

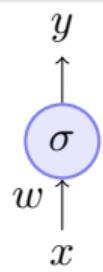
# CS7015 (Deep Learning) : Lecture 9

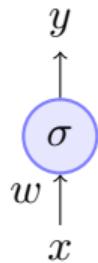
Greedy Layerwise Pre-training, Better activation functions, Better weight initialization methods, Batch Normalization

Mitesh M. Khapra

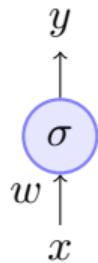
Department of Computer Science and Engineering  
Indian Institute of Technology Madras

# Module 9.1 : A quick recap of training deep neural networks



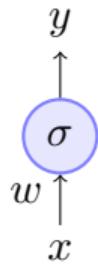


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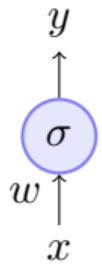
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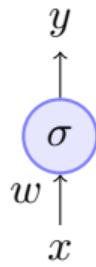


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$$= (f(\mathbf{x}) - y) * f(\mathbf{x}) * (1 - f(\mathbf{x})) * x$$

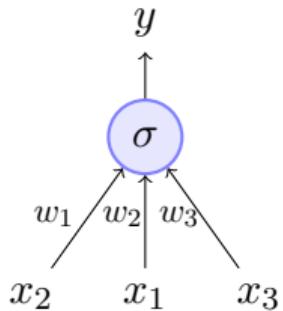


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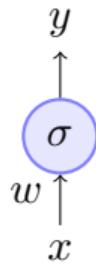
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- What about a wider network with more inputs:

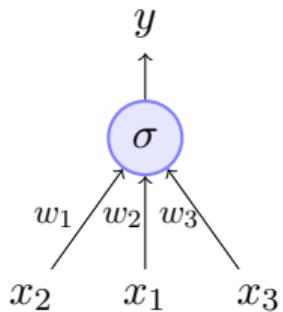


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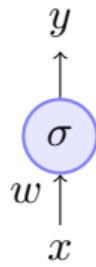
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$$w_1 = w_1 - \eta \nabla w_1$$

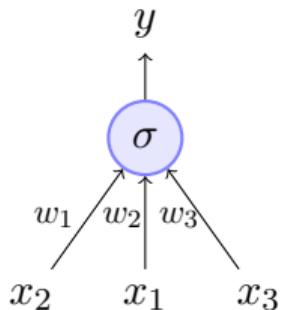


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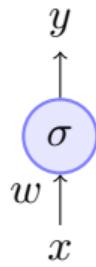
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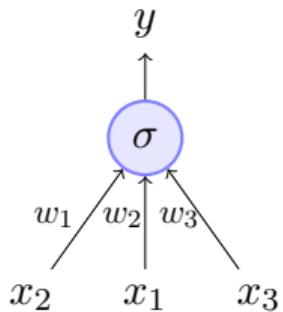


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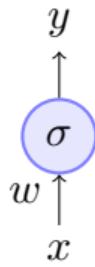


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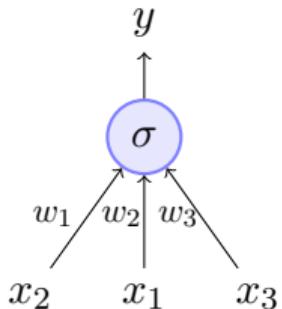


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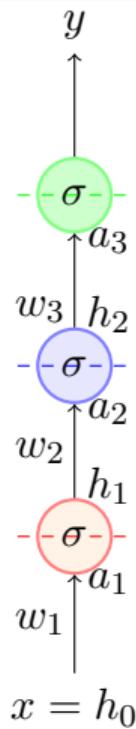
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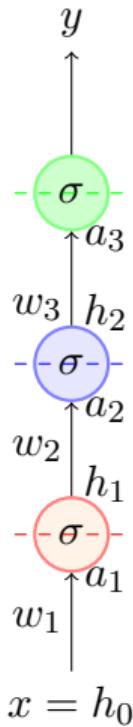
where,  $\nabla w_i = (f(\mathbf{x}) - y) * f(\mathbf{x}) * (1 - f(\mathbf{x})) * \mathbf{x}_i$

- What if we have a deeper network ?



$$a_i = w_i h_{i-1}; h_i = \sigma(a_i)$$

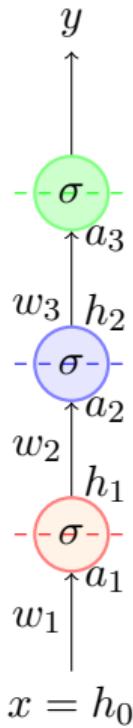
$$a_1 = w_1 * x = w_1 * h_0$$



- What if we have a deeper network ?
- We can now calculate  $\nabla w_1$  using chain rule:

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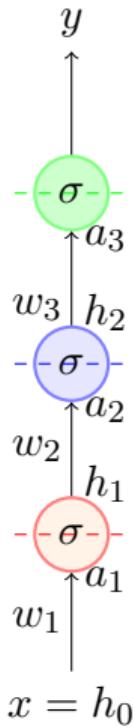


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$$\frac{\partial \mathcal{L}(\mathbf{w})}{\partial w_1} = \frac{\partial \mathcal{L}(\mathbf{w})}{\partial y} \cdot \frac{\partial y}{\partial a_3} \cdot \frac{\partial a_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial a_2} \cdot \frac{\partial a_2}{\partial h_1} \cdot \frac{\partial h_1}{\partial a_1} \cdot \frac{\partial a_1}{\partial w_1}$$

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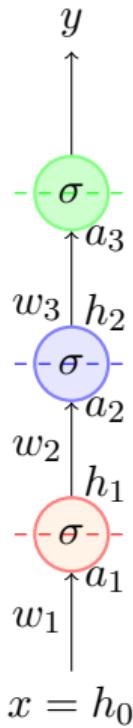


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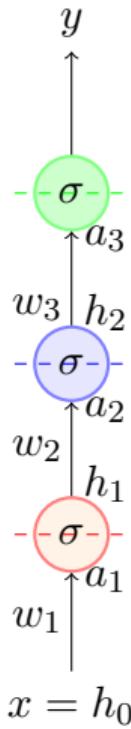
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- In general,

$$\nabla w_i = \frac{\partial \mathcal{L}(\mathbf{w})}{\partial y} * ..... * h_{i-1}$$

