Probabilitas: pengertian dan contoh

Abdullah Basuki Rahmat

abdullahbasuki@trunojoyo.ac.id
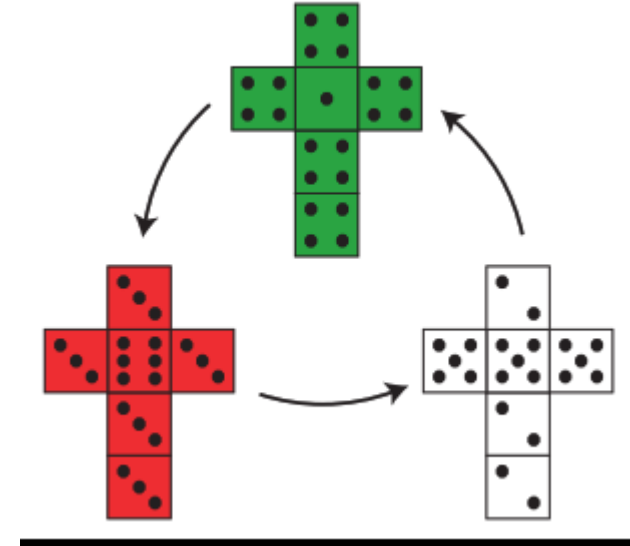
Lab. Multimedia Computing

Campus Telang

Trunojoyo University of Madura UTM

Outline

1. Pendekatan Permutasi
2. Contoh-contoh
3. Clicker test
4. Peran probabilitas
5. Ruang sampel diskret
6. Kejadian (Events)
7. Kejadian, Himpunan dan kata
8. Aturan Probabilitas dalam Notasi Matematika
9. Probabiltas dan Operasi himpunan dalam kejadian
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Soal

Board Question

Deck of 52 cards
- 13 *ranks*: 2, 3, ..., 9, 10, J, Q, K, A
- 4 *suits*: ♡, ♠, ◇, ♣,

Poker hands
- Consists of 5 cards
- A *one-pair* hand consists of two cards having one rank and the remaining three cards having three other ranks
- Example: {2♡, 2♠, 5♡, 8♣, K◇}

Question

(a) How many different 5 card hands have exactly one pair?

Hint: practice with how many 2 card hands have exactly one pair.
Hint for hint: use the rule of product.

(b) What is the probability of getting a one pair poker hand?

Jawaban

Answer to board question

We can do this two ways as combinations or permutations. The keys are:
1. be consistent
2. break the problem into a sequence of actions and use the rule of product.

Note, there are many ways to organize this. We will break it into very small steps in order to make the process clear.

Combinations approach

a) Count the number of one-pair hands, where the order they are dealt doesn't matter.

Action 1. *Choose* the rank of the pair: 13 different ranks, choosing 1, so $\binom{13}{1}$ ways to do this.

Action 2. *Choose* 2 cards from this rank: 4 cards in a rank, choosing 2, so $\binom{4}{2}$ ways to do this.

Action 3. *Choose* the 3 cards of different ranks: 12 remaining ranks, so $\binom{12}{3}$ ways to do this.

(Continued on next slide.)

Penyelesaian Kombinasi

Action 4. *Choose* 1 card from each of these ranks: 4 cards in *each* rank so $\binom{4}{1}^3$ ways to do this.

answer: (Using the rule of product.)

$$\binom{13}{1} \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot 4^3 = 1098240$$

b) To compute the probability we have to stay consistent and count combinations. To make a 5 card hand we *choose* 5 cards out of 52, so there are

$$\binom{52}{5} = 2598960$$

possible hands. Since each hand is equally likely the probability of a one-pair hand is

$$1098240/2598960 = 0.42257.$$

On the next slide we redo this problem using permutations.

Pendekatan Permutasi

Permutation approach

This approach is a little trickier. We include it to show that there is usually more than one way to count something.

a) Count the number of one-pair hands, where we keep track of the order they are dealt.

Action 1. (This one is tricky.) *Choose* the positions in the hand that will hold the pair: 5 different positions, so $\binom{5}{2}$ ways to do this.

Action 2. Put a card in the first position of the pair: 52 cards, so 52 ways to do this.

Action 3. Put a card in the second position of the pair: since this has to match the first card, there are only 3 ways to do this.

Action 4. Put a card in the first open slot: this can't match the pair so there are 48 ways to do this.

