Derivatives of some important functions.

$\frac{d}{dx}\, e^x = e^x$

$\frac{d}{dx}\, e^u = e^u\ \frac{du}{dx}$

$\frac{d}{dx}\, \log_a x = \frac{1}{(\ln a)\, x}$

$\frac{d}{dx}\, \log_a u = \frac{1}{(\ln a)\, u}\ \frac{du}{dx}$

$\frac{d}{dx}\, \ln x = \frac{1}{x}$

$\frac{d}{dx}\, \ln u = \frac{1}{u}\ \frac{du}{dx}$

$\frac{d}{dx}\, \sin x = \cos x$

$\frac{d}{dx}\, \sin u = \cos u\ \frac{du}{dx}$

$\frac{d}{dx}\, \cos x = -\sin x$

$\frac{d}{dx}\, \cos u = -\sin u\ \frac{du}{dx}$

$\frac{d}{dx}\, \tan x = \sec^2 x$

$\frac{d}{dx}\, \tan u = \sec^2 u\ \frac{du}{dx}$

$\frac{d}{dx}\, \cot x = -\csc^2 x$

$\frac{d}{dx}\, \cot u = -\csc^2 u\ \frac{du}{dx}$

$\frac{d}{dx}\, \sec x = \sec x \tan x$

$\frac{d}{dx}\, \sec u = \sec u \tan u\ \frac{du}{dx}$

$\frac{d}{dx}\, \csc x = -\csc x \cot x$

$\frac{d}{dx}\, \csc u = -\csc u \cot u\ \frac{du}{dx}$

Problems:

1. $y = \sqrt[3]{x}\sin(x)$

2. $f(r) = 5r - \cos(r) + \frac{1}{r}$

3. $f(z) = \sin^2(z)$

4. $y = x^4 \tan(x)$

5. $y = x^2 \sec(x)$

6. $f(r) = 3e^r - \frac{1}{r^2} + \sin(r)$

7. $y = \tan(x) + \frac{1}{x^2} + e^2 + 3$

8. $f(\theta) = 5\theta - \cot(\theta) + \sqrt{\theta}$

9. $f(s) = \tan(s) - \frac{3}{s^2} + 2e^s$

10. $y = \tan^2(x)$

11. $y = \frac{\sqrt{x}\cos(x)}{x^3 + 1}$

12. $y = \frac{x\sin(x)}{e^x}$

13. $y = \frac{x^2 + 5}{x + \sec(x)}$

14. $y = \frac{x\cos(x)}{\sin(x) + 1}$

Solutions:

1. $y = \sqrt[3]{x}\sin(x) = x^{1/3}\sin(x)$ $\qquad y' = \frac{1}{3}x^{1/3-1}\sin(x) + x^{1/3}\cos(x) = \frac{\sin(x)}{3\sqrt[3]{x^2}} + \sqrt[3]{x}\cos(x)$

3. $f(z) = \sin^2(z) = \sin(z)\sin(z)$ $\qquad f'(z) = \cos(z)\sin(z) + \sin(z)\cos(z) = 2\sin(z)\cos(z)$

5. $y = x^2\sec(x)$ $\qquad$ By product rule: $y' = 2x\sec(x) + x^2\sec(x)\tan(x)$

7. $y = \tan(x) + \frac{1}{x^2} + e^2 + 3 = \tan(x) + x^{-2} + e^2 + 3$ $\qquad y' = \sec^2(x) - 2x^{-3} + 0 + 0 = \sec^2(x) - \frac{2}{x^3}$

9. $f(s) = \tan(s) - \frac{3}{s^2} + 2e^s$ $\qquad f'(s) = \sec^2(s) + \frac{6}{s^3} + 2e^s$

11. $y = \frac{\sqrt{x}\cos(x)}{x^3+1} = \frac{x^{1/2}\cos(x)}{x^3+1}$ $\qquad y' = \frac{D_x\left[x^{1/2}\cos(x)\right](x^3+1) - x^{1/2}\cos(x)\cdot D_x\left[x^3+1\right]}{(x^3+1)^2}$

$$= \frac{\left(\frac{1}{2}x^{-1/2}\cos(x) + x^{1/2}(-\sin(x))\right)(x^3+1) - x^{1/2}\cos(x)3x^2}{(x^3+1)^2}$$

$$= \frac{\left(\frac{\cos(x)}{2\sqrt{x}} - \sqrt{x}\sin(x)\right)(x^3+1) - \sqrt{x}\cos(x)3x^2}{(x^3+1)^2}$$