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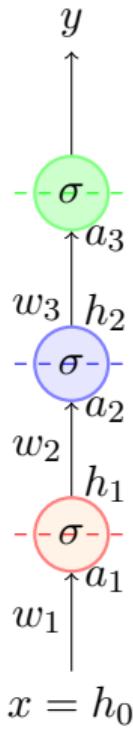
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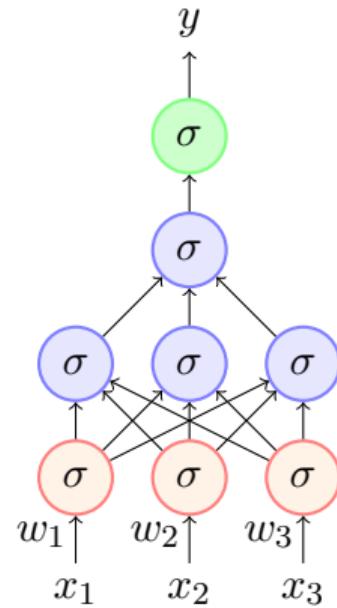
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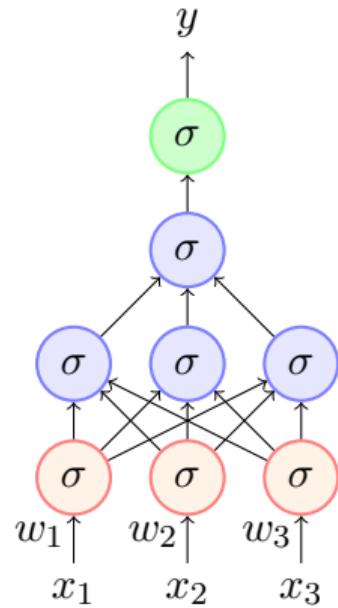
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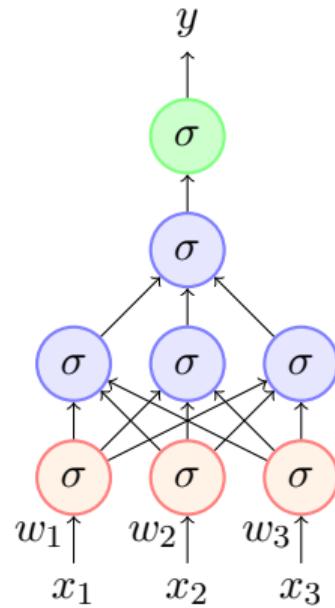
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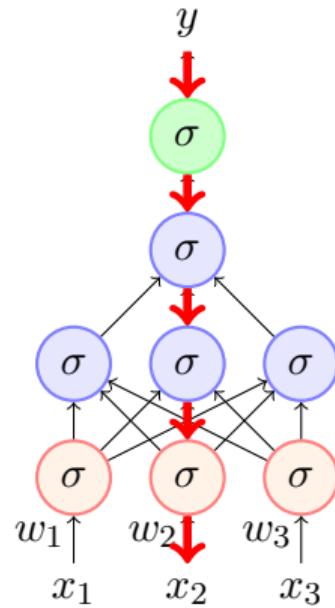
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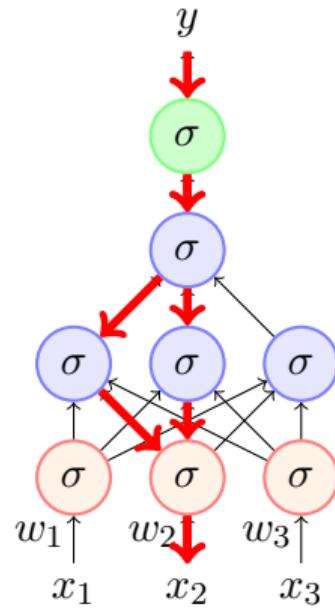
- What happens if we have a network which is deep and wide?
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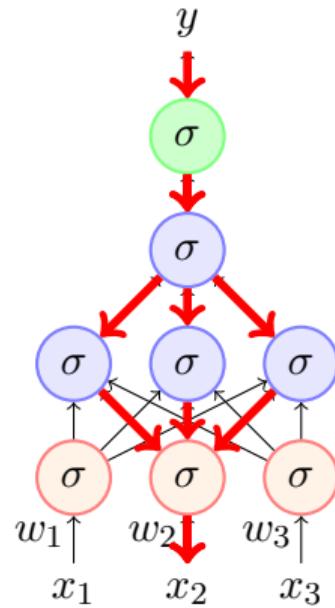
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- It will be given by chain rule applied across multiple paths



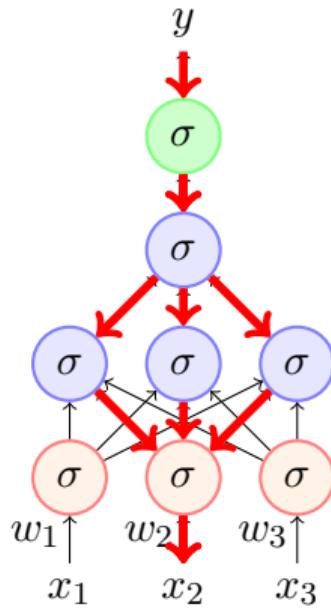
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## Things to remember

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- The gradient tells us the responsibility of a parameter towards the loss
- The gradient w.r.t. a parameter is proportional to the input to the parameters (recall the “..... \*  $x$ ” term or the “.... \*  $h_i$ ” term in the formula for  $\nabla w_i$ )

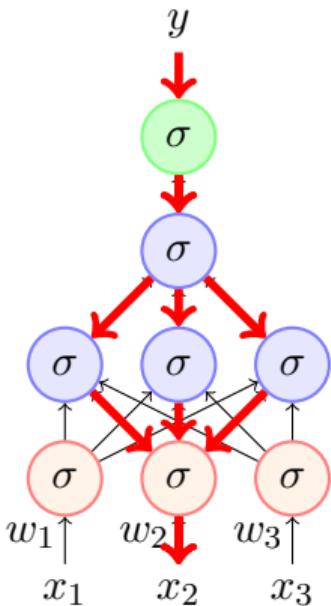
## Learning representations by back-propagating errors

David E. Rumelhart\*, Geoffrey E. Hinton†  
& Ronald J. Williams\*

\* Institute for Cognitive Science, C-015, University of California,  
San Diego, La Jolla, California 92093, USA

† Department of Computer Science, Carnegie-Mellon University,  
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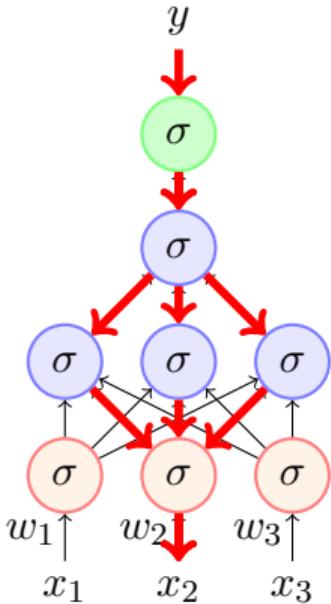
We describe a new learning procedure, back-propagation, for networks of neuron-like units. The procedure repeatedly adjusts the weights of the connections in the network so as to minimize a measure of the difference between the actual output vector of the net and the desired output vector. As a result of the weight adjustments, internal 'hidden' units which are not part of the input or output come to represent important features of the task domain, and the regularities in the task are captured by the interactions of these units. The ability to create useful new features distinguishes back-propagation from earlier, simpler methods such as the perceptron-convergence procedure<sup>1</sup>.



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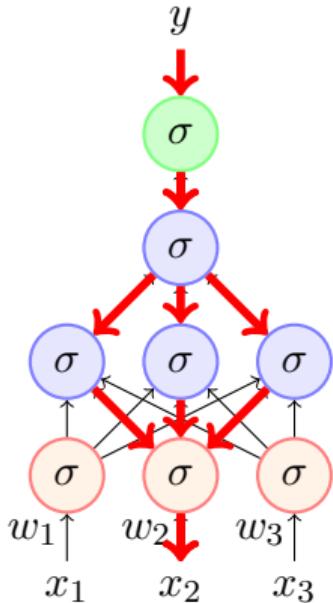
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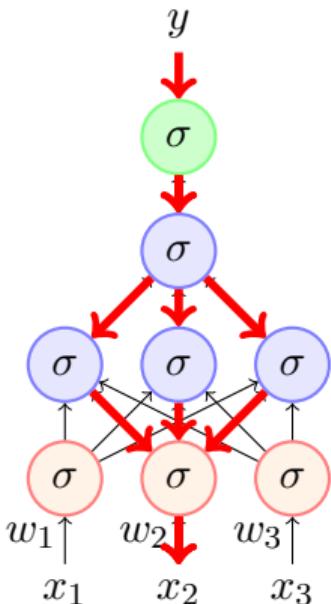
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- However when used for really deep networks it was not very successful
- In fact, till 2006 it was very hard to train very deep networks

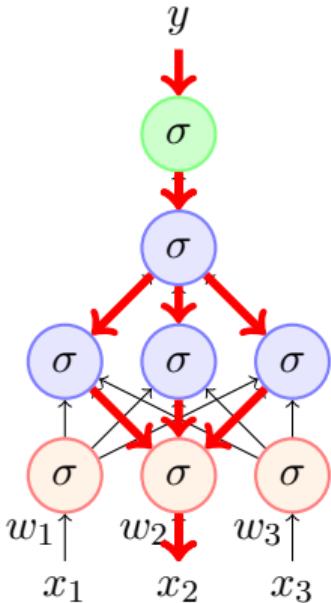
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- Backpropagation was made popular by Rumelhart et.al in 1986
- However when used for really deep networks it was not very successful
- In fact, till 2006 it was very hard to train very deep networks
- Typically, even after a large number of epochs the training did not converge

## Module 9.2 : Unsupervised pre-training

- What has changed now? How did Deep Learning become so popular despite this problem with training large networks?

