

# Adversarial Search

## Lecture 4

# Adversarial Search

**Adversarial search** is search when there is an "enemy" or "opponent" changing the state of the problem every step in a direction you do not want.

Examples: Chess, business, trading, war.

You change state, but then you don't control the next state.

Opponent will change the next state in a way:

1. unpredictable
2. *hostile* to you

# Environment

## Multi-agent environment:

- any given agent needs to consider the actions of other agents and how they affect its own welfare introduce possible contingencies into the agent's problem-solving process cooperative vs. competitive

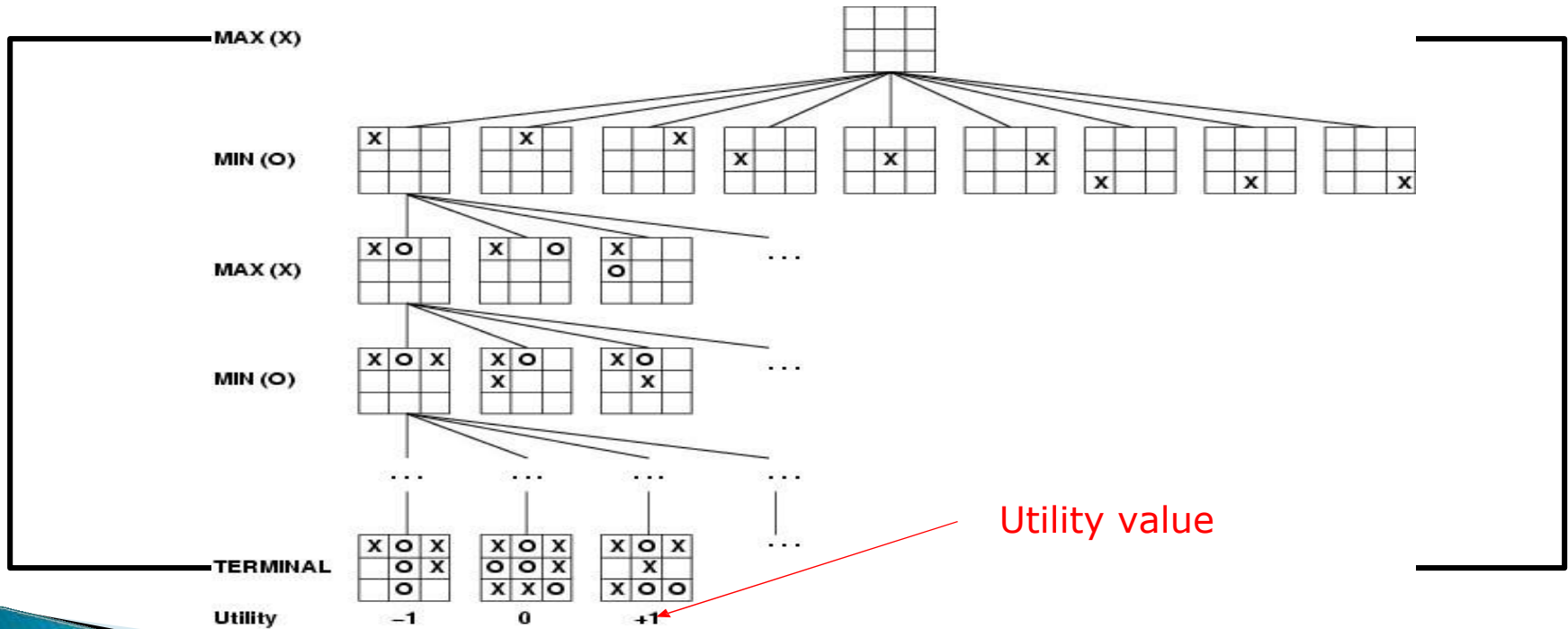
# Primary Assumptions:

- “Game” in AI:
  - A multi-agent, non-cooperative environment
  - Zero Sum Result: describes a situation in which a participant’s gain or loss is exactly balanced by the losses or gains of the other participant(s)
  - Turn Taking.
  - Deterministic.
  - *Two Player*

