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Enrollment Number AMENU4AIE20143

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21MAT301—Mathematics for Intelligent Systems 5

Mid Term Examination, November 2022

Time: 2 hours | Maximum Marks: 50

(4 marks for Q1–Q3; 6 marks for Q4, Q5; 8 marks for Q6, Q7; 10 marks for Q8)

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1. Let X, Y, Z be binary random variables with joint distribution

X	Y	Z	$P(X,Y,Y)$
$-$	$-$	$-$	1/15
$+$	$-$	$-$	2/15
$-$	$+$	$-$	1/15
$+$	$+$	$-$	1/15
$-$	$-$	$+$	2/15
$+$	$-$	$+$	4/15
$-$	$+$	$+$	2/15
$+$	$+$	$+$	2/15

Are X and Y conditionally independent given Z? Justify your answer.

2. Fifty-two percent of the students at a certain college are females. Five percent of the students in this college are majoring in computer science. Two percent of the students are women majoring in computer science. If a student is selected at random, find the conditional probability that

   a) the student is female given that the student is majoring in computer science

   b) this student is majoring in computer science given that the student is female.

3. Suppose that an insurance company classifies people into one of three classes: good risks, average risks, and bad risks. The company's records indicate that the probabilities that good-, average-, and bad-risk persons will be involved in an accident over a 1-year span are, respectively, 0.05, 0.15, and 0.30. If 20 percent of the population is a good risk, 50 percent an average risk, and 30 percent a bad risk, what proportion of people have accidents in a fixed year? If policyholder A had no accidents in 1997, what is the probability that he or she is in a good or average risk class?

4. Show that $P(X, Y \mid Z) = P(X \mid Y, Z)P(Y \mid Z)$ for any discrete random variables X, Y, Z.

5. Let X and Y be binary random variables. Complete the below table with their joint distribution in such a way that X and Y are independent.

X	Y	$P(X, Y)$
$-$	$-$	1/15
$+$	$-$	2/15
$-$	$+$	a
$+$	$+$	b

6. In a Markov Model, we have $X_3 \perp\!\!\!\perp X_1 \mid X_2$ and $X_4 \perp\!\!\!\perp X_1, X_2 \mid X_3$. Does this imply that $X_1 \perp\!\!\!\perp X_3, X_4 \mid X_2$? Justify your answer.

7. In a stationary Markov process $X_1, X_2, \ldots$, the transition probabilities are as following:

X_1	X_2	$P(X_2 \mid X_1)$
1	1	2/3
1	2	1/3
2	2	2/3
2	3	1/3
3	3	2/3
3	1	1/3

2 1 $\frac{1}{3}$

Calculate the stationary distribution of the process.

8. Consider a HMM with transition probabilities given in the previous question, an initial distribution

$$P(X_1 = 1) = P(X_2 = 2) = P(X_3 = 3) = 1/3$$

and emission probabilities

$$P(E_k = +e_k \mid X_k = s) = 1 - P(E_k = -e_k \mid X_k = s) = 1/s \quad \text{for } s = 1, 2, 3.$$

Use the Forward algorithm to calculate the distribution of $X_2 \mid -e_1, -e_2$.