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#### 1. Introduction

- 2 The Capital Asset Pricing Model (CAPM) made its debut on the financial scene in the
- a early 1960s by Treynor<sup>1</sup>, Sharpe<sup>2</sup>, Lintner<sup>3</sup> and Mossin<sup>4</sup>. This model stands as one of
- 4 the foundational pillars of modern finance theory, offering a framework to
- 5 understand the relationship between risk and return in financial markets. Since its
- 6 inception, the CAPM has been extensively studied and applied in various market
- 7 contexts worldwide. However, the validity of its assumptions and the robustness of
- 8 its predictions have been subject to ongoing debate and empirical scrutiny.

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### 1.2 Objectives

- In this thesis, the primary objective is to empirically test the CAPM framework using
- data from the FTSEMIB Index from 01.01.2018 to 12.31.2023. Specifically, the study
- 13 aims to:
  - Estimate beta coefficients for individual stocks listed on the FTSEMIB Index
- Analyze the relationship between returns and CAPM posits to assess the
   model's power.
  - Investigate potential deviations from the CAPM predictions and their implications for investors.
  - Through these objectives, this research seeks to provide insights into the validity and reliability of the CAPM in explaining asset pricing dynamics within the Italian market context.

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#### 1.3 Structure of the Thesis

<sup>&</sup>lt;sup>1</sup> Treynor, Jack. (1961). "Market value, time, and risk." *Unpublished manuscript, presented at the 1961 meeting of the Western Finance Association.* 

<sup>&</sup>lt;sup>2</sup> Sharpe, W.F. (1964) "Capital Asset Prices: A Theory of market Equilibrium under Condition of Risk." *Journal of Finance*. 19: 425-442.

<sup>&</sup>lt;sup>3</sup> Lintner, J. (1965) "Security Prices, Risk and Maximal Gains from Diversification." *Journal of Finance*. 20: 587-615.

<sup>&</sup>lt;sup>4</sup> Mossin, J. (1966). "Equilibrium in a capital asset market". *Econometrica* 34: 768-783.

1 The remainder of this thesis is organized as follows:

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- Chapter 2 provides a comprehensive review of the relevant literature on the CAPM, previous empirical studies examining its performance in different market settings, and any identified gaps in the existing literature.
- Chapter 3 outlines the data collection procedures employed in this study, with relevant choices on which data gather.
  - Chapter 4 explains the procedure used to test and evaluate the CAPM, showing also the statistical tools used.
    - Chapter 5 presents the data analysis of the first period, including an overview
      of the Italian stock market during the specified period, analysis of beta
      coefficients, and all the tests used to assess CAPM validity.
    - Chapter 6 shows the other periods' results, discusses the results of the empirical analysis as a whole and interprets the findings in the context of the CAPM, highlighting any deviations.
    - Chapter 7 offers concluding remarks, highlights the key contributions of the study, identifies its limitations, and suggests directions for future research.

#### 2. Literature Review

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### 2.1 Capital Asset Pricing Model (CAPM)

- 4 The Capital Asset Pricing Model (CAPM) has been a cornerstone of modern finance
- 5 theory since its introduction. According to the CAPM, the expected return on an asset
- 6 is determined by its beta (systematic risk), the risk-free rate, and the market risk
- 7 premium.

$$E[R_i] = R_f + \beta_i (E[R_m] - Rf) \tag{1}$$

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- 10 Where:
- 11  $R_f$  signifies the risk-free rate of return,
- 12  $\beta_i$  denotes the beta of security i,
- $-E[R_m]$  represents the expected return on the market, and
- 14  $(R_m R_f)$  indicates the market premium.

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- 16 It is useful to underline three key concepts:
- Expected return is the return that an investor expects to obtain from a security in the next period. Since it is an expectation, the return that will actually be realized may be higher or lower.
  - Systematic risk (or market risk) is any risk that affects, to a greater or lesser extent, a large number of assets (e.g., news regarding GDP, interest rates, general economic conditions), it is implied in the investment on a specific asset, and it can't be eliminated.
    - Unsystematic risk (or specific risk) is a risk that specifically affects a single
      asset or a limited number of assets (e.g., announcement of a strike within a
      small company), it is measured by the variance, and it can be eliminated by
      investing in a diversified portfolio.

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Now it's the moment to explain what the famous beta is: The beta measures the sensitivity of a stock to changes occurring in the market portfolio.

The formula of the beta is:

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$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \tag{2}$$

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- 5 Where:
- 6  $Cov(R_i, R_m)$  is the covariance between the return on stock i and the return on
- 7 market portfolio.
- 8  $Var(R_m)$  represents the variance ( $\sigma^2$ ) of the return on the market portfolio.

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Moreover, it's very important to notice that:

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$$\sum_{i=1}^{N} X_i \beta_i = 1 \tag{3}$$

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where  $X_i$  is the proportion of the market value of stock i relative to the entire market. Therefore, the beta of the market portfolio is, by definition, equal to 1.

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- 18 If an individual, instead of holding a single security, maintains a diversified portfolio,
- 19 they would still regard the variance (or the root mean square deviation) of the
- 20 portfolio's returns as the appropriate measure of portfolio risk, but they wouldn't
- 21 focus on the variance of each individual security's returns anymore. Instead, they
- 22 would concentrate on the contribution each individual security makes to the
- 23 portfolio's variance.

- 25 Under the assumption of homogeneous expectations, all individuals hold the market
- 26 portfolio. Therefore, we measure risk based on the contribution of a single security
- 27 to the variance of the market portfolio. This contribution, once properly standardized,
- represents the security's beta. While very few investors hold exactly the market
- 29 portfolio, many hold reasonably diversified portfolios. These portfolios are sufficiently
- 30 close to the market portfolio that a security's beta is a reasonable measure of its risk.

- 2 We can directly visualize the CAPM postulates with his graphical representation, the
- 3 Security Market Line (SML). An example of a hypothetical SML is represented in figure
- 4 1, obtained using Python<sup>5</sup> with Numpy<sup>6</sup> and Matplotlib<sup>7</sup> libraries.

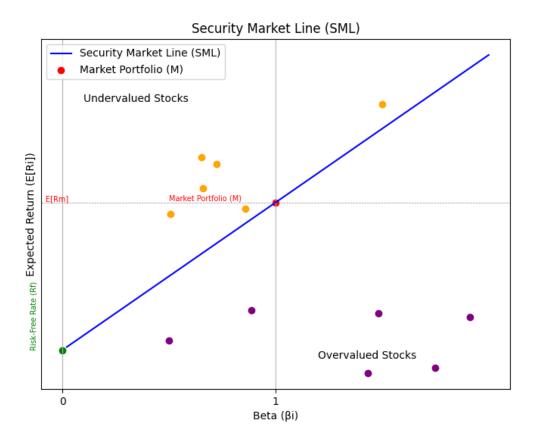


Figure 1: An example of a Security Market Line

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<sup>&</sup>lt;sup>5</sup> Python Software Foundation. Python Language Reference, version 3.10.10. Available at <a href="http://www.python.org">http://www.python.org</a>

<sup>&</sup>lt;sup>6</sup> Harris, C.R., Millman, K.J., van der Walt, S.J. et al. *Array programming with NumPy*. Nature 585, 357–362 (2020). DOI: <u>10.1038/s41586-020-2649-2</u>. (<u>Publisher link</u>).

<sup>&</sup>lt;sup>7</sup> J. D. Hunter, "Matplotlib: A 2D Graphics Environment", Computing in Science & Engineering, vol. 9, no. 3, pp. 90-95, 2007.

- 1 The SML is characterized by a linear relationship between beta and expected return.
- 2 It starts from the risk-free rate on the y-axis, denoting the return of a risk-free
- 3 investment. The slope of the SML represents the market risk premium, indicating the
- 4 additional return investors require for bearing one unit of systematic risk. Moreover,
- 5 the SML serves as a benchmark for evaluating investment opportunities.
- 6 Investments lying above the SML are considered undervalued, offering higher
- 7 expected returns relative to their systematic risk. Conversely, investments below the
- 8 SML are deemed overvalued, as they fail to provide adequate compensation for the
- 9 level of risk assumed. By comparing individual investments to the SML, investors can
- assess whether they offer sufficient returns given their risk profile.

