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# Analytical Inverse Kinematic Solution Using the D-H Method for a 6-DOF Robot

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***Abstract*** - This paper proposes an analytical approach to solve inverse kinematics of a 6-DOF robot. The robot discussed here is designed with 6R configuration. The D-H model of the six-axis robot is built. The forward analytical solution is deduced using the D-H method. Then, analysis of inverse kinematics for the robot is presented. And finally, the validity of the inverse kinematic equations is verified. Forward kinematic equations and inverse kinematic equations are suitable for all kinds of robots with configuration similar to those of the robot in this paper.

***Keywords*** - Analytical solution, D-H Coordinates, inverse kinematic, 6-DOF Robot.

## 1. Introduction

Inverse kinematics of 6-DOFs (degrees of freedom) manipulators is to solve the joints position according to the relative position and orientation between the base and the end effector. It is the base of trajectory planning and motion control [1]. In fact, computing based on position level is solving the nonlinear equations [2, 3]. According to Pieper's law, analytic solutions exist only in manipulators with three adjacent axes intersect at one point or parallel to each other) [4, 5]. Manipulators with other structure can only be solved using numerical methods, while the methods are computationally intensive and unsuitable for real-time control [6].

Analytical solution can obtain all feasible solutions, and many algorithms are developed to solve inverse kinematics. Ahuactzin, J. built the kinematic roadmap and obtained the closed-form solution [7]. The neural networks such as the LRNNs [8], the MDNN [4], and screw theory [8, 9] are all applied for solving inverse kinematics of manipulators, but the specific algorithms depend on the configuration of robot [10, 11].

This paper proposes one methodology for deriving the closed form inverse kinematic solutions of the 6-DOF robot on position level. Forward kinematic equations and inverse kinematic equations are suitable for all kinds of robots with structure similar to those of the robot in this paper.

## 2. The D-H Model of the 6-DOF Robot

The D-H model of the six-axis robot is given in this part. The forward solution on the position is carried out utilizing the D-H method.

It is more concise to build the D-H model in zero position of the robot. Figure 1 shows the zero position of the welding robot. Local reference coordinate of each joint is created according to the zero position as show in figure 2. The D-H parameters of the 6-DOF robot are shown in table 1. As can be seen from the table, $a_1$, $a_4$, $a_5$, $a_6$, $d_2$, $d_3$ of this kind of robot is equal to zero.

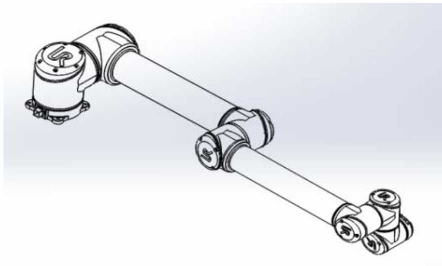

Fig. 1. Zero position of the robot

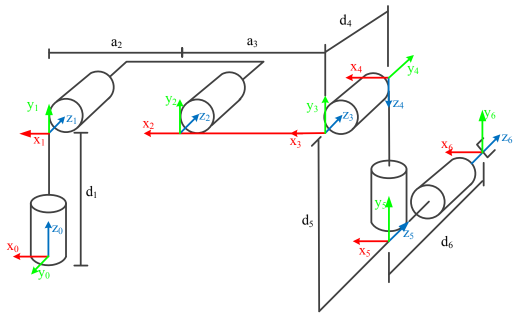

Fig. 2. Local reference coordinate of each joint

Table 1 The D-H parameters of the 6-DOF robot

| Link | θ | d | a | α |
|---|---|---|---|---|
| 1 | $\theta_1$ | $d_1$ | 0 | π/2 |
| 2 | $\theta_2$ | 0 | $a_2$ | 0 |
| 3 | $\theta_3$ | 0 | $a_3$ | 0 |
| 4 | $\theta_4$ | $d_4$ | 0 | π/2 |
| 5 | $\theta_5$ | $d_5$ | 0 | -π/2 |
| 6 | $\theta_6$ | $d_6$ | 0 | 0 |

After the linkage coordinate system was established by D-H rules, then, combined with the table of D-H parameters, homogeneous transformation matrices between adjacent link coordinate system were given:

$$A_n = \begin{bmatrix} C\theta_n & -S\theta_n C\alpha_n & S\theta_n S\alpha_n & a_n C\theta_n \\ S\theta_n & C\theta_n C\alpha_n & -C\theta_n S\alpha_n & a_n S\theta_n \\ 0 & S\alpha_n & C\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where:

$C\theta_n = \cos(\theta_n)$, $C\alpha_n = \cos\alpha$ (2)

$S\theta_n = \sin(\theta_n)$, $S\alpha_n = \sin\alpha$ (3)

${}^{B}T_H$ is the homogeneous transformation matrix between the base and the end axis:

$${}^{B}T_H = A_1 A_2 A_3 \cdots A_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$${}^{b}R_t = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix} \quad (5)$$

$${}^{b}P_t = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \quad (6)$$

${}^{b}R_t$ and ${}^{b}P_t$ is the position matrix and attitude matrix of the end coordinate in robot base respectively. When all θ are known, these two matrices can be obtained according to equation (4).

## 3. Inverse kinematic equations

The second, third and fourth joints of the robot parallel to each other, which satisfies the second condition of the Pieper's law, so the inverse solution can be obtained by analytic method. The specific derivation process is as follows.

