Constructions using Python

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Contents

***Abstract*—This book introduces constructions in high school geometry using Python. The content and exercises are based on NCERT textbooks from Class 6-12. A simple introduction to Python and LaTeXfigures is provided in the process.**

Download all python codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/constructions/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/constructions/figs
```

1 Triangle

1. Draw Fig. 1.1 for $a = 4, c = 3$.
Solution: The vertices of $\triangle ABC$ are

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{1.1.1}$$

The python code for Fig. 1.1 is

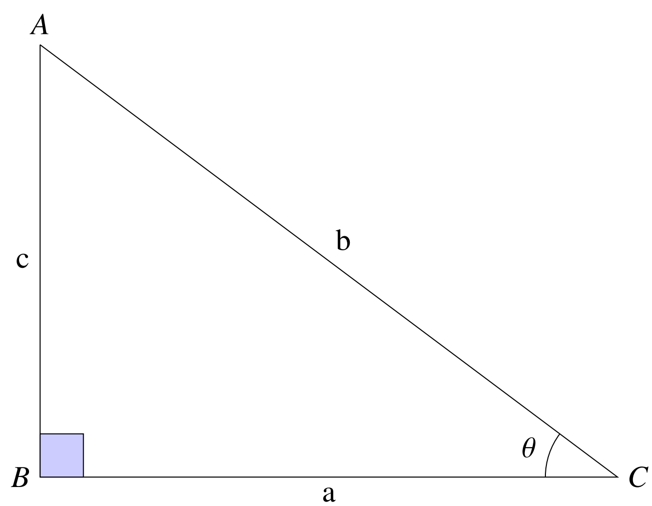

Fig. 1.1: Right Angled Triangle

```
codes/triangle/tri_right_angle.py
```

and the equivalent latex-tikz code is

```
figs/triangle/tri_right_angle.tex
```

The above latex code can be compiled as a standalone document as