- 12 This simple yet powerful model equips investors with a tool to evaluate investments.
- 13 If the expected return fails to meet or exceed the required return, the investment is
- 14 deemed unfavorable.

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- Operating within a set of assumptions, the CAPM framework posits several key conditions:
- All investors are risk-averse, striving to maximize the expected utility of their end-of-period wealth, thus implying a one-period model.
  - Investors hold homogeneous expectations regarding asset returns and utilize the same expected return and covariance matrix of stock returns.
  - A fixed risk-free rate enables investors to borrow or lend without restriction at a uniform interest rate.
  - In a perfectly competitive market, all stocks are divisible and priced, with nontradable assets such as education, private enterprises, and governmentfunded assets.
  - Market imperfections such as taxes, regulations, or trading costs are assumed to be absent.
  - The number and quantities of stocks are predetermined within the one-period world.

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- 1 While these assumptions may pose challenges in real-world applications, CAPM offers
- 2 a reasonably accurate depiction of reality as a financial theory.
- 3 Despite its widespread use in financial practice, the CAPM has faced scrutiny
- 4 regarding its assumptions and empirical validity.

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### 2.2 Empirical Validation of the CAPM

- 7 Since its inception in the early 1960s, the Capital Asset Pricing Model (CAPM) has
- 8 remained a focal point of discussion in financial economics.
- 9 The primary objective of the CAPM is to explain variations in the risk premium among
- assets. It posits that these variations stem from differences in the riskiness of asset
- returns, with beta serving as the key measure of asset risk. According to the CAPM,
- the risk premium per unit of risk is consistent across all assets. By leveraging the
- risk-free rate and beta, the model predicts the expected risk premium for each asset.
- In practical applications, the CAPM serves as a guiding framework for portfolio
- management and asset selection. Investors often utilize CAPM-derived metrics to
- identify undervalued or overvalued assets, informing their buying and selling
- decisions. Similarly, in corporate finance, estimated beta coefficients are instrumental
- in assessing the riskiness of investment projects and establishing minimum hurdle
- rates for project viability, or the cost of equity.
- However, the CAPM has faced criticism over the years, sparking extensive academic
- 21 debate regarding its validity and usefulness. Empirical testing of the CAPM serves
- 22 two main purposes: (i) to determine whether the model should be rejected and (ii)
- 23 to provide insights for financial decision-making. Statistical methods are employed to
- 24 rigorously evaluate whether the data support or refute the model's predictions.
- 25 Furthermore, is useful to underline that the CAPM model is just a construct that
- 26 needs to be tested empirically and that it can give some insights for the reality.

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### 2.3 The Roll's critique

- 1 The Roll's critique (1977)<sup>8</sup> of the market portfolio and the Capital Asset Pricing Model
- 2 (CAPM) revolves around two central arguments.
- 3 Firstly, Roll posits the mean-variance tautology, which asserts that any portfolio
- 4 achieving mean-variance efficiency automatically satisfies the CAPM equation. This
- 5 equation establishes a linear relationship between an asset's expected return, the
- 6 risk-free rate, and its beta coefficient multiplied by the expected excess return of the
- 7 market portfolio. Essentially, Roll argues that the CAPM equation is inherently true
- 8 for any portfolio meeting the criteria of mean-variance efficiency, without
- 9 necessitating specific model assumptions. Consequently, testing the CAPM equation
- 10 becomes synonymous with evaluating the mean-variance efficiency of the portfolio,
- especially when using a proxy for the market portfolio. Therefore, if one assumes the
- market to be mean-variance efficient, the CAPM equation becomes self-referential
- and loses its empirical testability.
- 14 Secondly, Roll highlights the unobservability of the market portfolio. He contends
- 15 that the true market portfolio would comprise every conceivable investment
- opportunity, ranging from traditional assets like stocks and bonds to alternative
- investments like real estate, commodities, and collectibles. However, the returns on
- these diverse investment options are practically impossible to observe
- comprehensively. Without the ability to observe the returns on all investment
- 20 opportunities, it becomes exceedingly challenging to ascertain whether any portfolio,
- including the market portfolio, is genuinely mean-variance efficient.
- 22 Consequently, Roll suggests that the inability to observe the returns on the entire
- 23 universe of investment opportunities poses a significant obstacle to empirically
- 24 testing the CAPM.

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# 2.4 Confirmation of Theory through Empirical Analysis

<sup>&</sup>lt;sup>8</sup> Roll, R. (1977). A critique of the asset pricing theory's tests: part I. *Journal of Financial Economics*, 4, 129-176.

- 1 The CAPM was developed in the early 1960s in its basic form, the model predicts that
- the expected return on an asset above the risk-free rate is directly proportional to its
- 3 non-diversifiable risk, measured by beta. In front of the huge popularity of the model
- 4 and considering the weaknesses of every empirical analysis, is useful to understand if
- 5 it can explain at least a part of the variations in the stocks yields.

- 7 One of the seminal empirical studies supporting the CAPM was conducted by Black,
- 8 Jensen, and Scholes (1972)<sup>9</sup>. Utilizing portfolio data rather than individual stocks,
- 9 they tested the linearity of the relationship between expected returns and beta. By
- aggregating securities into portfolios, they mitigated firm-specific risk, enhancing the
- precision of beta estimates and expected returns. The study found strong empirical
- support for the CAPM, demonstrating a linear relationship between average returns
- and beta across portfolios.
- 14 Similarly, Fama and McBeth (1973)<sup>10</sup> examined the relationship between average
- returns and beta, along with the influence of beta squared and asset return volatility
- on residual return variations. Their findings provided further validation for the CAPM,
- affirming the positive linear association between returns and beta.

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# 2.5 Critiques and Challenges to CAPM

- In the early 1980s, research began to identify deviations from the linear risk-return
- 21 tradeoff predicted by the CAPM. Banz (1981)<sup>11</sup> challenged the model by
- demonstrating that firm size could better explain variations in average returns than
- beta alone, leading to the discovery of the size effect. At its core, the size effect
- 24 unveils that smaller companies, often termed small-cap stocks, exhibit a tendency to

<sup>&</sup>lt;sup>9</sup> M.C. JENSEN, Michael C., BLACK, Fischer, & SCHOLES, Myron S. (1972). The Capital Asset Pricing Model: Some Empirical Tests. In Michael C. Jensen (Ed.), Studies in the Theory of Capital Markets (pp. 1-78). Praeger Publishers Inc.

<sup>&</sup>lt;sup>10</sup> Fama, E. F., & MacBeth, J. D. (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy*, 81(3), 607-636.

<sup>&</sup>lt;sup>11</sup> Banz, R. W., "The relationship between return and market value of common stocks," Journal of Financial Economics, vol. 9, no. 1, 1981, pp. 3-18.

- outperform their larger counterparts, known as large-cap stocks. This phenomenon
- 2 persists even after adjusting for risk factors like beta, suggesting that size plays a
- 3 significant role in driving stock returns.
- 4 However, the CAPM was seriously guestioned when in the 1990s Fama<sup>12</sup> and
- 5 French<sup>13</sup> presented findings that raised significant questions about the CAPM's
- 6 validity. Firstly, they conclude that for US firms, the relationship between average
- 7 return and beta is weak between 1941 and 1990 and practically non-existent
- 8 between 1963 and 1990. They assert that the average return of a stock is negatively
- 9 correlated with both the price-earnings ratio (P/E) and the market value-to-book
- value ratio (P/BV) of the firm, so the value effect underscores the notion that stocks
- with lower prices relative to their book value, or high book-to-market ratios, tend to
- 12 yield superior returns compared to stocks with higher prices relative to book value.
- 13 These high book-to-market ratio stocks, often categorized as value stocks,
- 14 consistently deliver higher returns than their low book-to-market ratio counterparts,
- 15 referred to as growth stocks.
- In summary, despite the extensive research efforts involving numerous papers and
- models, along with rigorous empirical testing, the outcomes have remained
- inconclusive. A clear verdict has yet to be reached regarding the effectiveness of the
- 19 Capital Asset Pricing Model (CAPM) and its alternatives. The ongoing debates within
- 20 academic and financial communities underscore the complexities inherent in
- 21 understanding stock returns and asset pricing mechanisms. This uncertainty
- 22 emphasizes the dynamic nature of financial markets and the persistent quest for a
- 23 more comprehensive understanding of these phenomena.

