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STAT 501 Regression Methods

13.1.1 - Weighted Least Squares Examples

Lesson

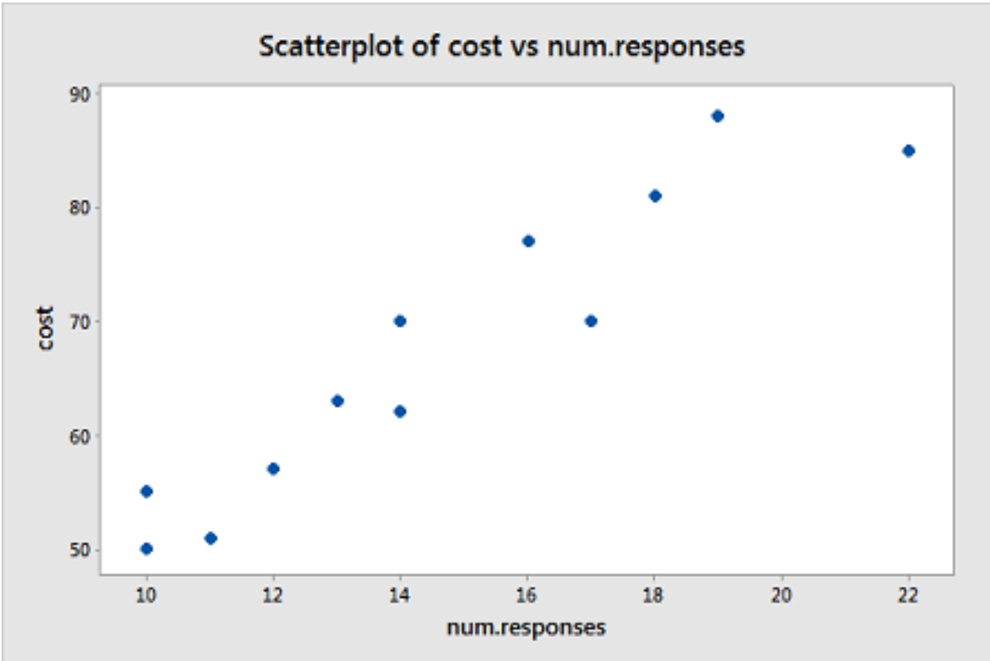
Example 13-1: Computer-Assisted Learning Dataset

The Computer-Assisted Learning New data was collected from a study of computer-assisted learning by n = 12 students.

i	Responses	Cost
1	16	77
2	14	70
3	22	85
4	10	50
5	14	62
6	17	70
7	10	55
8	13	63
9	19	88
10	12	57
11	18	81
12	11	51

The response is the cost of the computer time (Y) and the predictor is the

The response is the cost of the computer time (Y) and the predictor is the total number of responses in completing a lesson (X). A scatterplot of the data is given below.



From this scatterplot, a simple linear regression seems appropriate for explaining this relationship.

First, an ordinary least squares line is fit to this data. Below is the summary of the simple linear regression fit for this data

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
4.59830	88.91%	87.80%	81.27%

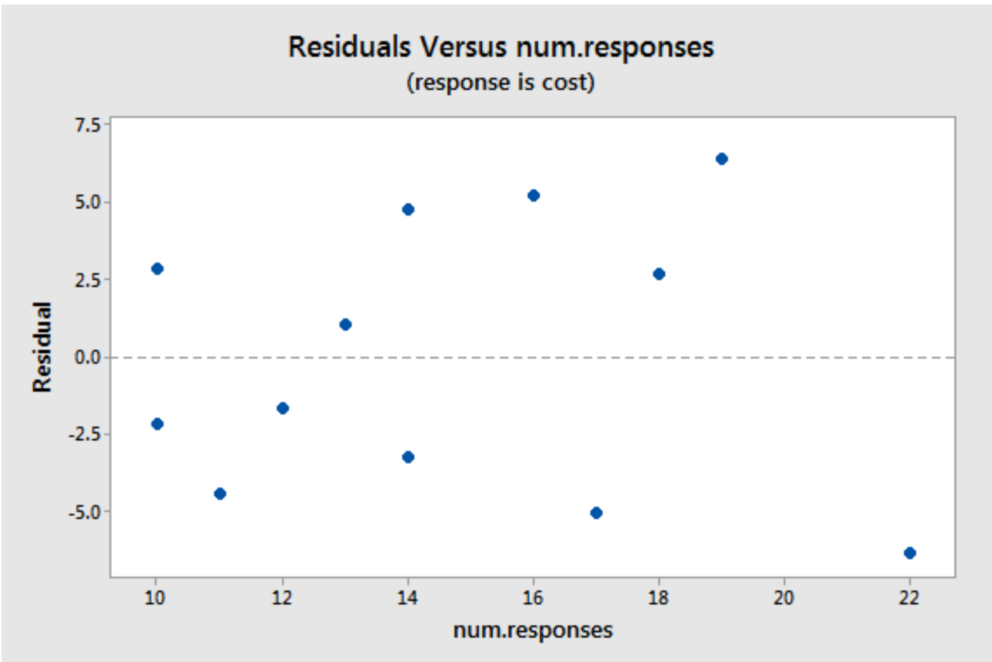
Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	19.47	5.52	3.53	0.005	
num.responses	3.269	0.365	8.95	0.000	1.00

