Steps to Convert to CNF (Conjunctive Normal Form)

Every sentence in Propositional Logic is logically equivalent to a conjunction of disjunctions of literals. A sentence expressed as a conjunction of disjunctions of literals is said to be in Conjunctive normal Form or CNF.

1. Eliminate implication '→'

   a → b = ~a v b

   ~ (a ^ b) = ~ a v ~ b …………………….. DeMorgan's Law

   ~ (a v b) = ~ a ^ ~ b ……………………... DeMorgan's Law

   ~ (~a) = a

2. Eliminate Existential Quantifier '∃'

   To eliminate an independent Existential Quantifier, replace the variable by a Skolem constant. This process is called as Skolemization.

   Example: ∃y: President (y)

   Here **'y'** is an independent quantifier so we can replace **'y'** by any name (say – George Bush).

   So, **∃y: President (y)** becomes **President (George Bush)**.

   To eliminate a dependent Existential Quantifier we replace its variable by Skolem Function that accepts the value of 'x' and returns the corresponding value of 'y.'

   Example: ∀x : ∃y : father_of (x, y)

   Here **'y'** is dependent on **'x',** so we replace **'y'** by **S(x).**

   So, **∀x : ∃y : father_of (x, y)** becomes **∀x : ∃y : father_of (x, S(x)).**

3. Eliminate Universal Quantifier '∀'

   To eliminate the Universal Quantifier, drop the prefix in PRENEX NORMAL FORM i.e. just drop ∀ and the sentence then becomes in PRENEX NORMAL FORM.

4. Eliminate AND '^'

   a ^ b splits the entire clause into two separate clauses i.e. a and b

   (a v b) ^ c splits the entire clause into two separate clauses a v b and c

   (a ^ b) v c splits the clause into two clauses i.e. a v c and b v c

To eliminate '^' break the clause into two, if you cannot break the clause, distribute the OR 'v' and then break the clause.

EXAMPLE

Now let us see an example which uses resolution.

Problem Statement:

1. Ravi likes all kind of food.
2. Apples and chicken are food
3. Anything anyone eats and is not killed is food
4. Ajay eats peanuts and is still alive
5. Rita eats everything that Ajay eats

Prove by resolution that Ravi likes peanuts using resolution.

Solution:

Step 1: Converting the given statements into Predicate/Propositional Logic

i. $\forall x$: food(x) $\rightarrow$ likes (Ravi, x)
ii. food (Apple) ^ food (chicken)
iii. $\forall a$: $\forall b$: eats (a, b) ^ killed (a) $\rightarrow$ food (b)
iv. eats (Ajay, Peanuts) ^ alive (Ajay)
v. $\forall c$: eats (Ajay, c) $\rightarrow$ eats (Rita, c)
vi. $\forall d$: alive(d) $\rightarrow$ ~killed (d)
vii. $\forall e$: ~killed(e) $\rightarrow$ alive(e)

Conclusion: likes (Ravi, Peanuts)

Step 2: Convert into CNF

i. ~food(x) v likes (Ravi, x)
ii. Food (apple)
iii. Food (chicken)
iv. ~ eats (a, b) v killed (a) v food (b)

v. Eats (Ajay, Peanuts)
vi. Alive (Ajay)
vii. ~eats (Ajay, c) V eats (Rita, c)
viii. ~alive (d) v ~ killed (d)
ix. Killed (e) v alive (e)

Conclusion: likes (Ravi, Peanuts)

<u>Step 3</u>: Negate the conclusion

~ likes (Ravi, Peanuts)

<u>Step 4</u>: Resolve using a resolution tree

~ likes (Ravi, Peanuts)~food(x) v likes (Ravi, x)

x | peanuts

~food (peanuts) ~ eats (a, b) v killed (a) v food (b)

b | peanuts

~eats (a, peanuts) v killed (a) eats (Ajay, peanuts)

a | Ajay

Killed (Ajay) ~alive(d) v ~killed (d)

d | Ajay

~alive (Ajay) alive (Ajay)

{ }

Hence we see that the negation of the conclusion has been proved as a complete contradiction with the given set of facts.

Hence the negation is completely invalid or false or the assertion is completely valid or true.

Hence Proved