13. $y = \frac{x^2+5}{x+\sec(x)}$ $\qquad y' = \frac{2x(x+\sec(x)) - (x^2+5)(1+\sec(x)\tan(x))}{(x+\sec(x))^2}$

Differentiate the following:

a. $\cos 3x$ $\qquad$ **b.** $\sin(4x+5)$ $\qquad$ **c.** $\sin^3 x$ $\qquad$ **d.** $\sin x \cos x$ $\qquad$ **e.** $x^2 \sin x$

f. $\cos(x^2+1)$ $\qquad$ **g.** $\frac{\sin x}{x}$ $\qquad$ **h.** $\sin\frac{1}{x}$ $\qquad$ **i.** $\tan(\sqrt{x})$ $\qquad$ **j.** $\frac{1}{x}\sin\frac{1}{x}$

Solutions to Exercise 1

a. $\dfrac{d}{dx}\cos 3x = -3\sin 3x$

b. $\dfrac{d}{dx}\sin(4x+5) = 4\cos(4x+5)$

c. $\dfrac{d}{dx}\sin^3 x = 3\sin^2 x\cos x$

d. $\dfrac{d}{dx}\sin x\cos x = \cos^2 x - \sin^2 x$

e. $\dfrac{d}{dx}x^2\sin x = 2x\sin x + x^2\cos x$

f. $\dfrac{d}{dx}\cos(x^2+1) = -2x\sin(x^2+1)$

g. $\dfrac{d}{dx}\left(\dfrac{\sin x}{x}\right) = \dfrac{x\cos x - \sin x}{x^2}$

h. $\dfrac{d}{dx}\sin\dfrac{1}{x} = -\dfrac{1}{x^2}\cos\dfrac{1}{x}$

i. $\dfrac{d}{dx}\tan\sqrt{x} = \dfrac{1}{2\sqrt{x}}\sec^2\sqrt{x}$

j. $\dfrac{d}{dx}\left(\dfrac{1}{x}\sin\dfrac{1}{x}\right) = -\dfrac{1}{x^2}\sin\dfrac{1}{x} - \dfrac{1}{x^3}\cos\dfrac{1}{x}$

1. Find the derivative of each of the following:

   a) $(3x-7)^{12}$ b) $\sin(5x+2)$ c) $\ln(2x-1)$ d) e^{2-3x}

   e) $\sqrt{5x-3}$ f) $(6x+5)^{5/3}$ g) $\dfrac{1}{(3-x)^4}$ h) $\cos(1-4x)$

2. Find the derivative of each of the following:

   a) $\ln(\sin x)$ b) $\sin(\ln x)$ c) $e^{-\cos x}$ d) $\cos\left(e^{-x}\right)$

   e) $(\sin x+\cos x)^3$ f) $\sqrt{1+x^2}$ g) $\dfrac{1}{\cos x}$ h) $\dfrac{1}{x^2+2x+1}$

3. Find the derivative of each of the following:

   a) $\ln\left(\sin^2 x\right)$ b) $\sin^2(\ln x)$ c) $\sqrt{\cos(3x-1)}$ d) $[1+\cos(x^2-1)]^{3/2}$

Answers

1. a) $36(3x-7)^{11}$ b) $5\cos(5x+2)$ c) $\dfrac{2}{2x-1}$ d) $-3e^{2-3x}$

   e) $\dfrac{5}{2\sqrt{5x-3}}$ f) $10(6x+5)^{2/3}$ g) $\dfrac{4}{(3-x)^5}$ h) $4\sin(1-4x)$

2. a) $\dfrac{\cos x}{\sin x} = \cot x$ b) $\dfrac{\cos(\ln x)}{x}$ c) $\sin x e^{-\cos x}$

   d) $e^{-x}\sin(e^{-x})$ e) $3(\cos x - \sin x)(\sin x + \cos x)^2$ f) $\dfrac{x}{\sqrt{1+x^2}}$

   g) $\dfrac{\sin x}{\cos^2 x} = \tan x \sec x$ h) $\dfrac{-2(x+1)}{(x^2+2x+1)^4} = \dfrac{-2}{(x+1)^3}$