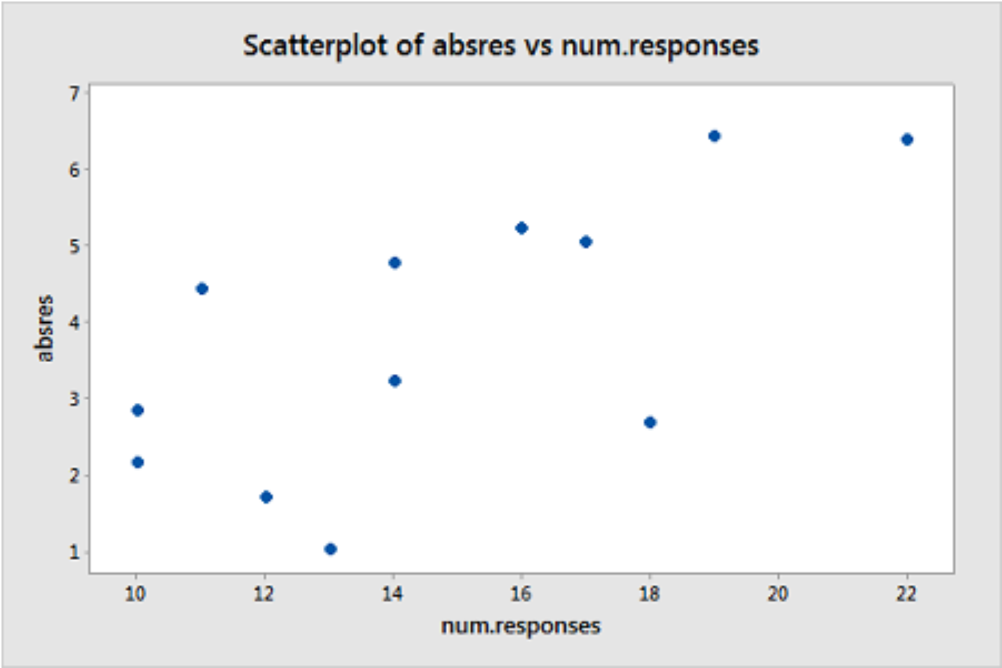
Regression Equation

cost = 19.47 + 3.269 num.responses

A plot of the residuals versus the predictor values indicates possible nonconstant variance since there is a very slight "megaphone" pattern:



We will turn to weighted least squares to address this possibility. The weights we will use will be based on regressing the absolute residuals versus the predictor. In Minitab, we can use the Storage button in the Regression Dialog to store the residuals. Then we can use Calc > Calculator to calculate the absolute residuals. A plot of the absolute residuals versus the predictor values is as follows:



The weights we will use will be based on regressing the absolute residuals versus the predictor. Specifically, we will fit this model, use the Storage button to store the fitted values, and then use Calc > Calculator to define the weights as 1 over the squared fitted values. Then we fit a weighted least squares regression model by fitting a linear regression model in the usual way but clicking "Options" in the Regression Dialog and selecting the just-created weights as "Weights."

The summary of this weighted least squares fit is as follows:

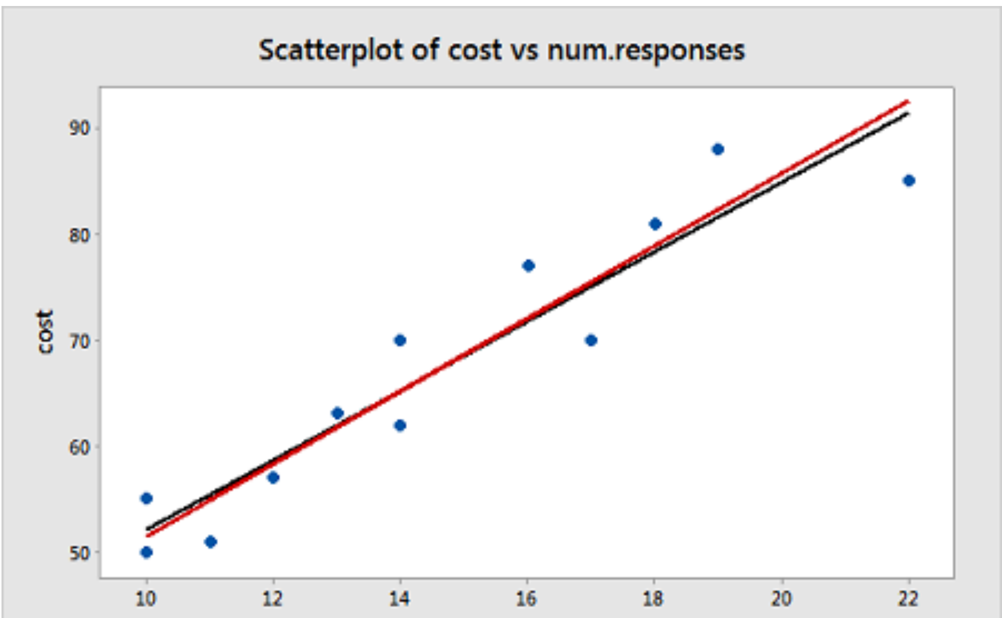
Model Summary

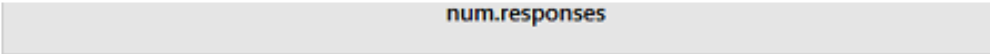
S	R-sq	R-sq(adj)	R-sq(pred)
1.15935	89.51%	88.46%	83.87%

Coefficients

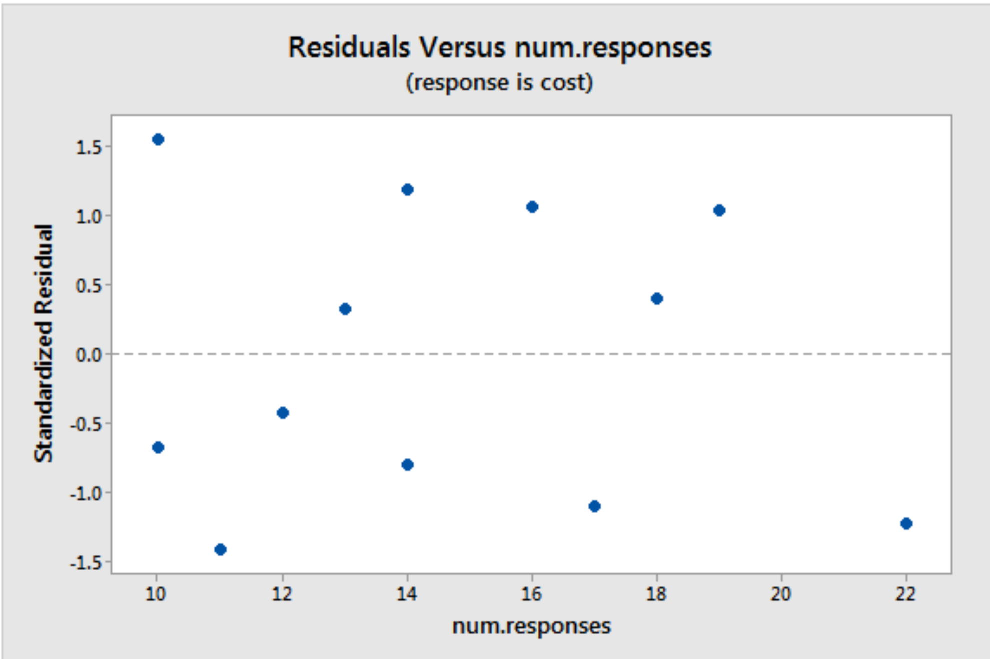
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	17.30	4.83	3.58	0.005	
num.responses	3.421	0.370	9.24	0.000	1.00

Notice that the regression estimates have not changed much from the ordinary least squares method. The following plot shows both the OLS fitted line (black) and WLS fitted line (red) overlaid on the same scatterplot.





A plot of the studentized residuals (remember Minitab calls these "standardized" residuals) versus the predictor values when using the weighted least squares method shows how we have corrected for the megaphone shape since the studentized residuals appear to be more randomly scattered about 0:



With weighted least squares, it is crucial that we use studentized residuals to evaluate the aptness of the model since these take into account the weights that are used to model the changing variance. The usual residuals don't do this and will maintain the same non-constant variance pattern no matter what weights have been used in the analysis.

