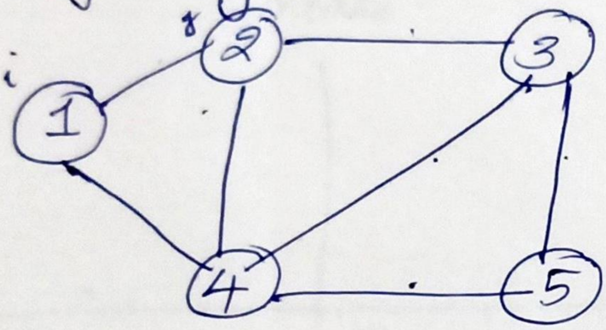
S_3 - AIE

Data Structures & Algorithms - 2

Graph Representation

(1) Adjacency Matrix. (3) incidence matrix

(2) Adjacency List



Matrix $m \times n$ → cols.
↓
rows

Adj. matrix

$n \times n$ where $n \rightarrow$ no: of vertices.

$$\begin{array}{c c}
i \rightarrow & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}
\end{array}$$

5×5

space complexity $\Rightarrow O(n^2)$

better → if dense graph.

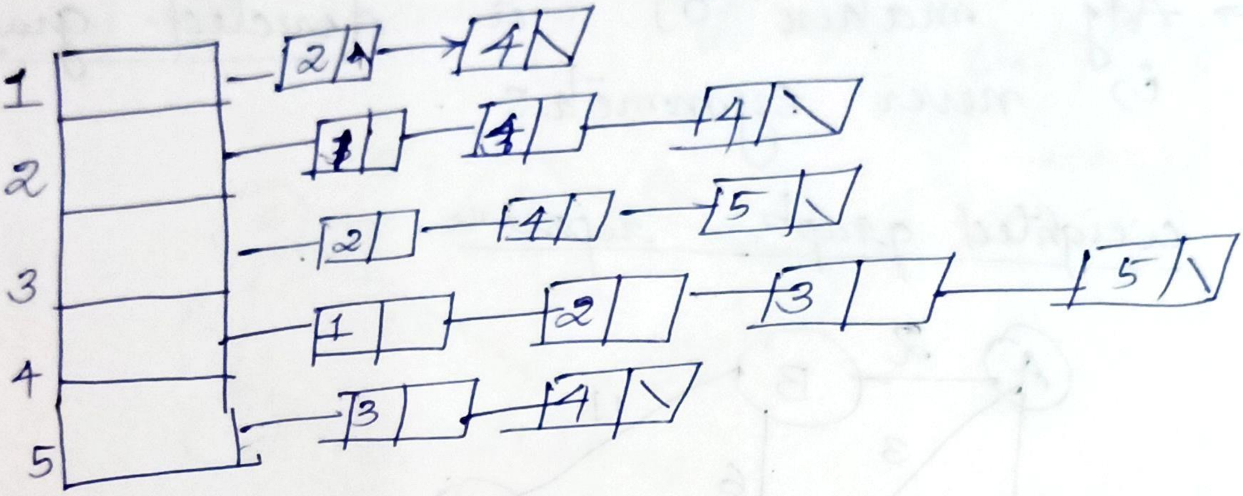
Defn

It is a matrix $A[n][n]$ where $n \rightarrow$ # vert

$$a[i][j] = \begin{cases} 1, & \text{if } i \,\&\, j \text{ are adjacent} \\ 0, & \text{otherwise} \end{cases}$$

Adjacency list

→ for each vertex one LL would be maintained. That LL will have contain the adj node to that node.



space complexity $\Theta(n+2e)$

$n \rightarrow$ vertices.

$e \rightarrow$ edges

eg:- ($1 \rightarrow 2$ & $2 \rightarrow 1$)

$2e \rightarrow$ edges we are taking 2 times.

dense → adjacency matrix

sparse → adjacency list

→ each every node is connected to every other nodes.

→ very few no: of edges.

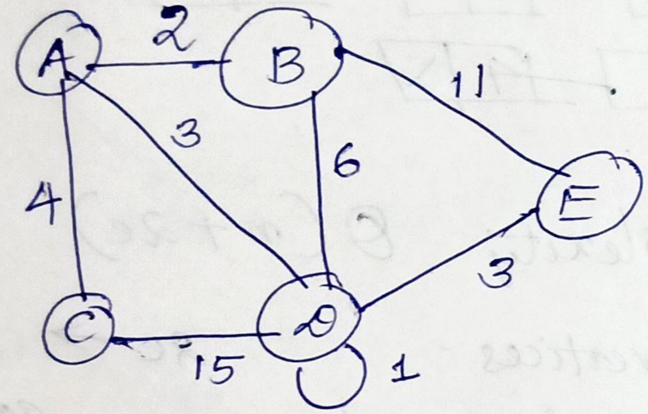
Adjacency matrix

A graph $G = (V, E)$, where $V = \{0, 1, \ldots n-1\}$ can be represented using dimensional array of size $n \times n$.

→ Adj matrix of an undirected graph is always a symmetric matrix.
ie, an edge (i, j) implies edge(j, i)

→ Adj matrix of a <u>directed graph</u> is never symmetric.

<u>weighted graph represn.</u>



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} &
\begin{bmatrix}
0 & 2 & 4 & 3 & 0 \\
2 & 0 & 0 & 6 & 11 \\
4 & 0 & 0 & 15 & 0 \\
3 & 6 & 15 & \cancel{1}\ 1 & 3 \\
0 & 11 & 0 & 3 & 0
\end{bmatrix}
\end{array}
$$

<u>Incidence matrix</u>

→ In this matrix; rows represent vertices, columns represent edges.

→ This matrix is filled with either 0 or 1 or −1

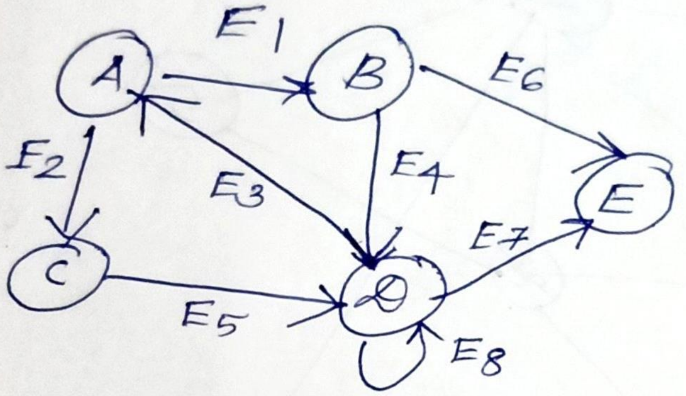
$0 \Rightarrow$ row edge is not connected to column.

$1 \Rightarrow$ row edge is connected to out going ~~edge~~ of column.

$-1 \Rightarrow$ incoming edge.

Outgoing +1

incom -1



$$
\begin{array}{c}
 \\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 & E_8 \end{array} \\
\begin{bmatrix}
1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & -1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & 0
\end{bmatrix}
\end{array}
$$