---

<sup>1</sup>G. E. Hinton and R. R. Salakhutdinov. Reducing the dimensionality of data with neural networks. *Science*, 313(5786):504–507, July 2006.

- What has changed now? How did Deep Learning become so popular despite this problem with training large networks?
- Well, until 2006 it wasn't so popular

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- What has changed now? How did Deep Learning become so popular despite this problem with training large networks?
- Well, until 2006 it wasn't so popular
- The field got revived after the seminal work of Hinton and Salakhutdinov in 2006

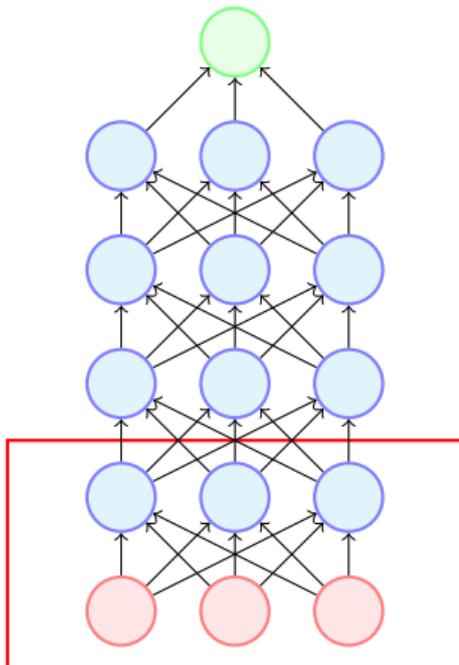
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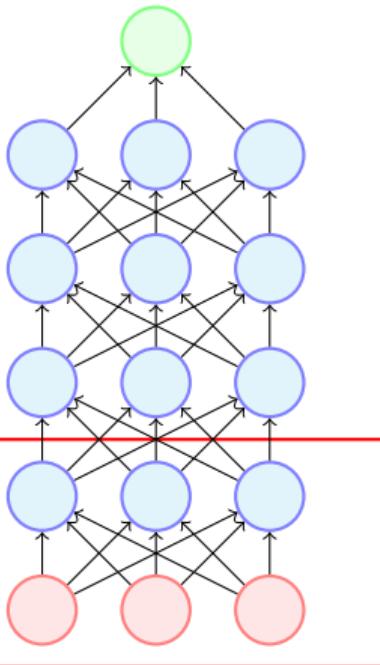
Let's look at the idea of unsupervised pre-training introduced in this paper ...

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(note that in this paper they introduced the idea in the context of RBMs but we  
will discuss it in the context of Autoencoders)

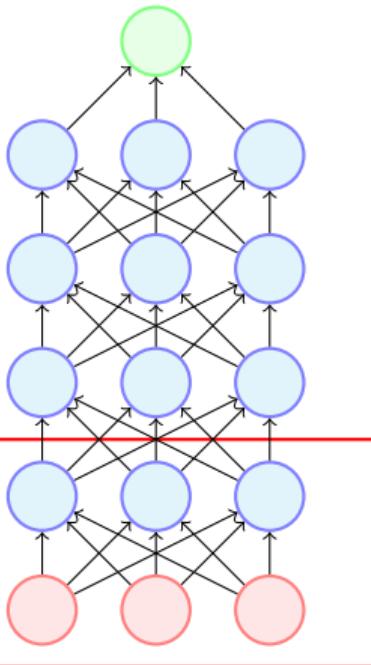
- Consider the deep neural network shown in this figure

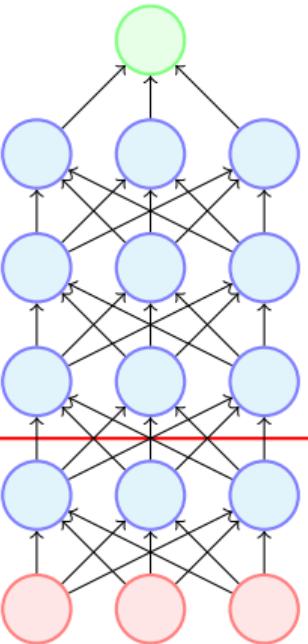


- Consider the deep neural network shown in this figure
- Let us focus on the first two layers of the network ( $x$  and  $h_1$ )

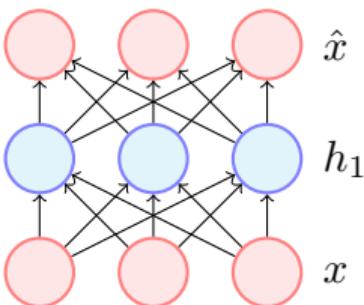


- Consider the deep neural network shown in this figure
- Let us focus on the first two layers of the network ( $x$  and  $h_1$ )
- We will first train the weights between these two layers using an **unsupervised objective**



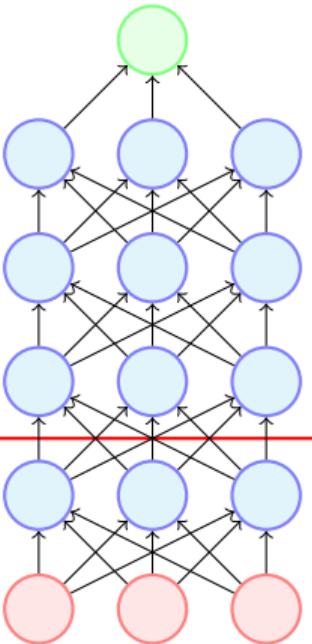


reconstruct  $x$

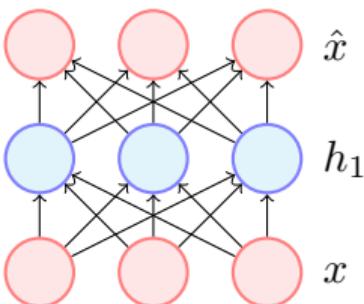


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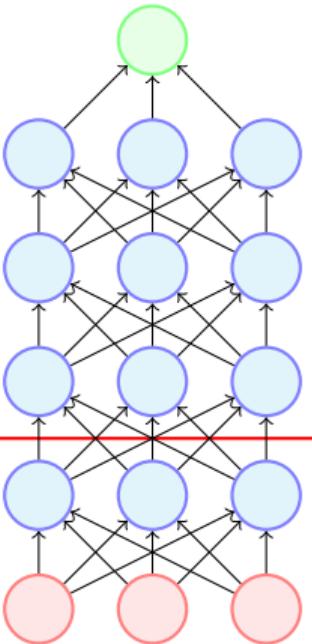


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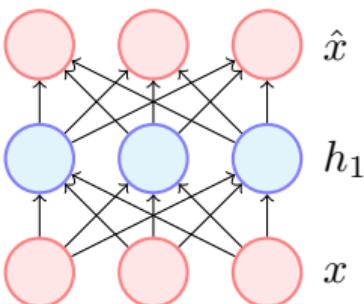


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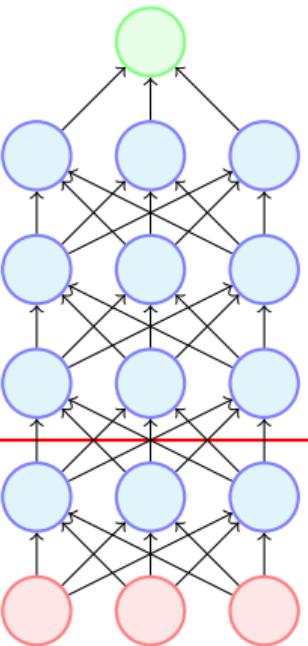


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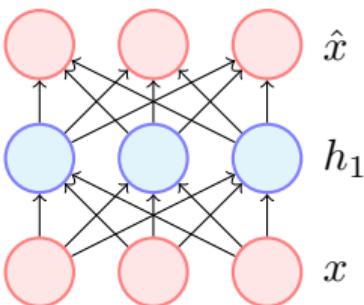


