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This thesis is dedicated to all of you. Thank you.

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1. Introduction

The Capital Asset Pricing Model (CAPM) made its debut on the financial scene in the early 1960s by Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966). This model stands as one of the foundational pillars of modern finance theory, offering a framework to understand the relationship between risk and return in financial markets. Since its inception, the CAPM has been extensively studied and applied in various market contexts worldwide. However, the validity of its assumptions and the robustness of its predictions have been subject to ongoing debate and empirical scrutiny.

1.2 Objectives

In this thesis, the primary objective is to empirically test the CAPM framework using data from the FTSEMIB Index from 01.01.2018 to 12.31.2023. Specifically, the study aims to:

- Estimate beta coefficients for individual stocks listed on the FTSE MIB Index.
- Analyze the relationship between returns and CAPM posits to assess the model's power.
- Investigate potential deviations from the CAPM predictions and their implications for investors.

Through these objectives, this research seeks to provide insights into the validity and reliability of the CAPM in explaining asset pricing dynamics within the Italian market context.

All the data used, and all the analysis can be found at the following link: https://github.com/GioviManto/Testing-CAPM

1.3 Structure of the Thesis

The remainder of this thesis is organized as follows:

- Chapter 2 provides a comprehensive review of the relevant literature on the CAPM, previous empirical studies examining its performance in different market settings, and any identified gaps in the existing literature.
- Chapter 3 outlines the data collection procedures employed in this study, with relevant choices on which data is gathered.
- Chapter 4 explains the procedure used to test and evaluate the CAPM, showing also the statistical tools used.
- Chapter 5 presents the data analysis of the first period, including an overview of the Italian stock market during the specified period, analysis of beta coefficients, and all the tests used to assess CAPM validity.
- Chapter 6 shows the other periods' results, discusses the results of the empirical analysis as a whole, and interprets the findings in the context of the CAPM, highlighting any deviations.
- Chapter 7 offers concluding remarks, highlights the key contributions of the study, emphasises the implications for investors and management, identifies its limitations, and suggests directions for future research.

2. Literature Review

2.1 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) has been a cornerstone of modern finance theory since its introduction. According to the CAPM, the expected return on an asset is determined by its beta (systematic risk), the risk-free rate, and the market risk premium.

$$E[R_i] = R_f + \beta_i(E[R_m] - Rf) \qquad (1)$$

Where:

- R_f signifies the risk-free rate of return,
- β_i denotes the beta of security i,
- $E[R_m]$ represents the expected return on the market, and
- $(R_m - R_f)$ indicates the market premium.

It is useful to underline three key concepts:

- The expected return is the return that an investor expects to obtain from a security in the next period. Since it is an expectation, the return that will be realized may be higher or lower.
- Systematic risk (or market risk) is any risk that affects, to a greater or lesser extent, a large number of assets (e.g., news regarding GDP, interest rates, and general economic conditions), it is implied in the investment in a specific asset, and it can't be eliminated.
- Unsystematic risk (or specific risk) is a risk that specifically affects a single asset or a limited number of assets (e.g., announcement of a strike within a small company), it is measured by the variance, and it can be eliminated by investing in a diversified portfolio.

Now it’s the moment to explain what the famous beta is. It measures the sensitivity of a stock to changes occurring in the market portfolio.

The formula of the beta is:

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \qquad (2)$$

Where:

- $Cov(R_i,\ R_m)$ is the covariance between the return on stock i and the return on the market portfolio.
- $Var(R_m)$ represents the variance (σ^2) of the return on the market portfolio.

Moreover, it's very important to note that:

$$\sum_{i=1}^{N} X_i \beta_i = 1 \qquad (3)$$

where X_i is the proportion of the market value of stock i relative to the entire market. Therefore, the beta of the market portfolio is, by definition, equal to 1.

If an individual, instead of holding a single security, maintains a diversified portfolio, they would still regard the variance (or the root mean square deviation) of the portfolio's returns as the appropriate measure of portfolio risk, but they wouldn't focus on the variance of each security's returns anymore. Instead, they would concentrate on the contribution each security makes to the portfolio's variance.

Under the assumption of homogeneous expectations, all individuals hold the market portfolio. Therefore, we measure risk based on the contribution of a single security to the variance of the market portfolio. This contribution, once properly standardized, represents the security's beta. While very few investors hold exactly the market portfolio, many hold reasonably diversified portfolios. These portfolios are sufficiently close to the market portfolio that a security's beta is a reasonable measure of its risk.

We can directly visualize the CAPM postulates with his graphical representation, the Security Market Line (SML). An example of a hypothetical SML is represented in Figure 1.

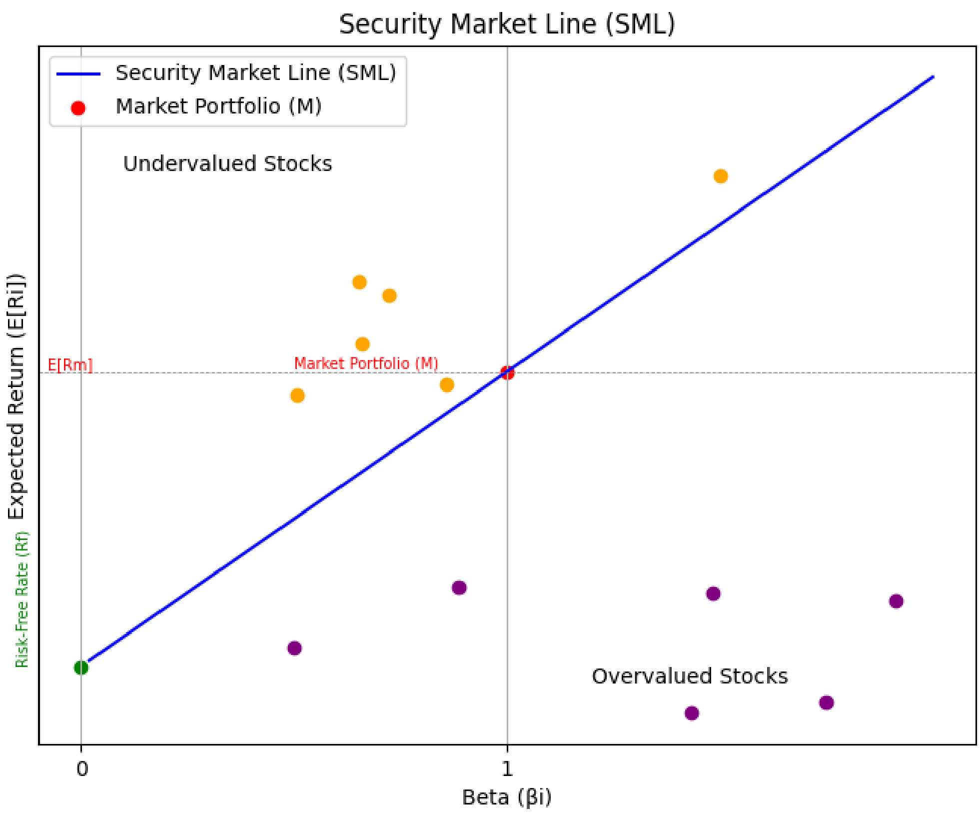

Figure 1: An example of a Security Market Line obtained using Python with Numpy and Matplotlib libraries.

The SML is characterized by a linear relationship between beta and expected return. It starts from the risk-free rate on the y-axis, denoting the return of a risk-free investment. The slope of the SML represents the market risk premium, indicating the additional return investors require for bearing one unit of systematic risk. Moreover, the SML serves as a benchmark for evaluating investment opportunities. Investments lying above the SML are considered undervalued, offering higher expected returns relative to their systematic risk. Conversely, investments below the SML are deemed overvalued, as they fail to provide adequate compensation for the level of risk assumed. By comparing individual investments to the SML, investors can assess whether they offer sufficient returns given their risk profile.

This simple yet powerful model equips investors with a tool to evaluate investments. If the expected return fails to meet or exceed the required return, the investment is deemed unfavorable.

Operating within a set of assumptions, the CAPM framework posits several key conditions:

- All investors are risk-averse, striving to maximize the expected utility of their end-of-period wealth, thus implying a one-period model.
- Investors hold homogeneous expectations regarding asset returns and utilize the same expected return and covariance matrix of stock returns.
- A fixed risk-free rate enables investors to borrow or lend without restriction at a uniform interest rate.
- In a perfectly competitive market, all stocks are divisible and priced, with non-tradable assets such as education, private enterprises, and government-funded assets.
- Market imperfections such as taxes, regulations, or trading costs are assumed to be absent.
- The number and quantities of stocks are predetermined within the one-period world.

While these assumptions may pose challenges in real-world applications, CAPM offers a reasonably accurate depiction of reality as a financial theory.
Despite its widespread use in financial practice, the CAPM has faced scrutiny regarding its assumptions and empirical validity.

2.2 Empirical Validation of the CAPM

Since its inception in the early 1960s, the Capital Asset Pricing Model (CAPM) has remained a focal point of discussion in financial economics.

The primary objective of the CAPM is to explain variations in the risk premium among assets. It posits that these variations stem from differences in the riskiness of asset returns, with beta serving as the key measure of asset risk. According to the CAPM, the risk premium per unit of risk is consistent across all assets. By leveraging the risk-free rate and beta, the model predicts the expected risk premium for each asset. In practical applications, the CAPM serves as a guiding framework for portfolio management and asset selection. Investors often utilize CAPM-derived metrics to identify undervalued or overvalued assets, informing their buying and selling decisions. Similarly, in corporate finance, estimated beta coefficients are instrumental in assessing the riskiness of investment projects and establishing minimum hurdle rates for project viability, or the cost of equity.