X_2X_3

Weighted Least

Example 13-2: Market Share Data

Here we have market share data for *n* = 36 consecutive months (Market Share data). Let *Y* = market share of the product; = price; = 1 if the discount promotion is in effect and 0 otherwise; = 1 if both discount and package promotions in effect and 0 otherwise. The regression results below are for a useful model in this situation:

Coefficients

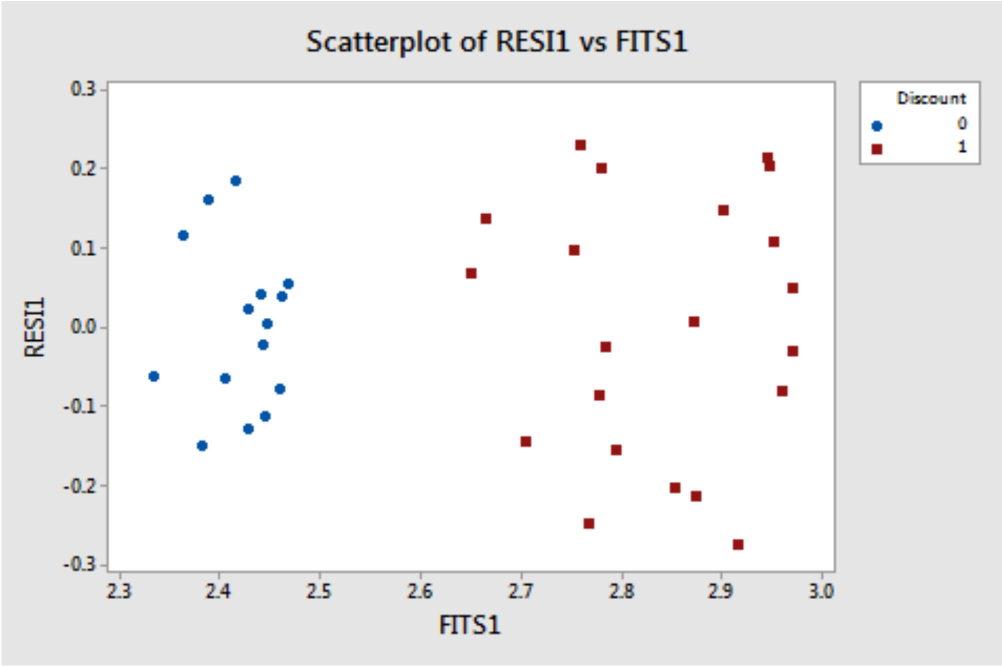
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	3.196	0.356	8.97	0.000	
Price	-0.334	0.152	-2.19	0.036	1.01
Discount	0.3081	0.0641	4.80	0.000	1.68
DiscountPromotion	0.1762	0.0660	2.67	0.012	1.69

This model represents three different scenarios:

1. Months in which there was no discount (and either a package promotion or not): X2 = 0 (and X3 = 0 or 1);
2. Months in which there was a discount but no package promotion: X2 = 1 and X3 = 0;
3. Months in which there was both a discount and a package promotion: X2 = 1 and X3 = 1.

So, it is fine for this model to break the hierarchy if there is no significant difference between the months in which there was no discount and no difference between the months in which there was no discount and no package promotion and months in which there was no discount but there was a package promotion.

A residual plot suggests nonconstant variance related to the value of :



From this plot, it is apparent that the values coded as 0 have a smaller variance than the values coded as 1. The residual variances for the two separate groups defined by the discount pricing variable are:

Discount	N	StDev	Variance	95% CI for StDevs
0	15	0.103	0.011	(0.077, 0.158)
1	21	0.164	0.027	(0.136, 0.217)

Because of this nonconstant variance, we will perform a weighted least squares analysis. For the weights, we use for *i* = 1, 2 (in Minitab use Calc > Calculator and define "weight" as 'Discount'/0.027 + (1-'Discount')/0.011 . The weighted least squares analysis (set the just-defined "weight" variable as "weights" under Options in the Regression dialog) is as follows:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	96.109	32.036	30.84	0.000
Price	1	4.688	4.688	4.51	0.041
Discount	1	23.039	23.039	22.18	0.000
DiscountPromotion	1	5.634	5.634	5.42	0.026
Error	32	33.246	1.039		
Total	35	129.354			

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	3.175	0.357	8.90	0.000	
Price	-0.325	0.153	-2.12	0.041	1.01
Discount	0.3083	0.0655	4.71	0.000	2.04
DiscountPromotion	0.1759	0.0755	2.33	0.026	2.05

