

## 1. How big of an array can I sort using merge-sort within one minute?

Merge sort runs in  $O(n \log n)$  time because at each “level” of recursion it does an  $O(n)$  merge, and there are  $\log n$  levels from repeatedly splitting the array in half.

A rough estimate of speed is that a program written in C or C++ would perform  $10^8$  high level operations such as addition, multiplication, comparison, reading memory, etc. in a second. Python, being an interpreted language, would be about 10 times slower ( $10^7$ ). Therefore, if I run this code on Python, in one minute I am able to do  $10^7 * 60$  high level operations, i.e.  $6 * 10^8$ , while in C or C++ I can do  $6 * 10^9$  high level operations.

So the equation to solve for Python is roughly  $n * \log_2(n) = 6 * 10^8$ . This equation cannot be solved algebraically in a simple closed form, but we can approximate the solution numerically to  $n = 2.44 * 10^7$  and so for C or C++ it is  $n = 2.44 * 10^8$ .

These are very rough ballpark figures and actual performance can vary significantly depending on hardware, compiler optimizations, memory speeds, and other implementation details.

So I proceed with an empirical research.

Empirical results in Python:

Sorting 1,000,000 elements took ~5 seconds on a local machine, and ~4.4–5 seconds on Google Colab.

Sorting 10,000,000 elements took ~65 seconds on Google Colab (very close to 1 minute).

Sorting 15,000,000 elements took ~100 seconds on Google Colab.

Sorting 24,400,000 elements took ~173 seconds on Google Colab.

On a local machine, attempting 24,400,000 elements caused memory constraints (swapping). This slowed the process considerably because the system had limited free RAM

Empirical results in C++:

Sorting 1,000,000 elements: ~0.14 seconds on a local machine, ~0.23 seconds on Google Colab.

Sorting 24,400,000 elements: ~4.26 seconds locally, ~6.73 seconds on Colab.

Sorting 244,000,000 elements: ~49.26 seconds locally, ~70.63 seconds on Colab.

Sorting 300,000,000 elements locally: ~67–86 seconds.

But Sorting 200,000,000 elements on Colab took ~57 seconds which is very close.

## 2. Gaussian elimination on an $n$ -by- $n$ matrix runs in time $O(n^3)$ . How long would it take to perform it on a matrix of size 2000x2000?

Gaussian elimination takes about  $O(n^3)$  time, so for a 2000 by 2000 matrix, that's roughly  $2000^3 = 8 \times 10^9$  operations. If Python handles about  $10^7$  operations per second, it would take around 800 seconds, which is over 13 minutes. In C or C++, which can do about  $10^8$  operations per second, it would take around 80 seconds. These are just rough estimates since actual speed depends on hardware and optimizations, but it gives a good idea of the difference.

So I conducted an empirical research.

Results in Python:

On a local machine, 2000x2000 elimination took ~363 seconds (~6 minutes).

On Google Colab, 2000x2000 elimination took ~268 seconds (~4.5 minutes).

This can be explained because not every “high-level” operation is equally expensive, some vectorized operations or hardware optimizations may come into play, real compilers/interpreters can do partial optimizations behind the scenes.

Results in C++:

A 2000×2000 matrix took ~0.80 seconds on a local machine—far faster than the naive 80-second estimate while on Cola bit took ~3.6 seconds.

Optimizations in the C++ compiler (like -O2) can greatly reduce overhead. CPU hardware instructions (e.g., vectorization, caching) make numeric loops more efficient than a simple “1 operation = 1 CPU cycle” approximation.