However, the CAPM has faced criticism over the years, sparking extensive academic debate regarding its validity and usefulness. Empirical testing of the CAPM serves two main purposes: (i) to determine whether the model should be rejected and (ii) to provide insights for financial decision-making. Statistical methods are employed to rigorously evaluate whether the data support or refute the model's predictions. Furthermore, is useful to underline that the CAPM model is just a construct that needs to be tested empirically and that it can give some insights into reality.

2.3 The Roll’s critique

The Roll's critique (Roll, 1977) of the market portfolio and the Capital Asset Pricing Model (CAPM) revolves around two central arguments.

Firstly, Roll posits the mean-variance tautology, which asserts that any portfolio achieving mean-variance efficiency automatically satisfies the CAPM equation. This equation establishes a linear relationship between an asset's expected return, the risk-free rate, and its beta coefficient multiplied by the expected excess return of the market portfolio. Essentially, Roll argues that the CAPM equation is inherently true for any portfolio meeting the criteria of mean-variance efficiency, without necessitating specific model assumptions. Consequently, testing the CAPM equation becomes synonymous with evaluating the mean-variance efficiency of the portfolio, especially when using a proxy for the market portfolio. Therefore, if one assumes the market to be mean-variance efficient, the CAPM equation becomes self-referential and loses its empirical testability.

Secondly, Roll highlights the unobservability of the market portfolio. He contends that the true market portfolio would comprise every conceivable investment opportunity, ranging from traditional assets like stocks and bonds to alternative investments like real estate, commodities, and collectibles. However, the returns on these diverse investment options are practically impossible to observe comprehensively. Without the ability to observe the returns on all investment opportunities, it becomes exceedingly challenging to ascertain whether any portfolio, including the market portfolio, is genuinely mean-variance efficient.

Consequently, Roll suggests that the inability to observe the returns on the entire universe of investment opportunities poses a significant obstacle to empirically testing the CAPM.

2.4 Confirmation of Theory through Empirical Analysis

In front of the huge popularity of the CAPM model and considering the weaknesses of every empirical analysis, is useful to understand if it can explain at least a part of the variations in the stock yields.

One of the seminal empirical studies supporting the CAPM was conducted by Black, Jensen, and Scholes (1972). Utilizing portfolio data rather than individual stocks, they tested the linearity of the relationship between expected returns and beta. By aggregating securities into portfolios, they mitigated firm-specific risk, enhancing the precision of beta estimates and expected returns. The study found strong empirical support for the CAPM, demonstrating a linear relationship between average returns and beta across portfolios.

Similarly, Fama and McBeth (1973) examined the relationship between average returns and beta, along with the influence of beta squared and asset return volatility on residual return variations. Their findings provided further validation for the CAPM, affirming the positive linear association between returns and beta.

2.5 Critiques and Challenges to CAPM

In the early 1980s, research began to identify deviations from the linear risk-return tradeoff predicted by the CAPM. Banz (1981) challenged the model by demonstrating that firm size could better explain variations in average returns than beta alone, leading to the discovery of the size effect. At its core, the size effect unveils that smaller companies, often termed small-cap stocks, tend to outperform their larger counterparts, known as large-cap stocks. This phenomenon persists even after adjusting for risk factors like beta, suggesting that size plays a significant role in driving stock returns.

However, the CAPM was seriously questioned when Fama and French (1992, 1993) presented findings that raised significant questions about the CAPM's validity. Firstly, they conclude that for US firms, the relationship between average return and beta was weak between 1941 and 1990 and practically non-existent between 1963 and 1990. They assert that the average return of a stock is negatively correlated with both the price-earnings ratio (P/E) and the market value-to-book value ratio (P/BV) of the firm, this is the value effect, which underscores the notion that stocks with lower prices relative to their book value, or high book-to-market ratios, tend to yield superior returns compared to stocks with higher prices relative to book value. These high book-to-market ratio stocks, often categorized as value stocks, consistently deliver higher returns than their low book-to-market ratio counterparts, referred to as growth stocks.

In summary, despite the extensive research efforts involving numerous papers and models, along with rigorous empirical testing, the outcomes have remained inconclusive. A clear verdict has yet to be reached regarding the effectiveness of the Capital Asset Pricing Model (CAPM) and its alternatives. The ongoing debates within academic and financial communities underscore the complexities inherent in understanding stock returns and asset pricing mechanisms. This uncertainty emphasizes the dynamic nature of financial markets and the persistent quest for a more comprehensive understanding of these phenomena.

2.6 Alternative Models

An alternative pricing model is the Arbitrage Pricing Theory (APT) proposed by economist Stephen Ross (1976). Unlike the Capital Asset Pricing Model (CAPM), which focuses solely on the relationship between the expected return of a single asset and its systematic risk (beta), APT takes a broader view.
At its core, APT posits that the expected return of an asset can be modeled as a linear function of various factors that influence its risk. These factors could be macroeconomic variables, industry-specific conditions, or any other relevant market forces.

A widely employed factorial model in finance is the Fama-French Three-Factor Model. It seeks to explain stock returns using three key factors: market risk, the performance discrepancy between small-cap and large-cap firms, and the performance discrepancy between high book-to-market value firms and low book-to-market value firms. The premise of this model lies in the observation that companies with high value and small market capitalization often demonstrate consistent outperformance compared to the broader market.

Thus, the model can be expressed through the following equation:

$$E[R_i] = R_f + \beta_1(E[R_m] - Rf) + \beta_2(SMB) + \beta_3(HML) + \varepsilon_i \quad (4)$$

Where:

- $E[R_i]$ represents the expected return of the asset i.
- R_f denotes the risk-free rate, which is the return on an investment with zero risk, often approximated by government bonds.
- $E[R_m]$ stands for the expected return of the market portfolio.
- β_1 is the beta coefficient of the asset concerning the market, indicating its sensitivity to market movements.
- SMB represents the size premium, which captures the excess return of small-cap stocks over large-cap stocks. A positive SMB coefficient indicates that small-cap stocks outperform large-cap stocks.
- HML denotes the value premium, which captures the excess return of high book-to-market value stocks over low book-to-market value stocks. A positive HML coefficient implies that value stocks outperform growth stocks.
- β_2 and β_3 are the respective coefficients for the size premium (SMB) and the value premium (HML).
- ε_i represents the error term, capturing any other factors not accounted for by the model that influence the return of asset i.

Moreover, the studies of Mark Carhart (1997) brought to the momentum factor, which is the tendency for assets that have performed well in the recent past to continue performing well in the short term, and vice versa. His empirical analysis demonstrated that momentum is a persistent anomaly in financial markets, challenging the efficient market hypothesis which suggests that all available information is quickly incorporated into asset prices. Carhart found that stocks that have exhibited strong performance over the past several months tend to continue performing well, while those with weak performance continue to underperform. Thus, the Carhart factorial model accounts for the momentum as the fourth factor, as expressed by the following equation:

$$E[R_i] = R_f + \beta_1(E[R_m] - Rf) + \beta_2(SMB) + \beta_3(HML) + \beta_3(HML) + \beta_4(MOM) + \varepsilon_i$$
(5)

Where:

- MOM captures the excess return of assets that have experienced recent positive performance relative to assets with weaker recent performance.
- β_4 is the coefficient that accounts for the MOM factor.

In addition, is crucial to say that in the APT model, the theory suggests that investors are rational and will exploit any arbitrage opportunities that arise due to mispricing of assets.

The beauty of APT lies in its flexibility. Unlike CAPM, which relies heavily on the market beta, APT allows for the inclusion of multiple factors, making it more adaptable to different market conditions and investment strategies. This flexibility also means that APT can better account for the unique characteristics of different assets and markets, leading to more accurate pricing predictions.

However, it's essential to note that APT is not without its challenges. One of the main criticisms is the difficulty in identifying and quantifying the relevant factors that drive asset prices. Additionally, the assumption of no arbitrage opportunities may not always hold in real-world markets, especially in less efficient or segmented markets.

2.7 Previous Studies on CAPM for the Italian Stock Market

Numerous empirical studies have examined the applicability of the CAPM in different market contexts, including the Italian stock market.

For instance, studies by Beltratti and Di Tria (2002), that compares multi-factor models with Italian stock market data for the period 1990-2000, using the CAPM as the relevant benchmark. The result of this research is that the presumably large size of shocks to returns in the Italian case and the presence of extra factors make it difficult to explore the relation among expected returns. Another important study is the one of De Chiara and Puopolo (2015), in which they intended to show that the CAPM, despite the heavy critical comments, still holds in the Italian market when

returns are measured at the monthly frequency and that the market portfolio fully explains the cross-section of stock returns with no need to appeal for additional determinants.
In addition, the Rega (2016) research outlines how market risk explains stock returns because all Jensen's alphas are around zero, it also partially proves that macroeconomic factors do not have a significant influence (APT) and finally states that the Fama and French Model (FFM) explains stock returns better than the Capital Asset Pricing Model (CAPM).

Thus, findings from these studies have been mixed, with some suggesting support for the model while others finding deviations from its predictions.

2.8 Gaps in the Literature

Despite the existing body of research on the CAPM and the Italian stock market, several gaps remain in the literature. First, there is a need for more comprehensive studies that consider the dynamics of both individual stocks and portfolios within the Italian market. Second, previous studies have often focused on specific periods or subsets of stocks, limiting the generalizability of their findings. Additionally, there is a lack of consensus regarding the factors that influence the performance of the CAPM in the Italian context, highlighting the need for further investigation.