```
figs/triangle/tri_right_angle_alone.tex
```

2. Draw Fig. 1.2 for $a = 4, c = 3$.
Solution: The vertex **A** can be expressed in

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polar coordinate form as

$$\mathbf{A} = b\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} \tag{1.2.1}$$

where

$$b = \sqrt{a^2 + c^2} = 5, \tan\theta = \frac{3}{4} \tag{1.2.2}$$

The python code for Fig. 1.2 is

codes/triangle/tri_polar.py

and the equivalent latex-tikz code is

figs/triangle/tri_polar.tex

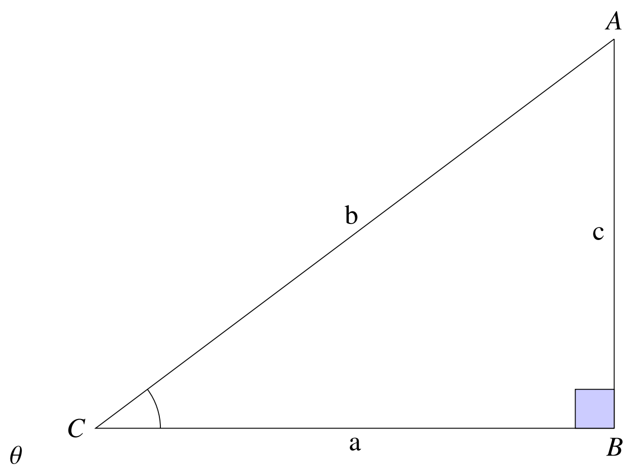

Fig. 1.2: Right Angled Triangle

3. Draw Fig. 1.3 with $a = 6$, $b = 5$ and $c = 4$.

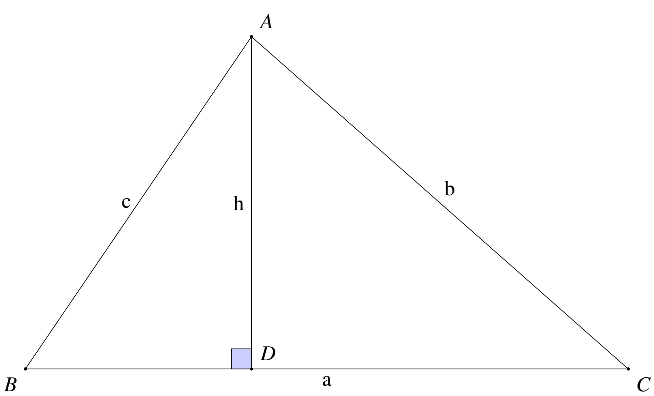

Fig. 1.3

Solution: Let the vertices of $\triangle ABC$ and $\mathbf{D}$ be

$$\mathbf{A} = \begin{pmatrix}p \\ q\end{pmatrix}, \mathbf{B} = \begin{pmatrix}0 \\ 0\end{pmatrix}, \mathbf{C} = \begin{pmatrix}a \\ 0\end{pmatrix}, \mathbf{D} = \begin{pmatrix}p \\ 0\end{pmatrix} \tag{1.3.1}$$

Then

$$AB = \|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \quad \because \mathbf{B} = \mathbf{0} \tag{1.3.2}$$

$$BC = \|\mathbf{C} - \mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \tag{1.3.3}$$

$$AC = \|\mathbf{A} - \mathbf{C}\|^2 = b^2 \tag{1.3.4}$$

From (1.3.4),

$$b^2 = \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{C}\|^T \|\mathbf{A} - \mathbf{C}\| \tag{1.3.5}$$

$$= \mathbf{A}^T\mathbf{A} + \mathbf{C}^T\mathbf{C} - \mathbf{A}^T\mathbf{C} - \mathbf{C}^T\mathbf{A} \tag{1.3.6}$$

$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \quad \left(\because \mathbf{A}^T\mathbf{C} = \mathbf{C}^T\mathbf{A}\right) \tag{1.3.7}$$

$$= a^2 + c^2 - 2ap \tag{1.3.8}$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \tag{1.3.9}$$

From (1.3.2),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \tag{1.3.10}$$

$$\implies q = \pm\sqrt{c^2 - p^2} \tag{1.3.11}$$

The python code for Fig. 1.3 is

codes/triangle/tri_sss.py

and the equivalent latex-tikz code is

figs/triangle/tri_sss.tex

2 Quadrilateral

1. Construct parallelogram $ABCD$ in Fig. 2.1 given that $BC = 5, AB = 6, \angle C = 85°$.