Action 5. Put a card in the next open slot: this can't match the pair or the previous card, so there 44 ways to do this.

Action 6. Put a card in the last open slot: there are 40 ways to do this.

Lanjutan

answer: (Using the rule of product.)

$$\binom{5}{2} \cdot 52 \cdot 3 \cdot 48 \cdot 44 \cdot 40 = 131788800$$

ways to deal a one-pair hand where we keep track of order.

b) There are

$$_{52}P_5 = 52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 = 311875200$$

five card hands where order is important.

Thus, the probability of a one-pair hand is

$$131788800/311875200 = 0.42257.$$

(Both approaches give the same answer.)

Test Klik

Set your clicker channel to 41.

Do you have your clicker with you?

$$No = 0$$
$$Yes = 1$$

Probabilitas:Pengertian dan Contoh

Peranan Probabilitas

Introduced so far

- Experiment: a repeatable procedure
- Sample space: set of all possible outcomes S (or Ω).
- Event: a subset of the sample space.
- Probability function, $P(\omega)$: gives the probability for each outcome $\omega \in S$
    1. Probability is between 0 and 1
    2. Total probability of all possible outcomes is 1.

Contoh:

Experiment: toss a fair coin, report heads or tails.

Sample space: $\Omega = \{H, T\}$.

Probability function: $P(H) = .5, \quad P(T) = .5$.

Use tables:

Outcomes	H	T
Probability	1/2	1/2

(Tables can really help in complicated examples)

Ruang Sampel Diskret

Discrete = listable

Examples:

$\{a, b, c, d\}$ (finite)

$\{0, 1, 2, \dots\}$ (infinite)

Kejadian (Events)

Events are sets:
- Can describe in words
- Can describe in notation
- Can describe with Venn diagrams

Experiment: toss a coin 3 times.

Event:
You get 2 or more heads = { HHH, HHT, HTH, THH}

Kejadian, Himpunan dan Kata

Experiment: toss a coin 3 times.

Which of following equals the event "exactly two heads"?

$A = \{THH, HTH, HHT, HHH\}$
$B = \{THH, HTH, HHT\}$
$C = \{HTH, THH\}$

(1) A   (2) B   (3) C   (4) A or B

answer: : 2) B.
The event "exactly two heads" determines a *unique subset*, containing *all* outcomes that have exactly two heads.

Lanjutan

Experiment: toss a coin 3 times.

Which of the following describes the event
$\{THH, HTH, HHT\}$?

(1) "exactly one head"
(2) "exactly one tail"
(3) "at most one tail"
(4) none of the above

answer: (2) "exactly one tail"
Notice that the same event $E \subset \Omega$ may be described in words in multiple ways ("exactly 2 heads" and "exactly 1 tail").

Lanjutan

Experiment: toss a coin 3 times.

The events "exactly 2 heads" and "exactly 2 tails" are disjoint.

(1) True   (2) False

answer: True: $\{THH, HTH, HHT\} \cap \{TTH, THT, HTT\} = \emptyset$.

Lanjutan

Experiment: toss a coin 3 times.

The event "at least 2 heads" implies the event "exactly two heads".

(1) True   (2) False

False. It's the other way around:
$\{THH, HTH, HHT\} \subset \{THH, HTH, HHT, HHH\}$.

Aturan Probabilitas dalam Notasi Matematis

Sample space: $S = \{\omega_1, \omega_2, \ldots, \omega_n\}$

Outcome: $\omega \in S$

Probability between 0 and 1: $0 \leq P(\omega) \leq 1$

Total probability is 1: $\displaystyle\sum_{j=1}^{n} P(\omega_j) = 1$, $\qquad \displaystyle\sum_{\omega \in S} P(\omega) = 1$

Event A: $P(A) = \displaystyle\sum_{\omega \in A} P(\omega)$

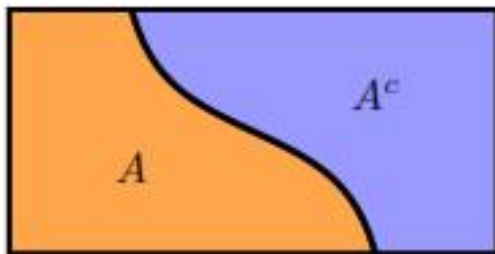# Probabilitas dan Operasi Himpunan pada Kejadian
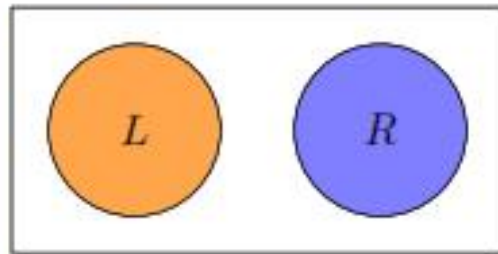
Events A, L, R

Rule 1. Complements: $P(A^c) = 1 - P(A)$.