3. Methodology

3.1 Data Collection

The first step in our analysis involves collecting data on stock prices, market indices, and risk-free rates for the Italian stock market over the specified period (1.1.2018 - 12.31.2023). Data is sourced from the reputable financial database of Bloomberg, to ensure accuracy and reliability. We will retrieve weekly closing prices for individual stocks listed on the Italian stock exchange, as well as corresponding market indices (FTSE MIB) and risk-free rates (Italian government bonds).

Therefore, we utilize the Italian 3-month Treasury Bill, known as "Buoni Ordinari del Tesoro (BOT)", converted into weekly returns (dividing for 52 weeks the annualized return) as a proxy for the risk-free rate (R_f). Additionally, we employ the price index "FTSE MIB", as a proxy of the market portfolio, to represent the market return (R_m), which consists of the 40 most-traded stock classes on the exchange.

3.2 Selection of Data Frequency

To obtain more precise estimates of the beta coefficient, our study adopts weekly stock returns as the focal point of analysis. This decision is grounded in the recognition that using returns calculated over longer periods, such as monthly returns, may introduce biases in beta estimation. These biases stem from potential changes in beta values over the examined period, compromising the accuracy of our estimates.

Conversely, employing high-frequency data, such as daily returns, poses challenges due to the inherent noise present in such data. Utilizing daily returns could result in the inclusion of excessive variability, leading to inefficient beta estimation.

Hence, weekly returns strike a balance between capturing meaningful trends in stock performance and mitigating the effects of excessive noise inherent in daily data.

3.3 Choice of Market Index

The FTSE MIB Index comprises the 40 most liquid and capitalized stocks listed on the Borsa Italiana (BIt) MTA and MIV markets. It is the primary benchmark index for the Italian equity market and represents approximately 80% of the domestic market capitalization. Stocks included in the index are weighted based on the free float, ensuring that only the investable opportunity set is represented. Constituents of the index are capped at 15% to prevent over-concentration, anyway the most capitalized company in the FTSE MIB is Unicredit with a weight on the index of about 11.43% on 29 March 2024. Moreover, referring on 30 April 2024, the top 10 Holdings constitute 72.59% of the Index market cap, and the ICB supersectors as banks (28.61%), automobiles and parts (18.57%), utilities (13.54%), energy (10.19%) and insurance (7.47%) weight the 78.38% of the Index. Additionally, in the chosen period, according to Bloomberg, the average P/E ratio was 11.060 and the P/BV ratio was 1.203.

Therefore, the "FTSE MIB" primarily comprises value and large-cap stocks, which serves to mitigate the influence of the Fama and French size and value effects discussed earlier, reducing potential confounding factors associated with these effects.

3.4 Data Exclusion

In the specified time frame, there were two notable changes in the composition of the FTSE MIB Index. The first occurred on June 24, 2019, with Nexi replacing Banca Generali in the index. The second change occurred on March 1, 2022, when Iveco entered the FTSE MIB following the spin-off of CNH Industrial's On-Highway activities. Consequently, data for Nexi is missing approximately 21.41%, while data for Iveco is missing around 66.77%. Due to the significant amount of missing data for these companies, they were excluded from the analysis.

4. Procedure for CAPM Testing and Evaluation

4.1 Testing Procedure

The study examines the period spanning from January 1, 2018, to December 31, 2023, focusing on testing the predictions of the Capital Asset Pricing Model (CAPM). Following the approach outlined by Black, Jensen, and Scholes (1972), as well as the methodology employed by Fama and MacBeth (1973), we divide the analysis into an initial estimation period and a testing period. During the initial estimation period, we estimate the beta coefficients of the portfolios, while the testing period is utilized to compute the results, as detailed in Table 1.

Black, Jensen, and Scholes (1972) introduced a time series test of the CAPM, which involves regressing excess portfolio returns on excess market returns. This regression equation is expressed as:

$$R_{it} - R_{ft} = \alpha_i + \beta_i\left(R_{mt} - R_{ft}\right) + \varepsilon_{it} \qquad (6)$$

Where:

- R_{it} represents the rate of return on asset i at time t,
- R_{ft} denotes the risk-free rate at time t,
- R_{mt} signifies the rate of return on the market portfolio at time t,
- β_i is the beta of stock i
- ε_{it} represents the random disturbance term in the regression equation at time t.
- α_i is the intercept term

We can think of:

- $(R_{it} - R_{ft})$ as the excess return of stock i, and
- $\left(R_{mt} - R_{ft}\right)$ as the average risk premium

The intercept term, α_i, is the so-called Jensen's alpha and represents the deviation between the expected return estimated through time series averaging and the expected return predicted by the CAPM. In an ideal scenario where the CAPM accurately portrays expected returns and a suitable market portfolio proxy is chosen, the regression intercepts for all portfolios (or assets) would ideally be zero. This suggests that there is no systemic bias in the CAPM predictions. However, any departure from zero indicates a disparity between the model's forecasts and the actual market returns observed. Therefore, analyzing the intercept term enables us to gauge the extent to which the CAPM effectively explains the dynamics of asset pricing.

4.2 Estimation of Beta Coefficients

Once the data collection phase is complete, we will proceed to estimate beta coefficients for each stock in our sample.

In the estimation of beta coefficients, we embark on a crucial step in our analysis aimed at quantifying the relationship between individual stock returns and overall market movements. Beta serves as a fundamental metric within the Capital Asset Pricing Model (CAPM), providing insights into the systematic risk inherent in each stock's returns.

To accomplish this, we employ regression analysis as our primary analytical tool. This method allows us to quantify the sensitivity of individual stock returns to changes in the broader market by assessing the historical relationship between a stock's returns and market returns.

Specifically, for each stock, the estimation of beta using weekly returns aligns with each Portfolio Formation Period. Beta was determined by regressing the weekly return of each stock against the market index, using the equation (6). Thus, a beta coefficient greater than 1 indicates that the stock is more volatile than the market, while a beta coefficient less than 1 suggests lower volatility compared to the market. A beta coefficient of 1 implies that the stock moves in perfect correlation with the market.

4.3 Portfolio Construction

In portfolio construction, it's important to utilize the true beta values of stocks. However, since we typically have access only to estimated betas, there's a potential for bias. Sorting stocks into portfolios based on estimated betas may introduce selection bias, particularly for stocks with high estimated betas, which are more prone to positive measurement errors. This could lead to a positive bias in high-beta portfolios and a negative bias in intercept estimates. To address this issue, Black, Fischer, Jensen, and Scholes (1972) employed a grouping method. They estimated the beta coefficients for stocks based on data from the previous year. These estimated betas were then used to group stocks into portfolios for the following year. By doing so, they aimed to reduce potential statistical errors associated with beta estimation. This approach allowed them to mitigate biases that may arise from using estimated betas directly, as the beta estimates from the previous year are likely to be more accurate and stable over time compared to estimates based on shorter time frames or more recent data (Elton and Gruber, 1995).

The initial step involves partitioning the six years into four intervals, with each interval spanning three years:

1) The first period is 2018.1.1 – 2020.12.31;
2) The second period is 2019.1.1 – 2021.12.31;
3) The third period is 2020.1.1 – 2022.12.31;
4) The fourth period is 2021.1.1 – 2023.12.31.

The study's structure is outlined in Table 1.

	Period 1	Period 2	Period 3	Period 4
	2018-2020	2019-2021	2020-2022	2021-2023
Portfolio Formation Period	2018	2019	2020	2021
Initial Estimation Period	2019	2020	2021	2022
Testing Period	2020	2021	2022	2023

Table 1: Portfolio Formation, Estimation, and Testing Periods.

Using the estimated betas computed for each stock, as illustrated above, we segmented the 38 stocks into 4 portfolios, each containing 10 or 9 stocks categorized by their beta values. Portfolio 1 consists of the 10 stocks with the lowest betas, while Portfolio 4 comprises the 10 stocks with the highest betas, so Portfolio 2 and Portfolio 3 each contain 9 stocks. By consolidating securities into portfolios, we effectively mitigate much of the firm-specific component of returns, thereby improving the accuracy of beta estimates and expected returns for the portfolios. Additionally, we have noticed a significant variation in portfolio composition during the periods.

4.4 Portfolio's Betas and Ex-Post SML

After that, we are ready to compute the betas of the portfolios using a similar equation of the (6) but with portfolios and no single stocks:

$$R_{pt} - R_{ft} = \alpha_p + \beta_p\left(R_{mt} - R_{ft}\right) + \varepsilon_{pt} \qquad (7)$$

Where:

- R_{pt} represents the rate of return on portfolio p at time t,
- R_{ft} denotes the risk-free rate at time t,
- R_{mt} signifies the rate of return on the market portfolio at time t,
- β_p is the beta of portfolio p
- ε_{pt} represents the random disturbance term in the regression equation at time t.
- α_p is the intercept term

To evaluate the Capital Asset Pricing Model (CAPM), Fama and MacBeth (1973) utilize a monthly cross-sectional regression (CSR) approach. This involves regressing the excess return of each portfolio against its estimated beta. Essentially, a simple cross-sectional regression entails regressing the average excess return against the market beta for various portfolios. The average excess return for each portfolio is calculated as the mean excess return over a specified period, while the market beta

(β_p) represents the slope in the time series regression of the average portfolio's excess return ($R_p - R_f$) on the average market's excess return (risk premium) $(R_{mt} - R_{ft})$.