# Game Problem Formulation

- A game with 2 players (MAX and MIN, MAX moves first, turn-taking) can be defined as a search problem with:
  - initial state: board position
  - player: player to move
  - successor function: a list of legal (move, state) pairs
  - goal test: whether the game is over – terminal states
  - utility function: gives a numeric value for the terminal states (win, loss, draw)
- Game tree = initial state + legal moves

# Game Tree (2-player, deterministic)



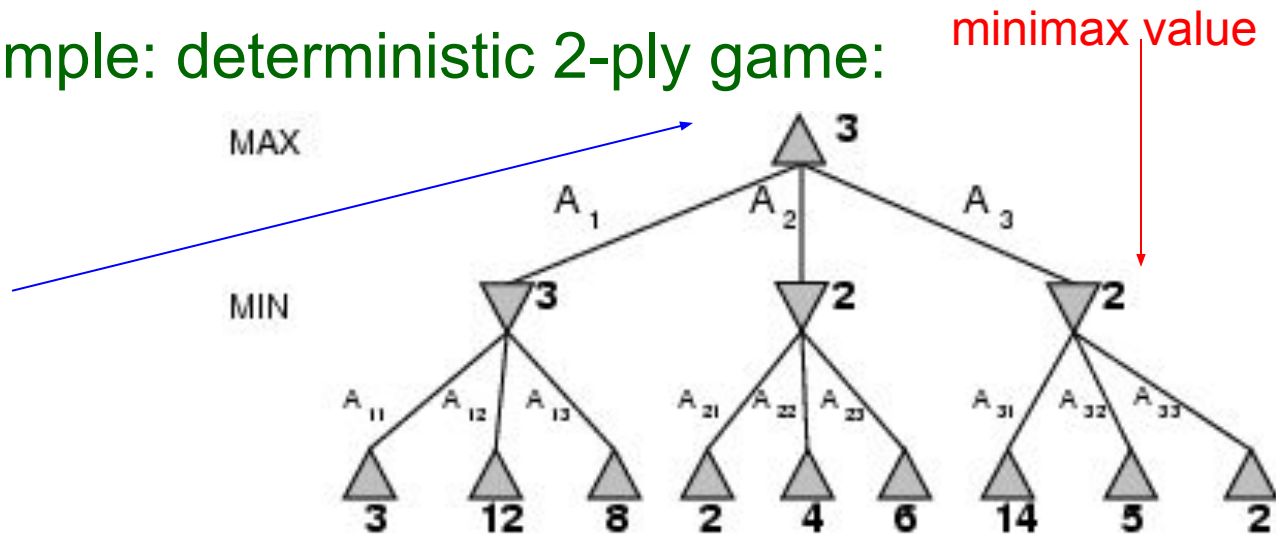
# Minimax

- Idea: choose a move to a position with the highest **minimax value** = best achievable payoff against a **rational** opponent.

Minimax value is  
computed  
bottom up:

Example: deterministic 2-ply game:

- Leaf values are given.
- 3 is the best outcome for MIN in this branch.
- 3 is the best outcome for MAX in this game.
- We explore this tree in **depth-first** manner.



# Properties of minimax

- Complete? Yes (if tree is finite)
- Optimal? Yes (against an rational opponent)
- Time complexity?  $O(b^m)$
- Space complexity?  $O(bm)$  (depth-first exploration)



# Minimax algorithm

**function** MINIMAX-DECISION(*state*) *returns an action*

$v \leftarrow \text{MAX-VALUE}(\textit{state})$

**return** the *action* in SUCCESSORS(*state*) with value *v*

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**function** MAX-VALUE(*state*) *returns a utility value*

**if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)

$v \leftarrow -\infty$

**for** *a, s* in SUCCESSORS(*state*) **do**

$v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s))$

**return** *v*

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**function** MIN-VALUE(*state*) *returns a utility value*

