AMRITA VISHWA VIDYAPEETHAM
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22BIO211: Intelligence of Biological Systems - 2

THE MANHATTAN TOURIST PROBLEM

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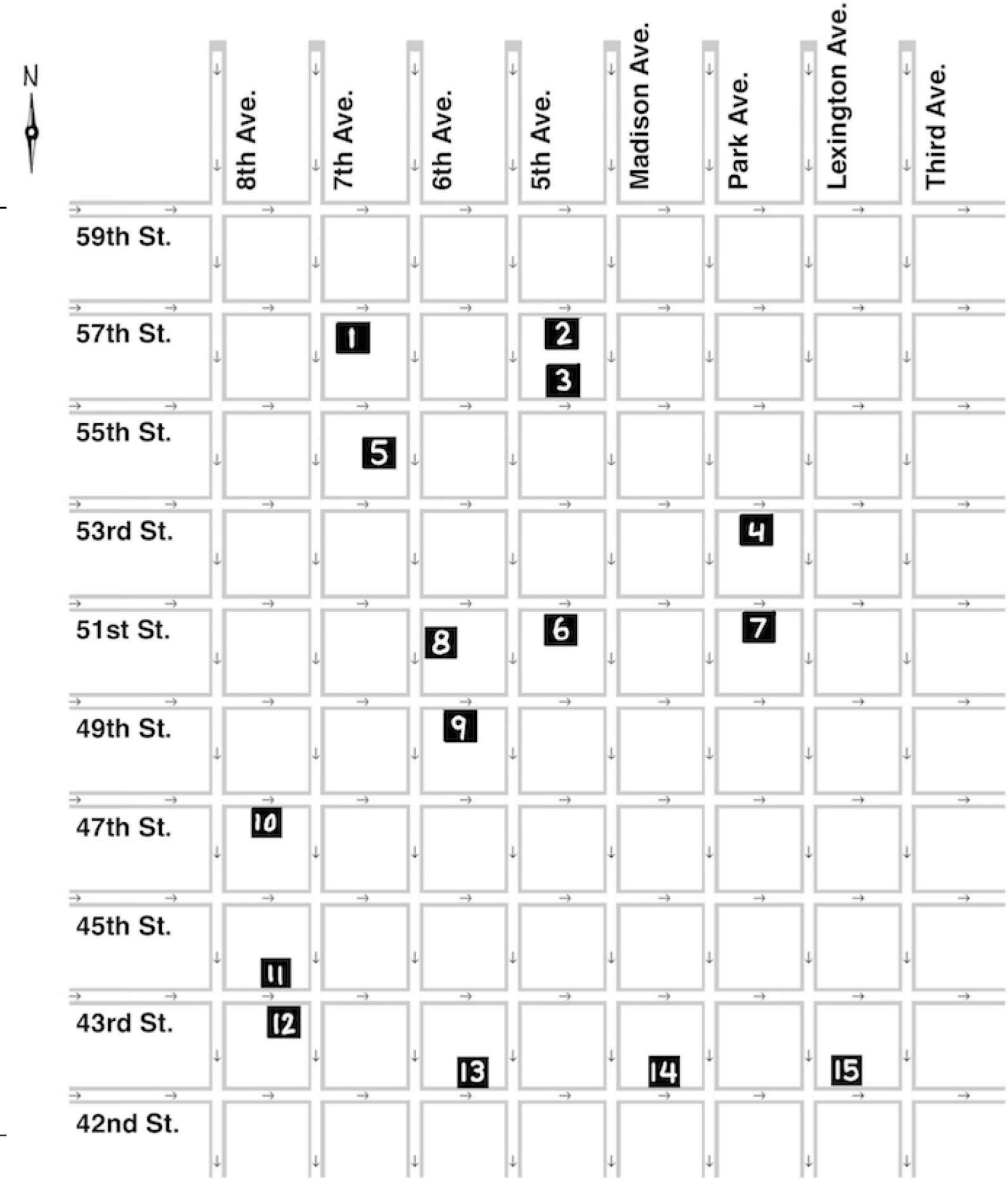
The City of Manhattan in New York