Given the worldwide events that happened during the chosen period and the consequently market volatility, we decided to use the beta values calculated in the initial estimation period just to have insights about their explanatory power and relation with portfolio excess returns (so we have one estimated beta for each portfolio in the year of initial estimation, for a total of 4). Subsequently, during the test period, we conducted regressions to determine the monthly beta for each portfolio. Subsequently, we calculated the adjusted beta for each one. Afterward, we computed the residual variance for each monthly portfolio excess return.

Thus, we have 48 observations for each variable researched (beta, residual variance, portfolio excess return) for the testing period (12 months multiplied by 4 portfolios).

Now we have to estimate the Security Market Line (SML), for every testing period, regressing the portfolio returns against the portfolio betas.

As we have said, the SML is the graphical representation of the CAPM model, expressed by the equation (1). Therefore, we can use the estimated beta that we've just obtained for the test period, adjust them, and estimate y0, and y1 as follows:

$$R_p - R_f = \gamma_0 + \gamma_1(Adj\beta_p) + \varepsilon_p \qquad (8)$$

Where:

- $R_p - R_f$ is the average excess return on a portfolio p,
- $Adj\beta_p$ is the adjusted beta of portfolio p,
- ε_p is the random disturbance term in the regression equation.

The adjusted beta provides an estimate of a security's future beta by considering its historical data while assuming a tendency for the security's beta to converge toward the market average over time. This adjustment involves weighting the historical raw beta of the security and the market beta. The formula for calculating the adjusted beta is as follows:

$$Adjusted\ \beta = 0.67 \times\ Raw\ \beta + 0.33\ \times\ 1.0 \qquad (9)$$

In the context of the Capital Asset Pricing Model (CAPM), the intercept γ_0 signifies a crucial aspect of the Security Market Line (SML). Ideally, if the CAPM accurately describes the relationship between risk and return in the market, this intercept should equal zero. The intercept essentially reflects the expected excess return on a portfolio when its beta (β_p) is zero, which implies that the portfolio bears no systematic risk relative to the market.

On the other hand, the slope of the Security Market Line, denoted by γ_1, holds significant importance as well. For the CAPM this slope should be equal to the average risk premium of the portfolio, capturing how the expected excess return on a portfolio changes concerning changes in its beta.

Thus, by CAPM, a zero intercept (γ_0) and a positive slope (γ_1) of the SML would indicate that investors are adequately compensated for the systematic risk they undertake, with higher-beta portfolios expected to yield higher average returns to offset the increased risk, so that there is a positive price of risk in the capital markets.

To investigate potential nonlinearity between total portfolio returns and betas, we employ the following regression equation:

$$R_p - R_f =\ \gamma_0 +\ \gamma_1(Adj\beta_p) +\ \gamma_2(Adj\beta_p^2) +\ \varepsilon_p \qquad (10)$$

If the underlying assumption of the Capital Asset Pricing Model (CAPM) holds—meaning that portfolio returns, and their adjusted betas exhibit a linear relationship—then the coefficient γ_2 should ideally be zero. Additionally, here we have first squared the betas and then we have adjusted them, using equation (9).

Subsequently, we explore whether the expected excess return on securities is solely determined by systematic risk, independent of nonsystematic risk, as measured by the residuals' variance:

$$R_p - R_f = \gamma_0 + \gamma_1(Adj\beta_p) + \gamma_2(Adj\beta_p^2) + \gamma_3\sigma^2(\varepsilon_{pt}) + \varepsilon_p \qquad (11)$$

Here, γ_2 gauges the potential nonlinearity of the return, while γ_3 assesses the explanatory power of nonsystemic risk. The term $\sigma^2(\varepsilon_{pt})$ represents the normalized residual variance of portfolio returns. The residual variance has been normalized, for numerical reasons, by subtracting the mean and dividing by the standard deviation. If the CAPM hypothesis holds, γ_3 should be zero, indicating that nonsystemic risk does not influence the expected excess return on securities.

4.5 t-test

In our statistical evaluation of the CAPM, we employ t-tests as our analytical tool. We set the significance level at 95%, indicating that a significant result at this level of confidence supports our conclusions with 95% certainty. However, it also implies a 5% probability of error. The critical value corresponding to a 95% confidence level from the t-distribution is 1.96. Consequently, we utilize this value in subsequent analyses to validate the accuracy of our estimation results. When interpreting the p-value derived from the t-test, a value less than 0.05 indicates statistical significance, suggesting that the observed results are unlikely to have occurred by chance.

5. Data Analysis

5.1 Overview of the Italian Stock Market (2018-2023)

Figure 2: Plot of the FTSEMIB compared with SP500 and BOT 3-month from 01.01.2018 to 12.31.2023

Looking at the plot we see some ups and downs in this period, the minimum is 15731.85 and the maximum is 30403.90 (1.93 times the minimum), however the coefficient of variation of the FTSEMIB is 13.27% while for the SP&500 is 19.99% suggesting that the data points in the FTSEMIB are less spread out relative to the mean compared to the S&P 500.

To provide context to these fluctuations, we can delve deeper into the circumstances.

2018: A Year of Turmoil and Economic Struggles

The Italian stock market faced significant challenges in 2018, with the FTSE MIB index starting the year at approximately 21,000 points and ending lower, around 18,000 points, representing a decline of approximately 14.3%.

One of the defining events of the year was the clash between Italy's populist government and the European Union over the country's proposed budget. This conflict contributed to heightened uncertainty and volatility in financial markets, with Italian government bond yields spiking to multi-year highs.

Global trade tensions also weighed on investor sentiment, with concerns over escalating tariffs and protectionist measures undermining confidence in the global economic outlook. These factors, combined with Italy's domestic political uncertainty, led to a risk-off environment in which investors sought safe-haven assets.

Despite these headwinds, Italy's economy continued to grow, albeit at a slower pace than initially anticipated. GDP growth for the year was approximately 0.9%, down from 1.6% in the previous year, reflecting the impact of heightened uncertainty and weaker external demand.

2019: Signs of Recovery Amidst Global Uncertainty

In 2019, the Italian stock market staged a partial recovery, with the FTSE MIB index starting the year at around 18,000 points and ending higher, around 20,000 points, representing a gain of approximately 11.1%.

Signs of progress in US-China trade negotiations and the European Central Bank's announcement of additional stimulus measures provided a much-needed boost to investor sentiment. Italy's economy showed signs of stabilization, with GDP growth rebounding to approximately 0.3% for the year, supported by resilient domestic demand and improving external conditions.

However, political uncertainty persisted, with Italy experiencing a change in government and ongoing tensions with the European Union over fiscal policy. These factors continued to weigh on investor confidence, limiting the extent of the market's recovery.

2020: The Unprecedented Impact of COVID-19

The outbreak of the COVID-19 pandemic in 2020 sent shockwaves through financial markets, leading to a widespread sell-off in equities and a flight to safety among investors. The FTSE MIB index plummeted from around 20,000 points at the beginning of the year to approximately 15,000 points at its lowest, representing a decline of approximately 25%.

Italy, one of the hardest-hit countries in Europe, faced a dual crisis as it grappled with the health and economic consequences of the pandemic. GDP contracted sharply by approximately 8.9% for the year, reflecting the severe impact of lockdown measures on economic activity.

In response, the Italian government implemented various fiscal measures, including financial support for businesses and individuals affected by the crisis. The European Central Bank also intervened to stabilize financial markets and provide liquidity, contributing to a partial recovery in equity prices towards the end of the year.

2021-2023: Navigating Geopolitical Challenges and Economic Recovery

The Italian stock market experienced a mix of geopolitical tensions, economic recovery efforts, and inflationary pressures from 2021 to 2023. Following the tumultuous period induced by the COVID-19 pandemic, the market saw gradual stabilization and growth, albeit against a backdrop of significant geopolitical events.

In February 2022, the Russian invasion of Ukraine sent shockwaves through global markets, including Italy's FTSE MIB index. The threat of conflict in Eastern Europe heightened uncertainty and risk aversion among investors, leading to bouts of market volatility. Italy, being a part of the European Union, closely monitored the situation and its potential economic ramifications.

Despite geopolitical tensions, Italy's economy continued its path to recovery, supported by fiscal stimulus measures and infrastructure projects. GDP growth averaged approximately 4.2% per year from 2021 to 2023, indicating a resilient domestic economy. However, alongside this recovery, inflationary pressures began to mount.

The post-pandemic economic rebound, coupled with supply chain disruptions and pent-up demand, contributed to inflationary pressures across various sectors of the economy. Rising prices for goods and services posed challenges for consumers and businesses alike, impacting purchasing power and production costs.

In October 2023, the Gaza-Israel conflict reignited, further adding to geopolitical concerns. The escalation of violence in the Middle East introduced another layer of uncertainty for global markets, including Italy's. Investor sentiment was affected as the conflict raised fears of broader regional instability and its potential impact on global trade dynamics.

Throughout this period, the FTSE MIB index navigated through geopolitical headwinds and inflationary pressures, reflecting the market's resilience amid challenging circumstances. While these events contributed to occasional market volatility, Italy's economy showed signs of strength, with the FTSE MIB index reaching approximately 30,000 points by the end of 2023.

Investors remained vigilant, closely monitoring geopolitical developments alongside economic indicators, including inflation, to gauge the market's trajectory. Despite the geopolitical challenges posed by the Russian invasion of Ukraine and the Gaza-Israel conflict, alongside rising inflation, there was a cautious sense of optimism as Italy's economy continued on its path to recovery, bolstered by domestic initiatives and global economic trends.