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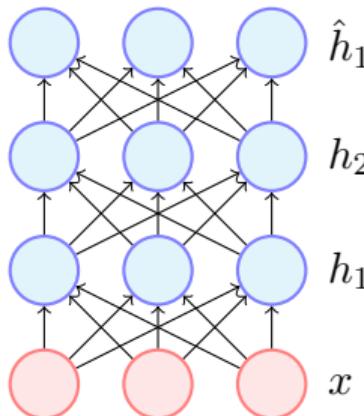
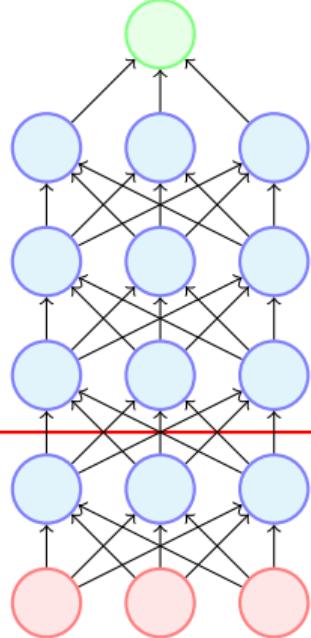


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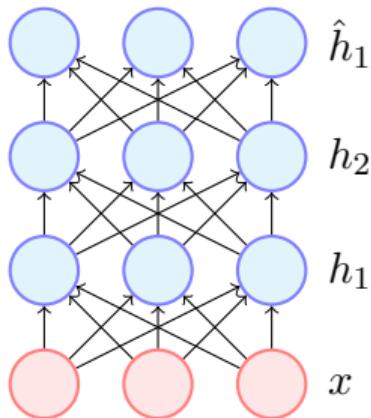
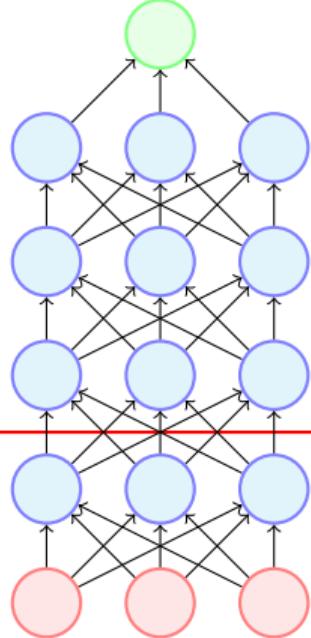
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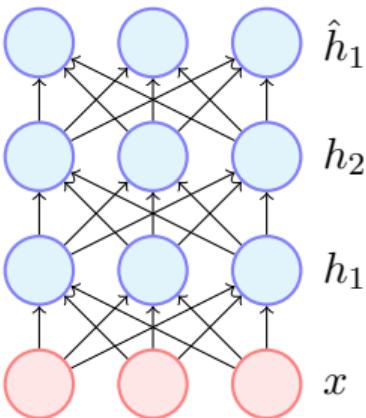
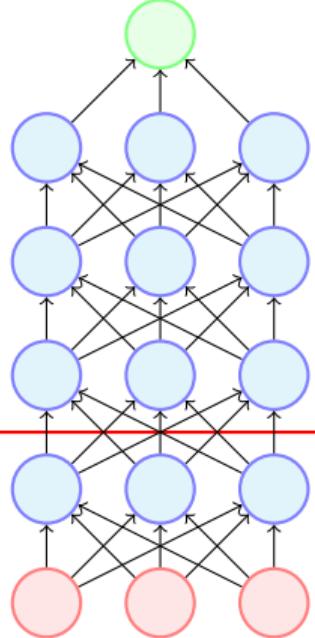
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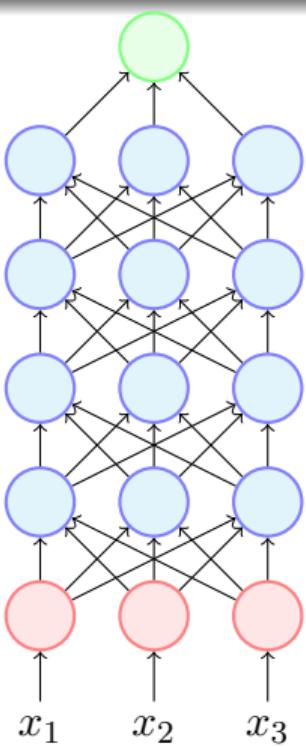
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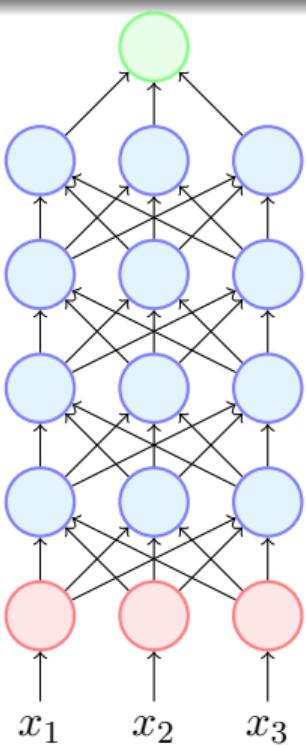
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- We continue this process till the last hidden layer (*i.e.*, the layer before the output layer) so that each successive layer captures an abstract representation of the previous layer



- After this layerwise pre-training, we add the output layer and train the whole network using the task specific objective

$$\min_{\theta} \frac{1}{m} \sum_{i=1}^m (y_i - f(x_i))^2$$



- After this layerwise pre-training, we add the output layer and train the whole network using the task specific objective
- Note that, in effect we have initialized the weights of the network using the greedy unsupervised objective and are now fine tuning these weights using the supervised objective

$$\min_{\theta} \frac{1}{m} \sum_{i=1}^m (y_i - f(x_i))^2$$

Why does this work better?

---

<sup>1</sup>The difficulty of training deep architectures and effect of unsupervised pre-training - Erhan et al,2009

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Let's see what these two questions mean and try to answer them based on some (among many) existing studies<sup>1,2</sup>

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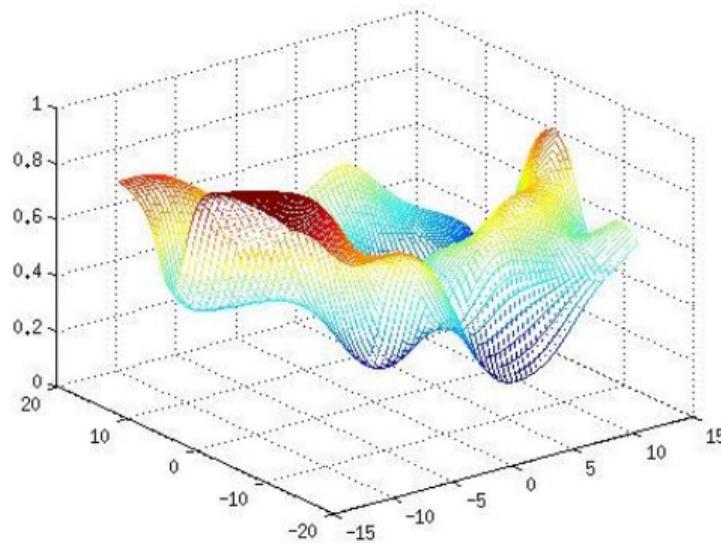
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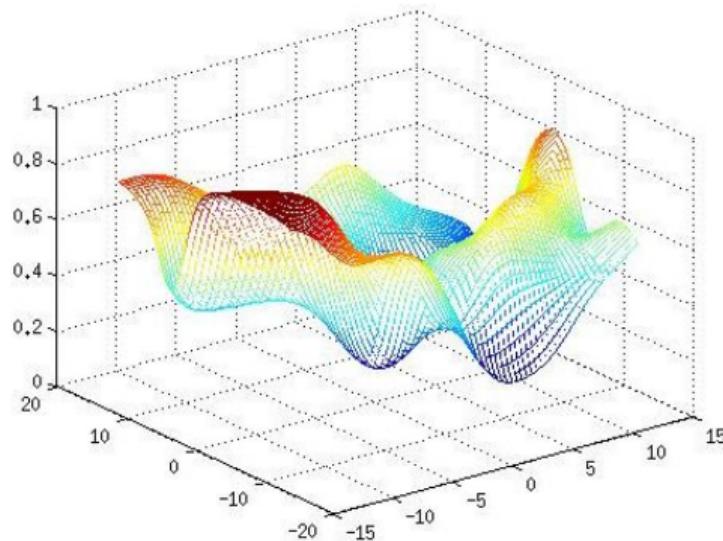
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- Let us see this in more detail ...

