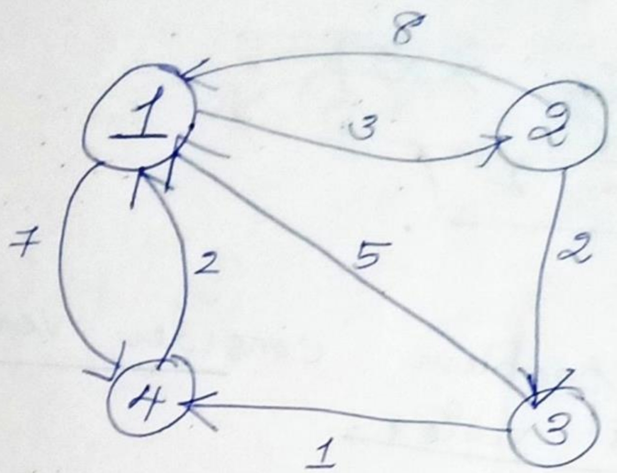
Floyd Warshall

→ All pairs shortest path
→ Dynamic programming approach

problem

Finding shortest path for every pair of vertices



1 → 2, 3, 4

2 → 1, 3, 4 etc

3 → 1, 2, 4

4 → 1, 2, 3

$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$

This is same as Dykstra's algm in the sense that we are finding the shortest path from every vertices as starting vertex

Dy → n^2 (for single vertex)

∴ All pairs → Dy → $n^2 \times n$

$O(n^3)$

greedy

- Dynamic programming says that the problem can be solved by taking decisions in each stage

$\Rightarrow$ sequence of decision.

What decision, we have to take.

$$\boxed{\begin{array}{l} \text{absence of edge} \rightarrow \infty \\ \text{no self loops} \rightarrow 0 \end{array}}$$

here in this problem consider vertex 1 as intermediate vertex.

$A^0 \Rightarrow$ Original matrix

$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

(2,3)

(2,1)

(1,3)

Then prepare matrix for vertex 1 matrix '0' as intermediate matrix

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{array}$$

$\left.\begin{array}{l} 1^{st} \text{ row} \\ 1^{st} \text{ col} \end{array}\right\}$ same as it is

no diagonals $\Rightarrow 0$

Then find $A^1(2,3)$

$A^0(2,3)$
$= 2$

$= A^0(2,1) + A^0(1,3)$
$= 8 + \infty$
$= \infty$

$2 < \infty$

$\therefore A^1(2,3) = \underline{\underline{2}}$

$A^0(2,4)$ $\quad$ $A^0(2,1) + A^0(1,4)$
∞ $\quad$ $8 + 7 = \underline{\underline{15}}$

$A^0(3,2)$ $\quad$ $A^0(3,1) + A^0(1,2)$
∞ $\quad$ $5 + 3 = \underline{\underline{8}}$

$A^0(3,4)$ $\quad$ $A^0(3,1) + A^0(1,4)$
1 $\quad$ $= 5 + 7 = 12$

$A^0(4,2)$ $\quad$ $A^0(4,1) + A^0(1,2)$
∞ $\quad$ $2 + 3 = \underline{\underline{5}}$

$A^0(4,3)$ $\quad$ $A^0(4,1) + A^0(1,3)$
∞ $\quad$ $2 + \infty = \infty$

$= \underline{\underline{\infty}}$

Then next find $\underline{A^2}$

Take $\underline{A^1}$ as intermediate matrix

$$A^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & \infty & 0 \end{array}$$

$$A^{2} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

2^{nd} R

2^{nd} Col a d

$A'(1,3)$

$(1,2)$

$A'(1,3)$ $\quad$ $A'(1,2) + A'(2,3)$

∞ $\quad$ $3 + 2 = \underline{\underline{5}}$

$A'(1,4)$ $\quad$ $A'(1,2) + A'(2,4)$

7 $\quad$ $3 + 15 = \underline{\underline{18}}$

$A'(3,1)$ $\quad$ $A'(3,2) + A'(2,1)$

5 $\quad$ $8 + 8 = 16$

$$A^{3} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 6 \\ 2 & 7 & 0 & 2 & 3 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array} \qquad A^{4} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 6 \\ 2 & 5 & 0 & 2 & 3 \\ 3 & 3 & 6 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

These are the shortest path

formula

prev. matrix

$$A^{k}[i,j] = \min\left\{A^{k-1}[i,j],\ A^{k-1}[i,k] + A^{k-1}(k,j)\right.$$

for getting $A(i,j)$ of any intermediate vertex.

k is introducing b/w and addition is done.

code

```
for (k=1; k<=n; k++)
{
 for(i=1; i<=n; i++)
 {
  for (j=1; j<=n; j++)
  {
   A[i,j] = min(A[i,j], A(i,k)+
                A(k,j))
```

$O(n^3)$