5.2 Beta Coefficients Analysis

The initial procedure involves computing the beta values using Equation (6). Below are the beta values for the 38 stocks:

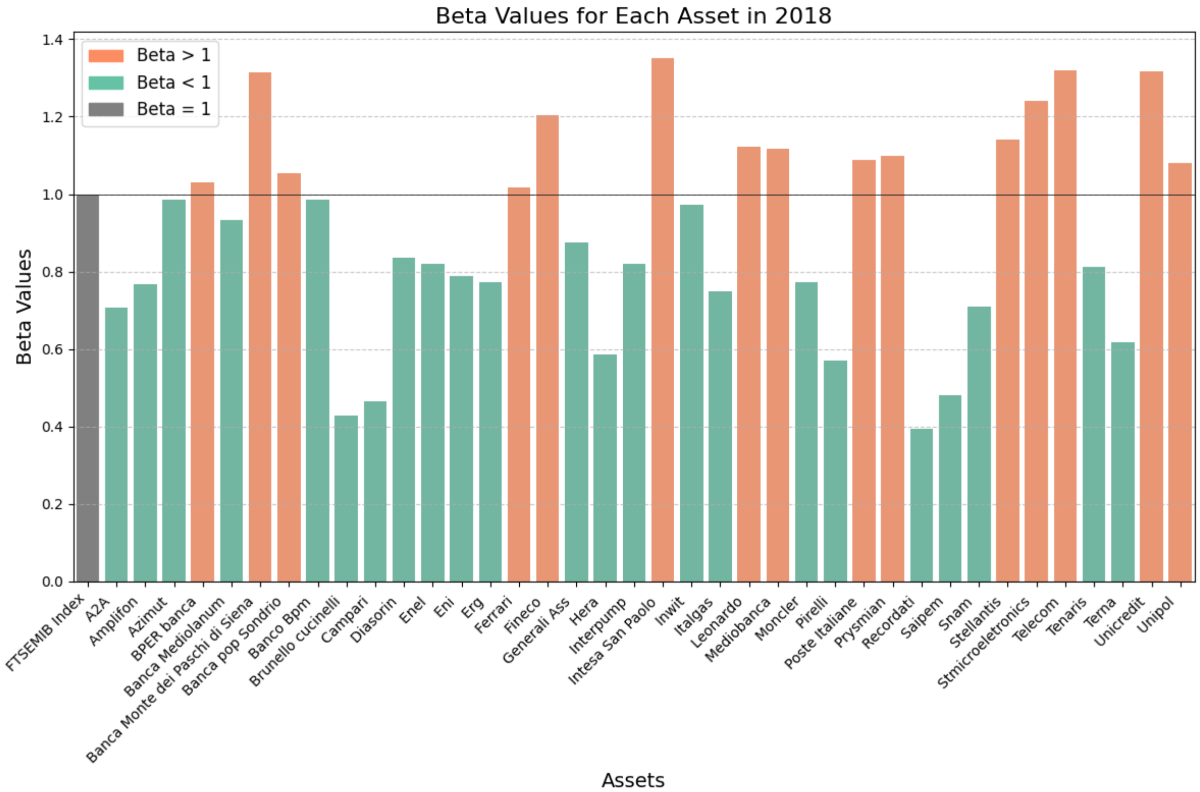

Figure 3: Stock beta estimates (Year 2018)

The beta estimates range for 2018 is from 0.393235 (minimum) with the beta of “Recordati” to 1.351083 (maximum) with the beta of "Intesa San Paolo", the average beta is 0.904038 and the standard deviation is 0.260878.

Subsequently, we divided the 38 stocks into 4 portfolios using their betas as the criterion, as described in Chapter 4.3.

5.3 Average excess returns and beta across the portfolios

Next, we proceed to determine the average excess return of the portfolios along with their corresponding betas by utilizing equation (7), the results are liste in Table 2 below.

Year 2019	$R_{pt} - R_{ft}$	β_p	t-value	Std. error	Adj. R^2	F-stat

Portfolio 1	0.009157	0.730137	9.009740	0.081039	0.611208	81.175406
Portfolio 2	0.011086	0.708754	10.847364	0.065338	0.695823	117.665315
Portfolio 3	0.014726	0.879047	9.285494	0.094669	0.625607	86.220396
Portfolio 4	0.011237	1.397507	17.455318	0.080062	0.856212	304.688142

Table 2: Average excess portfolio returns and betas (The year 2019)

By the CAPM, which posits that higher risk should be compensated with higher returns, we would expect to see a positive relationship between excess returns and betas. That is, portfolios with higher betas should yield higher excess returns, reflecting the additional risk they bear compared to the market.
However, upon examining the results, this expected trend appears to be ambiguous and not evident in these observations.
Furthermore, the t-value associated with the betas provides insights into their statistical significance. In this analysis, all the portfolios have t-values indicating a strong statistical significance, suggesting that the observed relationships between excess returns and betas are unlikely to have occurred by chance.
Additionally, the Adjusted R-squared values indicate how well the portfolios' excess returns are explained by their betas. Higher R-squared values indicate a better fit of the data to the CAPM framework, implying that a larger proportion of the variability in excess returns can be attributed to market risk as captured by betas, and in this case, this indicator is quite good.

Overall, these results suggest that the portfolios' excess returns are systematically related to their betas, in line with the predictions of the CAPM. However, further analysis and consideration of other factors may be necessary to fully evaluate the model's performance and its ability to explain asset pricing dynamics in the market.

5.4 Determination of the Security Market Line (SML)

Next, employ the findings to calculate the Security Market Line (SML). To derive the SML, Equation (8) was employed. According to the CAPM, it is anticipated that γ_0 equals zero, and γ_1 equals the average risk premium of the portfolio:

	Coefficients	Std. error	t-value	P-value
γ_0	0.019941	0.017032	1.187361	0.241180
γ_1	-0.013692	0.016794	-0.803858	0.425614

Table 3: Statistics of the SML's estimation (Year 2020)

Table 3 above displays the outcomes obtained from estimating the Security Market Line (SML).

The t-tests indicate that the null hypothesis is not rejected for the intercept (γ_0), as the absolute t-value doesn't exceed 1.96, suggesting no statistical significance. Therefore, γ_0 is not significantly different from zero, which doesn't contradict the CAPM hypothesis. Additionally, the null hypothesis for the slope (γ_1) is rejected, given the absolute t-value (0.804) smaller than 1.96, indicating that γ_1 is not significantly different from zero. According to the CAPM, γ_1 should be equal to the average risk premium, which is expected to be greater than zero (the average risk premium for the testing period, thus 2020, is 0.0659).

Consequently, only the first result is in line with the CAPM hypothesis for the first period.

5.5 Non-linearity Test

Equation (10) was employed to assess potential nonlinearity between the returns of portfolios and their respective adjusted betas. As discussed previously, under the assumption of the Capital Asset Pricing Model (CAPM), the intercept (γ_0) and the coefficient for the squared beta term (γ_2) should ideally be zero, while (γ_1) should equal the average risk premium.

	Coefficients	Std. error	t-value	P-value
γ_0	0.058623	0.032655	1.795434	0.079302
γ_1	-0.112193	0.073507	-1.526282	0.133938
γ_2	0.055876	0.040585	1.376764	0.175397

Table 4: Non-linearity test (Year 2002)

1. (γ_0) (Intercept): The coefficient of approximately 0.0586 suggests that even though it's not statistically different from zero at conventional levels of significance, there might be some systematic deviation from the CAPM prediction. However, even though the p-value is near the threshold of rejecting the null hypothesis, the coefficient is still not statistically significant, implying that the model is not severely biased.
2. (γ_1) (Coefficient for Adjusted Beta): The coefficient of approximately -0.1122 indicates a negative relationship between adjusted beta and excess returns, suggesting that higher adjusted betas are associated with lower excess returns. However, this coefficient is not statistically significant at conventional levels, so we cannot conclude a significant deviation from 0, thus, we can't accept the CAPM thesis where this coefficient should be equal to the risk premium.
3. (γ_2) (Coefficient for Adjusted Beta Squared): The coefficient of approximately 0.0559 suggests a potential non-linear relationship between excess returns and adjusted betas. However, like γ_1, it is not statistically significant, so we cannot definitively conclude the presence of non-linearity, according to CAPM.

In summary, while the coefficients show some deviation from the CAPM predictions, the data may not provide sufficient evidence to reject the CAPM hypothesis.

However, the results hint at potential complexities in the relationship between portfolio returns and betas that may deserve further investigation.

5.6 Non-Systematic Risk Test

Equation (11) was employed to gauge the significance of the non-systematic risk explaining the returns of portfolios.

	Coefficients	Std. error	t-value	P-value
γ_0	0.036836	0.028716	1.282782	0.206285
γ_1	-0.060238	0.064779	-0.929904	0.357495
γ_2	0.027792	0.035736	0.77771	0.440902
γ_3	-0.013806	0.003417	-4.040027	0.000211

Table 5: Non-Systematic risk test (Year 2002)

The intercept (γ_0) is still showing no statistically significant deviation from zero, as the CAPM predicts. The coefficient for adjusted beta (γ_1) indicates a non-significant relationship between beta and excess return. Additionally, the coefficient for adjusted beta squared (γ_2) does not demonstrate a significant nonlinear relationship with excess return. However, the coefficient for residual variance (γ_3) is statistically significant, suggesting that nonsystematic risk, as captured by residual variance, has explanatory power in determining excess returns. This finding challenges the strict assumption of the CAPM, which suggests that only systematic risk should influence excess returns, implying potential limitations in its applicability.

Thus, we conclude that CAPM is not fully valid in period 1.