- The error surface of the supervised objective of a Deep Neural Network is highly non-convex



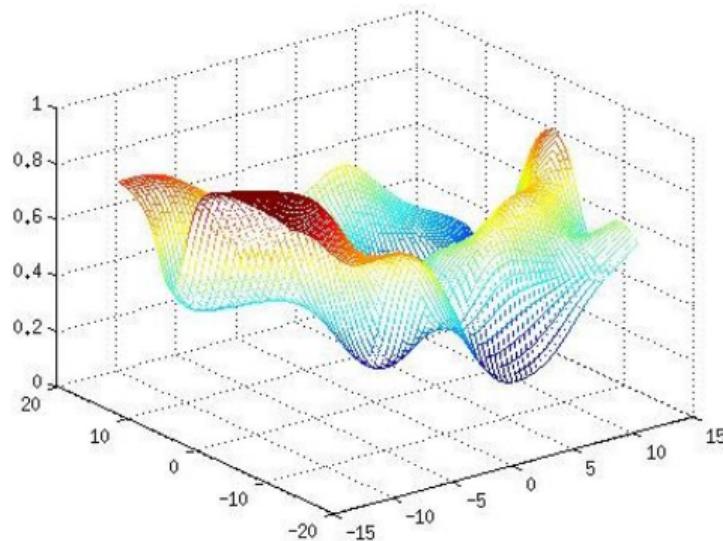
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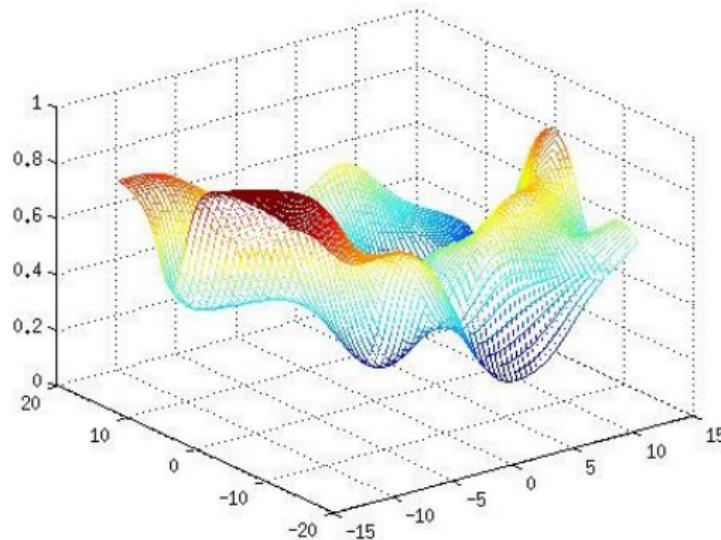


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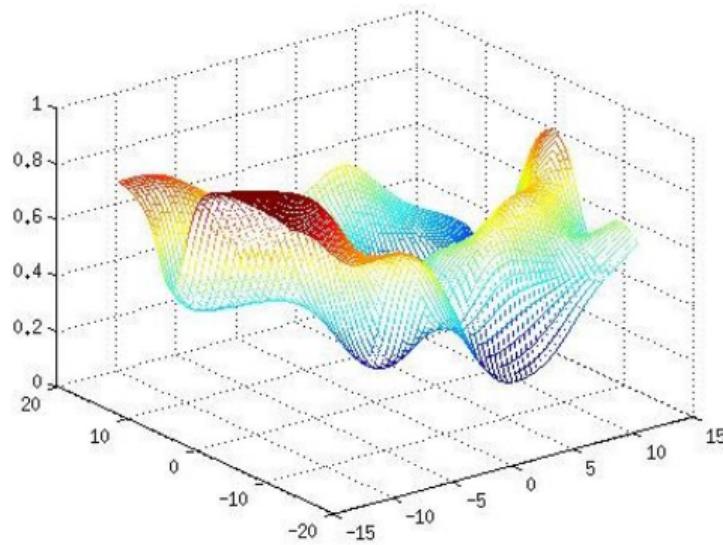


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- Indeed Larochelle et.al.<sup>1</sup> show that if the last layer has large capacity then  $\mathcal{L}(\theta)$  goes to 0 even without pre-training
- However, if the capacity of the network is small, unsupervised pre-training helps

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- Is it because of better optimization?
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- What does regularization do?

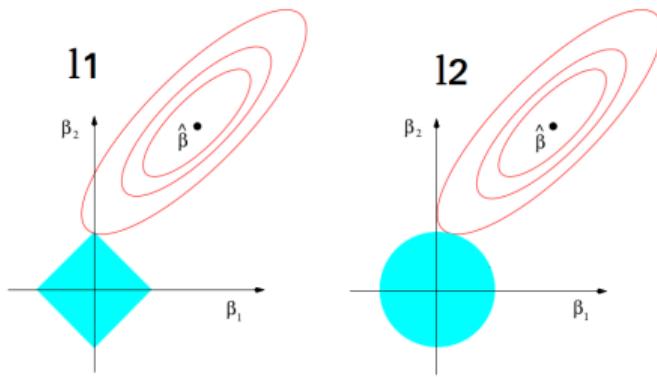
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<sup>1</sup>Image Source:The Elements of Statistical Learning-T. Hastie, R. Tibshirani, and J. Friedman,  
Pg 71

- What does regularization do? It constrains the weights to certain regions of the parameter space

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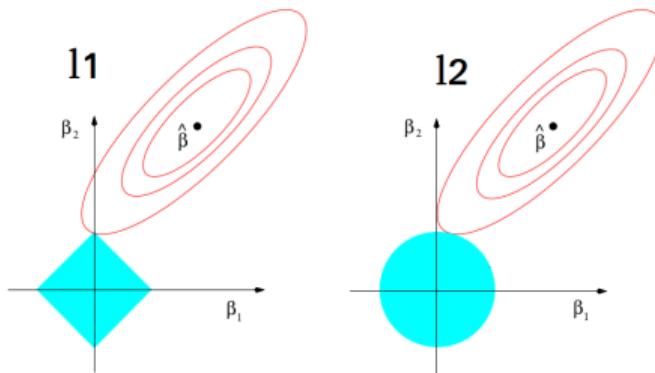
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- What does regularization do? It constrains the weights to certain regions of the parameter space
- L-1 regularization: constrains most weights to be 0
- L-2 regularization: prevents most weights from taking large values

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- **Supervised objective:**

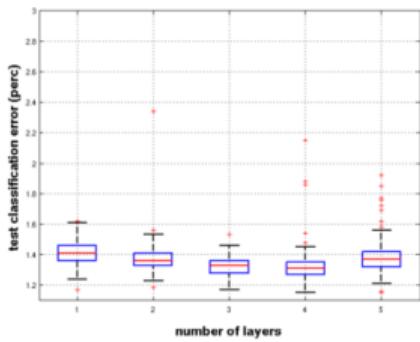
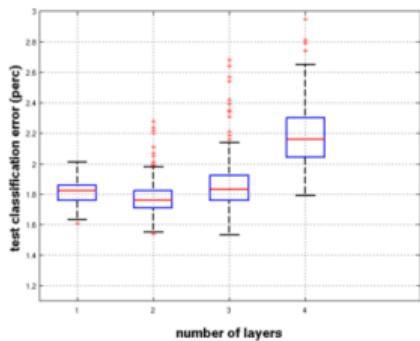
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- Specifically, it constrains the weights to lie in regions where the characteristics of the data are captured well (as governed by the unsupervised objective)
- This unsupervised objective ensures that the learning is not greedy w.r.t. the supervised objective (and also satisfies the unsupervised objective)

- Some other experiments have also shown that pre-training is more robust to random initializations

---

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- Some other experiments have also shown that pre-training is more robust to random initializations
- One accepted hypothesis is that pre-training leads to better weight initializations (so that the layers capture the internal characteristics of the data)

---

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So what has happened since 2006-2009?

