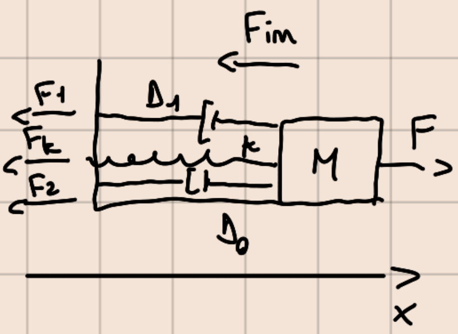
MODELLO MECCANICO

MKD --> Massa - Molla - Smorzatore



Forze

$F, \; F_1 \cdot D_1, \; F_2 \cdot D_2, \; F_k \cdot k, \; F_{im} \cdot M$

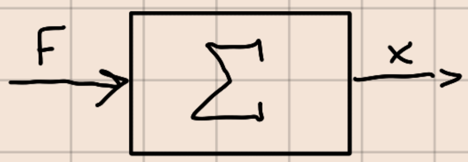
$F_{im} = M\ddot{x}$

$F_k = kx$

$$\left.\begin{aligned} F_1 &= D_1\dot{x} \\ F_2 &= D_2\dot{x} \end{aligned}\right\} F_D = (D_1 + D_2)\dot{x}$$

M spostandosi cambia posizione, velocità e accelerazione

$x, \dot{x}, \ddot{x}$



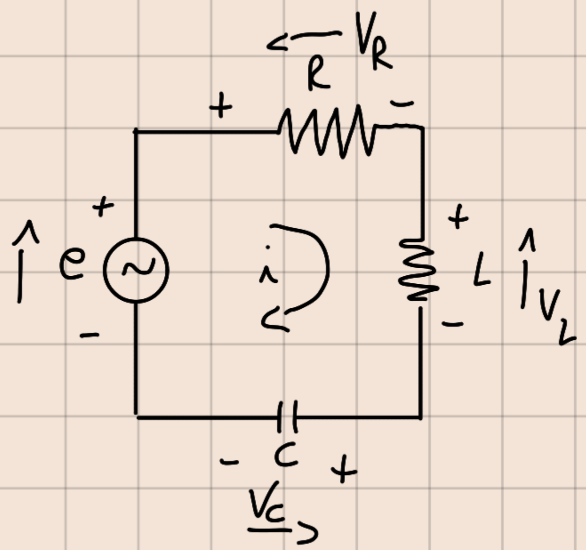
$F - F_D - F_k = F_{im}$ si dividono input con output

output input

$M\ddot{x} + \dot{x}(D_1 + D_2) + kx = \cancel{F} \to 0$ $n = 2$ $m = 0$

$$\sum_{i=0}^{n} a_i \frac{d^i y(t)}{dt^i} = \sum_{j=0}^{m} b_j \frac{d^i u(t)}{dt}$$

MODELLO ELETTRICO



$V_R = R\,i(t)$

$V_L = L\dfrac{di(t)}{dt}$

V_C

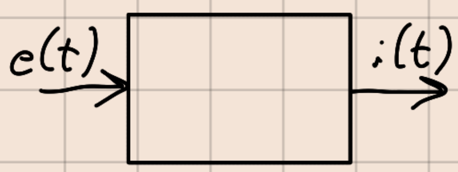
$\to i(t) = C\dfrac{dV_C}{dt}$

$V_C = \dfrac{1}{C}\displaystyle\int_0^t i(\tau)\,d\tau$

$i(t)$ + --> -

$V_i(t)$ - - -> +

cadute di tensione



derivate

$i(t), \dot{i}(t), \ddot{i}(t)$

$$L\,\dot{i}(t)$$

$$e(t) = R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(\tau)\,d\tau$$

derivo per togliere integrale e sistemo ordine

$$L\,\ddot{i}(t) + R\,\dot{i}(t) + \frac{1}{C}\,i(t) = \cancel{\dot{e}(t)} \qquad n=2 \quad m=1$$

0 perchè è una risposta libera

Condizioni iniziali = ordine dell'equazione

Condizioni iniziali

$n = 2$

$D_1 + D_2 = D = 3$

$k = 1$

$x(0) = 1$

$\dot{x}(0) = 0$

$$2\ddot{x}(t) + 3\dot{x}(t) + x(t) = 0$$

1) Polinomio caratteristico

$$s \quad t.c. \quad \frac{d^n}{dt^n} \mapsto s^n$$

$\boxed{\text{1) radici distinte e reali}}$

$$2s^2 + 3s + 1 = 0 \qquad s_1 = \lambda_1 = -\frac{1}{2}$$

$$s_2 = \lambda_2 = -1$$

AS => BIBO

$r = 2 \to$ n° radici

$\underbrace{\mu_1 = 1 \quad \mu_2 = 1}_{}$

molteplicità algebrica delle radici, compaiono 1 sola volta

$$v(t) = \sum_{i=1}^{r} \sum_{\ell=0}^{\mu_i - 1} c_{i,\ell} \, e^{\lambda_i t} \frac{t^\ell}{\ell!}$$

$$v_\ell(t) = \underbrace{c_1}_{A} e^{\lambda_1 t} + \underbrace{c_2}_{B} e^{\lambda_2 t}$$

$$= A e^{-\frac{1}{2}t} + B e^{-t}$$

Per trovare A e B occorre sfruttare le condizioni iniziali

C.I.

$$\begin{cases} v_\ell(0) = x(0) = Ae^0 + Be^0 = A+B = 1 \\ \dot{v}_\ell(0) = \dot{x}(0) = \qquad = -\frac{1}{2}A - B = 0 \end{cases}$$

$A=2$

$B=-1$

sostituisco $t=0$

$$\dot{v}_\ell(t) = -\frac{1}{2}Ae^{-\frac{3}{2}t}(-1)Be^{-t}$$

Si può riscrivere $v_\ell(t)$ con i valori di A e B

$$v_\ell(t) = 2e^{-\frac{1}{2}t} - e^{-t}$$ ✓

Sverse C.I.

