Code No: 133BD **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November/December - 2018

MATHEMATICS – IV

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, ETM, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) State the necessary and sufficient conditions for a function $f(z)$ to be analytic. [2]

b) Show that the function $f(z) = xy + iy$ is everywhere continuous but is not analytic. [3]

c) Show that $f(z) = \frac{1}{1-e^z}$ has a simple pole at $z = 2\pi i$. [2]

d) State Cauchy's integral formula and use it to evaluate $\oint_C \frac{z^2+4}{z-3}dz$ where C is the circle $|z| = 5$. [3]

e) Find the fixed points of the mapping $w = z + 2i$. [2]

f) Find the residues at the poles of the function $f(z) = \frac{2z+1}{(z-1)^2}$, C: $|z| \leq 4$. [3]

g) If $f(x) = x^3$ in $[-\pi, \pi]$, find the Fourier coefficient b_n. [2]

h) Find $f(x)$ if its finite sine transform is given by $\bar{f}_s(s) = \frac{1+\cos s\pi}{s\pi}$ where $0 < x < \pi$, $s = 1, 2, 3, \ldots$ [3]

i) Classify the PDE: $xu_{xx} - u_{xy} + yu_{yy} = 1$. [2]

j) Write the possible three solutions of the partial differential equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$. [3]

PART-B

(50 Marks)

2.a) Define analyticity of a function. Show that the function defined by $f(z) = \sqrt{|xy|}$ is not analytic at the origin although the C-R equations are satisfied at that point.

b) Find the analytic function $f(z) = u(r,\theta) + iv(r,\theta)$, when $v(r,\theta) = r^2\cos 2\theta - r\cos\theta + 2$. [5+5]

OR

3.a) Show that both the real and imaginary parts of an analytic function are harmonic.

b) If $f(z) = u + iv$ be an analytic function of z and if $u - v = (x - y)(x^2 + 4xy + y^2)$ find $f(z)$ in terms of z. [5+5]

4.a) State Cauchy integral theorem and use it to evaluate the integral $\int_C \frac{e^{2z}}{(z-1)^2(z-3)}\, dz$ where C is the circle $|z| = 4$.

b) If $\Phi(a) = \int_c \frac{3z^2+7z+1}{z-a}\, dz$, where C is the circle $x^2 + y^2 = 4$, find $\Phi(3)$, $\Phi'(1-i)$ and $\Phi''(1-i)$. [5+5]

OR

5.a) Expand $f(z) = \frac{1}{z^2-4z+3}$ in the region $1 < |z| < 3$. Also name the series so obtained.

b) Find the nature and location of the singularities of the function $f(z) = \frac{e^{2z}}{(z-2)^4}$ by finding its Laurent's series expansion. [5+5]

6. State Residues theorem. Evaluate the integral by contour integration: $\int_0^\pi \frac{d\theta}{13+5\cos\theta}$. [10]

OR

7.a) Find the residue of $f(z) = \frac{z^3}{z^2-1}$ at $z = \infty$.

b) Define bilinear transformation. Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$ and hence find the image of $|z| < 1$. [5+5]

8.a) Find the Fourier series for the function $f(x) = \frac{\pi x}{2}$ in $0 \le x \le 2$.

b) Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| \le a \\ 0, & |x| > a \end{cases}$. Hence prove that $\int_0^\infty \frac{\sin^2 ax}{x^2}\, dx = \frac{\pi a}{2}$. [5+5]

OR

9.a) Develop $f(x) = \begin{cases} 2, & -2 < x < 0, \\ x, & 0 < x < 2 \end{cases}$ in a series of sines and cosines and deduce the series for π^2.

b) Find the Fourier cosine transform of $f(x) = e^{-x}$, $x > 0$. [5+5]

10. The ends A and B of a rod 20 cm long have the temperature at 30^0C and 80^0C until steady state conditions prevail. The temperature at the ends are suddenly changed to 40^0 C and 60^0C respectively. Find the temperature distribution in the rod at time t. [10]

OR

11. Write down one dimensional wave equation. A string is stretched and fastened to two points l cm apart. Motion is started by displacing the string in a sinusoidal arch of height y_0 and then released from rest at time $t = 0$. Find the displacement at point x and at any time t. [10]

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