3. a) $\dfrac{2\cos x}{\sin x} = 2\cot x$ b) $\dfrac{2\sin(\ln x)\cos(\ln x)}{x}$

   c) $\dfrac{-3\sin(3x-1)}{2\sqrt{\cos(3x-1)}}$ d) $-3x\sin(x^2-1)\left[1+\cos(x^2-1)\right]^{1/2}$

Differentiate the functions with respect to x in Exercises 1 to 8.

1. $\sin(x^2+5)$ **2.** $\cos(\sin x)$ **3.** $\sin(ax+b)$

4. $\sec(\tan(\sqrt{x}))$ **5.** $\dfrac{\sin(ax+b)}{\cos(cx+d)}$ **6.** $\cos x^3 \cdot \sin^2(x^5)$

7. $2\sqrt{\cot(x^2)}$ **8.** $\cos(\sqrt{x})$

Answers: Set-I

1. $2x\cos(x^2+5)$

2. $-\cos x \cdot \sin(\sin x)$

3. $a.\cos(ax+b)$

4. $\dfrac{\sec(\tan\sqrt{x})\tan(\tan\sqrt{x})\cdot\sec^2(\sqrt{x})}{2\sqrt{x}}$

5. $\dfrac{a\cdot\cos(ax+b)\cdot\cos(cx+d)+c\cdot\sin(ax+b)\sin(cx+d)}{\cos^2(cx+d)}$

6. $-3x^2\cdot\sin x^3\cdot\sin^2 x^5+10x^4\cdot\cos x^3\cdot\sin x^5\cdot\cos x^5$

7. $\dfrac{-2x\cdot\operatorname{cosec}^2 x^2}{\sqrt{\cot x^2}}$

8. $\dfrac{-\sin\sqrt{x}}{2\sqrt{x}}$

Find $\dfrac{dy}{dx}$ in the following:

1. $2x + 3y = \sin x$
2. $2x + 3y = \sin y$
3. $ax + by^2 = \cos y$
4. $xy + y^2 = \tan x + y$
5. $x^2 + xy + y^2 = 100$
6. $x^3 + x^2y + xy^2 + y^3 = 81$
7. $\sin^2 y + \cos xy = \kappa$
8. $\sin^2 x + \cos^2 y = 1$
9. $y = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

Set-II

1. $\frac{dy}{dx} = \frac{\cos x - 2}{3}$ 2. $\frac{dy}{dx} = \frac{2}{\cos y - 3}$ 3. $\frac{dy}{dx} = \frac{-a}{2by + \sin y}$

4. $\frac{dy}{dx} = \frac{\sec^2 x - y}{(x + 2y - 1)}$ 5. $\frac{dy}{dx} = \frac{-(2x+y)}{(x+2y)}$ 6. $\frac{dy}{dx} = \frac{-(3x^2+2xy+y^2)}{(x^2+2xy+3y^2)}$

7. $\frac{dy}{dx} = \frac{y \sin xy}{\sin 2y - x \sin xy}$ 8. $\frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$

9. $\frac{dy}{dx} = \frac{2(1-x^2)}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}\,(1+x^2)^2}$ or $\frac{dy}{dx} = \frac{2}{1+x^2}$

Differentiate the following w.r.t. x:

1. $\frac{e^x}{\sin x}$ 2. $e^{\sin^{-1} x}$ 3. e^{x^3}

4. $\sin(\tan^{-1} e^{-x})$ 5. $\log(\cos e^x)$ 6. $e^x + e^{x^2} + \ldots + e^{x^5}$

7. $\sqrt{e^{\sqrt{x}}}$, $x > 0$ 8. $\log(\log x)$, $x > 1$ 9. $\frac{\cos x}{\log x}$, $x > 0$

Answers: Set-I

1. $\frac{e^x(\sin x - \cos x)}{\sin^2 x}$ 2. $\frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}$ 3. $3e^{x^3}x^2$

4. $\frac{e^{-x}\cos(\tan^{-1} e^{-x})}{1+e^{-2x}}$ 5. $-e^x \tan e^x$

6. $e^x + 2xe^{x^2} + 3x^2e^{x^3} + \ldots + 5x^4e^{x^5}$

7. $\frac{e^{\sqrt{x}}}{4\sqrt{e^{\sqrt{x}}}\sqrt{x}}$ 8. $\frac{1}{x \log x}$ 9. $\frac{-x \sin x \log x - \cos x}{x \log^2 x}$

Logarithmic differentiation.

Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

1. $\cos x \cdot \cos 2x \cdot \cos 3x$

2. $\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

3. $(\log x)^{\cos x}$

4. $x^x - 2^{\sin x}$

5. $(x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$

6. $\left(x+\dfrac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

7. $(\log x)^x + x^{\log x}$

8. $(\sin x)^x + \sin^{-1}\sqrt{x}$

9. $x^{\sin x} + (\sin x)^{\cos x}$

10. $x^{x\cos x} + \dfrac{x^2+1}{x^2-1}$

11. $(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

Logarithmic differentiation:

1. $\dfrac{dy}{dx} = \cos x \cdot \cos 2x \cdot \cos 3x \left[-\tan x - 2\tan 2x - 3\tan 3x\right]$

2. $\dfrac{dy}{dx} = \dfrac{1}{2}\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} \left[\dfrac{1}{(x-1)} + \dfrac{1}{(x-2)} - \dfrac{1}{(x-3)} - \dfrac{1}{(x-4)} - \dfrac{1}{(x-5)}\right]$

3. $\dfrac{dy}{dx} = (\log x)^{\cos x} \left[\dfrac{\cos x}{x\log x} - \sin x \log(\log x)\right]$

4. $\dfrac{dy}{dx} = x^x(1+\log x) - 2^{\sin x}(\cos x \cdot \log 2)$

5. $\dfrac{dy}{dx} = (x+3)\cdot(x+4)^2\cdot(x+5)^3(9x^2+70x+133)$

6. $\frac{dy}{dx} = \left(x+\frac{1}{x}\right)\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] + x^{\left(1+\frac{1}{x}\right)} \frac{(1+x^2)\log x}{x^2}$

7. $\frac{dy}{dx} = (\log x)^{x+1}\left(1+\log x \cdot \log\log x\right) + x^{\log x - 1} \cdot (2\log x)$

8. $\frac{dy}{dx} = (\sin x)^x (x\cot x + \log\sin x) + \frac{1}{2\sqrt{x-x^2}}$

9. $\frac{dy}{dx} = x^{\sin x - 1}(\sin x + x\log x\cos x) + \sin x^{\cos x}(\cos x\cot x - \sin x\log\sin x)$

10. $\frac{dy}{dx} = x^{x\cos x}\left[\cos x - x\sin x\log x + \cos x\log x\right] - \frac{4x}{(x^2-1)^2}$

11. $\frac{dy}{dx} = (x\cos x)^x\left[1 - x\tan x + \log(x\cos x)\right] + (\sin x \cdot x)^{1/x}\left[\frac{x\cot x + 1 - \log(x\sin x)}{x^2}\right]$

Functions in parametric form

If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

1. $x = 2at^2, y = at^4$
2. $x = a\cos\theta, y = b\cos\theta$
3. $x = \sin t, y = \cos 2t$
4. $x = 4t, y = \frac{4}{t}$
5. $x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$
6. $x = a(\theta - \sin\theta), y = a(1+\cos\theta)$
7. $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

Find the second order derivatives of the functions given in Exercises

1. $x^2 + 3x + 2$
2. x^{20}
3. $x \cdot \cos x$
4. $\log x$
5. $x^3 \log x$
6. $e^x \sin 5x$

Answers :

Parametric form

1. t^2 2. b/a 3. $-4\sin t$ 4. $-1/t^2$

5. $\dfrac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$ 6. $-\cot\theta/2$ 7. $-\cot 3t$

Second order derivatives

1. 2 2. $380 \cdot x^{18}$ 3. $-(x \cdot \cos x + 2\sin x)$

4. $-1/x^2$ 5. $x(5 + 6\log x)$ 6. $e^x(10\cos 5x - 24\sin 5x)$