Exercise: I

Eliminate the arbitrary constants from the following.

a) $z = (x-a)^2 + (y-b)^2$ b) $z = a(x+y) + b$ c) $z = ax + by + ab$

d) $z = ax + a^2y^2 + b$ e) $z = axe^y + \frac{1}{2}a^2e^{2y} + b$

f) $(x-h)^2 + (y-k)^2 + z^2 = \lambda^2$ (h & k are arb. const.)   g) $ax^2 + by^2 + z^2 = 1$

h) $2z = x^2/a^2 + y^2/b^2$ i) $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ j) $ax + bxy + cy^2 = z$

k) $z = ax + by + cxy$

Answers:

a) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 4z$ b) $\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$ c) $z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + \frac{\partial z}{\partial x}\frac{\partial z}{\partial y}$

d) $\frac{\partial z}{\partial y} = 2y\left(\frac{\partial z}{\partial x}\right)^2$ e) $\frac{\partial z}{\partial y} = x\frac{\partial z}{\partial x} + \left(\frac{\partial z}{\partial x}\right)^2$ f) $x^2 = z^2\left(1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right)$

g) $z\left(z - x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}\right) = 1$ h) $2z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$

i) $zx\frac{\partial^2 z}{\partial x^2} + x\left(\frac{\partial z}{\partial x}\right)^2 - z\left(\frac{\partial z}{\partial x}\right) = 0$, $zy\left(\frac{\partial^2 z}{\partial y^2}\right) + y\left(\frac{\partial z}{\partial y}\right)^2 - z\left(\frac{\partial z}{\partial y}\right) = 0$,

$z\frac{\partial^2 z}{\partial x \partial y} + \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right) = 0$

j) $2z = x^2\left(\frac{\partial^2 z}{\partial x^2}\right) + 2xy\left(\frac{\partial^2 z}{\partial x \partial y}\right) + y^2\left(\frac{\partial^2 z}{\partial y^2}\right)$

k) $z + xy\left(\frac{\partial^2 z}{\partial x \partial y}\right) = x\left(\frac{\partial z}{\partial x}\right) + y\left(\frac{\partial z}{\partial y}\right)$