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### 2.6 Alternative Models

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<sup>&</sup>lt;sup>12</sup> Fama, E. F., & French, K. R. "The Cross-Section of Expected Stock Returns." Journal of Finance, vol. 47, no. 2, 1992, pp. 427-465.

<sup>&</sup>lt;sup>13</sup> Fama, E. F., & French, K. R. "Common Risk Factors in the Returns on Stocks and Bonds." Journal of Financial Economics, vol. 33, no. 1, 1993, pp. 3-56.

- 1 An alternative pricing model is the Arbitrage Pricing Theory (APT) proposed by
- 2 economist Stephen Ross in 1976<sup>14</sup>. Unlike the Capital Asset Pricing Model (CAPM),
- which focuses solely on the relationship between the expected return of a single
- 4 asset and its systematic risk (beta), APT takes a broader view.
- 5 At its core, APT posits that the expected return of an asset can be modeled as a
- 6 linear function of various factors that influence its risk. These factors could be
- 7 macroeconomic variables, industry-specific conditions, or any other relevant market
- 8 forces.
- 9 A widely employed factorial model in finance is the Fama-French Three-Factor Model.
- 10 It seeks to explain stock returns using three key factors: market risk, the
- performance discrepancy between small-cap and large-cap firms, and the
- 12 performance discrepancy between high book-to-market value firms and low book-to-
- market value firms. The premise of this model lies in the observation that companies
- with high value and small market capitalization often demonstrate consistent
- outperformance compared to the broader market.
- 16 Thus, the model can be expressed through the following equation:

18 
$$E[R_i] = R_f + \beta_1(E[R_m] - R_f) + \beta_2(SMB) + \beta_3(HML) + \varepsilon_i$$
 (4)

Where:

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- $E[R_i]$  represents the expected return of the asset ii.
- $R_f$  denotes the risk-free rate, which is the return on an investment with zero risk, often approximated by government bonds.
  - $E[R_m]$  stands for the expected return of the market portfolio.
- $\beta_1$  is the beta coefficient of the asset with respect to the market, indicating its sensitivity to market movements..

<sup>14</sup> Ross, Stephen A (1976-12-01). "The arbitrage theory of capital asset pricing". *Journal of Economic Theory*. **13** (3): 341–360. doi:10.1016/0022-0531(76)90046-6. ISSN 0022-0531.

- SMB represents the size premium, which captures the excess return of smallcap stocks over large-cap stocks. A positive SMB coefficient indicates that small-cap stocks outperform large-cap stocks.
  - HML denotes the value premium, which captures the excess return of high book-to-market value stocks over low book-to-market value stocks. A positive HML coefficient implies that value stocks outperform growth stocks.
  - $\beta_2$  and  $\beta_3$  are the respective coefficients for the size premium (SMB) and the value premium (HML).
  - $\varepsilon_i$  represents the error term, capturing any other factors not accounted for by the model that influence the return of asset i.
- Moreover, the studies of Mark Carhart<sup>15</sup> in 1997 brought to the momentum factor,
- which is the tendency for assets that have performed well in the recent past to
- continue performing well in the short term, and vice versa. His empirical analysis
- demonstrated that momentum is a persistent anomaly in financial markets,
- challenging the efficient market hypothesis which suggests that all available
- information is quickly incorporated into asset prices. Carhart found that stocks that
- 17 have exhibited strong performance over the past several months tend to continue
- performing well, while those with weak performance continue to underperform.
- 19 Thus, the Carhart factorial model accounts the momentum as the fourth factor, as
- 20 expressed by the following equation:

22 
$$E[R_i] = R_f + \beta_1(E[R_m] - R_f) + \beta_2(SMB) + \beta_3(HML) + \beta_3(HML) + \beta_4(MOM) + \varepsilon_i$$
  
23 (5)

24 Where:

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• *MOM* captures the excess return of assets that have experienced recent positive performance relative to assets with weaker recent performance.

<sup>15</sup> Carhart, M. M. (1997). "On Persistence in Mutual Fund Performance." The Journal of Finance, 52(1), 57-82.

- In addition, is crucial to say that in the APT model the theory suggests that investors
- 2 are rational and will exploit any arbitrage opportunities that arise due to mispricing of
- 3 assets.
- 4 The beauty of APT lies in its flexibility. Unlike CAPM, which relies heavily on the
- 5 market beta, APT allows for the inclusion of multiple factors, making it more
- 6 adaptable to different market conditions and investment strategies. This flexibility
- 7 also means that APT can better account for the unique characteristics of different
- 8 assets and markets, leading to more accurate pricing predictions.
- 9 However, it's essential to note that APT is not without its challenges. One of the main
- criticisms is the difficulty in identifying and quantifying the relevant factors that drive
- asset prices. Additionally, the assumption of no arbitrage opportunities may not
- 12 always hold true in real-world markets, especially in less efficient or segmented
- 13 markets.

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#### 2.7 Previous Studies on CAPM for the Italian Stock Market

- Numerous empirical studies have examined the applicability of the CAPM in different
- market contexts, including the Italian stock market. For instance, studies by Beltratti
- and Di Tria (2002)<sup>16</sup>, that compares multi-factor models with Italian stock market
- data for the period 1990-2000, using the CAPM as the relevant benchmark. The
- result of this research is that the presumably large size of shocks to returns in the
- 21 Italian case and the presence of extra factors makes it difficult to explore the relation
- among expected returns. Another important study is the one of De Chiara and
- Puopolo (2015)<sup>17</sup>, in which they intended to show that the CAPM, despite the heavy
- critical comments, still holds in the Italian market when returns are measured at the
- 25 monthly frequency and that the market portfolio fully explains the cross section of
- stock returns with no need to appeal for additional determinants.

<sup>&</sup>lt;sup>16</sup> Beltratti, A. and Di Tria, M. (2002), The Cross–section of Risk Premia in the Italian Stock Market. Economic Notes, 31: 389-416. <a href="https://doi.org/10.1111/1468-0300.00092">https://doi.org/10.1111/1468-0300.00092</a>

<sup>&</sup>lt;sup>17</sup> De Chiara, C., & Puopolo, G. W. (2015). Testing the capital asset pricing model in the Italian market. *Corporate Ownership & Control*, *12*(3), 40-47. https://doi.org/10.22495/cocv12i3p4

- In addition, the Rega (2016)<sup>18</sup> research outlines how market risk explains stock
- 2 returns because all the Jensen's alphas are around zero, it also partially proves that
- 3 macroeconomic factors do not have a significant influence (APT) and finally states
- 4 that the Fama and French Model (FFM) explains stock returns better than the Capital
- 5 Asset Pricing Model (CAPM).
- 6 Thus, findings from these studies have been mixed, with some suggesting support
- 7 for the model while others finding deviations from its predictions.

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### 2.8 Gaps in the Literature

- 10 Despite the existing body of research on the CAPM and the Italian stock market,
- several gaps remain in the literature. First, there is a need for more comprehensive
- studies that consider the dynamics of both individual stocks and portfolios within the
- 13 Italian market. Second, previous studies have often focused on specific time periods
- or subsets of stocks, limiting the generalizability of their findings. Additionally, there
- is a lack of consensus regarding the factors that influence the performance of the
- 16 CAPM in the Italian context, highlighting the need for further investigation.

