Lab - 2

1. LU Decomposition of Matrices

$$A_1 = \begin{bmatrix} 2 & 1 \\ 6 & 7 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \qquad \cancel{U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

$$L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$$

2. LU Decomposition of Matrix

$$A_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

3. <u>LU Decomposition of Matrix</u>

$$A_3 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & -\frac{2}{3} & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & -1 & 0 \\ 0 & \frac{3}{2} & -1 \\ 0 & 0 & \frac{4}{3} \end{bmatrix} \quad \begin{aligned} R_2 &\to R_2 + \frac{R_1}{2} \\ R_3 &\to R_3 + \frac{2}{3} R_2 \end{aligned}$$

$$A_3 = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & -\frac{2}{3} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 2 & -1 & 0 \\ 0 & \frac{3}{2} & -1 \\ 0 & 0 & \frac{4}{3} \end{bmatrix}}_{U}$$

4. Use LU Decomposition to solve linear eqn $Ax = b$, where $A = \begin{bmatrix} 2 & 1 \\ 6 & 7 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$A = LU \Rightarrow U = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$Ax = b$

$\Rightarrow A = LU$

$\Rightarrow L \cdot U x = b$

$L \cdot y = b$ where $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \Rightarrow Ux$

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$y_1 + 0y_2 = 1 \Rightarrow \boxed{y_1 = 1}$

$3y_1 + y_2 = 1 \Rightarrow y_2 = 1 - 3y_1$

$y_2 = 1 - 3 \cdot 1 = -2$

$\boxed{y_2 = -2}$

$Ux = y$

$$\begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$2x_1 + x_2 = 1$

$0x_1\ 4x_2 = -2$

$\boxed{x_2 = -1/2}$ $\quad x_1 = \frac{1 - x_2}{2} = \frac{1+1/2}{2}$

$$\boxed{x_1 = \frac{3}{4}}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \cancel{\begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix}} \rightarrow \begin{bmatrix} 3/4 \\ -1/2 \end{bmatrix}$$ is the solution to the system of linear equations $Ax = b$

4. $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$

Here 0 appears in a pivotal position

At these times a permutation matrix is required

$A = P^{-1}LU$

or $\boxed{PA = LU}$

normal LU Decomposition

$$[A] = [L][U] \Rightarrow \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l & 1 \end{bmatrix} \begin{bmatrix} d & e \\ 0 & f \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} d & e \\ ld & le+f \end{bmatrix}$$

$d = 0$ $\quad$ $e = 1$

$l.d = 2$ $\quad$ $le + f = 3$

$l.0 = 2$

$0 = 2 \Rightarrow$ Contradiction, thus normal LU Decomposition is not possible

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$