An important note is that Minitab's ANOVA will be in terms of the weighted SS. When doing a weighted least squares analysis, you should note how different the SS values of the weighted case are from the SS

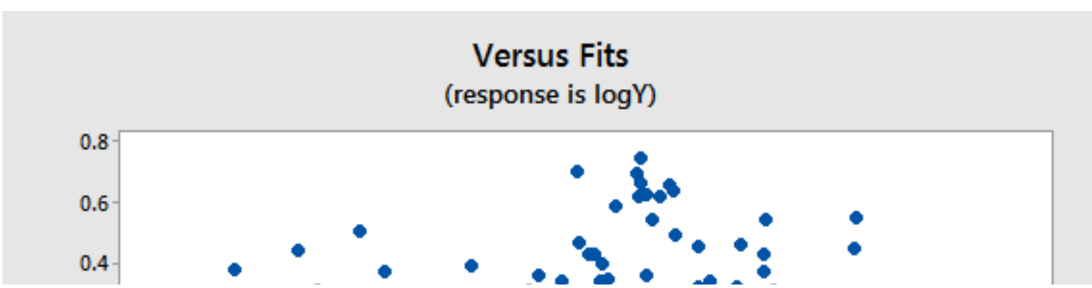
Example 13-3: Home Price Dataset

The Home Price data set has the following variables:

Y = sale price of a home
X_1 = square footage of the home
X_2 = square footage of the lot

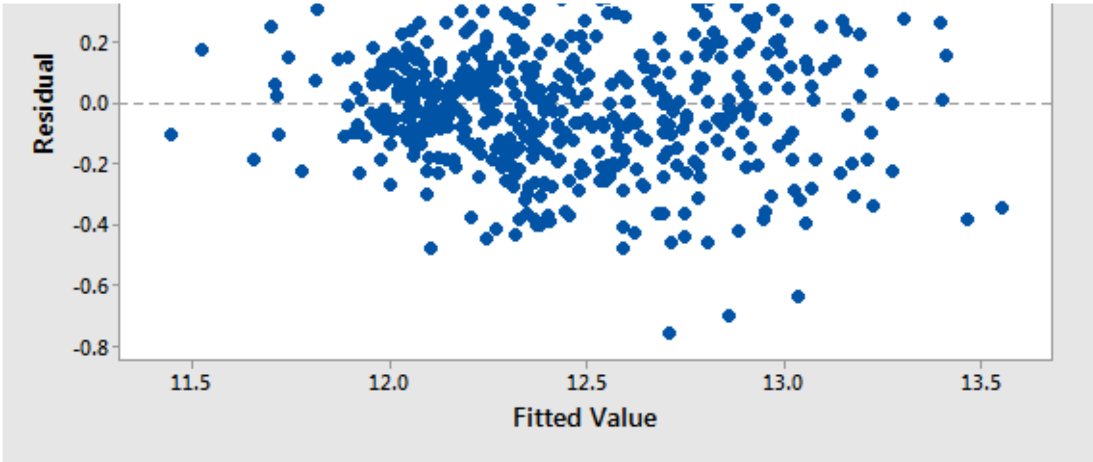
Since all the variables are highly skewed we first transform each variable to its natural logarithm. Then when we perform a regression analysis and look at a plot of the residuals versus the fitted values (see below), we note a slight "megaphone" or "conic" shape of the residuals.

Term	Coef	SE Coeff	T-Value	P-Value	VIF
Constant	1.964	0.313	6.28	0.000	
logX1	1.2198	0.0340	35.87	0.000	1.05
logX2	0.1103	0.0241	4.57	0.000	1.05



Optional Content

Resource Menu



We interpret this plot as having a mild pattern of nonconstant variance in which the amount of variation is related to the size of the mean (which are the fits).

So, we use the following procedure to determine appropriate weights:

- Store the residuals and the fitted values from the ordinary least squares (OLS) regression.
- Calculate the absolute values of the OLS residuals.
- Regress the absolute values of the OLS residuals versus the OLS fitted values and store the fitted values from this regression. These fitted values are estimates of the error standard deviations.
- Calculate weights equal to $1/fits^2$, where "fits" are the fitted values from the regression in the last step.

We then refit the original regression model but use these weights this time in a weighted least squares (WLS) regression.

Results and a residual plot for this WLS model:

Results and a residual plot for this WLS model:

Term	Coef	SE Coeff	T-Value	P-Value	VIF
Constant	2.377	0.284	8.38	0.000	
logX1	1.2014	0.0336	35.72	0.000	1.08
logX2	0.0831	0.0217	3.83	0.000	1.08