Assignment 6

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***Abstract*—This document explains the the concept of finding the angle between the two straight lines from given second degree equation**

Download all latex-tikz codes from

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https://github.com/venkateshelangovan/IIT-Hyderabad-Assignments/tree/master/Assignment6_Matrix_Theory
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1 Problem

Prove that the equation $12x^2 + 7xy - 10y^2 + 13x + 45y - 35 = 0$ represents two straight lines and find the angle between them

2 Pair of staraight lines

The general second order equation is given by ,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.1}$$

the above equation (2.0.1) can be expressed as

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

where

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \tag{2.0.4}$$

the above equation (2.0.2) represents a pair of straight lines if

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \tag{2.0.5}$$

3 Solution

Given,

$$12x^2 + 7xy - 10y^2 + 13x + 45y - 35 = 0 \tag{3.0.1}$$

The above equation (3.0.1) can be expressed as shown in equations (2.0.2), (2.0.3), (2.0.4)

$$\mathbf{x}^T \begin{pmatrix} 12 & \frac{7}{2} \\ \frac{7}{2} & -10 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} \frac{13}{2} & \frac{45}{2} \end{pmatrix} \mathbf{x} - 35 = 0 \tag{3.0.2}$$

Comparing equation (3.0.2) with (2.0.2) we get

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} 12 & \frac{7}{2} \\ \frac{7}{2} & -10 \end{pmatrix} \tag{3.0.3}$$

$$\mathbf{u} = \begin{pmatrix} \frac{13}{2} \\ \frac{45}{2} \end{pmatrix} \tag{3.0.4}$$

$$f = -35 \tag{3.0.5}$$

Substituting the above equations (3.0.3), (3.0.4), (3.0.5) in LHS of equation (2.0.5) to verify the given equation is pair of straight lines

$$\delta = \begin{vmatrix} 12 & \frac{7}{2} & \frac{13}{2} \\ \frac{7}{2} & -10 & \frac{45}{2} \\ \frac{13}{2} & \frac{45}{2} & -35 \end{vmatrix} \tag{3.0.6}$$

Expanding the above determinant , we get

$$\delta = 0 \tag{3.0.7}$$

Since equation (2.0.5) is satisfied, we could say that the given equation (3.0.1) represents two straight lines From equation (3.0.1)

Consider ,

$$12x^2 + 7xy - 10y^2 \tag{3.0.8}$$
$$\implies 12x^2 + 15xy - 8xy - 10y^2 \tag{3.0.9}$$
$$\implies 3x(4x + 5y) - 2y(4x + 5y) \tag{3.0.10}$$
$$\implies (3x - 2y)(4x + 5y) \tag{3.0.11}$$

Therefore equation (3.0.1) can be modified as

$$(3x - 2y + l)(4x + 5y + m) = 0 \tag{3.0.12}$$

$$\begin{aligned} 12x^2 + 7xy - 10y^2 + (3m + 4l)x \\ + (-2m + 5l)y + lm = 0 \end{aligned} \tag{3.0.13}$$

Equating x and y co-efficients of the equations (3.0.1) and (3.0.13) , we get ,

$$3m + 4l = 13 \tag{3.0.14}$$
$$-2m + 5l = 45 \tag{3.0.15}$$

Solving equations (3.0.14), (3.0.15) we get ,

$$l = 7 \tag{3.0.16}$$

$$m = -5 \tag{3.0.17}$$

Substituting the equations (3.0.16), (3.0.17) in (3.0.12) we get

$$(3x - 2y + 7)(4x + 5y - 5) = 0 \tag{3.0.18}$$

The above equation (3.0.18) represents two straights and straight line equation is given by

$$3x - 2y + 7 = 0 \tag{3.0.19}$$

$$4x + 5y - 5 = 0 \tag{3.0.20}$$

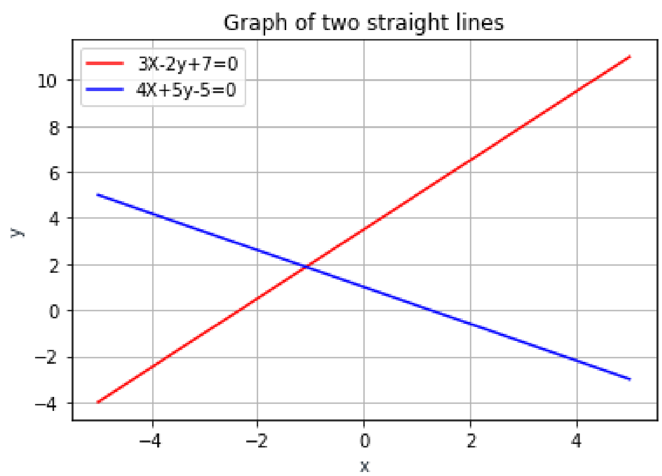

Fig. 1: Pair of straight lines

4 Angle between the straight lines

From the straight line equation (3.0.19), (3.0.20)

$$\mathbf{n_1} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{4.0.1}$$

$$\mathbf{n_2} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \tag{4.0.2}$$

Angle between the two straight lines is given by

$$\theta = \cos^{-1}\left(\frac{\mathbf{n_1}^T\mathbf{n_2}}{\|\mathbf{n_1}\|\,\|\mathbf{n_2}\|}\right) \tag{4.0.3}$$

$$\mathbf{n_1}^T\mathbf{n_2} = \begin{pmatrix} 3 & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 5 \end{pmatrix} = 2 \tag{4.0.4}$$

$$\|\mathbf{n_1}\| = \sqrt{3^2 + (-2)^2} = \sqrt{13} \tag{4.0.5}$$

$$\|\mathbf{n_2}\| = \sqrt{4^2 + 5^2} = \sqrt{41} \tag{4.0.6}$$

Substituting equations (4.0.4), (4.0.5) ,(4.0.6) in equation (4.0.3), we get

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{13}\,\sqrt{41}}\right) \tag{4.0.7}$$

$$\theta = 85° \tag{4.0.8}$$

***Result* :**
Angle between the two straight line is given by

$$\theta = 85° \tag{4.0.9}$$