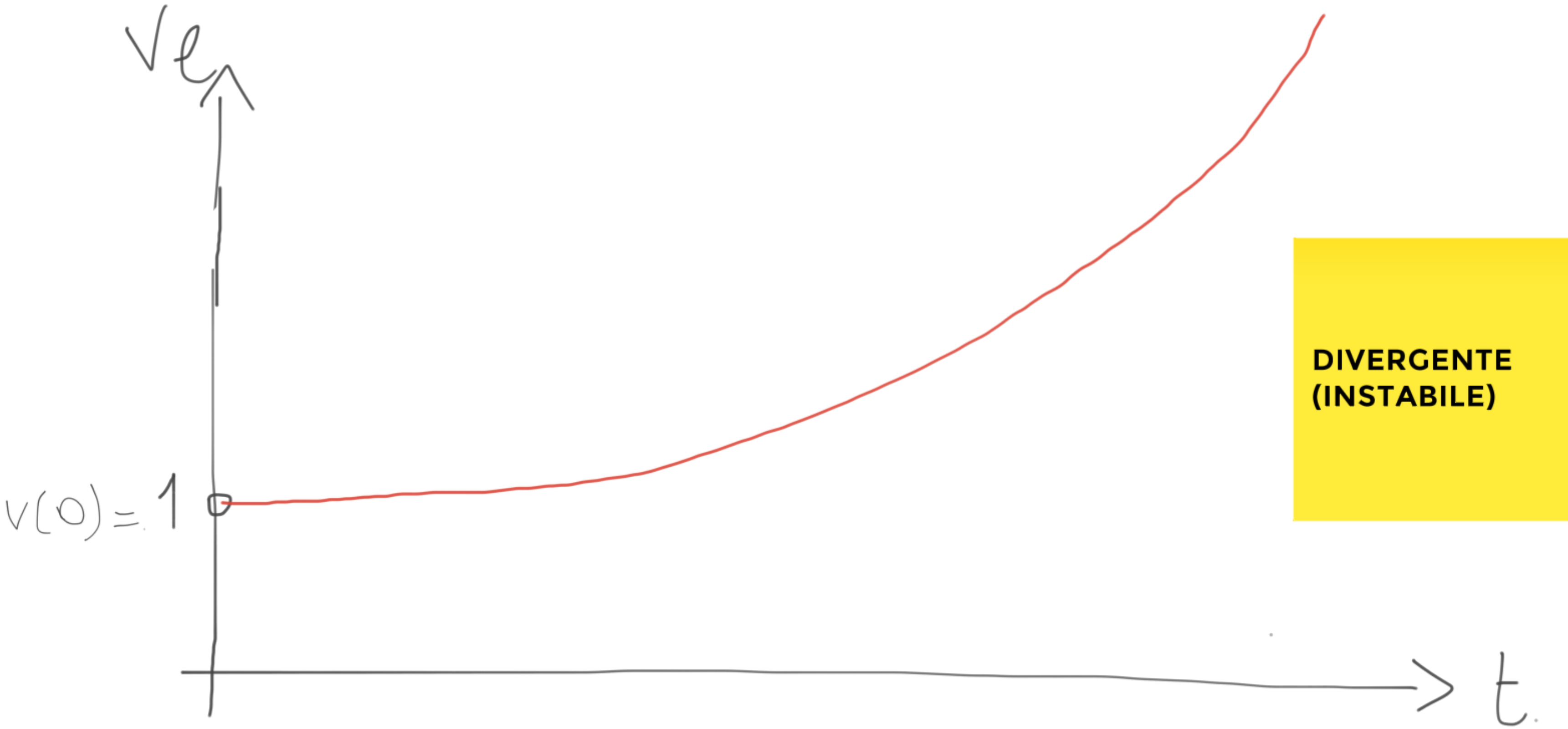
RISPOSTA LIBERA

$$\begin{cases} v(0) = 1 \\ \dot{v}(0) = 0 \end{cases}$$

$$s^2 - 4 = 0 \Rightarrow s = \pm 2$$

$$\begin{cases} v_\ell(t) = A e^{-2t} + B e^{2t} \\ \dot{v}_\ell(t) = -2Ae^{-2t} + 2Be^{2t} \end{cases} \Rightarrow \begin{cases} A + B = 1 \\ 2B - 2A = 0 \end{cases} \Rightarrow A = B = \frac{1}{2}$$