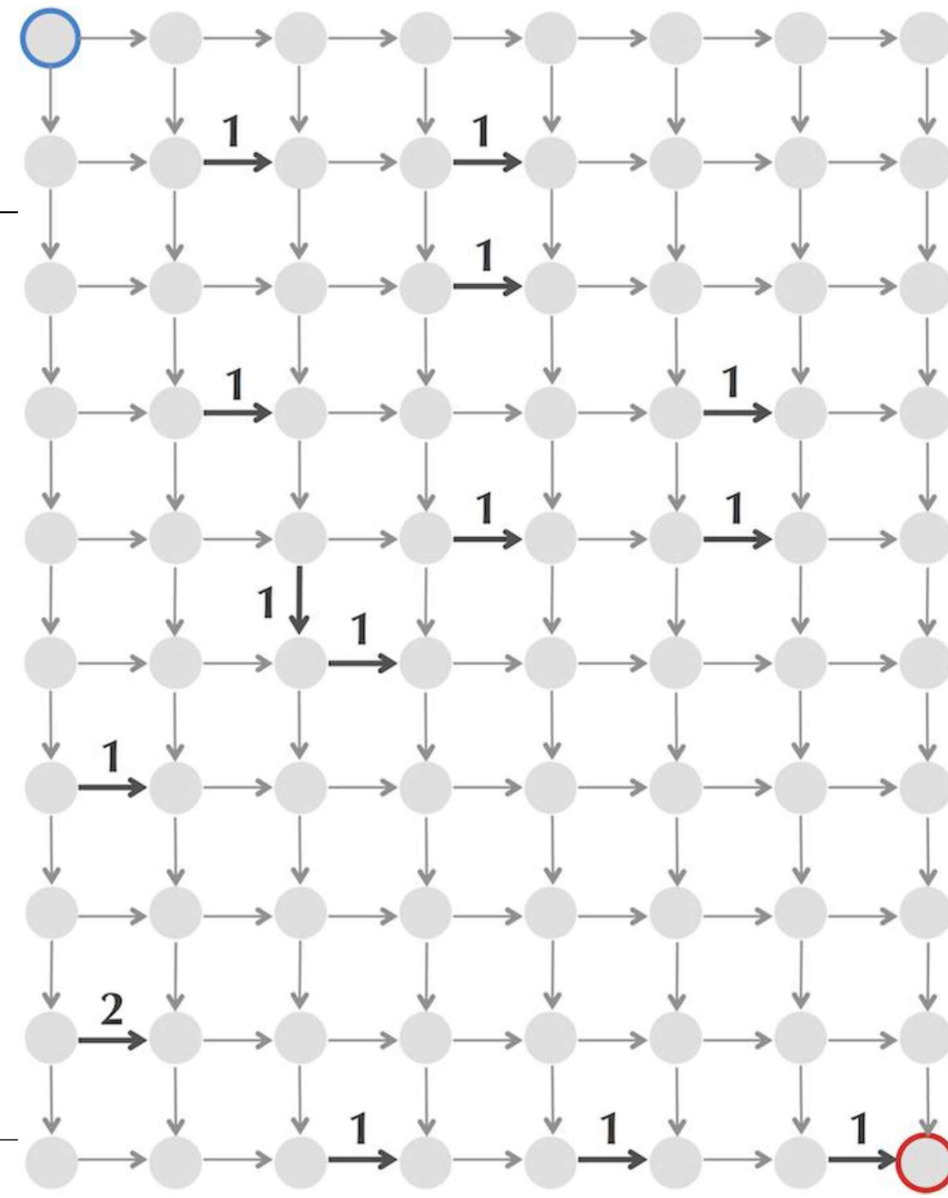
Manhattan Tourist Problem

- Goal:
  - *You want to see as many sights as possible on your way from the corner of 59th Street and 8th Avenue to the corner of 42nd Street and 3rd Avenue*
- Constraints
  - *You are short on time, and at each intersection, you can only move south (↓) or east (→)*
- You can choose from many different paths through the map, but **no path will visit all the sights.**



Manhattan Graph

- The challenge of finding a legal path through the city that visits the most sights is called the **Manhattan Tourist Problem**.
- Manhattan Graph
  - *Directed graph*
  - *Node – each intersection*
  - *directed edge - each city block between two intersections*
    - -indicating the legal direction of travel (↓ or →)
  - *assign each directed edge a **weight** equal to the number of attractions along the corresponding block.*