According to

$$A_2 A_3 A_4 A_5 = A_1^{-1}\,{}^{B}T_H A_6^{-1} \quad (7)$$

it can be obtained:

$$\left(P_y - d_6 A_y\right) C_1 - \left(P_x - d_6 A_x\right) S_1 = -d_4 \quad (8)$$

$$\theta_1 = \text{Atan2}\left( \pm\sqrt{1-\left(\frac{d_4}{\rho}\right)^2}, -\frac{d_4}{\rho} \right) - \text{Atan2}\left(\frac{b}{\rho}, \frac{a}{\rho}\right) \quad (9)$$

where

$$a = P_y - d_6 A_y,\quad b = P_x - d_6 A_x,\quad \rho = \sqrt{a^2 + b^2} \quad (10)$$

According to

$$A_2 A_3 A_4 A_5 A_6 = A_1^{-1}\,{}^{B}T_H \quad (11)$$

it can be extracted from equation (9):

$$\begin{aligned} n_x S_1 - n_y C_1 &= C_6 S_5 \\ o_x S_1 - o_y C_1 &= -S_5 S_6 \end{aligned} \quad (12)$$

Simplify equation (10),

$$S_5 = \pm\sqrt{(n_x S_1 - n_y C_1)^2 + (o_x S_1 - o_y C_1)^2} \quad (13)$$

$$C_5 = A_x S_1 - A_y C_1 \quad (14)$$

Then the angle of joint 5 can be calculated from

$$\theta_5 = \text{Atan2}\left(S_5, C_5\right) \quad (15)$$

Thus

$$C_6 = \frac{n_x S_1 - n_y C_1}{S_5}, \quad S_6 = \frac{o_x S_1 - o_y C_1}{-S_5} \quad (16)$$

$$\theta_6 = \text{Atan2}\left(\frac{o_x S_1 - o_y C_1}{-S_5}, \frac{n_x S_1 - n_y C_1}{S_5}\right) \quad (17)$$

Similarly, find the corresponding terms on both sides of equation (18):

$$A_2 A_3 A_4 = A_1^{-1}\,{}^{B}T_H A_6^{-1} A_5^{-1} \quad (18)$$

then $\theta_{234}$ (=$\theta_2+\theta_3+\theta_4$) is calculated from

$$\begin{aligned} \theta_{234} &= \text{Atan2}\left(S_{234},\ C_{234}\right) \\ C_{234} &= C_5\left[C_6\left(C_1 n_x + n_y S_1\right) - S_6\left(C_1 o_x + o_y S_1\right)\right] - S_5\left(A_y S_1 + A_x C_1\right) \\ S_{234} &= C_5\left(C_6 n_z - o_z S_6\right) - a_z S_5 \end{aligned} \quad (19)$$

$\theta_3$ is given by

$$\theta_3 = A\tan 2\left(S_3, C_3\right) \quad (20)$$

$\theta_2$ is given by

$$\theta_2 = A\tan 2\left(\frac{y}{\sqrt{k_1^2 + k_2^2}}, \frac{x}{\sqrt{k_1^2 + k_2^2}}\right) - A\tan 2\left(a_3 S_3, a_2 + a_3 C_3\right) \quad (21)$$

Finally, $\theta_4$ is given by

$$\theta_4 = \theta_{234} - \theta_2 - \theta_3 \tag{22}$$

Then the analytical inverse solution of the six joint is all given out.

## 4. Verification of Inverse Kinematic Equations

In order to verify the correctness of the analytical inverse solution.

When $\theta$ is

$$\theta = [\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}] \tag{23}$$

Take it into equation (4) to caculate the forward solution :

$$T = \begin{bmatrix} -0.2165 & -0.8750 & 0.4330 & -0.4850 \\ -0.6250 & -0.2165 & -0.7500 & -0.5614 \\ 0.7500 & -0.4330 & -0.5000 & -0.7195 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{24}$$

Inverse solutions are computed by the inverse kinematic equations above, specific value is show in table 2. As can be seen from Table 2, the first group solution is the right one, and the other seven groups are other possible poses in the workspace of the robot.

Table 2 The result of inverse solution

| i | $\theta_1(rad)$ | $\theta_2(rad)$ | $\theta_3(rad)$ | $\theta_4(rad)$ | $\theta_5(rad)$ | $\theta_6(rad)$ |
|---|---|---|---|---|---|---|
| 1 | 0.5236 | 0.5236 | 0.5236 | 0.5236 | 0.5236 | 0.5236 |
| 2 | 0.5236 | 1.0286 | -0.5236 | 1.0658 | 0.5236 | 0.5236 |
| 3 | 0.5236 | 0.3849 | 1.1560 | -3.1116 | 0.5236 | -2.6180 |
| 4 | 0.5236 | 1.4956 | -1.1560 | -1.9104 | 0.5236 | -2.6180 |
| 5 | -2.1584 | 1.7416 | 0.9119 | -0.4276 | 2.4594 | -1.8377 |
| 6 | -2.1584 | 2.6194 | -0.9119 | 0.5183 | 2.4594 | -1.8377 |
| 7 | -2.1584 | 1.9853 | 0.8542 | 2.5280 | -2.4594 | 1.3039 |
| 8 | -2.1584 | 2.8079 | -0.8542 | -2.8694 | 2.4594 | 1.3039 |

## 5. Conclusion

This paper has derived the closed form inverse kinematic solutions of the 6-DOF robot on position level. Forward kinematic equations and inverse kinematic equations are suitable for all kinds of robots with configuration similar to those of the robot in this paper. Finally, the validity of the analytical inverse solutions is verified.


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