band matrix

Tridiagonal & Tri band matrix

Tridiagonal matrix

$$M = \begin{array}{c|c} & j \rightarrow \; 1 \quad 2 \quad 3 \quad 4 \quad 5 \\ \begin{array}{c} i \downarrow \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & a_{54} & a_{55} \end{bmatrix} \end{array}$$

$n = 5$

→ elts are in the main diagonal, lower diagonal and upper diagonal. Rest of the elts are zero.

property → main diagonal $i-j=0$
lower diagonal $i-j=1$
upper diagonal $i-j=-1$
} These elts are non zero

→ All other elts are zero.

$$|i-j| \leq 1$$

$M[i,j]$ = non zero if $|i-j| \leq 1$
$M[i,j] = 0$; if $|i-j| > 1$

How many non ~~diagonal~~ zero elts are there?

→ $5+4+4 \Rightarrow n + n-1 + n-1 = 3n-2$

So don't need to take 2D array for representing this matrix. We can represent using 1D array, depending on the no. of nonzero elts.

lower diag				main diag					upper diag			
a_{21}	a_{32}	a_{43}	a_{54}	a_{11}	a_{22}	a_{33}	a_{44}	a_{55}	a_{12}	a_{23}	a_{34}	a_{45}
0	1	2	3	4	5	6	7	8	9	10	11	12

Then how to map these elts in single dimensional array

→ row by row / col by col

~~row by row~~ ⟹ but rows and cols don't have uniform number of elts.

row	# col
2	2
3	3
3	3
3	3
2	2

nonzero

∴ we can represent them diagonal by diagonal.

① lower
② main
③ upper

or

① upper
② main
③ lower

index $A[i][j]$

case 1

if $i - j = 1$, index $= i - 2$ ($i-2$)

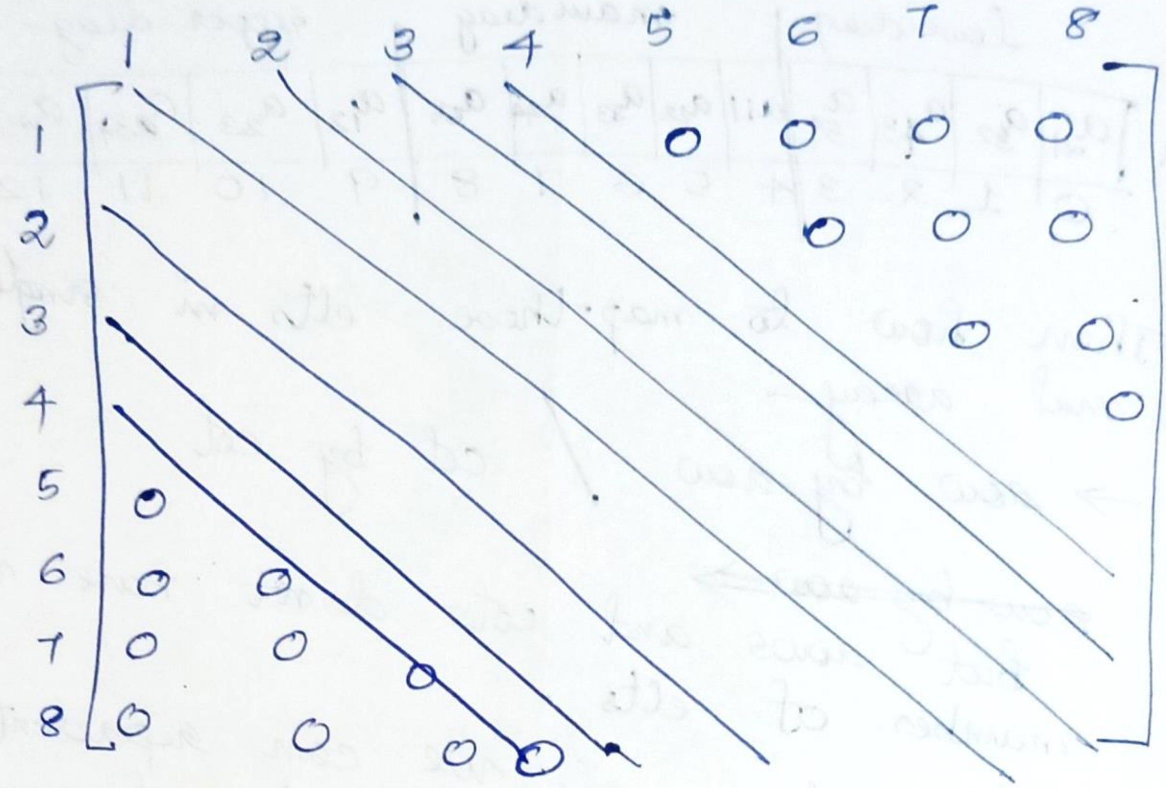
case 2 if $i - j = 0$, index $= (n-1) + (i-1)$

case 3 if $i - j = -1$, index $= (2n-1) +$

b.