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- Better optimization algorithms

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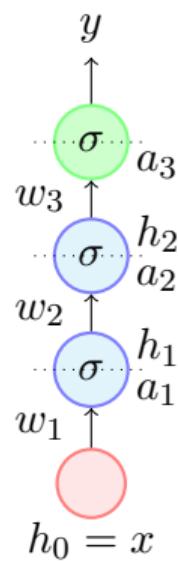
## Module 9.3 : Better activation functions

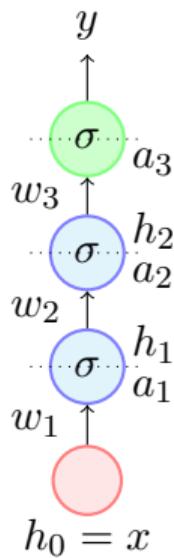
## Deep Learning has evolved

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- **Better activation functions**
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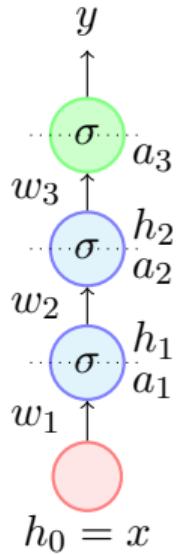
- Before we look at activation functions, let's try to answer the following question:  
“What makes Deep Neural Networks powerful ?”

- Consider this deep neural network



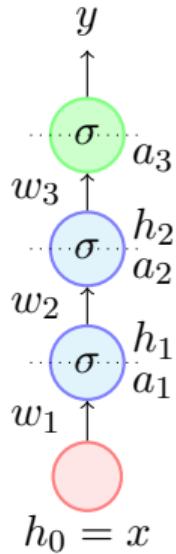


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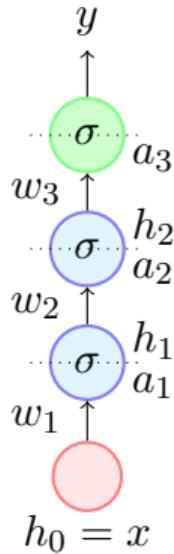
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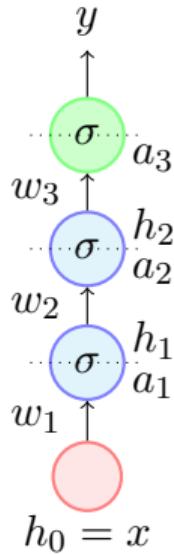
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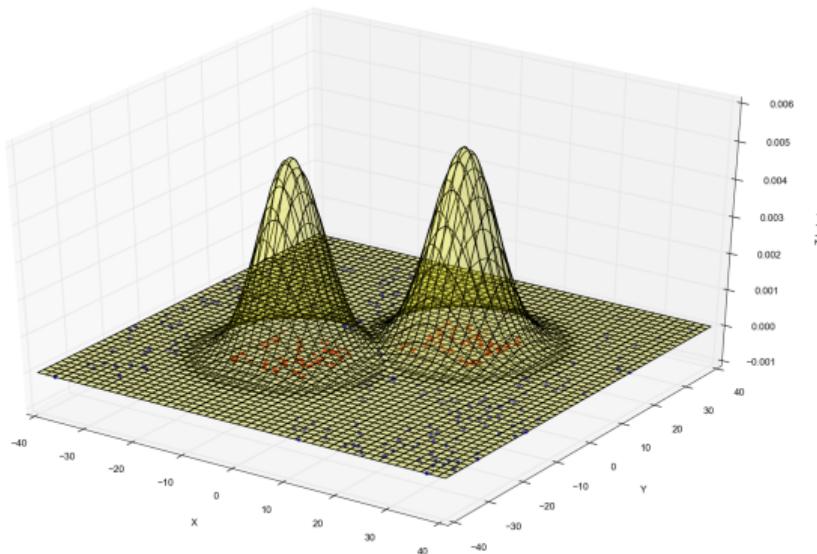


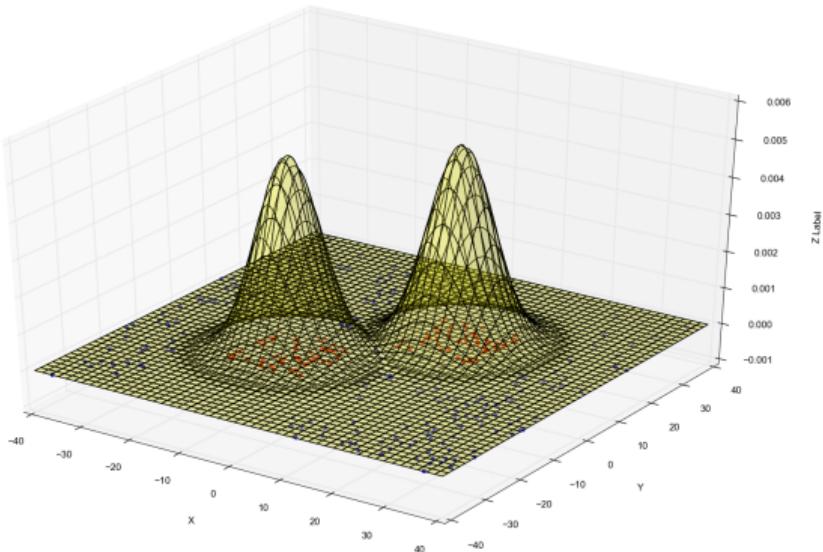
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- Then we will just learn  $y$  as a linear transformation of  $x$
- In other words we will be constrained to learning linear decision boundaries
- We cannot learn arbitrary decision boundaries

- In particular, a deep linear neural network cannot learn such boundaries





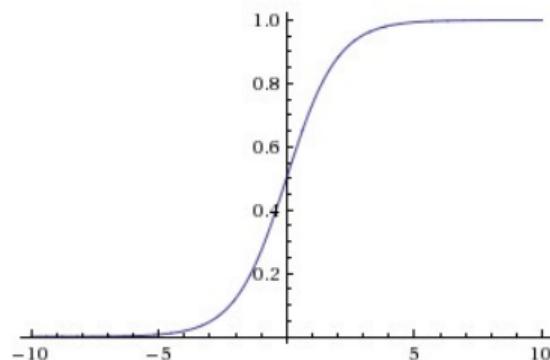
- In particular, a deep linear neural network cannot learn such boundaries
- But a deep non linear neural network can indeed learn such boundaries (recall Universal Approximation Theorem)

- Now let's look at some non-linear activation functions that are typically used in deep neural networks (Much of this material is taken from Andrej Karpathy's lecture notes <sup>1</sup>)

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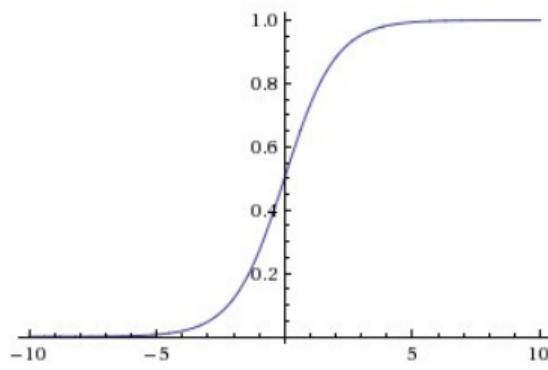
<sup>1</sup><http://cs231n.github.io>

- $\sigma(x) = \frac{1}{1+e^{-x}}$



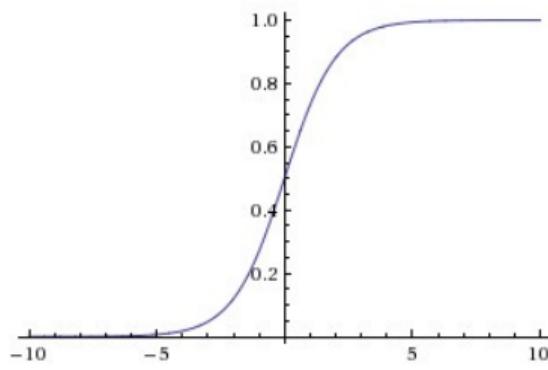
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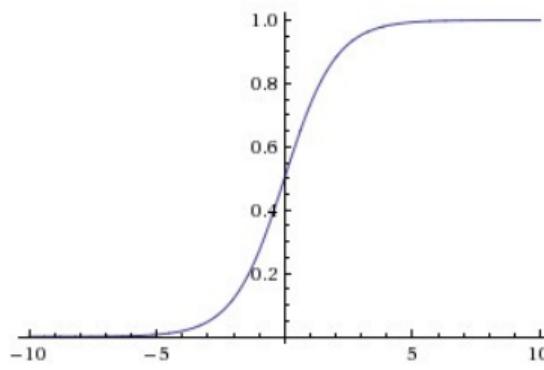