$M = 1$

$D = 4$ $\qquad \ddot{x}(t) + 4\dot{x}(t) + 4x(t) = 0$

$k = 4$

2) Radici reali coincidenti

$s^2 + 4s + 4 = 0 = (s+2)^2$

$s_1 = s_2 = \lambda_1 = \lambda_2 = -2$

$r = 1$

$\mu_i = 2$

modi elementari

AS => BIBO

$$v_\ell(t) = Ae^{-2t}\left(\frac{t^0}{0!}\right) + Be^{-2t}\left(\frac{t^1}{1!}\right) =$$

$$= Ae^{-2t} + Bte^{-2t} = e^{-2} + 2te^{-2t}$$

$$\begin{cases} v_\ell(0) = x(0) = Ae^0 + B\cdot 0 \cdot e^0 = A = 1 \\ \dot{v}_\ell(0) = \dot{x}(0) = -2A + B = 0 \end{cases}$$

$A = 1$

$B = 2$

$$\dot{v}_\ell(t) = -2Ae^{-2t} + B\cdot 1 \cdot e^{-2} - 2Bte^{-2t}$$

$M = 1$ Sistema C.1.

$D = 2$ $\ddot{x}(t) + 2\dot{x}(t) + 3x(t) = 0$

$k = 3$

3) Radice complesse coniugate

$$s^2 + 2s + 3 = 0 \rightarrow \Delta < 0 \rightarrow 4 - 4(3) = -8 = 8i^2$$

$$\lambda_1 = -1 - i\sqrt{2} \qquad AS \Rightarrow BIBO$$
$$\lambda_2 = -1 + i\sqrt{2}$$

$$v_\ell(t) = Ae^{(-1-\sqrt{2}i)t} + Be^{(-1+\sqrt{2}i)t} \rightarrow \text{formula di Eulero}$$

$$e^{(\alpha \pm J\omega)} = e^{\alpha}(\cos\omega t \pm J\sin\omega t)$$

$$\begin{cases} v_\ell(0) = x(0) = Ae^0 + Be^0 = A + B = 1 \\ \dot{v}_\ell(0) = \dot{x}(0) = (-1-\sqrt{2}i)A + (-1+\sqrt{2}i)B = 0 \end{cases} \quad \begin{cases} A = \frac{1}{2} + \frac{1}{4}\sqrt{2}i \\ B = \frac{1}{2} - \frac{1}{4}\sqrt{2}i \end{cases}$$

$$\dot{v}_\ell(t) = (-1-\sqrt{2}i)Ae^{(-1-\sqrt{2}i)t} + (-1+\sqrt{2}i)Be^{(-1+\sqrt{2}i)t}$$

[Conviene derivare come al solito e poi sostituire A e B tramite formula di Eulero]

$$v_\ell(t) = \left(\frac{1}{2} + \frac{\sqrt{2}}{4}i\right)e^{-t}\left(\cos(\sqrt{2}t) - i\sin(\sqrt{2}t)\right) + \left(\frac{1}{2} - \frac{\sqrt{2}}{4}i\right)e^{-t}\left(\cos(\sqrt{2}t) + i\sin(\sqrt{2}t)\right)$$

$$e^{(-1-\sqrt{2}i)t} = e^{-1}\left(\cos(\sqrt{2}t) - i\sin(\sqrt{2}t)\right)$$

$\alpha = -1$

$J = 1$

$\omega = \sqrt{2}$

$$= e^{-t}\left(\cos(\sqrt{2}t) + \frac{\sqrt{2}}{2}\sin(\sqrt{2}t)\right)$$

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stesse C.I.

4) Radici pure immaginarie

$M = 1$

$D = 0$ $\qquad \ddot{x}(t) + 3x(t) = 0$

$k = 3$

$s^2 + 3 = 0 \qquad s_1 = \lambda_1 = \sqrt{3}i \qquad$ AS instabile

$s_2 = \lambda_2 = -\sqrt{3}i$

esempio

$\left.\begin{matrix} -2+\sqrt{3}i \\ -\sqrt{3} \end{matrix}\right\}$ stabile $\quad \forall i \; Re(\lambda_i) < 0$ e se

$\exists \lambda_i$ t.c. $Re(\lambda_i) = 0$

$\Rightarrow \mu(\lambda_i) = 1$

$$v_\ell(t) \overset{*}{=} Ae^{\sqrt{3}it} + Be^{-\sqrt{3}it}$$

$$\dot{v}_\ell(t) = (\sqrt{3}it)Ae^{(\sqrt{3}it)} + (-\sqrt{3}i)B^{(-\sqrt{3}it)}$$

$$\begin{cases} v_\ell(0) = x(0) = A + B = 1 \\ \dot{v}_\ell(0) = \dot{x}(0) = (\sqrt{3}i)A - (\sqrt{3}i)B = 0 \end{cases} \qquad \begin{matrix} A = 1/2 \\ B = 1/2 \end{matrix}$$

$$e^{\pm\sqrt{3}it} = e^{0}\left(\cos(\sqrt{3}t) \pm i\sin(\sqrt{3}t)\right)$$

$\alpha = 0$

$J = i$

$\omega = \sqrt{3}$

$$v_\ell(t) = \frac{1}{2}\left(\cos(\sqrt{3}t) + i\sin(\sqrt{3}t) + \frac{1}{2}(\cos(\sqrt{3}t) - i\sin(\sqrt{3}t))\right)$$

$$= \cos(\sqrt{3}t)$$