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<sup>18</sup> Rega, Federico Giovanni, Dal CAPM ai modelli multifattoriali: una verifica empirica (An Empirical Test to Single and Multifactor CAPM Models in the Italian Stock Market) (November 1, 2016). Available at SSRN: <a href="https://ssrn.com/abstract=2956044">https://ssrn.com/abstract=2956044</a> or <a href="http://dx.doi.org/10.2139/ssrn.2956044">https://dx.doi.org/10.2139/ssrn.2956044</a>

### 3. Methodology

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#### 3.1 Data Collection

- 4 The first step in our analysis involves collecting data on stock prices, market indices,
- 5 and risk-free rates for the Italian stock market over the specified time period
- 6 (1.1.2018 12.31.2023). Data is sourced from the reputable financial database of
- 7 Bloomberg, to ensure accuracy and reliability. We will retrieve weekly closing prices
- 8 for individual stocks listed on the Italian stock exchange, as well as corresponding
- 9 market indices (FTSE MIB) and risk-free rates (Italian government bonds).
- 10 Therefore, we utilize the Italian 3-month Treasury Bill, known as "Buoni Ordinari del
- Tesoro (BOT)", converted in weekly returns (dividing for 52 weeks the annualized
- return) as a proxy for the risk-free rate  $(R_f)$ . Additionally, we employ the price index
- "FTSE MIB", as proxy of the market portfolio, to represent the market return  $(R_m)$ ,
- which consists of the 40 most-traded stock classes on the exchange.

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# 3.2 Selection of Data Frequency

- 17 In order to obtain more precise estimates of the beta coefficient, our study adopts
- weekly stock returns as the focal point of analysis. This decision is grounded in the
- 19 recognition that using returns calculated over longer time periods, such as monthly
- 20 returns, may introduce biases in beta estimation. These biases stem from potential
- changes in beta values over the examined period, compromising the accuracy of our
- 22 estimates.
- Conversely, employing high-frequency data, such as daily returns, poses challenges
- due to the inherent noise present in such data. Utilizing daily returns could result in
- 25 the inclusion of excessive variability, leading to inefficient beta estimation. Hence,
- weekly returns strike a balance between capturing meaningful trends in stock
- 27 performance and mitigating the effects of excessive noise inherent in daily data.

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#### 3.3 Choice of Market Index

- 30 The FTSE MIB Index comprises the 40 most liquid and capitalized stocks listed on the
- Borsa Italiana (BIt) MTA and MIV markets. It is the primary benchmark index for the

- 1 Italian equity market and represents the approximately 80% of the domestic market
- 2 capitalisation. Stocks included in the index are weighted based on free-float,
- 3 ensuring that only the investable opportunity set is represented. Constituents of the
- 4 index are capped at 15% to prevent over-concentration, anyway the most capitalized
- 5 company in the FTSE MIB is Unicredit with a weight on the index of about 11.43%
- on 29 March 2024. Moreover, referring on 30 April 2024, the top 10 Holdings
- 7 constitute the 72.59% of the Index market cap and the ICB supersectors as banks
- 8 (28.61%), autmobiles and parts (18.57%), utilities (13.54%), energy (10.19%) and
- 9 insurance (7.47%) weight the 78.38% of the Index<sup>19</sup>.
- 10 Therefore, the "FTSE MIB" primarily comprises value and large-cap stocks, which
- serves to mitigate the influence of the Fama and French size and value effects,
- discussed earlier, reducing potential confounding factors associated with these
- 13 effects.

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### 3.4 Data Exclusion

- In the specified time frame, there were two notable changes in the composition of
- the FTSEMIB Index. The first occurred on June 24, 2019, with Nexi replacing Banca
- Generali in the index. The second change occurred on March 1, 2022, when Iveco
- 19 entered the FTSE MIB following the spin-off of CNH Industrial's On-Highway
- activities. Consequently, data for Nexi is missing approximately 21.41%, while data
- for Iveco is missing around 66.77%. Due to the significant amount of missing data
- for these companies, they were excluded from the analysis.

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### 4. Procedure for CAPM Testing and Evaluation

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### 4.1 Testing Procedure

- 4 The study examines the period spanning from January 1, 2018, to December 31,
- 5 2023, focusing on testing the predictions of the Capital Asset Pricing Model (CAPM).
- 6 Following the approach outlined by Black, Jensen, and Scholes in 1972, as well as
- 7 the methodology employed by Fama and MacBeth in 1973, we divide the analysis
- 8 into an Initial Estimation Period and a Testing Period. During the Initial Estimation
- 9 Period, we estimate the beta coefficients of the portfolios, while the Testing Period is
- utilized to compute the results, as detailed in Table 3.
- 11 Black, Jensen, and Scholes introduced a time series test of the CAPM, which involves
- regressing excess portfolio returns on excess market returns. This regression
- 13 equation is expressed as:

$$R_{it} - R_{ft} = \alpha_i + \beta_i (R_{mt} - R_{ft}) + \varepsilon_{it}$$
 (6)

- 15 Where:
- $R_{it}$  represents the rate of return on asset i at time t,
- R<sub>ft</sub> denotes the risk-free rate at time t,
- $R_{mt}$  signifies the rate of return on the market portfolio at time t,
- $\beta_i$  is the beta of stock i
- $\epsilon_{it}$  represents the random disturbance term in the regression equation at time
- 21 t.
- $\alpha_i$  is the intercept term
- 23 We can think of:
- $(R_{it} R_{ft})$  as the excess return of stock i, and
- 25  $(R_{mt} R_{ft})$  as the average risk premium
- The intercept term,  $\alpha_i$ , is the so-called Jensen's alpha and represents the deviation
- between the expected return estimated through time series averaging and the
- 28 expected return predicted by the CAPM. In an ideal scenario where the CAPM
- 29 accurately portrays expected returns and a suitable market portfolio proxy is chosen,
- the regression intercepts for all portfolios (or assets) would ideally be zero. This

- suggests that there is no systemic bias in the CAPM predictions. However, any
- 2 departure from zero indicates a disparity between the model's forecasts and the
- actual market returns observed. Therefore, analyzing the intercept term enables us
- 4 to gauge the extent to which the CAPM effectively explains the dynamics of asset
- 5 pricing.

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#### 4.2 Estimation of Beta Coefficients

- 8 Once the data collection phase is complete, we will proceed to estimate beta
- 9 coefficients for each individual stock in our sample.
- In the estimation of beta coefficients, we embark on a crucial step in our analysis
- aimed at quantifying the relationship between individual stock returns and overall
- market movements. Beta serves as a fundamental metric within the Capital Asset
- 13 Pricing Model (CAPM), providing insights into the systematic risk inherent in each
- 14 stock's returns.
- 15 To accomplish this, we employ regression analysis as our primary analytical tool. This
- method allows us to quantify the sensitivity of individual stock returns to changes in
- the broader market by assessing the historical relationship between a stock's returns
- 18 and market returns.
- 19 Specifically, for each stock, the estimation of beta using weekly returns aligns with
- 20 each Portfolio Formation Period. Beta was determined by regressing the weekly
- return of each stock against the market index, using the equation (6). Thus, a beta
- coefficient greater than 1 indicates that the stock is more volatile than the market,
- while a beta coefficient less than 1 suggests lower volatility compared to the market.
- 24 A beta coefficient of 1 implies that the stock moves in perfect correlation with the
- 25 market.