Rule 2. Disjoint events: If L and R are disjoint then
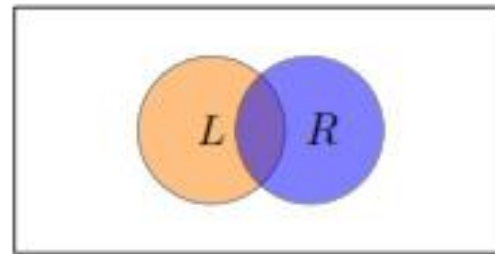$$P(L \cup R) = P(L) + P(R).$$

Rule 3. Inclusion-exclusion principle: For any L and R:
$$P(L \cup R) = P(L) + P(R) - P(L \cap R).$$

$\Omega = A \cup A^c$, no overlap — $L \cup R$, no overlap — $L \cup R$, overlap $= L \cap R$

Pertanyaan

- Class has 50 students
- 20 male (M), 25 brown-eyed (B)

For a randomly chosen student what is the range of possible values for $p = P(M \cup B)$?

(a) $p \le .4$
(b) $.4 \le p \le .5$
(c) $.4 \le p \le .9$
(d) $.5 \le p \le .9$
(e) $.5 \le p$

answer: (d) $.5 \le p \le .9$
Explanation on next slide.

Penyelesaian

The easy way to answer this is that $A \cup B$ has a minumum of 25 members (when all males are brown-eyed) and a maximum of 45 members (when no males have brown-eyes). So, the probability ranges from .5 to .9
Thinking about it in terms of the inclusion-exclusion principle we have

$$P(M \cup B) = P(M) + P(B) - P(M \cap B) = .9 - P(M \cap B).$$

So the maximum possible value of $P(M \cup B)$ happens if M and B are disjoint, so $P(M \cap B) = 0$. The minimum happens when $M \subset B$, so $P(M \cap B) = P(M) = .4$.

Soal

Table Question

Experiment:

1. Your table should make 9 rolls of a 20-sided die (one each if the table is full).

2. Check if all rolls at your table are distinct.

Repeat the experiment five times and record the results.

For this experiment, how would you define the sample space, probability function, and event?

Compute the true probability that all rolls (in one trial) are distinct and compare with your experimental result.

answer: $1 - (20 \cdot 19 \cdots 13 \cdot 12)/(20^9) = 0.881.$ (The explanation is on the next frame.)

Board Question Solution

For the sample space S we will take all sequences of 9 numbers between 1 and 20. (We are assuming there are 9 people at table.) We find the size of S using the rule of product. There are 20 ways to choose the first number in the sequence, followed by 20 ways to choose the second, etc. Thus, $|S| = 20^9$.

It is sometimes easier to calculate the probability of an event indirectly by calculating the probability of the complement and using the formula

$$P(A) = 1 - P(A^c).$$

In our case, A is the event 'there is a match', so A^c is the event 'there is no match'. We can use the rule of product to compute $|A^c|$ as follows. There are 20 ways to choose the first number, then 19 ways to choose the second, etc. down to 12 ways to choose the ninth number. Thus, we have

$$|A^c| = 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12$$
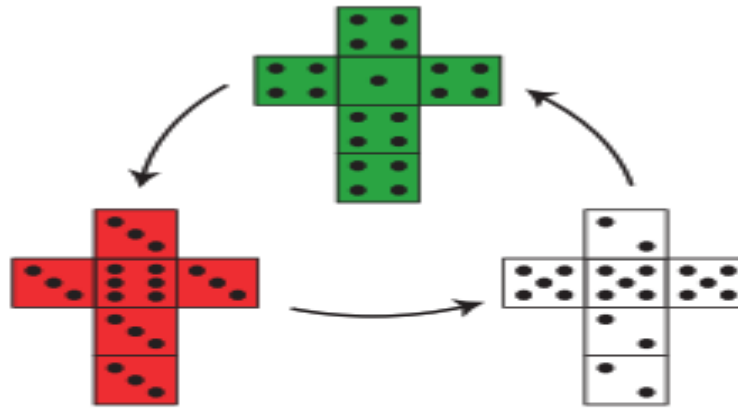
That is $|A^c| = {}_{20}P_9$.

