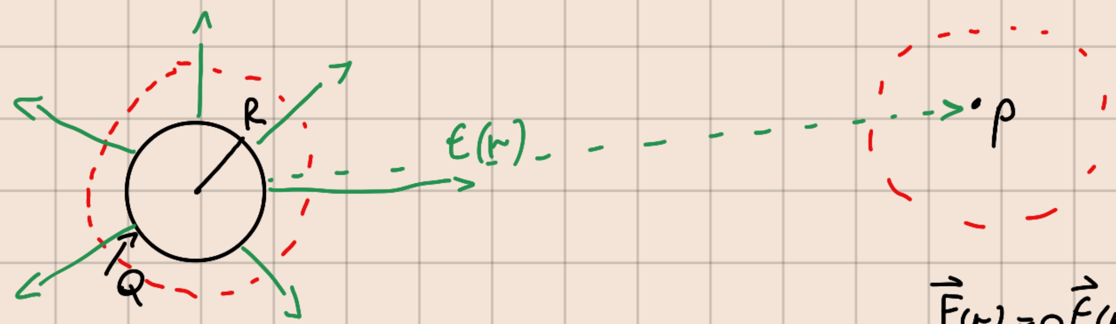
Esercizio T. Gauss nei dielettrici (calcolo $\vec{E}$)



$\vec{F}(r)_{su\ q} = q\vec{E}(r)$ [N]

$L_{r_q \to r_{finale}} = -q\left(V(r_p) - V(r_q)\right)$ [J]

conduttore → $\sigma = \underbrace{\frac{Q}{4\pi R^2}}_{sup.} = \left[C/m^2\right]$

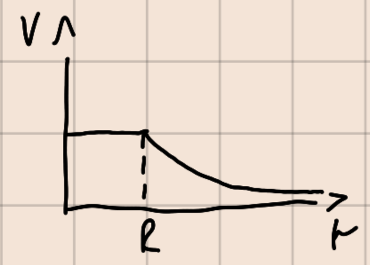
$\oint_{sup} E\,dS = \frac{Q}{\varepsilon_0}$ $\left[V/m\right]$ → $\vec{E}_{nel\ vuoto} = \frac{Q}{4\pi\varepsilon_0 r^2}$ $\quad r \geq R$

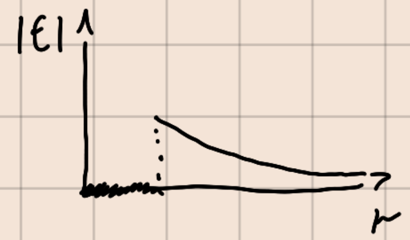
$V_r - V_{RIF\,\infty} = -\int_\infty^r E(r)\,dr$ * $[V]$ → integro lungo radiale perché campo conservativo

$C = \frac{Q}{V_{sup.\ del\ conduttore}}$ $[F]$ → $4\pi\varepsilon_0 R$

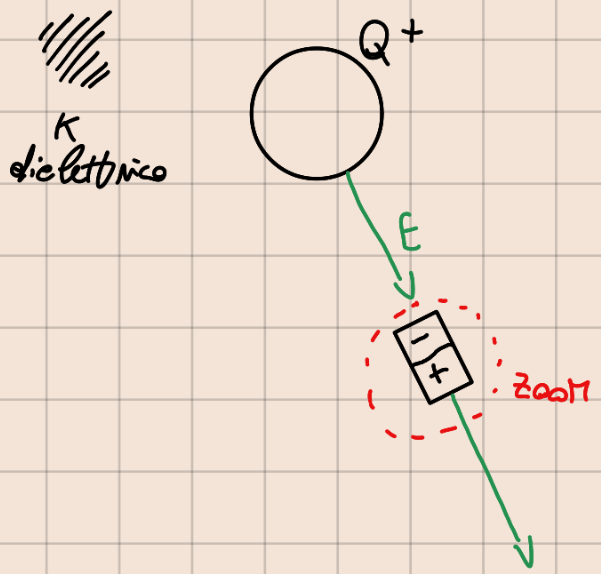
→ non è un condensatore

* $V(r) = \frac{Q}{4\pi\varepsilon_0 r}$ $\quad r \geq R \qquad V_{COND} = costante \quad r < R$





STESSA SITUAZIONE MA CON DIELETTRICO AGGIUNTO



GAUSS NEI DIELETTRICI

$$\oint \vec{D}\cdot\hat{n}\, dS = Q_{LIBERE}$$

$$\downarrow$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

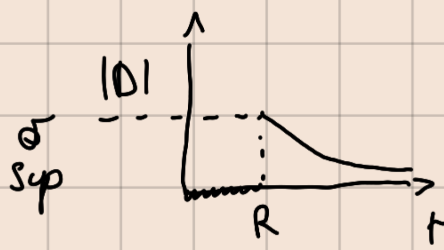
$$\downarrow$$

$\vec{P} = qd$ momento di dipolo

$$\vec{P} = \varepsilon_0 \chi \vec{E}$$

$$K = 1 + \chi$$

$$\oint_{S(r)} D(r)\, dS = D(r) 4\pi r^2 = Q_{LIBERE} \begin{cases} 0 & r<R \\ Q & r \geq R \end{cases} \Rightarrow \vec{D}(r) = \frac{Q}{4\pi r^2}\hat{r}$$



$$\vec{D} = \varepsilon_0 k \vec{E}_{TOT} \to E = \frac{D}{\varepsilon_0 k} = \frac{Q}{4\pi\varepsilon_0 r^2 k} < E_{VUOTO} \quad \left[V/m\right]$$

$$\vec{P} = \frac{k-1}{k}\vec{D} = \frac{k-1}{k}\frac{Q}{4\pi\varepsilon r^2} \left[C/m^2\right]$$

$$\sigma_{POL} = \vec{P}\hat{n}$$

$$\sigma_{POL}(r=R) = -\frac{k-1}{k}\frac{Q}{4\pi R^2} = -\frac{k-1}{k}\sigma^{+}_{LIBERE}$$

CAPACITÀ NEI DIELETTRICI

$$\underset{\text{CON DIELETTRICO}}{C} = k\, C_{vuoto} = k\, 4\pi \varepsilon_0 R \quad [F]$$