Solution: BD is found using the cosine formula and $\triangle BDC$ is drawn using the approach in Construction 1.3 with

$$\mathbf{B} = \begin{pmatrix}0 \\ 0\end{pmatrix}, \mathbf{C} = \begin{pmatrix}5 \\ 0\end{pmatrix}, \tag{2.1.1}$$

Since the diagonals bisect each other,

$$\mathbf{O} = \frac{\mathbf{B} + \mathbf{D}}{2} \tag{2.1.2}$$

$$\mathbf{A} = 2\mathbf{O} - \mathbf{C}. \tag{2.1.3}$$

AB and AD are then joined to complete the ||gm. The python code for Fig. 2.1 is

codes/quad/pgm_sas.py

and The equivalent latex-tikz code is

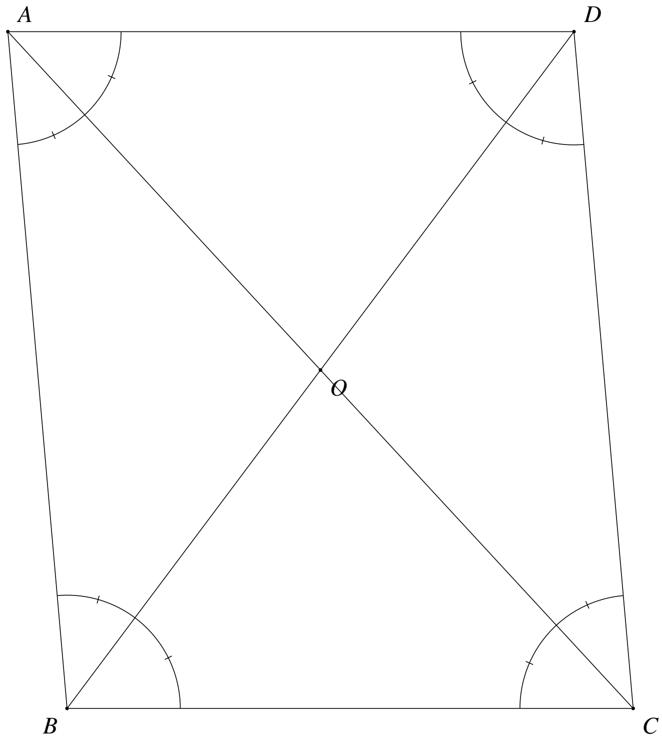

Fig. 2.1: Parallelogram Properties

figs/quad/pgm_sas.tex

2. Draw the ||gm $ABCD$ in Fig. 2.2 with $BC = 6, CD = 4.5$ and $BD = 7.5$. Show that it is a rectangle.

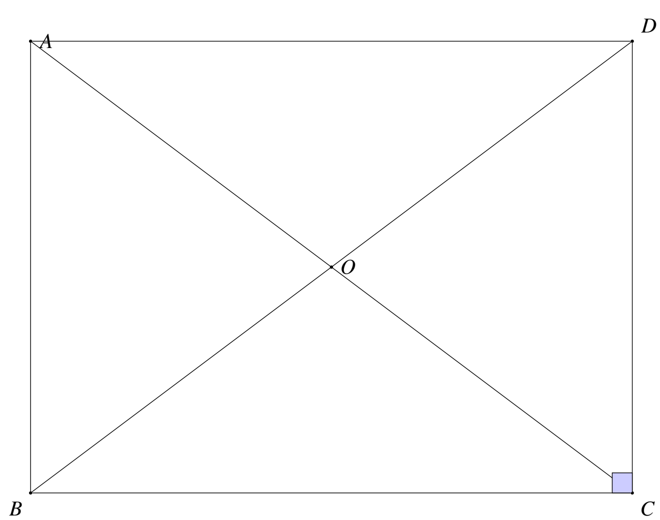

Fig. 2.2: Rectangle

Solution: It is easy to verify that

$$BD^2 = BC^2 + C^2 \tag{2.2.1}$$

Hence, using Baudhayana theorem,

$$\angle BCD = 90° \tag{2.2.2}$$

and $ABCD$ is a rectangle.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4.5 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \tag{2.2.3}$$

The python code for Fig. 2.2 is

codes/quad/pgm_sss.py

and the equivalent latex-tikz code is

figs/quad/pgm_sss.tex

3. Draw the rhombus $BEST$ with $BE = 4.5$ and $ET = 6$.

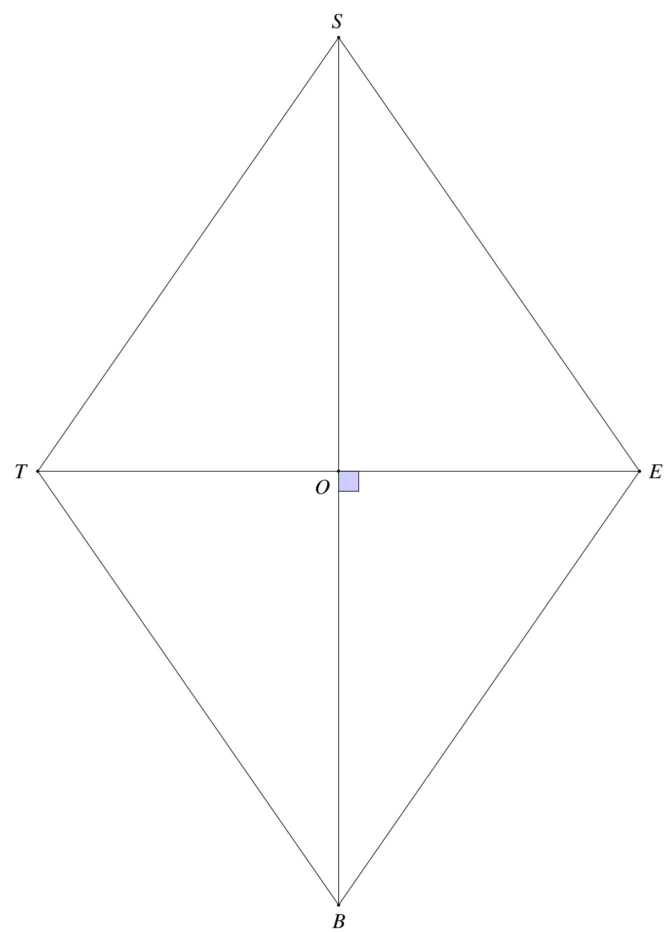

Fig. 2.3: Rhombus

Solution: The coordinates of the various points in Fig. 2.3 are obtained as

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ -4.5 \end{pmatrix} \tag{2.3.1}$$

$$\mathbf{E} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \tag{2.3.2}$$

4. A square is a rectangle whose sides are equal. Draw a square of side 4.5.
   Solution: The coordinates of the various points in Fig. 2.4 are obtained as

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4.5 \end{pmatrix} \tag{2.4.1}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 4.5 \\ 4.5 \end{pmatrix} \mathbf{O} = \frac{\mathbf{B} + \mathbf{C}}{2} \tag{2.4.2}$$

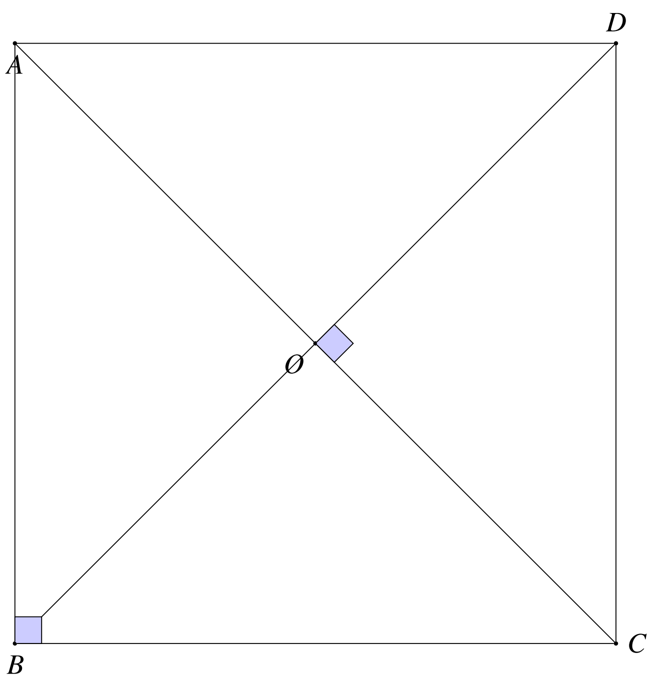

Fig. 2.4: Square