Manhattan Graph

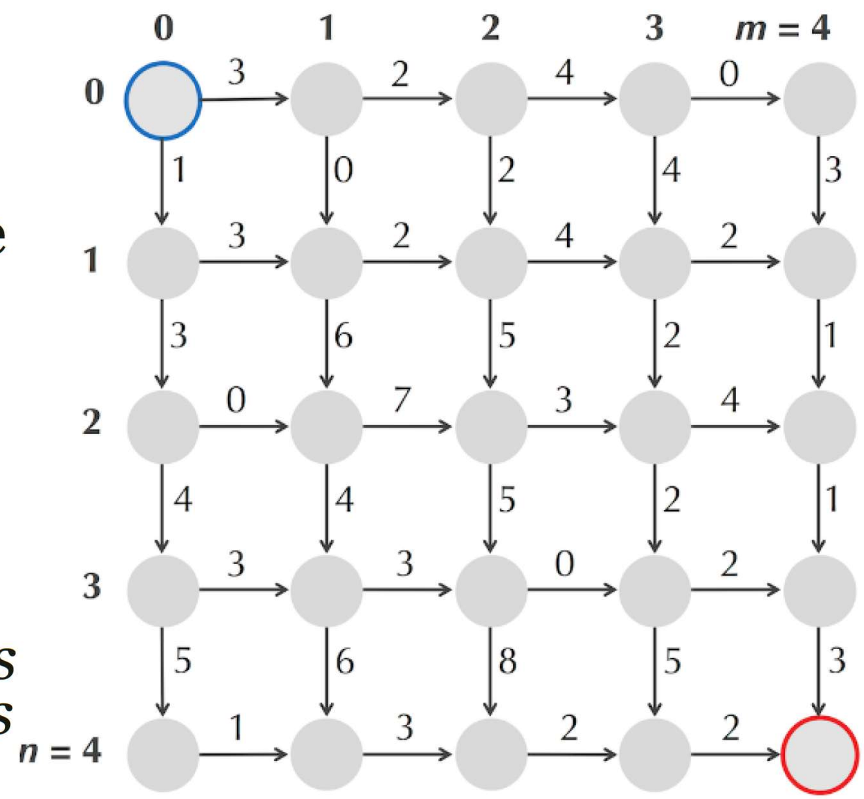
- The starting (blue) node is called the **source node**
- The ending (red) node is called the **sink node**.
- Adding the weights along a path from the source to the sink yields the number of attractions along that path;
  - *therefore, to solve the Manhattan Tourist Problem, we need to find a* ***maximum-weight path*** *connecting the source to the sink (also called a* ***longest path****) in ManhattanGraph.*

Manhattan Tourist Problem

- **Manhattan Tourist Problem:** *Find a longest path in a rectangular city.*
  - ***Input:*** *A weighted $n \times m$ rectangular grid with $n + 1$ rows and $m + 1$ columns.*
  - ***Output:*** *A longest path from source (0,0) to sink (n, m) in the grid.*

Manhattan Tourist Problem

- Figure shows the graph for a hypothetical city with even more attractions.
- In contrast to the Cartesian plane, we orient the axes of this grid down and to the right.
- Therefore, the blue source node is assigned the coordinates (0, 0), and the red sink node is assigned the coordinates (n, m).
  - *The bottom left node is indexed as (4, 0), and the upper right node is indexed as (0, 4).*



An $n \times m$ city grid is represented as a graph with weighted edges for $n = m = 4$.

