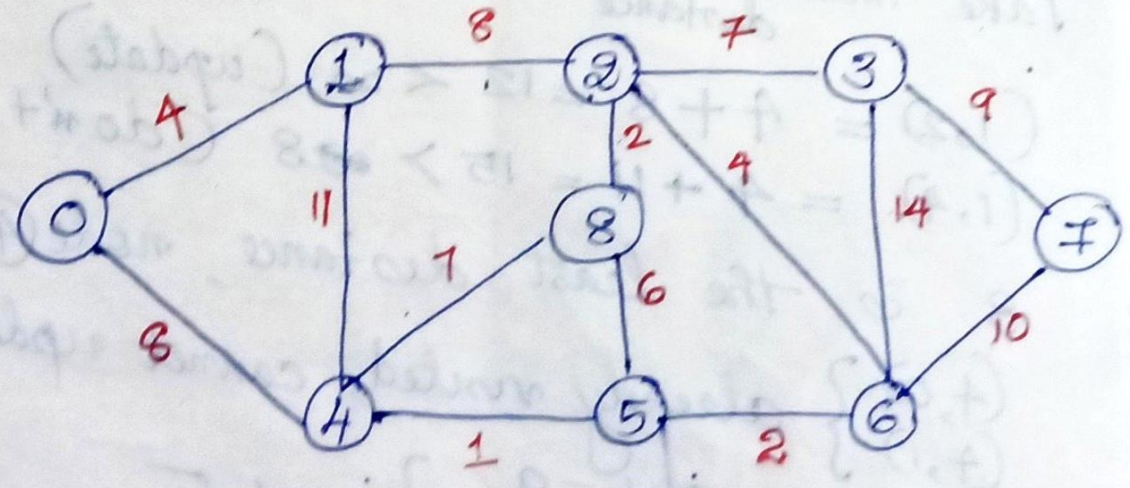
Dykstra's Algorithm

→ Greedy approach
→ Works on directed and undirected graph.
→ Single source shortest path.
  ↳ Find shortest path from one source to all other vertices.

Step 1: Assign all vertices distance as ∞ other than source

Qn:



step2: if $d(u) + c(u,v) < d(v)$
then assign $d(v) = d(u) + c(u,v)$

selected vertex	0	1	2	3	4	5	6	7	8
0	[0]	∞	∞	∞	∞	∞	∞	∞	∞
1	-	[4]	∞	∞	8	∞	∞	∞	∞
4	-	-	12	∞	[8]	∞	∞	∞	∞
5	-	-	12	∞	-	[9]	∞	∞	15
6	-	-	12	∞	-	-	[11]	∞	15
2	-	-	[12]	25	-	-	-	21	15
8	-	-	-	[19]	-	-	-	21	[14]
3	-	-	-	-	-	-	-	[21]	-
7									

Vertex 0

step2 0 → (0 is visited)

$(0,4) = 8 < \infty$

$(0,1) = 4 < \infty$ (Take minimum from entire vertex)

update

step3 1 is visited

Take minimum = 4 distance

$(1,2) = 4 + 8 = 12 < \infty$ (update)

$(1,4) = 4 + 11 = 15 > 8$ (don't update)

step4 8 is the least distance, node (4)

(4,0) } already visited, cannot update
(4,1) }

(4,5) = 8+1 = 9 } update

(4,8) = 8+7 = 15 }

step5

select (5)

(5,4) × → already visited

(5,8) = 9+6 = 15 (same value)

(5,6) = 9+2 = 11 (update)

step6

select node (6)

(6,5) ×

(6,2) = 11+4 = 15 > 12 (don't update)

(6,3) = 11+14 = 25 (update)

(6,7) = 11+10 = 21 (update)

step7

select node (2)

(2,1) ×

(2,3) = 12+7 = 19 < 25 (update)

(2,6) ×

(2,8) = 12+2 = 14 < 15 (update)

step8

select node (8)

(8,5) ×

(8,4) ×

(8,2) ×

All nodes are visited adjacent to 8.

∴ ~~visit~~ no updations

~~mark visit~~ mark (8) as visited

step9

Take min from 19 & 21

min = 19, node is (3)

(3,2) ×

(3,7) = 19+9 = 28 > 21 (don't update)

(3,6) ×

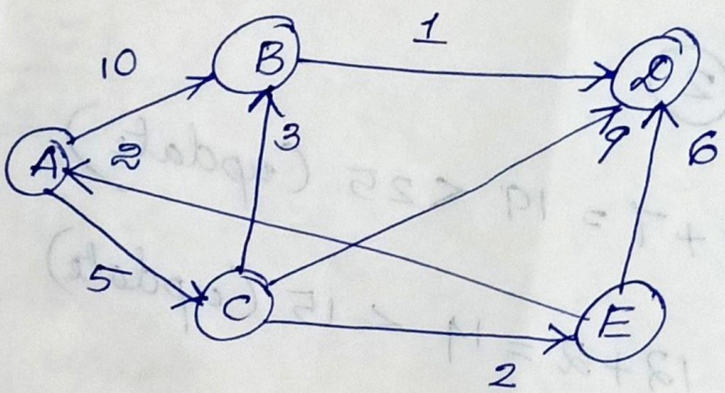
*select (7) as visited

Ans

cost

(0,1) = 4	(0,5) = 9
(0,2) = 12	(0,6) = 11
(0,3) = 19	(0,7) = 21
(0,4) = 8	(0,8) = 14

Qn)



selected vertex ↓	A	B	C	D	E
A	0	∞	∞	∞	∞
C	—	10	5	∞	∞
E	—	8	—	14	7
B	—	8	—	13	—
D	—	—	—	9	—

Find path from A to D?

