FUNDAMENTAL PRINCIPLE OF COUNTING

If an operation can be performed in 'm' different ways and another operation in 'n' different ways then these two operations can be performed one after the other in 'mn' ways If an operation can be performed in 'm' different ways and another operation in 'n' different ways then either of these two operations can be performed in 'm+n' ways.(provided only one has to be done)

FACTORIAL 'n' The continuous product of the first 'n' natural numbers is called factorial n and is denoted by n!

PERMUTATION

An arrangement that can be formed by taking some or all a finite set of things (or objects) is called a **Permutation**. Order of the things is very important in case of permutation. A permutation is said to be a **Linear Permutation** if the objects are arranged in a line. A linear permutation is simply called as a permutation. A permutation is said to be a **Circular Permutation** if the objects are arranged in the form of a circle. The number of (linear) permutations that can be formed by taking r things at a time from a set of n distinct things $(r \leq n)$ is denoted by $^{n}P_{r} \quad or \quad P(n,r)$.

$^{n}P_{r} = n(n-1)(n-2)(n-3)......(n-r+1) = \frac{n!}{(n-r)!}$

NUMBER OF PERMUTATIONS UNDER CERTAIN CONDITIONS

1. Number of permutations of n different things, taken r at a time, when a particular thing is to be always included in each arrangement, is $r\left(^{n-1}P_{r-1}\right)$.

2. Number of permutations of n different things, taken r at a time, when a particular thing is never taken in each arrangement is $^{n-1}P_{r}$.

3. Number of permutations of n different things, taken all at a time, when m specified things always come together is $m!(n-m+1)!$.

4. Number of permutations of n different things, taken all at a time, when m specified never come together is $n! - [m!(n-m+1)!]$.

5. The number of permutations of n dissimilar things taken r at a time when k (< r) particular things always occur is $\left[^{n-k}P_{r-k}\right].\left[^{r}P_{k}\right]$.

6. The number of permutations of n dissimilar things taken r at a time when k particular things never occur is $^{n-k}P_{r}$.

7. The number of permutations of n dissimilar things taken r at a time when repetition of things is allowed any number of times is n^{r}

8. The number of permutations of n different things, taken not more than r at a time, when each thing may occur any number of times is $n + n^{2} + n^{3} + ... + n^{r} = \frac{n(n^{r}-1)}{n-1}$.

9. The number of permutations of n different things taken not more than r at a time $^{n}P_{1} + ^{n}P_{2} + ^{n}P_{3} + ... + ^{n}P_{r}$.

PERMUTATIONS OF SIMILAR THINGS

The number of permutations of n things taken all tat a time when p of them are all alike and the rest are all different is $\frac{n!}{p!}$ .If p things are alike of one type, q things are alike of other type, r things are alike of another type, then the number of permutations with p + q + r things is $\frac{(p+q+r)!}{p!.q!.r!}$.

CIRCULAR PERMUTATIONS

1. The number of circular permutations of n dissimilar things taken r at a time is $\frac{^nP_r}{r}$.

2. The number of circular permutations of n dissimilar things taken all at a time is $(n-1)!$.

3. The number of circular permutations of n things taken r at a time in one direction is $\frac{^nP_r}{2r}$.

4. The number of circular permutations of n dissimilar things in clock-wise direction = Number of permutations in anti clock-wise direction = $\frac{(n-1)!}{2}$.

COMBINATION

A selection that can be formed by taking some or all of a finite set of things (or objects) is called a **Combination**

The number of combinations of n dissimilar things taken r at a time is denoted by $^nC_r \quad or \quad C(n,r) \quad or \quad \binom{n}{r}$.

1. $^nC_r = \frac{n!}{r!(n-r)!}$ 2. nCr = nCn-r 3. $^nC_r +^n C_{r-1} =^{n+1} C_r$

4. $If \quad ^nC_r =^n C_s \Rightarrow r = s \quad or \quad n = r + s$

5. The number of combinations of n things taken r at a time in which

a) s particular things will always occur is $^{n-s}C_{r-s}$.

b) s particular things will never occur is $^{n-s}C_r$.

c) s particular things always occurs and p particular things never occur is $^{n-p-s}C_{r-s}$.

DISTRIBUTION OF THINGS INTO GROUPS

1. Number of ways in which (m+n) items can be divided into two unequal groups containing m and n items is $^{m+n}C_m = \frac{(m+n)!}{m!n!}$.

2. The number of ways in which mn different items can be divided equally into m groups, each containing n objects and the order of the groups is not important is $\left[\frac{(mn)!}{(n!)^m}\right]\frac{1}{m!}$

3. The number of ways in which mn different items can be divided equally into m groups, each containing n objects and the order of the groups is important is $\frac{(mn)!}{(n!)^m}$.

