(a)

Use the multinomial idea to obtain that the joint PMF is simply

$$P(X=x,Y=y,Z=z)=\frac{n!}{x!y!z!}\left(\frac{1}{3}\right)^x\left(\frac{1}{3}\right)^y\left(\frac{1}{3}\right)^z=\frac{n!}{x!y!z!}\left(\frac{1}{3}\right)^n$$

for $x+y+z=n$, otherwise it is equal to zero.

(b)

First of all, observe that the game is decisive if and only if there is one and only one random variable (out of X,Y,Z) that is equal to zero. So, let's consider the case where $X=0$. Then we have to have that $Y=k$ for some $k=1,\dots,n-1$. Hence, $Z=n-k$. The probability in this case is

$$\begin{aligned}P(\text{decisive}, X=0) &= \sum_{k=1}^{n-1}P(X=0,Y=k,Z=n-k)=\sum_{k=1}^{n-1}\frac{n!}{k!(n-k)!}\left(\frac{1}{3}\right)^n\\ &=\left(\frac{1}{3}\right)^n\cdot(2^n-2)\end{aligned}$$

where we have used that $\sum_{k=1}^{n-1}\frac{n!}{k!(n-k)!}=\sum_{k=1}^{n-1}\binom{n}{k}=\sum_{k=0}^{n}\binom{n}{k}-\binom{n}{0}-\binom{n}{n}=2^n-2$. Using the symmetry argument, we have that the probabilities for case where $Y=0$ and $Z=0$ are the same. Hence, we have that the required probability is

$$P(\text{decisive})=3P(\text{decisive}, X=0)=\frac{2^n-2}{3^{n-1}}$$

(c)

From the part (b), plugging $n = 5$ we see that the probability that the game is decisive is equal to $\frac{2^5-2}{3^5-1} = \frac{10}{27}$. Now, we are interested what is happening with the probability when $n \to \infty$.

$$\lim_{n\to\infty} P(\text{decisive}) = \lim_{n\to\infty} \frac{2^n - 2}{3^{n-1}} = \lim_{n\to\infty} \frac{2^n}{3^{n-1}} - \lim_{n\to\infty} \frac{2}{3^{n-1}}$$
$$= 3 \lim_{n\to\infty} \frac{2^n}{3^n} - \lim_{n\to\infty} \frac{2}{3^n} = 0 - 0 = 0$$

We have used the fact that $0 < \frac{2}{3} < 1$ so we have that the limit is zero. And it has sense intuitively because large amount of players gives great probability that every possible outcome will be presented.