$_0$

6. Results

6.1 Other periods result

	Coefficients	Value	t-value
Estimation of SML	γ_0	0.016146	4.203844
	γ_1	0.000202	0.050852
Non-linearity test	γ_0	0.015528	3.443595
	γ_1	0.002497	0.265050
	γ_2	-0.001471	-0.269202
Non-systemic risk test	γ_0	0.015542	3.409466
	γ_1	0.002486	0.261072
	γ_2	-0.001475	-0.26713
	γ_3	0.000279	0.204057

Table 6: Test results of Period 2 (2019.1.1 to 2021.12.31)

	Coefficients	Value	t-value
Estimation of SML	γ_0	-0.009074	-1.375659
	γ_1	0.005207	0.872956

Non-linearity test	γ_0	-0.008661	-1.132047
	γ_1	0.004266	0.407569
	γ_2	0.000414	0.110043
Non-systemic risk test	γ_0	-0.008567	-1.107264
	γ_1	0.00409	0.386092
	γ_2	0.000479	0.125695
	γ_3	0.000747	0.280726

Table 7: Test results of Period 3 (2020.1.1 to 2022.12.31)

	Coefficients	Value	t-value
Estimation of SML	γ_0	-0.049257	-4.900404
	γ_1	-0.008807	-0.837141
Non-linearity test	γ_0	-0.048619	-3.448482
	γ_1	-0.010686	-0.348983
	γ_2	0.001151	0.065422

Non-systemic risk test	γ_0	-0.049044	-3.355071
	γ_1	-0.009559	-0.297534
	γ_2	0.000504	0.027315
	γ_3	0.000376	0.131303

Table 8: Test results of Period 4 (2021.1.1 to 2023.12.31)

The estimated beta used for the portfolio creation, in the whole period empirical analysis, is the one regressed considering the entire period. Thus, the initial estimation period and the test period are the same as the portfolio formation period. In this case, the sample size is way bigger.

	Coefficients	Value	t-value
Estimation of SML	γ_0	-0.003209	-0.685067
	γ_1	-0.000596	-0.129613
Non-linearity test	γ_0	-0.000140	-0.027572
	γ_1	-0.009408	-1.281797
	γ_2	0.004667	1.537663
Non-systemic risk test	γ_0	-0.00013	-0.025498
	γ_1	-0.009441	-1.284134

	γ_2	0.004685	1.541038
	γ_3	0.000581	0.321643

Table 9: Test results of the entire period (2018.1.1 to 2023.12.31)

6.2 Summary of Findings

This thesis investigated the applicability of the Capital Asset Pricing Model (CAPM) in the context of the FTSEMIB Index. The research utilized weekly stock returns data from 38 companies listed on the chosen index spanning from January 1, 2018, to December 31, 2023.

The primary aim of this study was to assess the validity of the CAPM in the Italian stock market by analyzing:

1. Whether the intercept is zero and if the slope of the Security Market Line (SML) equals the average risk premium.
2. Whether there exists a linear relationship between the rate of return and its beta.
3. Whether non-systemic risk impacts the returns of portfolios.

Through the methodologies discussed in Chapter 4, the study generated results summarized in Table 10.

		Period 1	Period 2	Period 3	Period 4	Entire period
SML	γ_0	Support	Reject	Support	Reject	Support

	γ_1	Reject	Reject	Reject	Reject	Reject
Non-Linearity	γ_0	Support	Reject	Support	Reject	Support
	γ_1	Reject	Reject	Reject	Reject	Reject
	γ_2	Support	Support	Support	Support	Support
Non-systematic risk	γ_0	Support	Reject	Support	Reject	Support
	γ_1	Reject	Reject	Reject	Reject	Reject
	γ_2	Support	Support	Support	Support	Support
	γ_3	Reject	Support	Support	Support	Support

Table 10: Summary of findings of the empirical analysis

As we can see from the table above the tests supporting the CAPM hypothesis are only 51,11%, but we can conclude that:

- For the intercept coefficient (γ_0) the research achieved mixed results, some results showed that this coefficient is not statistically significant, as theorized by the model, while others showed the opposite.
- According to CAPM, the γ_1 coefficient should be equal to the average risk premium showing the linear relationship between beta and portfolio excess returns, which in this period has been different from 0. However, the tests showed unanimously that it is not significantly different from 0.
- For the γ_2 coefficient, which, if different from 0, shows a non-linear relationship between the beta and expected excess return on securities, the

tests have supported the no statistical significance, in line with the CAPM predictions.

- For the γ_3 coefficient, which according to the CAPM should be 0, as it gauges the influence of nonsystemic risk on the expected excess return on securities, the tests have supported that it is not significantly different from 0, but not in period 1.

7. Conclusion

7.1 Interpretation of Results

The results of our empirical analysis present a nuanced perspective on the applicability of the Capital Asset Pricing Model (CAPM) within the FTSEMIB index during the period spanning from January 1, 2018, to December 31, 2023. The outcome could be succinctly summarized as a "Yes and No" verdict. This verdict underscores the complexity of financial markets and the challenges inherent in modeling real-world phenomena with simplified theoretical frameworks. While the CAPM retains its status as a foundational theory in finance, its efficacy in explaining the intricacies of asset pricing dynamics within the FTSE MIB context appears to be mixed.

Nonetheless, it is essential to recognize that the CAPM, with its elegant simplicity, continues to offer valuable insights into the relationship between risk and expected returns. Even in instances where it may fall short of providing a comprehensive explanation, it serves as a valuable benchmark against which more sophisticated models can be compared.

Therefore, while acknowledging its limitations, we assert that the CAPM retains some degree of explanatory power within the FTSE MIB context, albeit perhaps in a more nuanced and partial manner than initially envisioned.

7.2 Limitations of the Study

In our rigorous evaluation of the Capital Asset Pricing Model (CAPM)'s performance within the context of the FTSE MIB over the period from January 1, 2018, to December 31, 2023, it is important to exercise caution in drawing definitive conclusions. A complete rejection of the model is not warranted based solely on our findings, as several key factors contribute to the nuanced nature of our assessment.

This discussion will delve into these factors in detail, highlighting the complexities and considerations that inform our interpretation.

Firstly, one of the primary considerations in our evaluation is the choice of the market index used as a proxy for the broader market. In this study, the FTSE MIB index was employed to represent the market portfolio. However, this choice introduces a significant layer of complexity and potential bias, as highlighted by Roll's critique. Roll (1977) pointed out that the true "market portfolio" in the CAPM framework is a theoretical construct encompassing all risky assets in the economy, not just a subset of publicly traded stocks. Consequently, any market index, including the FTSE MIB, is likely to deviate from this idealized market portfolio. This discrepancy introduces potential sources of bias and uncertainty, as the index may not fully capture the diversification benefits and risk exposures inherent in the theoretical market portfolio. As a result, our findings must be interpreted with an understanding that the proxy used may not perfectly align with the CAPM's assumptions.

Additionally, the estimation of betas for individual securities, which are crucial components of the CAPM equation, poses another layer of complexity. Betas are typically derived from historical data using statistical techniques such as regression analysis. While these estimates provide valuable insights into the relationship between individual asset returns and market returns, they are inherently subject to a degree of uncertainty and approximation. Factors such as the chosen time period for estimation, the frequency of data (e.g., daily, weekly, or monthly returns), and market conditions during the estimation period can all influence the resulting beta estimates. This inherent uncertainty can impact the robustness of our analysis and must be considered when interpreting the performance of the CAPM.

Moreover, the limited sample size and relatively brief observation period of our study constrain the scope and generalizability of our findings. The period from January 1, 2018, to December 31, 2023, includes significant market events, such as the COVID-19 pandemic, geopolitical tensions, and economic policy changes, which may have introduced anomalies and heightened volatility in the data. A more extensive dataset spanning a longer time horizon would provide a more comprehensive understanding of asset pricing dynamics and potentially mitigate the impact of measurement inaccuracies associated with a constrained sample size and observation period. A longer time frame would also help to smooth out short-term fluctuations and offer a clearer picture of long-term trends and relationships.

The market conditions and economic events during the study period further complicate the evaluation of CAPM. Economic shocks such as the COVID-19 pandemic had a profound impact on financial markets, causing unprecedented volatility and market anomalies. Geopolitical tensions and economic policy changes during this period also contributed to market instability. High volatility in particular can skew results and may not reflect normal market conditions, thereby affecting the validity of CAPM's predictions during such turbulent times. These events underscore the importance of considering the broader economic context when interpreting our findings.

Model assumptions also play a critical role in the assessment of CAPM's performance. The choice of the risk-free rate, often represented by government bonds, may not be truly risk-free, especially during times of economic uncertainty. CAPM's assumption of homogeneity of investor expectations—where all investors are presumed to have the same expectations for future returns—is another potential source of bias, as this is often unrealistic in practice. Investor behavior can vary significantly based on individual risk preferences, access to information, and market outlooks, leading to deviations from the model's assumptions.

Data quality and availability are additional factors that impact our analysis. There may be gaps or inconsistencies in the data, especially if there were any trading suspensions or irregularities. Survivorship bias is another concern, as the analysis may only include companies that survived the entire period, potentially skewing results. Companies that went bankrupt or were delisted during the study period are excluded, which can lead to an overestimation of market performance and an underestimation of risk.

Statistical and methodological issues also pose challenges. Estimation errors in calculating beta can be sensitive to the time period and frequency of data used. For instance, using daily data versus monthly data can yield different beta estimates. Additionally, financial time series data often exhibit non-stationarity, violating the assumptions of many statistical techniques used in CAPM tests. Non-stationarity refers to changes in the statistical properties of the data over time, which can lead to misleading results if not properly accounted for in the analysis.