4. The number of ways in which (m+n+p) things can be divided into three different groups of m,n, an p things respectively is $\frac{(m+n+p)!}{m!n!p!}$

5. The required number of ways of dividing 3n things into three groups of n each = $\frac{1}{3!}\frac{(3n)!}{n!.n!.n!}$.When the order of groups has importance then the required number of ways= $\frac{(3n)!}{(n!)^3}$

DIVISION OF IDENTICAL OBJECTS INTO GROUPS

The total number of ways of dividing n identical items among r persons, each one of whom, can receive 0, 1, 2 or more items $(\leq n)$ is ${}^{n+r-1}C_{r-1}$

The number of non-negative integral solutions of the equation $x_1 + x_2 + x_3 + ... + x_r = n \quad is \quad {}^{n+r-1}C_{r-1}$.

The total number of ways of dividing n identical items among r persons, each one of whom receives at least one item is ${}^{n-1}C_{r-1}$

The number of positive integral solutions of the equation $x_1 + x_2 + x_3 + ... + x_r = n \quad is \quad {}^{n-1}C_{r-1}$.

TOTAL NUMBER OF COMBINATIONS

The total number of combinations of $(p_1 + p_2 + ... + p_k)$ things taken any number at a time when p_1 things are alike of one kind, p_2 things are alike of second kind... p_k things are alike of k^{th} kind, is $(p_1 + 1)(p_2 + 1).....(p_k + 1)$.

2.The total number of combinations of $(p_1 + p_2 + ... + p_k)$ things taken one or more at a time when p_1 things are alike of one kind, p_2 things are alike of second kind... p_k things are alike of k^{th} kind, is $(p_1 + 1)(p_2 + 1).....(p_k + 1) - 1$.

SUM OF THE NUMBERS (repetition Not allowed)

Sum of the numbers formed by taking all the given n digits (excluding 0) is
$(Sum \quad of \quad all \quad the \quad n \quad digits)(n-1)!(111..n \quad times)$

Sum of the numbers formed by taking all the given n digits (including 0) is
$(Sum \quad of \quad all \quad the \quad n \quad digits)(n-1)!(111..n \quad times) -$

$(Sum \quad of \quad all \quad the \quad n \quad digits)(n-2)!(111..(n-1) \quad times)$

Sum of all the r-digit numbers formed by taking the given n digits (excluding 0) is
$(Sum \quad of \quad all \quad the \quad n \quad digits)^{n-1}P_{r-1}(11111.......r \quad times)$

Sum of all the r-digit numbers formed by taking the given n digits (including 0) is
$(Sum \quad of \quad all \quad the \quad n \quad digits)^{n-1}P_{r-1}(11111.......r \quad times) -$
$(Sum \quad of \quad all \quad the \quad n \quad digits)^{n-2}P_{r-2}(11111.......(r-1) \quad times)$

DE-ARRANGEMENT:

The number of ways in which exactly r letters can be placed in wrongly addressed envelopes when n letters are placed in n addressed envelopes is ${}^{n}P_r[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ... + (-1)^r \frac{1}{r!}]$.

The number of ways in which n different letters can be placed in their n addressed envelopes so that all the letters are in the wrong envelopes is $n![1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ... + (-1)^n \frac{1}{n!}]$.

*IMPORTANT RESULTS **TO** REMEMBER*

In a plane if there are n points of which no three are collinear, then

1. The number of straight lines that can be formed by joining them is $^nC_2$. 44

2. The number of triangles that can be formed by joining them is $^nC_3$.

3. The number of polygons with k sides that can be formed by joining them is ^nC_k.

In a plane if there are n points out of which m points are collinear, then

1. The number of straight lines that can be formed by joining them is $^nC_2 -^m C_2 + 1$.

2. The number of triangles that can be formed by joining them is $^nC_3 -^m C_3$.

3. The number of polygons with k sides that can be formed by joining them is $^nC_k -^m C_k$.

1. Number of rectangles of any size in a square of n x n is $\sum_{r=1}^{n} r^3$

2. In a rectangle of p x q (p < q) number of rectangles of any size is $\frac{pq}{4}(p+1)(q+1)$

3. In a rectangle of p x q (p < q) number of squares of any size is $\sum_{r=1}^{n}(p+1-r)(q+1-r)$

4. n straight lines are drawn in the plane such that no two lines are parallel and no three lines three lines are concurrent. Then the number of parts into which these lines divide the plane is equal to $1 + \frac{n(n+1)}{2}$.

5. When n is even, nCr is greatest when r = n/2 and when n is odd, nCr is greatest when r = (n-1)/2 or (n + 1)/2.