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#### 4.3 Portfolio Construction

- 28 In portfolio construction, it's important to utilize the true beta values of stocks.
- However, since we typically have access only to estimated betas, there's a potential
- for bias. Sorting stocks into portfolios based on estimated betas may introduce
- 31 selection bias, particularly for stocks with high estimated betas, which are more
- 32 prone to positive measurement errors. This could lead to a positive bias in high-beta

- portfolios and a negative bias in intercept estimates<sup>9</sup>. To address this issue, Black,
- 2 Fischer, Jensen, and Scholes<sup>10</sup> employed a grouping method. They estimated the
- 3 beta coefficients for stocks based on data from the previous year. These estimated
- 4 betas were then used to group stocks into portfolios for the following year. By doing
- 5 so, they aimed to reduce potential statistical errors associated with beta estimation.
- 6 This approach allowed them to mitigate biases that may arise from using estimated
- betas directly, as the beta estimates from the previous year are likely to be more
- 8 accurate and stable over time compared to estimates based on shorter time frames
- 9 or more recent data<sup>20</sup>.
- 10 The initial step involves partitioning the six-year period into four intervals, with each
- interval spanning three years:
- 1) The first period is: 2018.1.1 2020.12.31;
- 2) The second period is: 2019.1.1 2021.12.31;
- 14 3) The third period is: 2020.1.1 2022.12.31;
- 15 4) The fourth period is: 2021.1.1 2023.12.31.
- 16 The study's structure is outlined in Table 4.

	Period 1	Period 2	Period 3	Period 4
	2018-2020	2019-2021	2020-2022	2021-2023
Portfolio Formation Period	2018	2019	2020	2021
Initial Estimation Period	2019	2020	2021	2022
Testing Period	2020	2021	2022	2023

- 17 **Table 1**: Portfolio Formation, Estimation and Testing Periods.
- 18 Using the estimated betas computed for each stock, as illustrated above, we
- segmented the 38 stocks into 4 portfolios, each containing 10 or 9 stocks categorized

<sup>9</sup> Elton, E. J. and Gruber, M. J. (1995) *Modern Portfolio Theory and Investment Analysis*. 5th edition, New York: John: Wiley & Sons, Inc. p.311-355.

<sup>10</sup> Black, Fischer, Michael C. Jensen, and Myron Scholes. "The Capital Asset Pricing Model: Some Empirical Tests." In *Studies in the Theory of Capital Markets*, edited by Michael C. Jensen, 79-121. Praeger Publishers, 1972.

- by their beta values. Portfolio 1 consists of the 10 stocks with the lowest betas, while
- 2 Portfolio 4 comprises the 10 stocks with the highest betas. By consolidating securities
- 3 into portfolios, we effectively mitigate much of the firm-specific component of
- 4 returns, thereby improving the accuracy of beta estimates and expected returns for
- 5 the portfolios. Additionally, I have noticed a significant variation of portfolios
- 6 composition during the periods.

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### 4.4 Portfolio's Betas and Ex-Post SML

- 9 After that, we are ready to compute the betas of the portfolios using a similar
- equation of the (6) but with portfolios and no single stocks:

$$R_{pt} - R_{ft} = \alpha_p + \beta_p (R_{mt} - R_{ft}) + \varepsilon_{pt}$$
 (7)

- 12 Where:
  - R<sub>pt</sub> represents the rate of return on portfolio p at time t,
- $R_{ft}$  denotes the risk-free rate at time t,
- $R_{mt}$  signifies the rate of return on the market portfolio at time t,
- $\beta_p$  is the beta of portfolio p
- $\epsilon_{pt}$  represents the random disturbance term in the regression equation at time
- 18 t.
- $\alpha_p$  is the intercept term
- 20 To evaluate the Capital Asset Pricing Model (CAPM), Fama and MacBeth utilize a
- 21 monthly cross-sectional regression (CSR) approach. This involves regressing the
- 22 excess return of each portfolio against its estimated beta. Essentially, a simple cross-
- 23 sectional regression entails regressing the average excess return against the market
- 24 beta for various portfolios. The average excess return for each portfolio is calculated
- as the mean excess return over a specified period, while the market beta  $(\beta_p)$
- 26 represents the slope in the time series regression of the average portfolio's excess
- return  $(R_p R_f)$  on the average market's excess return (risk premium)  $(R_{mt} R_{ft})$ .
- 28 Given the worldwide events happened during the chosen period, and the
- 29 consequently market volatility, I decided to use the beta values calculated in the
- initial estimation period just to have insights about their explanatory power and

- relation with portfolio excess returns (so we have one estimated beta for each
- 2 portfolio in the year of initial estimation, for a total of 4). Subsequently, in the test
- period I have regressed the monthly beta for each portfolio, then I calculated the
- 4 adjusted beta for each one and I calculated the residual variance for each monthly
- 5 portfolio excess return. Thus, we have 48 observations for each variable researched
- 6 (beta, residual variance, portfolio excess return) for the testing period (12 months
- 7 multiplied by 4 portfolios).
- 8 Now we have to estimate the Security Market Line (SML), for every Testing Period,
- 9 regressing the portfolio returns against the portfolio betas.
- 10 As we have said, the SML is the graphical representation of the CAPM model,
- expressed by the equation (1). Therefore, we can use the estimated beta that we've
- just obtained for the test period, adjust them, and estimate y0, y1 as follows:

$$R_{n} - R_{f} = \gamma 0 + \gamma 1 (Adj \beta_{n}) + \varepsilon_{n}$$
 (8)

- 14 Where:
- 15  $R_p R_f$  is the average excess return on a portfolio p,
- 16  $Adj\beta_p$  is the adjusted beta of portfolio p,
- 17  $\varepsilon_p$  is the is random disturbance term in the regression equation.
- 18 The adjusted beta provides an estimate of a security's future beta by considering its
- 19 historical data while assuming a tendency for the security's beta to converge towards
- 20 the market average over time. This adjustment involves weighting the historical raw
- beta of the security and the market beta. The formula for calculating the adjusted
- 22 beta is as follows:

23 
$$Adjusted \beta = 0.67 \times Raw \beta + 0.33 \times 1.0$$
 (9)

- In the context of the Capital Asset Pricing Model (CAPM), the intercept  $\gamma 0$  signifies a
- crucial aspect of the Security Market Line (SML). Ideally, if the CAPM accurately
- describes the relationship between risk and return in the market, this intercept
- 27 should equal zero. The intercept essentially reflects the expected excess return on a
- portfolio when its beta  $(\beta_n)$  is zero, which implies that the portfolio bears no
- 29 systematic risk relative to the market.
- 30 On the other hand, the slope of the Security Market Line, denoted by  $\gamma_1$ , holds
- 31 significant importance as well. For the CAPM this slope represents should be equal to

- the average risk premium of the portfolio, capturing how the expected excess return
- 2 on a portfolio change with respect to changes in its beta.
- Thus, in accordance with CAPM, a zero intercept  $(\gamma 0)$  and a positive slope  $(\gamma 1)$  of the
- 4 SML would indicate that investors are adequately compensated for the systematic
- 5 risk they undertake, with higher-beta portfolios expected to yield higher average
- 6 returns to offset the increased risk, so that there is a positive price of risk in the
- 7 capital markets.
- 8 To investigate potential nonlinearity between total portfolio returns and betas, we
- 9 employ the following regression equation:

$$R_p - R_f = \gamma 0 + \gamma 1 (Adj\beta_p) + \gamma 2 (Adj\beta_p^2) + \varepsilon_p \tag{9}$$

- 11 If the underlying assumption of the Capital Asset Pricing Model (CAPM) holds true—
- meaning that portfolio returns, and their adjusted betas exhibit a linear relationship—
- then the coefficient  $\gamma^2$  should ideally be zero. Additionally, here I have firstly
- squared the betas and then I have adjusted them.
- Subsequently, we explore whether the expected excess return on securities is solely
- determined by systematic risk, independent of nonsystematic risk, as measured by
- 17 the residuals' variance:

$$R_p - R_f = \gamma 0 + \gamma 1 (Adj\beta_p) + \gamma 2 (Adj\beta_p^2) + \gamma 3\sigma^2(\varepsilon_{pt}) + \varepsilon_p$$
 (10)

- Here,  $\gamma^2$  gauges the potential nonlinearity of the return, while  $\gamma^3$  assesses the
- 21 explanatory power of nonsystemic risk. The term  $\sigma^2(\varepsilon_{nt})$  represents the normalized
- residual variance of portfolio returns. The residual variance has been normalized
- 23 subtracting the mean and dividing by standard deviation. If the CAPM hypothesis
- 24 holds true,  $\gamma$ 3 should be zero, indicating that nonsystemic risk does not influence the
- 25 expected excess return on securities.