Square band matrix

↳ Similar to tridiagonal



8x8

~~main~~ → forming a band of diagonals equal no: of diagonals above and below the main diagonals.

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Block matrices

Often times, there are clear "patterns" in the entries of a large matrix and it might be useful to break that matrix down into smaller chunks based on some partition of its rows and cols.

eg:-

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 & -2 & 3 \end{bmatrix}$$

6×6

6×4

3×3 Identity matrix

(zero matrix)

$$B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$A = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 & -2 & 3 \end{array}\right] \qquad A = \begin{bmatrix} I_3 & I_3 \\ O_{2,3} & C \end{bmatrix}$$

where,

$$C = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & 3 \end{bmatrix}$$

$$B = \left[\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & +1 & 0 & 0 \\ \hline 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & 1 \end{array}\right] \qquad B = \begin{bmatrix} \mathcal{D} & O_{3,2} \\ O_{3,2} & \mathcal{D} \end{bmatrix}$$

where

$$\mathcal{D} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -1 & 1 \end{bmatrix}$$

&

→ A and B are matrices whose elts are themselves matrices. They are called <u>block matrices</u>.

→ Viewing matrices in this way often simplifies calculations and reveals structure; especially when the matrix has a lot of zeros.

$$AB = \begin{bmatrix} I & I \\ 0 & C \end{bmatrix}\begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix}$$

$$= \begin{bmatrix} ID + I0 & I0 + ID \\ 0D + C0 & 00 + CD \end{bmatrix}$$

11 - 16(6)
~~10~~ - ~~20~~
Dec 7 - 6(4)
Jan 28 - 30
~~Dec~~ 28 (4)
Jans - 12

$$= \begin{bmatrix} D & D \\ 0 & CD \end{bmatrix}$$

pa

$$CD = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2+1 & 4+1-1 \\ -4-3 & -2+3 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -7 & 1 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} A & 0 \\ 0 & CD \end{bmatrix} = \left[\begin{array}{cc|cc} 1 & 3 & 1 & 3 \\ 2 & 1 & 2 & 1 \\ -1 & 1 & -1 & 1 \\ \hline 0 & 0 & 3 & 7 \\ 0 & 0 & -7 & 1 \end{array}\right].$$

which of the following block matrix multi-plications make sense ?

<u>example</u>

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & -1 \\ -1 & 0 & 2 \end{bmatrix}$$

① $\begin{bmatrix} A & B \\ B & A \end{bmatrix}^2$

$$\left[\begin{array}{cc|ccc} 1 & 2 & 1 & 3 & -1 \\ -1 & 0 & -1 & 0 & 2 \\ \hline 1 & 3 & -1 & 1 & 2 \\ -1 & 0 & 2 & -1 & 0 \end{array}\right].$$

we can ~~take~~ only take powers of square matrices.

②

$$\begin{bmatrix} A & B \\ 0 & I_3 \end{bmatrix} \begin{bmatrix} A & A \\ 0 & A \\ I_2 & 0 \end{bmatrix}$$

$2 \times 2 \qquad 3 \times 2$

inner dimensions do not match

b) $\rightarrow \begin{bmatrix} A & B \\ 0 & I_3 \end{bmatrix} \begin{bmatrix} A & A \\ 0 & A \end{bmatrix} = \begin{bmatrix} A^2 & A^2 + BA \\ 0 & A \end{bmatrix}$

$\rightarrow$ BA doesn't make sense.

BA $\quad B \rightarrow 2 \times 3$

$A \rightarrow 2 \times 2 \quad X$

$\rightarrow I_3 A$ X $\rightarrow$ not correct

(3) $\rightarrow \begin{bmatrix} A & B \\ 0 & I_3 \end{bmatrix} \begin{bmatrix} B & 0 \\ I_3 & I_3 \end{bmatrix} = \begin{bmatrix} AB+B & B \\ I_3 & I_3 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 3 & 3 \\ -1 & -3 & 1 \end{bmatrix}$$

$$AB + B = \begin{bmatrix} 0 & 6 & 2 \\ -2 & -3 & 3 \end{bmatrix}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 0 & 6 & -2 & 1 & 3 & -1 \\ -2 & -3 & 3 & -1 & 0 & 2 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$