$$V_e(t) = \frac{1}{2}\left(e^{-2t} + e^{2t}\right)$$



$$\ddot{v}(t) - \dot{v}(t) + v(t) = 4\,u(t)$$

RISPOSTA LIBERA

$$\begin{cases} v(0) = -1 \\ \dot{v}(0) = 1 \end{cases}$$

$$\ddot{v}(t) - \dot{v}(t) + v(t) = 0 \implies s^2 - s + 1 = 0 \implies s_{1/2} = \frac{1 \pm j\sqrt{3}}{2}$$

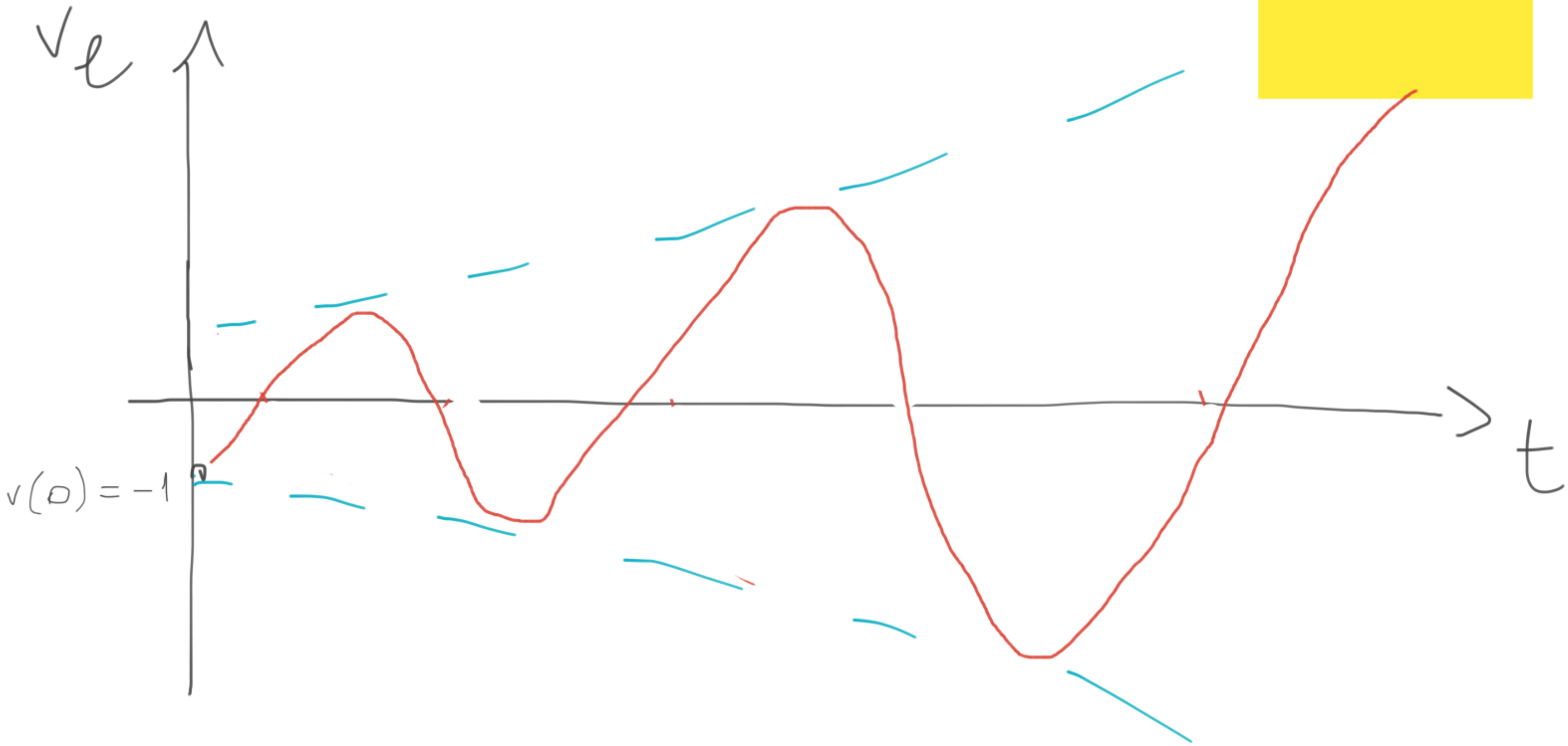
$$\begin{cases} v_e(t) = e^{t/2}\left(A\cos\frac{\sqrt{3}}{2}t + B\sin\frac{\sqrt{3}}{2}t\right) \\ \dot{v}_e(t) = \frac{1}{2}e^{t/2}\left(A\cos\frac{\sqrt{3}}{2}t + B\sin\frac{\sqrt{3}}{2}t\right) + e^{t/2}\frac{\sqrt{3}}{2}\left(-A\sin\frac{\sqrt{3}}{2}t + B\cos\frac{\sqrt{3}}{2}t\right) \end{cases}$$