	A	B	C	D	E
			step 6		
A	0	∞	∞	∞	∞
C	—	10	5	∞	∞
E	—	8	—	14	7
B	—	8	—	13	—
D	—	—	—	9	—

step 5, step 4, step 3, step 2, step 1

D B C A

~~select~~ D B C A

∴ path = A → C → B → D

cost = 5 + 3 + 1 = 9

path from A to B

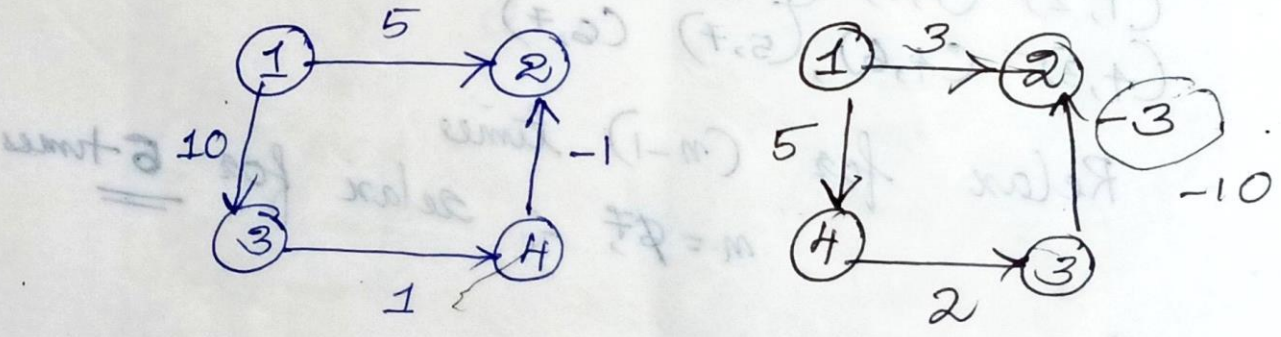
= B C A

→ A → C → B

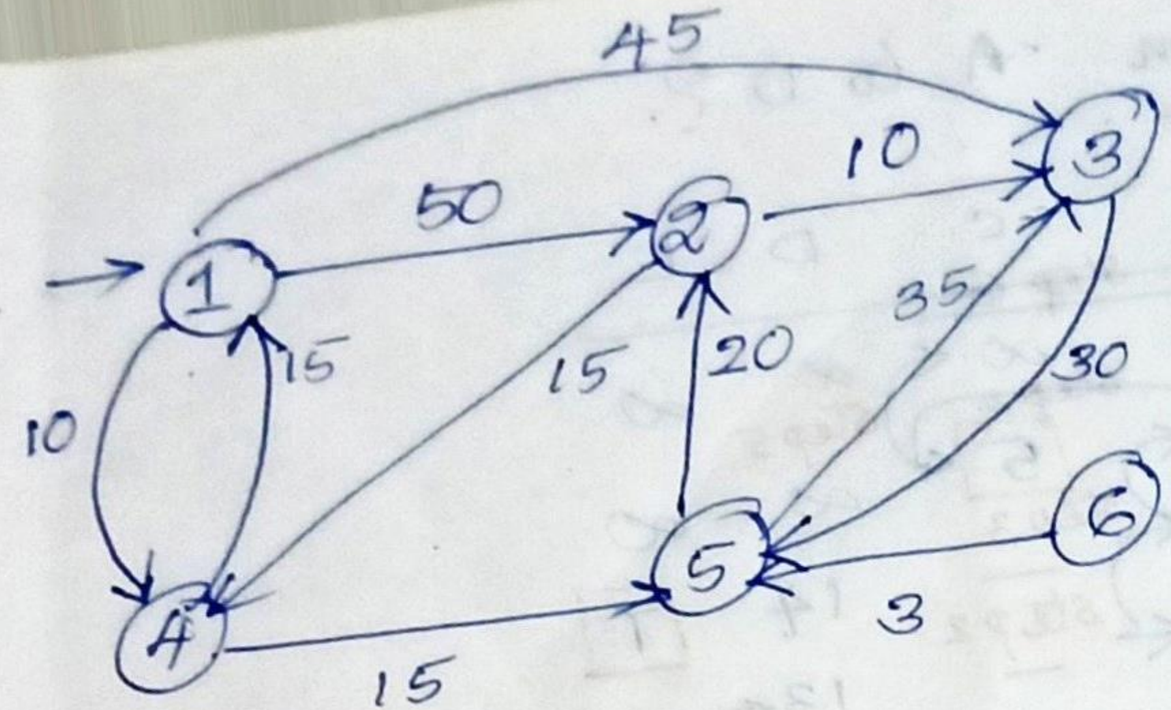
= 5 + 3 = 8

Disadv of Dykstra

→ If negative value of weight, shortest distance calculation is wrong in Dykstra.



Qn:



2	45
3	45
4	10
5	25
6	∞