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*(you can easily derive it)*



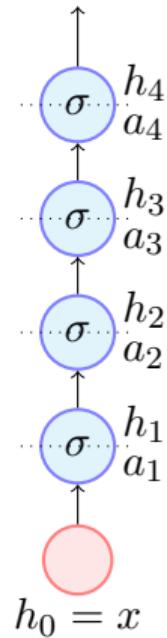
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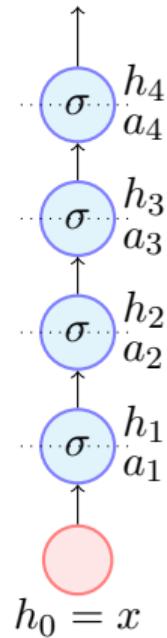
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- Let us see what happens if we use sigmoid in a deep network



$$a_3 = w_2 h_2$$
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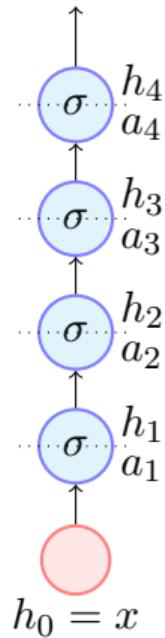


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- While calculating  $\nabla w_2$  at some point in the chain rule we will encounter

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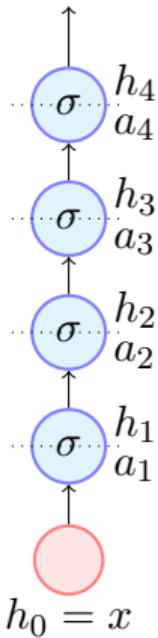
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- What is the consequence of this ?



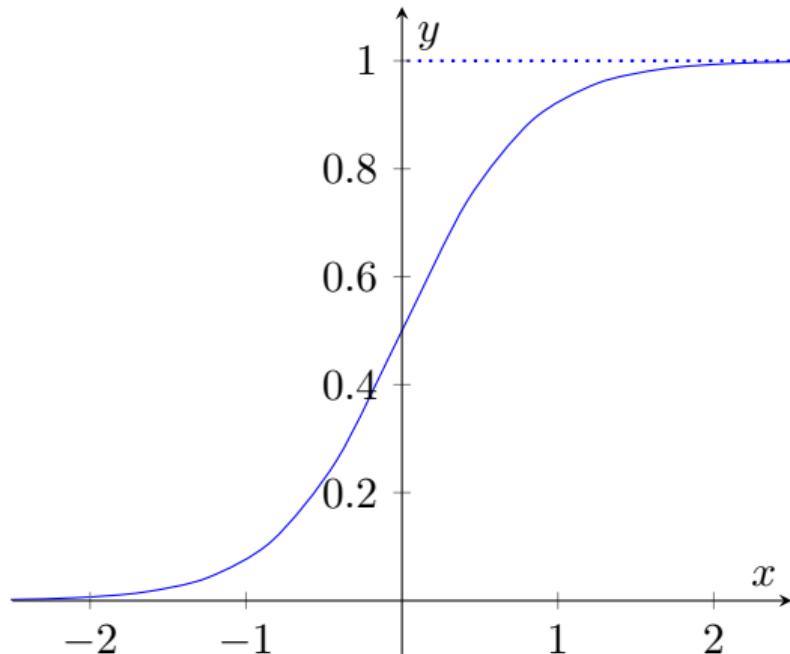
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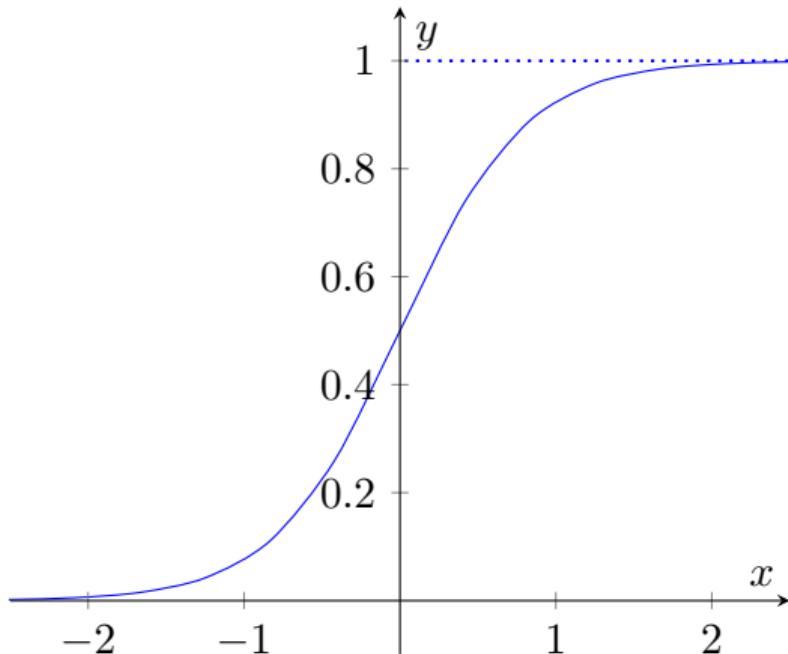
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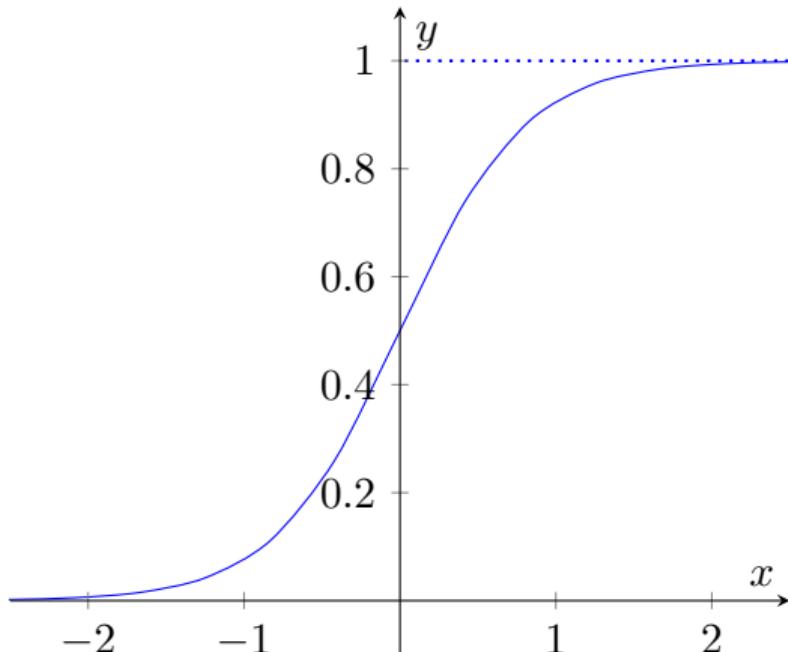
- What is the consequence of this ?
- To answer this question let us first understand the concept of saturated neuron ?



