(a)

Define random variable X that marks the number of students that take the same seat in both classes. If we denote with S_j that jth student has the same seat, we have following

$$P(X=0) = 1 - P(X \geq 1) = 1 - P\left(\cup_j S_J\right)$$

Using the inclusion-exclusion formula and the symmetry, we have

$$P\left(\bigcup_j S_J\right) = \sum_j (-1)^{j-1} \binom{100}{j} P\left(\bigcap_{k=1}^{j} S_k\right)$$

The probability that first j students sit on their seats is simply $\dfrac{(100-j)!}{100!}$. Thus we have

$$P\left(\bigcup_j S_J\right) = \sum_j (-1)^{j-1} \binom{100}{j} \frac{(100-j)!}{100!} = \sum_{j=1}^{100} \frac{(-1)^{j-1}}{j!}$$

Finally,

$$P(X=0) = 1 - \sum_{j=1}^{100} \frac{(-1)^{j-1}}{j!} = \sum_{j=0}^{100} \frac{(-1)^{j}}{j!}$$

(b)

Defining indicator random varibles I_j that indicates if S_j has occurred, we have

$$X = \sum_{j=1}^{100} I_j$$

We know that $P(I_j) = \frac{1}{100}$ and that we can approximate

$$P((I_j = 1) \cap (I_k = 1)) = \frac{1}{100} \cdot \frac{1}{99} \approx \left(\frac{1}{100}\right)^2 = P(I_j = 1)P(I_k = 1)$$

so we can consider them as independent random variables. Next, we can approximate X with Poisson distribution with parameter $\lambda = E(X) = 100E(I_1) = 1$. So, we have

$$P(X = 0) \approx \frac{1^0}{0!}e^{-1} \approx 0.37$$

(c)

Use Poisson approximation to finally obtain that

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) \approx 1 - e^{-1} - e^{-1} = 1 - 2e^{-1} \approx 0.26$$