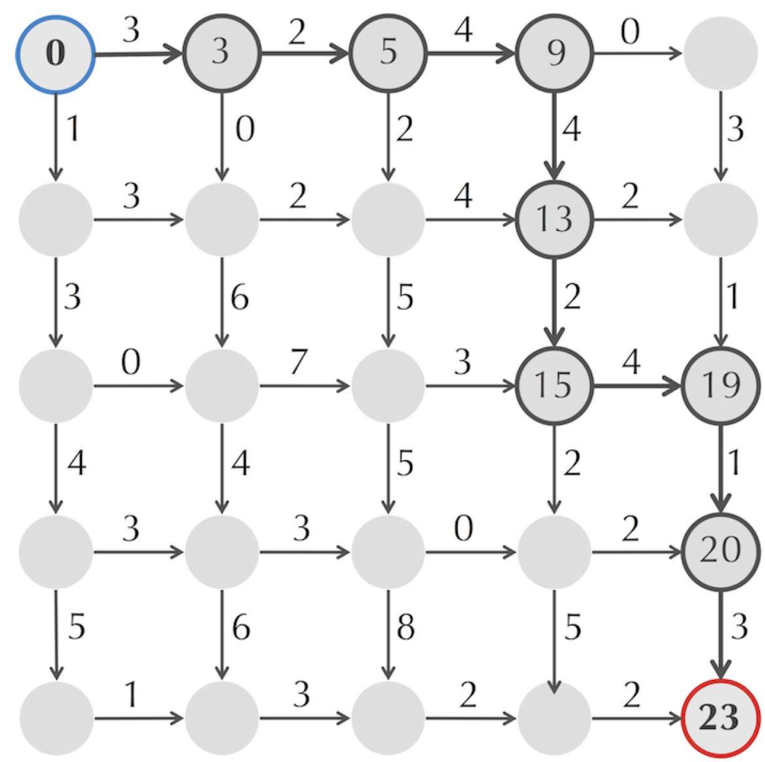
Manhattan Tourist Problem

- How many different paths are there from source to sink in a 16×12 rectangular grid?
  - *According to our notation, a 16×12 grid contains $17 \times 13 = 221$ nodes.*
- Applying a **brute force approach** to the Manhattan Tourist Problem is impractical
  - *because the number of paths is huge.*
- A sensible **greedy approach** would choose between the two possible directions at each node ($\rightarrow$ or $\downarrow$)
  - *according to how many attractions you would see if you moved only one block south versus moving one block east.*

Manhattan Tourist Problem - Greedy Solution

- For example, in the previous figure, we start by moving east from (0, 0) rather than south
  - because the horizontal edge has three attractions, while the vertical edge has only one.
- Unfortunately, this greedy strategy may miss the longest path in the long run.
- The highlighted path through this graph, which is found by the greedy algorithm, is not the longest.

How to find a longer path in the graph

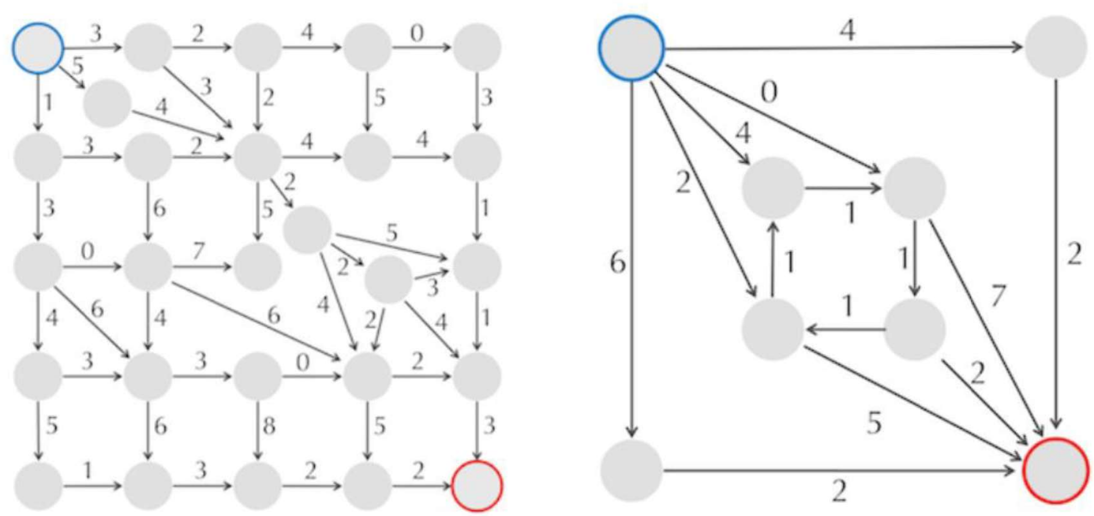


Longest Path in a Directed Graph Problem

- The Manhattan Tourist Problem is just a special case of the more general problem of finding the longest path in an arbitrary directed graph.
- **Longest Path in a Directed Graph Problem:** Find a longest path between two nodes in an edge-weighted directed graph.
  - *Input: An edge-weighted directed graph with source and sink nodes.*
  - *Output: A longest path from source to sink in the directed graph.*

Longest Path in a Directed Graph Problem

- What is the length of a longest path between the source and sink in the directed graph shown on the right in the figure below?



Directed graphs corresponding to hypothetical irregular city grids.

Directed Acyclic Graph - DAG

- If a directed graph contained a directed cycle (e.g., the four central edges of weight 1 in the graph)
  - *then a tourist could traverse this cycle indefinitely, revisiting the same attractions over and over again and creating a path of huge length.*
- For this reason, the graphs that we will consider in this chapter do not contain directed cycles
- Such graphs are called **directed acyclic graphs (DAGs).**

Longest Path in a DAG Problem

- **Longest Path in a DAG Problem:** Find a longest path between two nodes in an edge-weighted DAG.
  - *Input: An edge-weighted DAG with source and sink nodes.*
  - *Output: A longest path from source to sink in the DAG.*

What are the similarities between the Longest Path in a DAG Problem and the Longest Common Subsequence Problem?

Summary

- The City of Manhattan in New York
- Manhattan Tourist Problem
- Manhattan Graph
- Greedy Solution
- Longest Path in a Directed Graph Problem
- Directed Acyclic Graph
- Longest Path in a DAG Problem