The sectoral composition of the FTSE MIB is another important consideration. The index may have a concentration in specific sectors, such as finance or energy, and the CAPM might not fully capture sector-specific risks. Sector-specific risks can significantly impact the returns of individual stocks within those sectors, leading to deviations from the expected returns predicted by CAPM. A diversified market portfolio, as envisioned by CAPM, would ideally spread these risks across a broader range of sectors.

Behavioral factors also play a crucial role in shaping market outcomes. Behavioral finance suggests that investors may not always act rationally, contrary to the assumptions of CAPM. Phenomena such as herd behavior, overreaction, and underreaction can lead to price movements that are not explained by traditional financial models. Investor sentiment and cognitive biases can drive market trends, contributing to deviations from the CAPM's predictions.

Global influences further complicate the evaluation of CAPM within the context of the FTSE MIB. The performance of the Italian stock market can be significantly influenced by global economic conditions, including international trade policies, global financial crises, and cross-border capital flows. These factors might not be fully accounted for in a CAPM model that focuses solely on the Italian market, leading to potential inaccuracies in the model's predictions.

Finally, the study does not compare CAPM with alternative asset pricing models, such as the Fama-French three-factor model. These alternative models incorporate additional factors, such as size and value, which have been shown to provide a better fit for empirical data in certain contexts. Comparing CAPM with these models could offer a more comprehensive understanding of the return patterns observed and highlight the strengths and limitations of each model.

In conclusion, while our study provides valuable insights into the limitations of the CAPM within the FTSE MIB context, it is crucial to interpret our findings within the broader context of the model's theoretical underpinnings and the inherent complexities of financial markets. The choice of the market index, the estimation of betas, the limited sample size, and the relatively brief observation period all contribute to the nuanced nature of our assessment. Furthermore, considering the broader theoretical and empirical context, including behavioral finance and alternative asset pricing models, enhances our understanding of the CAPM's performance and limitations. Therefore, our evaluation should be viewed as part of an ongoing dialogue in the field of finance, contributing to a more comprehensive understanding of asset pricing dynamics while acknowledging the model's theoretical and practical complexities.

7.3 Implications for Investors

The Capital Asset Pricing Model (CAPM) has long been heralded as a valuable tool for investors seeking to forecast expected returns and assess the risk-return trade-off of individual assets. However, our analysis highlights the need for caution in relying solely on CAPM-derived estimates, as numerous external factors can influence asset pricing dynamics beyond the scope of the model. While CAPM provides a convenient framework for estimating expected returns based on systematic risk, it may overlook additional sources of risk and fail to account for the full spectrum of market dynamics.

In response to these limitations, multi-factor models have emerged as promising alternatives, aiming to incorporate a broader array of risk factors into the asset pricing equation. Nevertheless, identifying the precise factors that drive asset returns remains a complex and ongoing challenge.

Moreover, behavioural finance theory (Kahneman and Tversky, 1979) introduces further complexities by acknowledging the role of investor sentiment, cognitive biases, and irrational behavior in shaping market outcomes. Concepts such as irrational exuberance (Shiller, 2000) and black swan events (Taleb, 2007), underscore the importance of considering psychological factors alongside traditional financial metrics when making investment decisions.

Irrational exuberance refers to the phenomenon where investors exhibit excessive optimism about the prospects of financial markets or specific assets, leading to inflated asset prices that are not supported by underlying fundamentals. This concept suggests that market participants may become overly enthusiastic during periods of economic expansion or speculative bubbles, driving prices to unsustainable levels. Thus, in times of irrational exuberance, investors may succumb to herd mentality, ignoring warning signs and rational analysis in favor of following the crowd.

Instead, black swan events refer to rare and unpredictable occurrences that have profound and widespread impacts on financial markets and society. These events are characterized by their extreme rarity, their unexpected nature, and their significant consequences, often defying conventional predictive models and expectations. Unlike standard market risks, which can be quantified and managed to some extent, black swan events are by definition unforeseeable and can catch investors off guard. Examples of black swan events include natural disasters, geopolitical crises, and technological disruptions that have far-reaching implications for global markets, in our chosen period COVID-19 can be classified as a blackswan.

Therefore, while CAPM offers valuable insights into asset pricing dynamics, investors should approach its predictions with caution and supplement their analysis with additional tools and perspectives to account for the complexities of real-world markets.

7.4 Cost of Equity

The cost of equity, representing the rate of return required by investors to compensate for the risk of holding a particular stock, is a crucial input in various financial decision-making processes, including investment evaluation, capital budgeting, and corporate valuation.

If a company finds itself with excess liquidity, it has two primary avenues for action. The first entails promptly distributing the surplus to shareholders in the form of dividends. The second option involves deploying the excess funds into an investment project, subsequently distributing the project's future cash flows as dividends. However, the decision to pursue the latter course hinges on the project's expected return surpassing that of a financial asset with comparable risk. Consequently, the discount rate applied to evaluate the project should align closely with the expected return of a financial asset possessing similar risk characteristics.

From the company's point of view, the expected return represents the cost of capital. According to the CAPM, the expected return on equity can be written as:

$$R_E = R_F + \beta \times (R_M - R_F) \qquad (12)$$

Where:

- R_E: This represents the expected return on equity.
- R_F: This stands for the risk-free rate of return.
- β: This denotes the beta coefficient.
- R_M: This refers to the return on the market portfolio.
- $(R_M - R_F)$: This represents the market risk premium.

As we've said earlier, the beta coefficient has to be estimated and it is the covariance between a stock and the market, divided by the variance of the yield of the market.

However, betas may vary over time, the sample size could be inadequate, and betas might also be influenced by changes in financial leverage and the firm's risk profile. Consequently, if the operational activities of a particular company closely resemble those of other companies in the same industry, it is advisable to use the industry's beta to minimize estimation errors.

Despite the critiques found in academic literature, the Capital Asset Pricing Model (CAPM) maintains its status as the favored model in managerial finance courses and remains in practical use by managers. According to Welch (2008), approximately 75.0% of finance professors endorse utilizing the CAPM for estimating the cost of capital in capital budgeting. Additionally, Graham and Harvey's (2001) survey of chief financial officers reveals that 73.5% of respondents rely on the CAPM. Additionally, is interesting to notice the findings of a survey made by Brounen, Jong, and Koedijk (2006), reported in the table 11:

Method	U.S.	U.K.	Netherlands	Germany	France
Using CAPM (the beta approach)	73.49	47.06	55.56	33.96	45.16
With average historical returns on common stock	39.41	31.25	30.77	18.00	27.27
Using CAPM but including some extra “risk factors”	34.29	27.27	15.38	16.07	30.30
Backout from discounted dividend/earnings model	15.74	10.00	10.71	10.42	10.34
Whatever our investors tell us they require	13.93	18.75	44.83	39.22	34.38

By regulatory decisions	7.04	16.13	3.70	0.00	16.39

Table 11: Percentage of firms that estimate the cost of equity with the different methods.

However, the findings of our empirical analysis on the applicability of the Capital Asset Pricing Model (CAPM) within the FTSEMIB index from 2018 to 2023 have significant implications for managers who rely on the cost of equity estimates derived from CAPM theory.

The empirical evidence presented in this study challenges the reliability of CAPM-derived cost of equity estimates as accurate measures of required returns. While the CAPM framework provides a systematic method for estimating the cost of equity based on market risk, our analysis suggests that the model may not fully capture the complexities of asset pricing dynamics within the FTSE MIB context. The mixed results regarding the intercept and slope of the Security Market Line (SML) indicate potential deviations from CAPM predictions, casting doubt on the model's ability to provide precise estimates of required returns.

For managers tasked with evaluating investment opportunities and assessing corporate performance, the implications of these findings are profound.

Firstly, the discrepancies between CAPM-derived cost of equity estimates and actual market returns underscore the importance of exercising caution in relying solely on CAPM-based valuations. Managers may need to supplement their analysis with alternative valuation methods or adjust their cost of equity estimates to account for the limitations of the CAPM model.

Moreover, the limitations of the CAPM model highlight the need for a more nuanced approach to risk management and capital allocation. Recognizing that CAPM-derived estimates may not fully capture all relevant sources of risk, managers may need to adopt more sophisticated risk assessment techniques too and consider a broader range of risk factors when evaluating investment opportunities. This could involve incorporating industry-specific risk factors, company-specific fundamentals, and qualitative assessments of market sentiment into the decision-making process.

Furthermore, the findings of our analysis suggest that managers should remain vigilant and adaptive in their approach to cost of equity estimation, particularly in light of changing market conditions and economic dynamics. While the CAPM provides a valuable framework for understanding the relationship between risk and return, its efficacy may vary over time and across different market environments. Therefore, managers should regularly review and update their cost of equity estimates based on the latest market data and economic insights to ensure they accurately reflect the prevailing risk-return dynamics.

7.5 Future Research Directions

Looking ahead, several intriguing avenues for future research emerge from our analysis of the limitations and implications of the Capital Asset Pricing Model (CAPM). One promising direction involves extending the testing of the CAPM model across a broader period and with a more extensive sample size, allowing for a more robust assessment of its efficacy in different market environments. Additionally, experimenting with various market indexes as proxies for the market portfolio could help identify the most suitable benchmark for CAPM analysis within specific contexts.

Moreover, exploring alternative factors that exhibit consistent significance in explaining asset returns over extended periods could enhance our understanding of asset pricing dynamics and inform the development of more comprehensive asset pricing models. Furthermore, incorporating insights from behavioral finance into traditional asset pricing frameworks represents a fertile area for future research. Understanding how investor sentiment, cognitive biases, and market anomalies influence asset prices and expected returns could lead to more accurate and nuanced models of asset valuation.