Putting this all together

$$P(A) = 1 - P(A^c) = 1 - \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12}{20^9} = .881$$

Jon's Dice
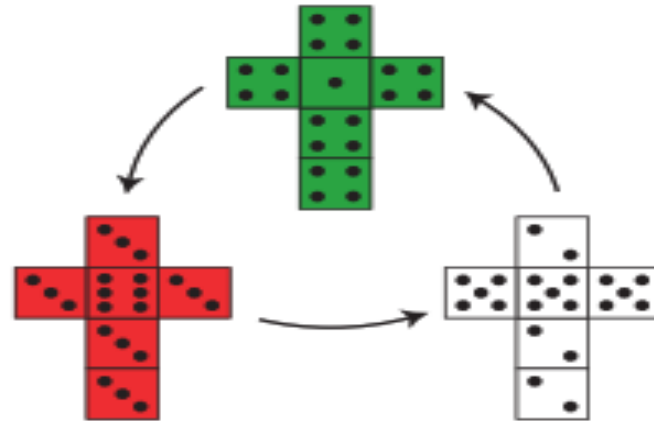
Jon has three six-sided dice with unusual numbering.

A game consists of two players each choosing a die. They roll once and the highest number wins.

Which die would you choose?

Soal

1. Make probability tables for the red and which dice.
2. Make a probability table for the product sample space of red and white.
3. Compute the probability that red beats white.

4. Pair up with another group. Have one group compare red vs. green and the other compare green vs. red. Based on the three comparisons rank the dice from best to worst.

Komputasi penyelesaian

Outcomes	Red die		White die		Green die	
	3	6	2	5	1	4
Probability	5/6	1/6	3/6	3/6	1/6	5/6

- The 2 × 2 tables show pairs of dice.
- Each entry is the probability of seeing the pair of numbers corresponding to that entry.
- The color gives the winning die for that pair of numbers. (We use black instead of white when the white die wins.)

		White		Green	
		2	5	1	4
Red	3	15/36	15/36	5/36	25/36
	6	3/36	3/36	1/36	5/36
Green	1	3/36	3/36		
	4	15/36	15/36		

Jawaban soal

		White		Green	
		2	5	1	4
Red	3	15/36	15/36	5/36	25/36
	6	3/36	3/36	1/36	5/36
Green	1	3/36	3/36		
	4	15/36	15/36		

The three comparisons are:

P(red beats white)   =   21/36 = 7/12
P(white beats green) =   21/36 = 7/12
P(green beats red)   =   25/36

Thus: red is better than white is better than green is better than red.

There is no best die: the property of being 'better than' is non-transitive.

Soal konsep

Lucky Larry has a coin that you're quite sure is not fair.

- He will flip the coin twice
- It's your job to bet whether the outcomes will be the same (HH, TT) or different (HT, TH).

Which should you choose?

1. Same
2. Different
3. It doesn't matter, same and different are equally likely

Soal

Lucky Larry has a coin that you're quite sure is not fair.

- He will flip the coin twice
- It's your job to bet whether the outcomes will be the same (HH, TT) or different (HT, TH).

Which should you choose?

1. Same   2. Different   3. Doesn't matter

Question: Let p be the probability of heads and use probability to answer the question.

(If you don't see the symbolic algebra try p = .2, p=.5)

Solusi

answer: 1. Same (same is more likely than different)

The key bit of arithmetic is that if $a \neq b$ then

$$(a - b)^2 > 0 \quad \Leftrightarrow \quad a^2 + b^2 > 2ab.$$

To keep the notation cleaner, let's use $P(T) = (1 - p) = q$. Since the flips are independent (we'll discuss this next week) the probabilities multiply. This gives the following 2×2 table.

		second flip	
		H	T
first flip	H	p^2	pq
	T	qp	q^2

So, $P(\text{same}) = p^2 + q^2$ and $P(\text{diff}) = 2pq$. Since the coin is unfair we know $p \neq q$. Now we use our key bit of arithmetic to say

$$p^2 + q^2 > 2pq \quad \Rightarrow \quad P(\text{same}) > P(\text{different}).$$

Probabilitas:Pengertian dan Contoh