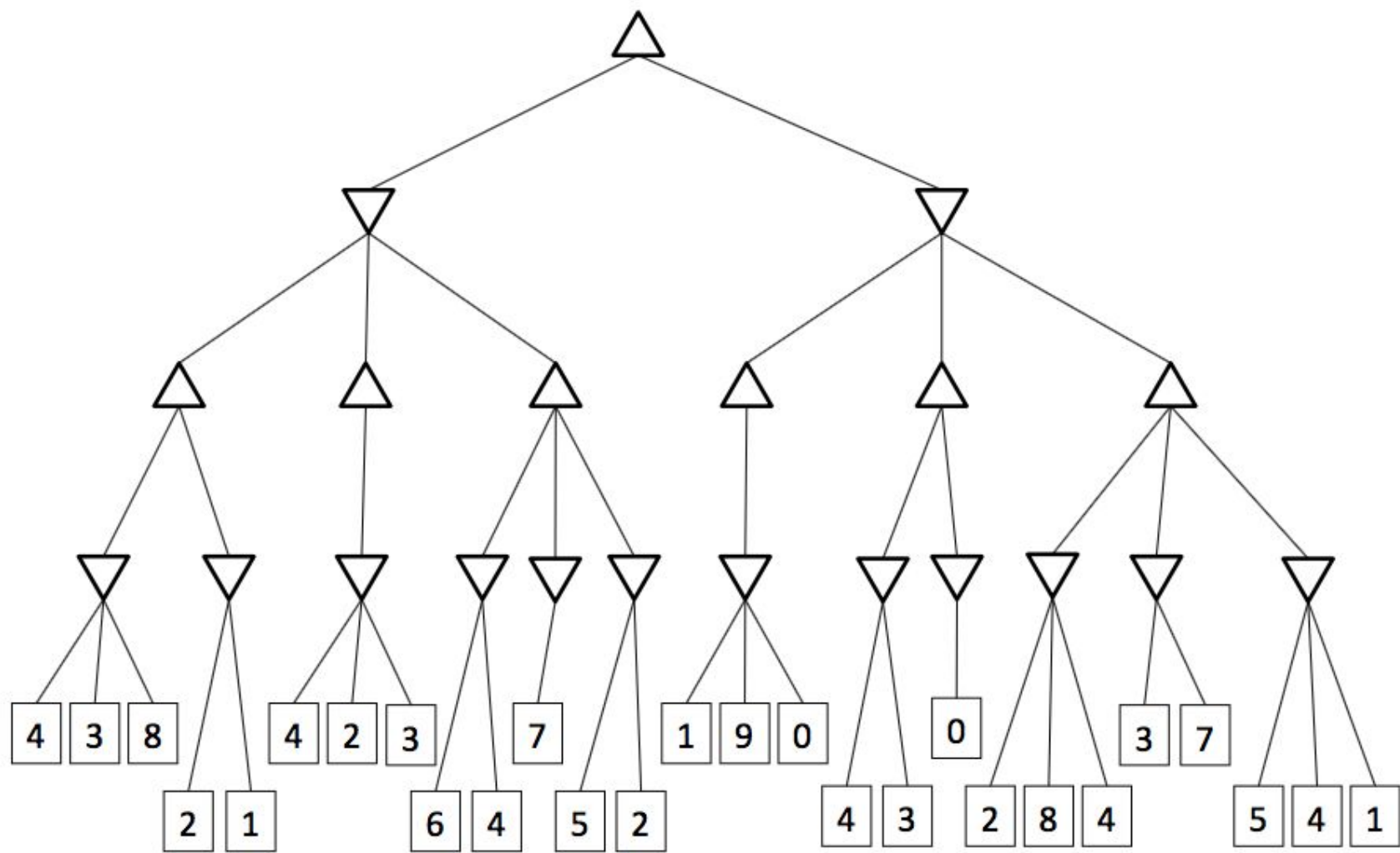
**if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)

$v \leftarrow \infty$

**for** *a, s* in SUCCESSORS(*state*) **do**

$v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s))$

**return** *v*



# Solution