Ultimately, by embracing interdisciplinary approaches and leveraging advances in data analytics and computational methods, future research endeavors have the potential to enrich our understanding of financial markets and refine our tools for investment decision-making.

8. References

8.1 Journals and Literature

1. Banz, R.W. (1981). "The relationship between return and market value of common stocks." *Journal of Financial Economics*, 9(1), 3-18.
2. Beltratti, A., & Di Tria, M. (2002). "The Cross–section of Risk Premia in the Italian Stock Market." *Economic Notes*, 31, 389-416. <https://doi.org/10.1111/1468-0300.00092> [last access: 05/21/24]
3. Black, F., Jensen, M.C., & Scholes, M. (1972). "The Capital Asset Pricing Model: Some Empirical Tests." In M.C. Jensen (Ed.), *Studies in the Theory of Capital Markets* (pp. 79-121). Praeger Publishers.
4. Brounen, D., de Jong, A., & Koedijk, K. (2006). "Corporate finance in Europe: confronting theory and practice." *Financial Management*, 33(4), 71-101.
5. Carhart, M.M. (1997). "On Persistence in Mutual Fund Performance." *The Journal of Finance*, 52(1), 57-82.
6. De Chiara, C., & Puopolo, G.W. (2015). "Testing the capital asset pricing model in the Italian market." *Corporate Ownership & Control*, 12(3), 40-47. <https://doi.org/10.22495/cocv12i3p4>
7. Elton, E.J., & Gruber, M.J. (1995). *Modern Portfolio Theory and Investment Analysis* (5th ed.). John Wiley & Sons, Inc.
8. Fama, E.F., & French, K.R. (1992). "The Cross-Section of Expected Stock Returns." *Journal of Finance*, 47(2), 427-465.
9. Fama, E.F., & French, K.R. (1993). "Common Risk Factors in the Returns on Stocks and Bonds." *Journal of Financial Economics*, 33(1), 3-56.
10. Fama, E.F., & MacBeth, J.D. (1973). "Risk, Return, and Equilibrium: Empirical Tests." *Journal of Political Economy*, 81(3), 607-636.
11. Graham, J.R., & Harvey, C. (2001). "The theory and practice of corporate finance: evidence from the field." *Journal of Financial Economics*, 60, 187-243.

12. Harris, C.R., Millman, K.J., van der Walt, S.J., et al. (2020). Array programming with NumPy. *Nature*, 585(7825), 357–362. <10.1038/s41586-020-2649-2> [last access: 05/21/24]
13. Hunter, J.D. (2007). "Matplotlib: A 2D Graphics Environment." *Computing in Science & Engineering*, 9(3), 90-95.
14. Kahneman, D., & Tversky, A. (1979). "Prospect Theory: An Analysis of Decision under Risk." *Econometrica*, 47(2), 263-291.
15. Lintner, J. (1965). "Security Prices, Risk and Maximal Gains from Diversification." *Journal of Finance*, 20(4), 587-615.
16. Mossin, J. (1966). "Equilibrium in a Capital Asset Market." *Econometrica*, 34(4), 768-783.
17. Rega, F.G. (2016). Dal CAPM ai modelli multifattoriali: una verifica empirica (An Empirical Test to Single and Multifactor CAPM Models in the Italian Stock Market). *Second University of Naples*. <https://ssrn.com/abstract=2956044>or <http://dx.doi.org/10.2139/ssrn.2956044> [last access: 05/21/24]
18. Roll, R. (1977). "A critique of the asset pricing theory"s tests: part I." *Journal of Financial Economics*, 4, 129-176.
19. Ross, S.A. (1976). "The arbitrage theory of capital asset pricing." *Journal of Economic Theory*, 13(3), 341–360. <10.1016/0022-0531(76)90046-6> [last access: 05/21/24]
20. Sharpe, W.F. (1964). "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *Journal of Finance*, 19(3), 425-442.
21. Shiller, R.J. (2000). *Irrational Exuberance*. Princeton University Press.
22. Taleb, N.N. (2007). *The Black Swan: The Impact of the Highly Improbable*. Random House.
23. Treynor, J. (1961). "*Market value, time, and risk*." Unpublished manuscript, presented at the 1961 meeting of the Western Finance Association.
24. Welch, I. (2008). "*The Consensus Estimate for the Equity Premium by Academic Financial Economists in December 2007*." Unpublished Working Paper, Brown University.

8.2 Internet Sources

1. FTSE Russell. Fact Sheets: FTSEMIB. <https://research.ftserussell.com/Analytics/FactSheets/Home/DownloadSingleIssue?issueName=FTSEMIB&isManual=False> [last access: 05/21/24]
2. GitHub. GioviManto/Testing-CAPM. <https://github.com/GioviManto/Testing-CAPM> [last access: 05/21/24]
3. Python Software Foundation. (2022). *Python Language Reference*, version 3.10.10. <http://www.python.org> [last access: 05/21/24]

9. Appendix

Appendix Table 1: Stock beta estimates. (Years: 2018-2019-2020-2021)

	Company	Beta 2018	Beta 2019	Beta 2020	Beta 2021
0	FTSEMIB Index	1.000000	1.000000	1.000000	1.000000
1	A2A	0.707333	0.572608	0.943380	0.907429
2	Amplifon	0.767687	0.142832	0.738002	1.096253
3	Azimut	0.986934	1.129163	1.197755	0.856314
4	BPER banca	1.031309	1.247849	1.155734	1.484485
5	Banca Mediolanum	0.934348	0.824005	1.130218	1.135823
6	Banca Monte dei Paschi di Siena	1.313966	1.922966	1.012127	1.231961
7	Banca pop Sondrio	1.053730	0.856272	1.205108	1.396168
8	Banco Bpm	0.986934	1.129163	1.197755	0.856314
9	Brunello cucinelli	0.428006	0.338281	0.664913	1.354659
10	Campari	0.465506	0.600058	0.667943	0.765316
11	Diasorin	0.835113	0.104282	-0.139913	-0.294731
12	Enel	0.819471	0.525617	1.026329	0.689178
13	Eni	0.788617	0.981083	1.239297	0.846176
14	Erg	0.773954	0.424123	0.720143	0.546355
15	Ferrari	1.017630	0.578417	0.546943	0.577323
16	Fineco	1.205031	1.354987	0.682070	1.203327
17	Generali Ass	0.874754	0.821140	1.003072	0.876757
18	Hera	0.585476	0.345686	0.661816	0.771398
19	Interpump	0.821302	0.992392	0.614499	1.273455
20	Intesa San Paolo	1.351083	1.295337	1.191645	1.334042
21	Inwit	0.971737	0.090477	0.412998	0.300971

22	Italgas	0.749921	0.496096	0.577299	0.575877
23	Leonardo	1.123506	1.229617	1.605905	0.726197
24	Mediobanca	1.116493	1.019197	1.408351	0.988110
25	Moncler	0.773354	0.892211	0.669733	1.176060
26	Pirelli	0.569951	1.725078	0.756518	1.073598
27	Poste Italiane	1.089143	0.928288	1.136797	1.168087
28	Prysmian	1.099756	1.284706	0.837578	0.991524
29	Recordati	0.393235	0.483568	0.416276	0.552578
30	Saipem	0.481572	1.665453	1.161933	1.313710
31	Snam	0.708777	0.533880	0.744483	0.558699
32	Stellantis	1.140059	1.567915	1.081512	1.820401
33	Stmicroeletronics	1.240147	1.419131	1.225969	0.896603
34	Telecom	1.319273	1.134180	0.856433	-0.129355
35	Tenaris	0.812125	1.495105	1.414110	1.146623
36	Terna	0.617907	0.540666	0.617340	0.581386
37	Unicredit	1.317415	1.747030	1.325598	1.430585
38	Unipol	1.080885	1.127788	1.174126	1.334526

Appendix Table 2: Betas and average excess portfolios returns. (Year 2020)

Year 2020	$R_{pt} - R_{ft}$	β_p	t-value	Std. error	Adj. R^2	F-stat
Portfolio 1	0.009647	0.582235	11.718196	0.049686	0.727733	137.3161123
Portfolio 2	0.007376	0.880060	27.540182	0.031955	0.936917	758.461642
Portfolio 3	0.003795	1.161095	17.940148	0.064721	0.862848	321.848911
Portfolio 4	0.006379	1.068906	23.968134	0.044597	0.918331	574.471430

Appendix Table 3: Betas and average excess portfolios returns. (Year 2021)

Year 2021	$R_{pt} - R_{ft}$	β_p	t-value	Std. error	Adj. R^2	F-stat
Portfolio 1	0.016060	0.645823	6.444869	0.100207	0.4380586	41.536341
Portfolio 2	0.016192	0.824877	9.211457	0.089549	0.617228	84.850943
Portfolio 3	0.013632	1.228325	13.590747	0.090380	0.779388	184.708428
Portfolio 4	0.017956	1.047713	23.968134	0.086705	0.736058	146.013614

Appendix Table 4: Betas and average excess portfolios returns. (Year 2022)

Year 2022	$R_{pt} - R_{ft}$	β_p	t-value	Std. error	Adj. R^2	F-stat
Portfolio 1	-0.005171	0.735765	9.535345	0.077162	0.638010	90.922811
Portfolio 2	-0.003843	0.790846	13.721543	0.057635	0.785967	188.280733
Portfolio 3	-0.002835	1.002764	17.897549	0.056028	0.862282	320.322264
Portfolio 4	-0.004761	1.274279	10.399268	0.122535	0.677511	108.144782