$$\Downarrow$$

$$\begin{cases} A = -1 \\ \frac{1}{2}A + \frac{\sqrt{3}}{2}B = 1 \end{cases} \implies A = -1,\ B = \sqrt{3} \implies v_e(t) = e^{t/2}\left(-\cos\frac{\sqrt{3}}{2}t + \sqrt{3}\sin\frac{\sqrt{3}}{2}t\right)$$

$$v_e(t) = e^{t/2}\left(-\cos\frac{\sqrt{3}}{2}t + \sqrt{3}\sin\frac{\sqrt{3}}{2}t\right)$$

OSCILLATORE DIVERGENTE



$$\dddot{v}(t) + 3\ddot{v}(t) + 3\dot{v}(t) + v(t) = 5u(t)$$

RISPOSTA LIBERA

$$\begin{cases} v(0) = 2 \\ \dot{v}(0) = 0 \\ \ddot{v}(0) = 0 \end{cases}$$

n condizioni iniziali

TEOREMA FONDAMENTALE DELL'ALGEBRA

$$s^3 + 3s^2 + 3s + 1 = 0 \implies (s+1)^3 = 0 \implies s_{1/2/3} = -1$$

MOLTEPLICITÀ ALGEBRICA

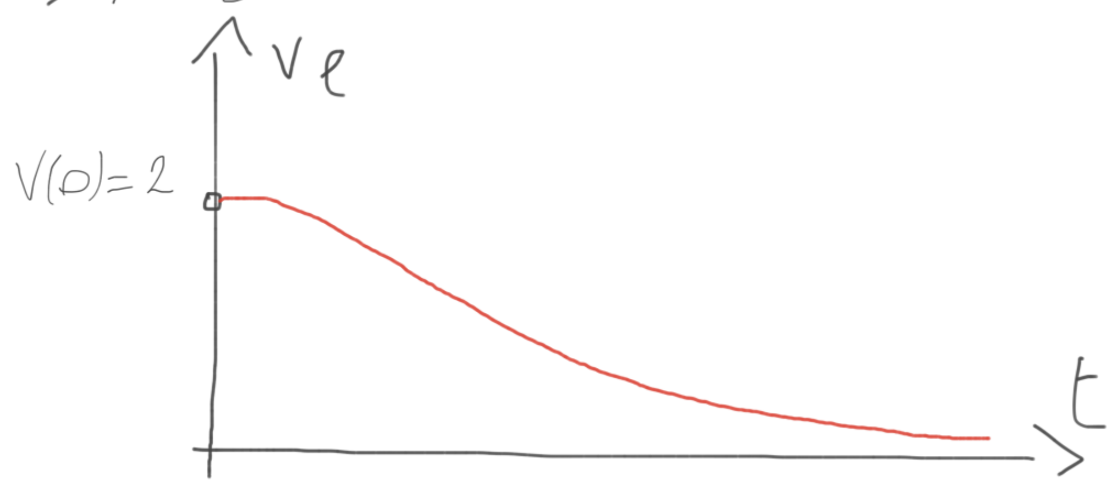
$$v_e(t) = Ae^{-t} + Bte^{-t} + C\frac{t^2}{2}e^{-t}$$