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#### 4.5 t-test

- In our statistical evaluation of the CAPM, we employ t-tests as our analytical tool. We
- set the significance level at 95%, indicating that a significant result at this level of
- 30 confidence supports our conclusions with 95% certainty. However, it also implies a
- 5% probability of error. The critical value corresponding to a 95% confidence level

- from the t-distribution is 1.96. Consequently, we utilize this value in subsequent
- 2 analyses to validate the accuracy of our estimation results. When interpreting the p-
- 3 value derived from the t-test, a value less than 0.05 indicates statistical significance,
- 4 suggesting that the observed results are unlikely to have occurred by chance.

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### 5. Data Analysis

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# 5.1 Overview of the Italian Stock Market (2018-2023)

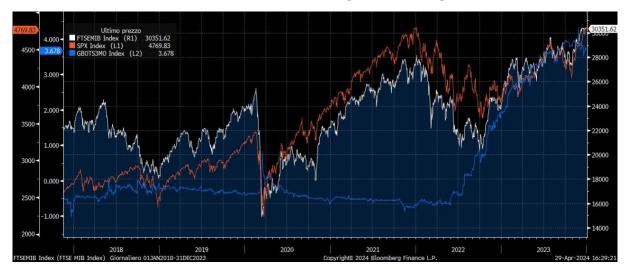


Figure 2: Plot of the FTSEMIB compared with SP500 and BOT 3-month from

6 01.01.2018 to 12.31.2023

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- 8 Looking at the plot we see some ups and downs in this period, in fact the minimum
- 9 is 15731.85 and the maximum is 30403.90 (1.93 times the minimum), however the
- coefficient of variation of the FTSEMIB is 13.27% while for the SP&500 is 19.99%
- suggesting that the data points in the FTSEMIB are less spread out relative to the
- mean compared to the S&P 500.
- To provide context to these fluctuations, we can delve deeper in the circumstances:

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### 2018: A Year of Turmoil and Economic Struggles

- 16 The Italian stock market faced significant challenges in 2018, with the FTSE MIB
- index starting the year at approximately 21,000 points and ending lower, around
- 18,000 points, representing a decline of approximately 14.3%.
- One of the defining events of the year was the clash between Italy's populist
- 20 government and the European Union over the country's proposed budget. This
- 21 conflict contributed to heightened uncertainty and volatility in financial markets, with
- 22 Italian government bond yields spiking to multi-year highs.

- 1 Global trade tensions also weighed on investor sentiment, with concerns over
- 2 escalating tariffs and protectionist measures undermining confidence in the global
- 3 economic outlook. These factors, combined with Italy's domestic political uncertainty,
- 4 led to a risk-off environment in which investors sought safe-haven assets.
- 5 Despite these headwinds, Italy's economy continued to grow, albeit at a slower pace
- 6 than initially anticipated. GDP growth for the year was approximately 0.9%, down
- 7 from 1.6% in the previous year, reflecting the impact of heightened uncertainty and
- 8 weaker external demand.

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### **2019: Signs of Recovery Amidst Global Uncertainty**

- In 2019, the Italian stock market staged a partial recovery, with the FTSE MIB index
- starting the year around 18,000 points and ending higher, around 20,000 points,
- representing a gain of approximately 11.1%.
- 14 Signs of progress in US-China trade negotiations and the European Central Bank's
- announcement of additional stimulus measures provided a much-needed boost to
- investor sentiment. Italy's economy showed signs of stabilization, with GDP growth
- 17 rebounding to approximately 0.3% for the year, supported by resilient domestic
- demand and improving external conditions.
- 19 However, political uncertainty persisted, with Italy experiencing a change in
- 20 government and ongoing tensions with the European Union over fiscal policy. These
- 21 factors continued to weigh on investor confidence, limiting the extent of the market's
- 22 recovery.

23

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#### 2020: The Unprecedented Impact of COVID-19

- 25 The outbreak of the COVID-19 pandemic in 2020 sent shockwaves through financial
- 26 markets, leading to a widespread sell-off in equities and a flight to safety among
- investors. The FTSE MIB index plummeted from around 20,000 points at the
- beginning of the year to approximately 15,000 points at its lowest, representing a
- 29 decline of approximately 25%.
- 30 Italy, one of the hardest-hit countries in Europe, faced a dual crisis as it grappled
- with the health and economic consequences of the pandemic. GDP contracted

- sharply by approximately 8.9% for the year, reflecting the severe impact of lockdown
- 2 measures on economic activity.
- 3 In response, the Italian government implemented various fiscal measures, including
- 4 financial support for businesses and individuals affected by the crisis. The European
- 5 Central Bank also intervened to stabilize financial markets and provide liquidity,
- 6 contributing to a partial recovery in equity prices towards the end of the year.

### 8 **2021-2023: Navigating Geopolitical Challenges and Economic Recovery**

- 9 The Italian stock market experienced a mix of geopolitical tensions, economic
- recovery efforts, and inflationary pressures from 2021 to 2023. Following the
- tumultuous period induced by the COVID-19 pandemic, the market saw gradual
- stabilization and growth, albeit against a backdrop of significant geopolitical events.
- 13 In February 2022, the Russian invasion of Ukraine sent shockwaves through global
- markets, including Italy's FTSE MIB index. The threat of conflict in Eastern Europe
- 15 heightened uncertainty and risk aversion among investors, leading to bouts of
- market volatility. Italy, being a part of the European Union, closely monitored the
- 17 situation and its potential economic ramifications.
- Despite geopolitical tensions, Italy's economy continued its path to recovery,
- supported by fiscal stimulus measures and infrastructure projects. GDP growth
- averaged approximately 4.2% per year from 2021 to 2023, indicating a resilient
- domestic economy. However, alongside this recovery, inflationary pressures began to
- 22 mount.
- 23 The post-pandemic economic rebound, coupled with supply chain disruptions and
- 24 pent-up demand, contributed to inflationary pressures across various sectors of the
- economy. Rising prices for goods and services posed challenges for consumers and
- businesses alike, impacting purchasing power and production costs.
- 27 In October 2023, the Gaza-Israel conflict reignited, further adding to geopolitical
- concerns. The escalation of violence in the Middle East introduced another layer of
- 29 uncertainty for global markets, including Italy's. Investor sentiment was affected as
- the conflict raised fears of broader regional instability and its potential impact on
- 31 global trade dynamics.

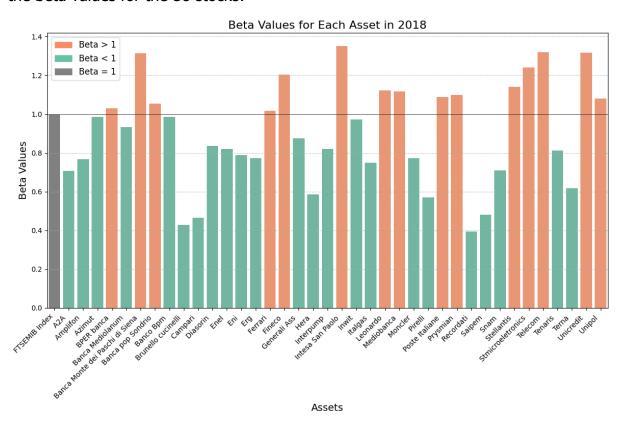
- 1 Throughout this period, the FTSE MIB index navigated through geopolitical
- 2 headwinds and inflationary pressures, reflecting the market's resilience amid
- 3 challenging circumstances. While these events contributed to occasional market
- 4 volatility, Italy's economy showed signs of strength, with the FTSE MIB index
- 5 reaching approximately 30,000 points by the end of 2023.
- 6 Investors remained vigilant, closely monitoring geopolitical developments alongside
- 7 economic indicators, including inflation, to gauge the market's trajectory. Despite the
- 8 geopolitical challenges posed by the Russian invasion of Ukraine and the Gaza-Israel
- 9 conflict, alongside rising inflation, there was a cautious sense of optimism as Italy's
- economy continued on its path to recovery, bolstered by domestic initiatives and
- 11 global economic trends.

