MAT220: Assignment 2
Due date: 5/3/24

Q1. The least squares criterion tries to minimize f(x)=$\frac{1}{2}||Ax-b||^2$. Find the gradient of this function and describe how it can be used to minimize the error in regression fitting using gradient descent algorithm.

Q2. Find the direction of increase of the function f(x,y,z) = $2y^2z - x^2z^2$ (as a vector) at the point p=(1,2,3)′.

Q3. Find the set of all points where the gradient vector of the function f(x,y,z) = x(y-3) + z(z^2-3) is zero.

Q4. Use Lagrangian multiplier method to find the global maximum, if any, of the function f(x,y) = $3x + xy + 2y^2$ subject to $x^2 - y$=3.

Q5. Obtain the KKT conditions for the following problem.
Minimise $x^2+6y^2-10z^2+4xy+6yz-2xz+5y$ subject to x+2y+3z $\geq$ 5. All variables are positive.

Q6. Find the solution, if any, and the optimal value of the problem to minimise ($\frac{1}{2}x^2 - y$) subject to x+2y$\leq$ 6, x$\geq$ 1 and $x^2 + y^2 \leq$25.

Q7. You want to design a cylindrical top-less water-tank (like a can without top) with minimum surface area that should hold maximum 500 cubic feet of water. Obtain the solution using KKT conditions.

Q8. Let $\{X_n, n \in N_0\}$ be a Markov chain with transition matrix

$$A = \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 1/5 & 2/5 & 1/5 \\ 1/4 & 1/2 & 1/4 \end{bmatrix},$$

Check if the Markov chain is stationary.