$$\left(\rho_s = n \implies e^{st} + te^{st} + \frac{t^2}{2}e^{st} + \ldots + \frac{t^{n-1}}{(n-1)!}e^{st}\right)$$

$$\begin{cases} v_e(t) = Ae^{-t} + Bte^{-t} + C\frac{t^2}{2}e^{-t} \\ \dot{v}_e(t) = -Ae^{-t} + Be^{-t} - Bte^{-t} + Cte^{-t} - C\frac{t^2}{2}e^{-t} = (B-A)e^{-t} + (C-B)te^{-t} - C\frac{t^2}{2}e^{-t} \\ \ddot{v}_e(t) = (A-B)e^{-t} + (C-B)e^{-t} - (C-B)te^{-t} - Cte^{-t} + C\frac{t^2}{2}e^{-t} = (A+C-2B)e^{-t} - (2C-B)te^{-t} + C\frac{t^2}{2}e^{-t} \end{cases}$$

$$\begin{cases} v_e(0) = 2 \\ \dot{v}_e(0) = 0 \\ \ddot{v}_e(0) = 0 \end{cases} \Rightarrow \begin{cases} A = 2 \\ B - A = 0 \\ A + C - 2B = 0 \end{cases} \Rightarrow A = B = C = 2$$

$$v_e(t) = 2e^{-t}\left(1 + t + \frac{t^2}{2}\right)$$



$$3\dddot{v}(t) + 2\ddot{v}(t) + 6\dot{v}(t) + 4v(t) = \dddot{u}(t) + 5\dot{u}(t) + 2u(t)$$

RISPOSTA LIBERA

$$\begin{cases} v(0) = 1 \\ \dot{v}(0) = 1 \\ \ddot{v}(0) = 0 \end{cases}$$

$$3s^3 + 2s^2 + 6s + 4 = 0 \Rightarrow (3s+2)s^2 + 2(3s+2) = 0 \Rightarrow s_1 = -\frac{2}{3},\ s_{2/3} = \pm j\sqrt{2}$$