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# **5.2 Beta Coefficients Analysis**

- The initial procedure involves computing the beta values using Equation (6). Below are
- the beta values for the 38 stocks:



**Figure 3:** Stock beta estimates (Year 2018)

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- 1 The beta estimates range for the 2018 is from 0.393235 (minimum) with the beta of
- 2 "Recordati" to 1.351083 (maximum) with the beta of "Intesa San Paolo", the average
- 3 beta is 0.904038 and the standard deviation is 0.260878.
- 4 Subsequently, we proceed to divide the 38 stocks in 4 portfolios accordingly to chapter
- 5 4.3.

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# 5.3 Average excess returns and beta across the portfolios

- 8 Next, we proceed to determine the average excess return of the portfolios along with
- 9 their corresponding betas by utilizing equation (7).

Year	$R_{pt}-R_{ft}$	$eta_p$	t-value	Std. error	Adj. R <sup>2</sup>	F-stat
2019						
Portfolio	0.009157	0.730137	9.009740	0.081039	0.611208	81.175406
1						
Portfolio	0.011086	0.708754	10.847364	0.065338	0.695823	117.665315
2						
Portfolio	0.014726	0.879047	9.285494	0.094669	0.625607	86.220396
3						
Portfolio	0.011237	1.397507	17.455318	0.080062	0.856212	304.688142
4						

**Table 2:** Average excess portfolio returns and betas (Year 2019)

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- 12 In accordance with the CAPM, which posits that higher risk should be compensated
- with higher returns, we would expect to see a positive relationship between excess
- returns and betas. That is, portfolios with higher betas should yield higher excess
- returns, reflecting the additional risk they bear compared to the market.
- 16 However, upon examining the results, this expected trend appears to be ambiguous
- and not clearly evident in these observations.
- Furthermore, the t-value associated with the betas provide insights into their
- statistical significance. In this analysis, all the portfolios have t-values indicating a
- 20 strong statistical significance, suggesting that the observed relationships between
- 21 excess returns and betas are unlikely to have occurred by chance.

- 2 Additionally, the Adjusted R-squared values provide an indication of how well the
- 3 portfolios' excess returns are explained by their betas. Higher R-squared values
- 4 indicate a better fit of the data to the CAPM framework, implying that a larger
- 5 proportion of the variability in excess returns can be attributed to market risk as
- 6 captured by betas, and in this case this indicator is quite good.
- 7 Overall, these results suggest that the portfolios' excess returns are systematically
- 8 related to their betas, in line with the predictions of the CAPM. However, further
- 9 analysis and consideration of other factors may be necessary to fully evaluate the
- model's performance and its ability to explain asset pricing dynamics in the market.

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# 5.4 Determination of the Security Market Line (SML)

- 13 Next, employ the findings to calculate the Security Market Line (SML). To derive the
- 14 SML, Equation (8) was employed. According to the CAPM, it is anticipated that
- 15  $\gamma_0$  equals zero, and  $\gamma_1$  equals the average risk premium of the portfolio:

	Coefficients	Std. error	t-value	P-value
$\gamma_0$	0.019941	0.017032	1.187361	0.241180
γ <sub>1</sub>	-0.013692	0.016794	-0.803858	0.425614

**Table 3:** Statistics of the SML's estimation (Year 2020)

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- 18 Table 3 above displays the outcomes obtained from estimating the Security Market
- 19 Line (SML).
- The t-tests indicate that the null hypothesis is not rejected for the intercept  $(\gamma_0)$ , as
- 21 the absolute t-value doesn't exceed 1.96, suggesting no statistical significance.
- Therefore,  $\gamma_0$  is not significantly different from zero, which doesn't contradict the
- CAPM hypothesis. Additionally, the null hypothesis for the slope  $(\gamma_1)$  is rejected,
- given the absolute t-value (0.804) smaller than 1.96, indicating that  $\gamma_1$  is not
- 25 significantly different from zero. According to the CAPM,  $\gamma_1$  should be equal to the

- average risk premium, which is expected to be greater than zero (the average risk
- 2 premium for the testing period, thus 2020, is 0.0659).
- 3 Consequently, only the first result is in line with the CAPM hypothesis for the first
- 4 period.

6

### 5.5 Non-linearity Test

- 7 Equation (9) was employed to assess potential nonlinearity between the returns of
- 8 portfolios and their respective adjusted betas. As discussed previously, under the
- 9 assumption of the Capital Asset Pricing Model (CAPM), the intercept  $(\gamma_0)$  and
- coefficient for the squared beta term ( $\gamma_2$ ) should ideally be zero, while ( $\gamma_1$ ) should
- 11 equal the average risk premium.

	Coefficients	Std. error	t-value	P-value
$\gamma_0$	0.058623	0.032655	1.795434	0.079302
γ <sub>1</sub>	-0.112193	0.073507	-1.526282	0.133938
γ <sub>2</sub>	0.055876	0.040585	1.376764	0.175397

**Table 4:** Non-linearity test (Year 2002)

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- 1.  $(\gamma_0)$  (Intercept): The coefficient of approximately 0.0586 suggests that even
- though it's not statistically different from zero at conventional levels of significance,
- there might be some systematic deviation from the CAPM prediction. However, even
- 17 though the p-value is near the threshold of rejecting the null hypothesis, the
- coefficient is still not statistically significant, implying that the model is not severely
- 19 biased.
- 20 2.  $(\gamma_1)$  (Coefficient for Adjusted Beta):The coefficient of approximately -0.1122
- 21 indicates a negative relationship between adjusted beta and excess returns,
- 22 suggesting that higher adjusted betas are associated with lower excess returns.
- However, this coefficient is not statistically significant at conventional levels, so we

- cannot conclude a significant deviation from 0, thus, we can't accept the CAPM thesis
- where this coefficient should be equal to the risk premium.
- 3.  $(\gamma_2)$  (Coefficient for Adjusted Beta Squared): The coefficient of approximately
- 4 0.0559 suggests a potential non-linear relationship between excess returns and
- adjusted betas. However, like  $\gamma_1$ , it is not statistically significant, so we cannot
- 6 definitively conclude the presence of non-linearity, according to CAPM.
- 7 In summary, while the coefficients show some deviation from the CAPM predictions,
- 8 the data may not provide sufficient evidence to reject the CAPM hypothesis.
- 9 However, the results hint at potential complexities in the relationship between
- portfolio returns and betas that may deserve further investigation.

### **5.6 Non-Systematic Risk Test**

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	Coefficients	Std. error	t-value	P-value
γ <sub>0</sub>	0.036836	0.028716	1.282782	0.206285
γ <sub>1</sub>	-0.060238	0.064779	-0.929904	0.357495
γ <sub>2</sub>	0.027792	0.035736	0.77771	0.440902
γ <sub>3</sub>	-0.013806	0.003417	-4.040027	0.000211

**Table 5:** Non-Systematic risk test (Year 2002)

15 The intercept  $(\gamma_0)$  is still showing no statistically significant deviation from zero, as

the CAPM predicts. The coefficient for adjusted beta  $(\gamma_1)$  indicates a non-significant

17 relationship between beta and excess return. Additionally, the coefficient for adjusted

beta squared  $(\gamma_2)$  does not demonstrate a significant nonlinear relationship with

19 excess return. However, the coefficient for residual variance ( $\gamma_3$ ) is statistically

20 significant, suggesting that nonsystematic risk, as captured by residual variance, has

explanatory power in determining excess returns. This finding challenges the strict

assumption of the CAPM, which suggests that only systematic risk should influence

excess returns, implying potential limitations in its applicability.