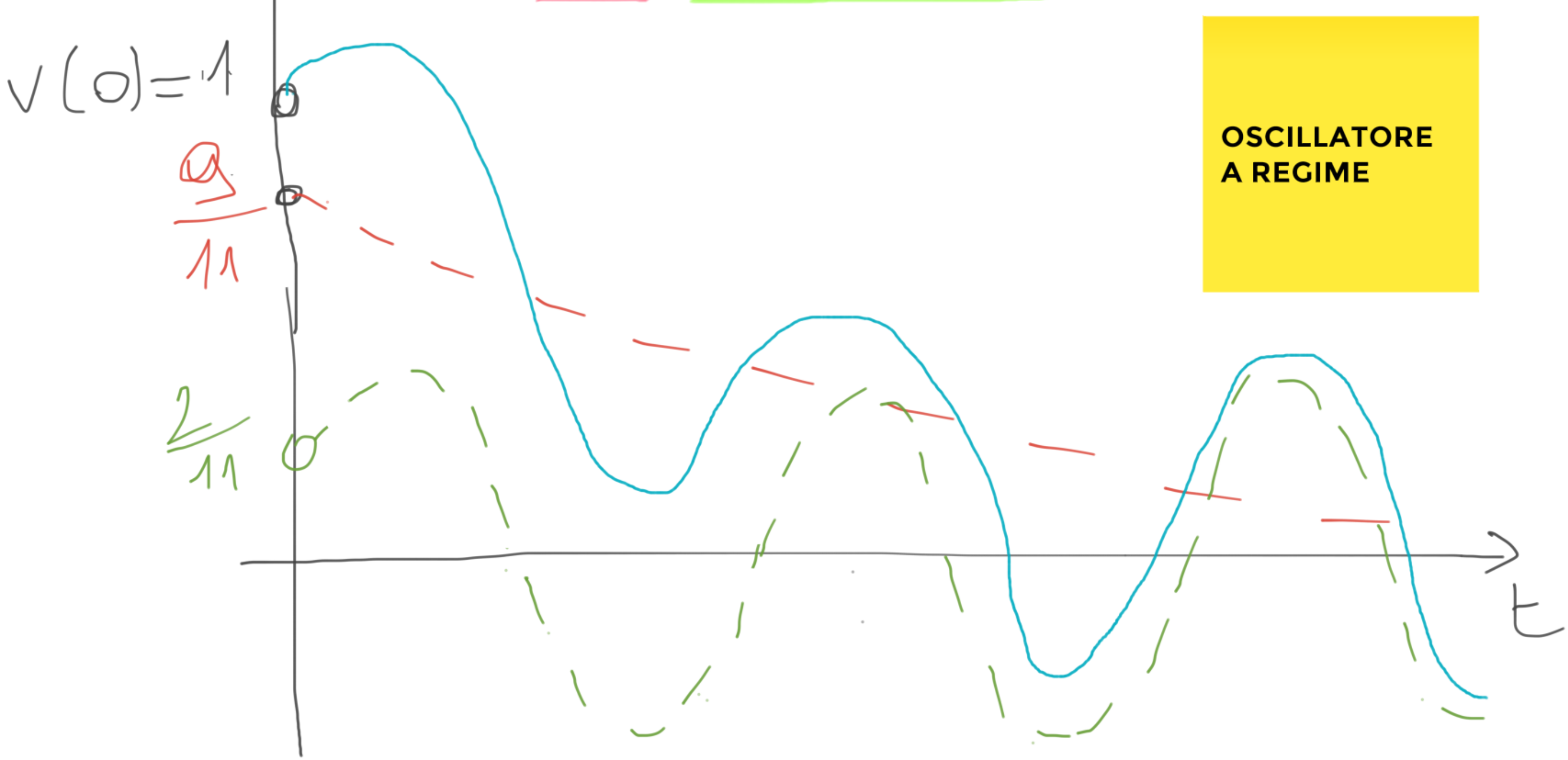
$$v_e(t) = Ae^{-\frac{2}{3}t} + B\cos\sqrt{2}t + C\sin\sqrt{2}t$$

$$\begin{cases} v_e(t) = Ae^{-\frac{2}{3}t} + B\cos\sqrt{2}t + C\sin\sqrt{2}t \\ \dot{v}_e(t) = -\frac{2}{3}Ae^{-2/3t} - \sqrt{2}B\sin\sqrt{2}t + \sqrt{2}C\cos\sqrt{2}t \\ \ddot{v}_e(t) = \frac{4A}{9}e^{-2/3t} - 2B\cos\sqrt{2}t - 2C\sin\sqrt{2}t \end{cases}$$

$$\begin{cases} v(0) = 1 \\ \dot{v}(0) = 1 \\ \ddot{v}(0) = 0 \end{cases}$$

$$\begin{cases} A + B = 1 \\ -\frac{2}{3}A + \sqrt{2}C = 1 \\ \frac{4}{9}A - 2B = 0 \Rightarrow A = \frac{9}{2}B \end{cases} \Rightarrow \begin{cases} B = \frac{2}{11} \\ C = \frac{17}{11} \cdot \frac{1}{\sqrt{2}} = \frac{17\sqrt{2}}{22} \\ A = \frac{9}{11} \end{cases}$$

$$V_e(t) = \frac{9}{11} e^{-\frac{2}{3}t} + \frac{2}{11}\cos\sqrt{2}t + \frac{17\sqrt{2}}{22}\sin\sqrt{2}t$$



OSCILLATORE A REGIME