24 Thus, we conclude that CAPM is not fully valid in period 1.

# **6. Results**

# **6.1 Other periods result**

	Coefficients	Value	t-value
Estimation of SML	$\gamma_0$	0.016146	4.203844
	γ <sub>1</sub>	0.000202	0.050852
Non-linearity test	$\gamma_0$	0.015528	3.443595
	γ <sub>1</sub>	0.002497	0.265050
	γ <sub>2</sub>	-0.001471	-0.269202
Non-systemic risk test	$\gamma_0$	0.015542	3.409466
	γ <sub>1</sub>	0.002486	0.261072
	γ <sub>2</sub>	-0.001475	-0.26713
	γ <sub>3</sub>	0.000279	0.204057

# **Table 6**: Test results of Period 2 (2019.1.1 to 2021.12.31)

	Coefficients	Value	t-value
Estimation of SML	$\gamma_0$	-0.009074	-1.375659
	$\gamma_1$	0.005207	0.872956

Non-linearity test	$\gamma_0$	-0.008661	-1.132047
	γ <sub>1</sub>	0.004266	0.407569
	$\gamma_2$	0.000414	0.110043
Non-systemic risk test	γο	-0.008567	-1.107264
	$\gamma_1$	0.00409	0.386092
	$\gamma_2$	0.000479	0.125695
	$\gamma_3$	0.000747	0.280726

**Table 7**: Test results of Period 3 (2020.1.1 to 2022.12.31)

	Coefficients	Value	t-value
Estimation of SML	γο	-0.049257	-4.900404
	γ <sub>1</sub>	-0.008807	-0.837141
Non-linearity test	$\gamma_0$	-0.048619	-3.448482
	$\gamma_1$	-0.010686	-0.348983
	$\gamma_2$	0.001151	0.065422
Non-systemic risk test	$\gamma_0$	-0.049044	-3.355071

$\gamma_1$	-0.009559	-0.297534
$\gamma_2$	0.000504	0.027315
$\gamma_3$	0.000376	0.131303

**Table 8**: Test results of Period 4 (2021.1.1 to 2023.12.31)

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- 3 The estimated beta used for the portfolio creation, in the whole period empirical
- 4 analysis, is the one regressed considering the entire period. Thus, the initial
- 5 estimation period and the test period are the same as the portfolio formation period.
- 6 In this case the sample size is way bigger.

	Coefficients	Value	t-value
Estimation of SML	$\gamma_0$	-0.003209	-0.685067
	γ <sub>1</sub>	-0.000596	-0.129613
Non-linearity test	$\gamma_0$	-0.000140	-0.027572
	$\gamma_1$	-0.009408	-1.281797
	γ <sub>2</sub>	0.004667	1.537663
Non-systemic risk test	γ <sub>0</sub>	-0.00013	-0.025498
	γ <sub>1</sub>	-0.009441	-1.284134
	γ <sub>2</sub>	0.004685	1.541038

$\gamma_3$	0.000581	0.321643

**Table 9**: Test results of the entire period (2018.1.1 to 2023.12.31)

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#### **6.2 Summary of Findings**

- 4 This thesis investigated the applicability of the Capital Asset Pricing Model (CAPM) in
- 5 the context of the FTSEMIB Index. The research utilized weekly stock returns data
- 6 from 38 companies listed on the chosen index spanning from January 1, 2018, to
- 7 December 31, 2023.
- 8 The primary aim of this study was to assess the validity of the CAPM in the Italian
- 9 stock market by analyzing:
- 1. Whether the intercept is zero and if the slope of the Security Market Line (SML) equals the average risk premium.
- 2. Whether there exists a linear relationship between the rate of return and its beta.
  - 3. Whether non-systemic risk impacts the returns of portfolios.
- 15 Through the methodologies discussed in chapter 4, the study generated results 16 summarized in Table 11.

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		Period 1	Period 2	Period 3	Period 4	Entire
						period
SML	$\gamma_0$	Support	Reject	Support	Reject	Support
	$\gamma_1$	Reject	Reject	Reject	Reject	Reject
Non-	$\gamma_0$	Support	Reject	Support	Reject	Support
Linearity						
	γ <sub>1</sub>	Reject	Reject	Reject	Reject	Reject

	$\gamma_2$	Support	Support	Support	Support	Support
Non- systematic risk	γ <sub>0</sub>	Support	Reject	Support	Reject	Support
	$\gamma_1$	Reject	Reject	Reject	Reject	Reject
	γ <sub>2</sub>	Support	Support	Support	Support	Support
	γ <sub>3</sub>	Reject	Support	Support	Support	Support

**Table 11:** Summary of findings of the empirical analysis

- As we can see from the table above the tests supporting the CAPM posits are only the 51,11%, but we can conclude that:
  - For the intercept coefficient ( $\gamma_0$ ) the research achieved mixed results, some results showed that this coefficient is not statistically significant, as theorized by the model, others showed the opposite.
  - According to CAPM the  $\gamma_1$  coefficient should be equal to the average risk premium showing the linear relationship between beta and portfolio excess returns, which in this period has been different from 0. However, the tests showed unanimously that it is not significantly different from 0.
  - For the  $\gamma_2$  coefficient, which, if different from 0, shows a non-linear relationship between the beta and expected excess return on securities, the tests have supported the not statistically significance, in line with the CAPM predictions.
  - For the  $\gamma_3$  coefficient, that accordingly to the CAPM should be 0, as it gauges the influence of nonsystemic risk to the expected excess return on securities, the tests have supported that it is not significantly different from 0, but not in period 1.

#### 7. Conclusion

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#### 7.1 Interpretation of Results

- 4 The results state that the validity of the CAPM in the FTSEMIB in the time interval
- 5 2018.1.1 to 2023.12.31 is a "Yes and No", however the CAPM remains a simple and
- 6 elegant finance theory in a complex reality but that can still have some partial
- 7 explanatory power.

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#### 7.2 Limitations of the Study

- 10 In our evaluation of the CAPM, complete rejection isn't warranted due to several
- factors. Firstly, the market index utilized in our assessment likely deviates from the
- ideal "market portfolio" as defined by CAPM. Additionally, the betas assigned to
- securities are estimates rather than true values. Moreover, the limited sample size
- and brief observation period could introduce measurement inaccuracies.

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#### 7.3 Implications for Investors

- 17 CAPM offers investors a convenient and efficient method to forecast an asset's
- 18 expected return. However, numerous external factors can sway this prediction. Multi-
- 19 factor models aim to address this complexity, though identifying the precise
- 20 influencing factors remains challenging. Additionally, behavioral finance theory
- introduces further nuances. Concepts like irrational exuberance, as observed by
- 22 Shiller<sup>21</sup>, and black swan events, as highlighted by Taleb<sup>22</sup>, can significantly impact
- the relationship between expected returns and market dynamics.

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#### 7.4 Future Research Directions

<sup>&</sup>lt;sup>21</sup> Shiller, Robert J. Irrational Exuberance. Princeton University Press, 2000.

<sup>&</sup>lt;sup>22</sup> Taleb, Nassim Nicholas. The Black Swan: The Impact of the Highly Improbable. Random House, 2007.

- 1 Exploring an intriguing avenue for future research could involve extending the testing
- 2 of the CAPM model across a broader time span and with a more extensive sample
- 3 size. It would also be compelling to experiment with various indexes in an attempt to
- 4 pinpoint the one that best approximates the market portfolio. Additionally, identifying
- 5 other factors exhibiting consistent significance over extended periods and devising
- 6 methods to incorporate diverse investor behaviors, biases, and irrationalities into the
- 7 decision-making process would be captivating endeavors.

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