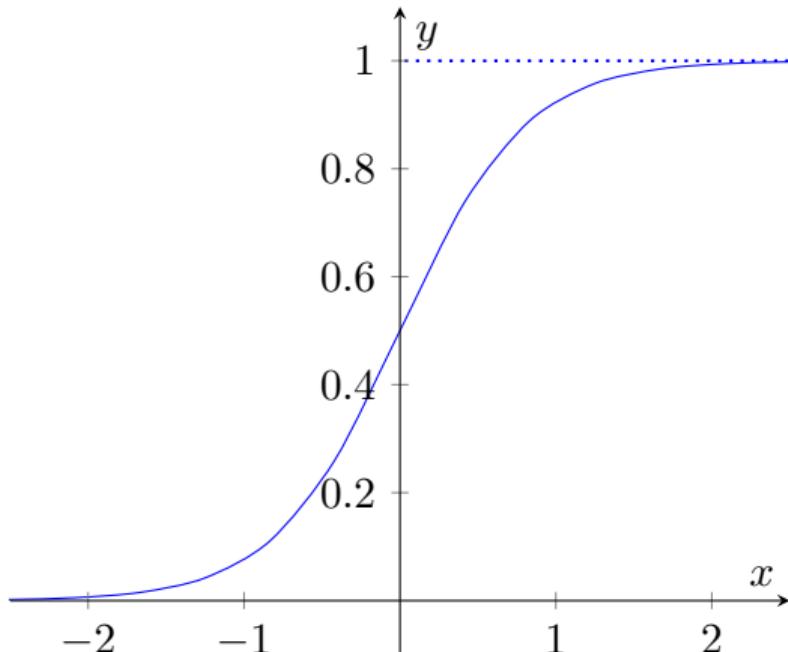
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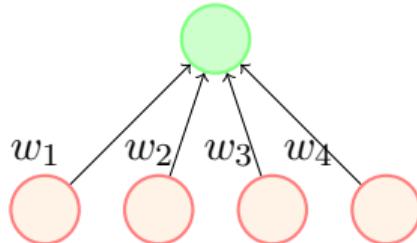
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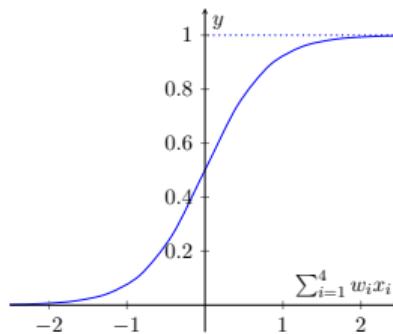
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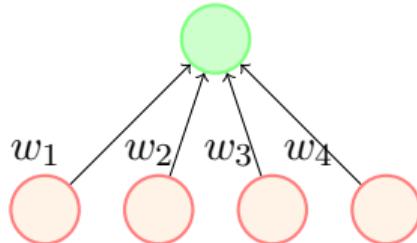


$$\sigma(\sum_{i=1}^4 w_i x_i)$$

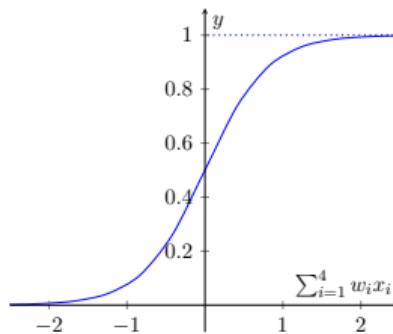


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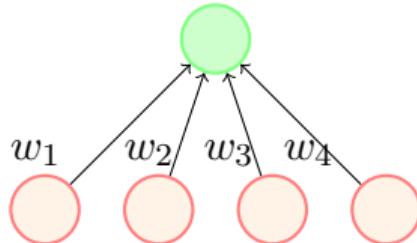


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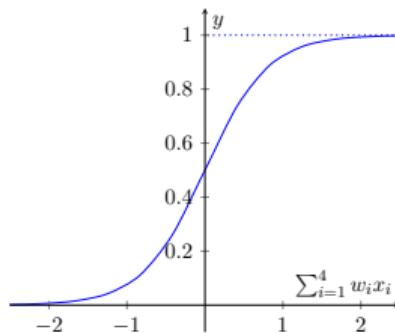


- But why would the neurons saturate ?
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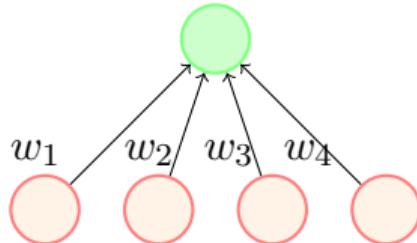


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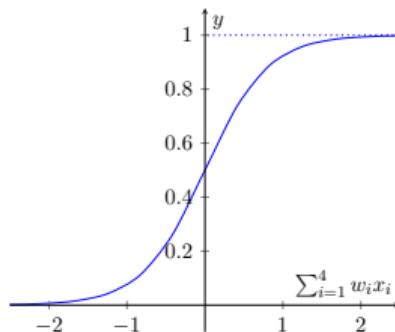


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- But why would the neurons saturate ?
- Consider what would happen if we use sigmoid neurons and initialize the weights to very high values ?
- The neurons will saturate very quickly
- The gradients will vanish and the training will stall (more on this later)

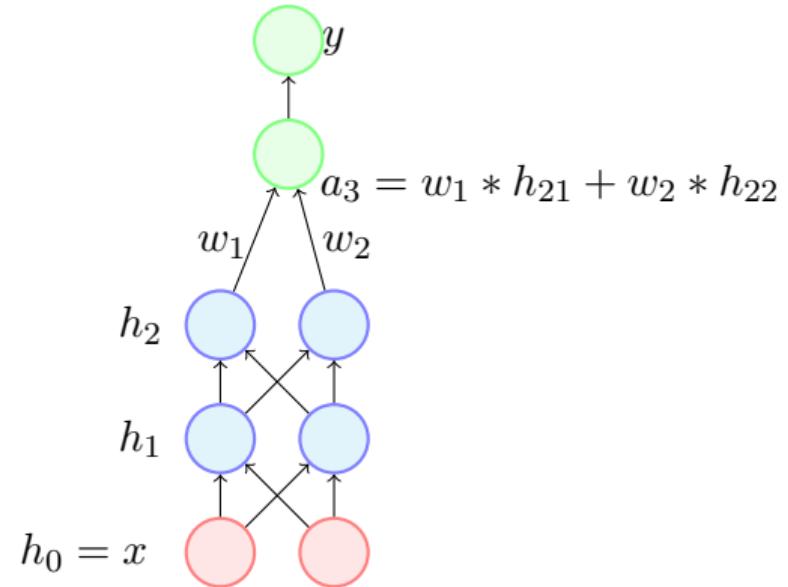
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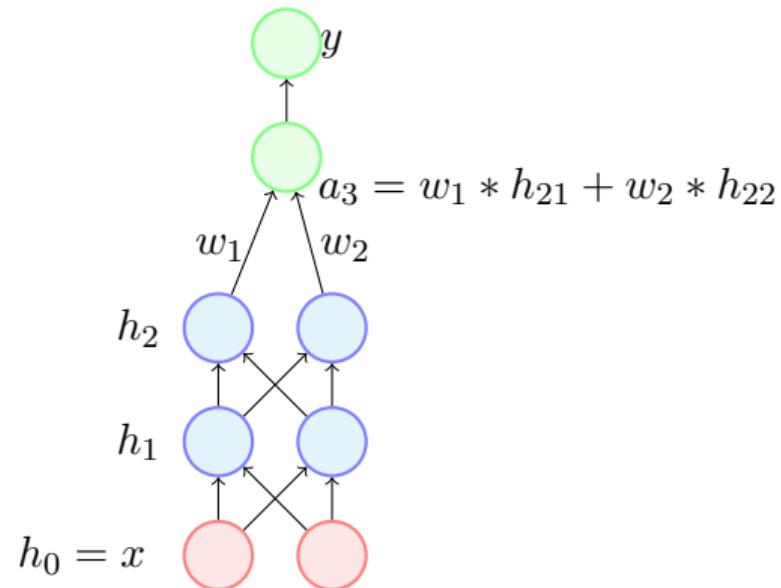


- Saturated neurons cause the gradient to vanish
- Sigmoids are not zero centered**
- Consider the gradient w.r.t.  $w_1$  and  $w_2$

$$\nabla w_1 = \frac{\partial \mathcal{L}(\mathbf{w})}{\partial y} \frac{\partial y}{\partial h_3} \frac{\partial h_3}{\partial a_3} \frac{\partial a_3}{\partial w_1}$$

$$\nabla w_2 = \frac{\partial \mathcal{L}(\mathbf{w})}{\partial y} \frac{\partial y}{\partial h_3} \frac{\partial h_3}{\partial a_3} \frac{\partial a_3}{\partial w_2}$$

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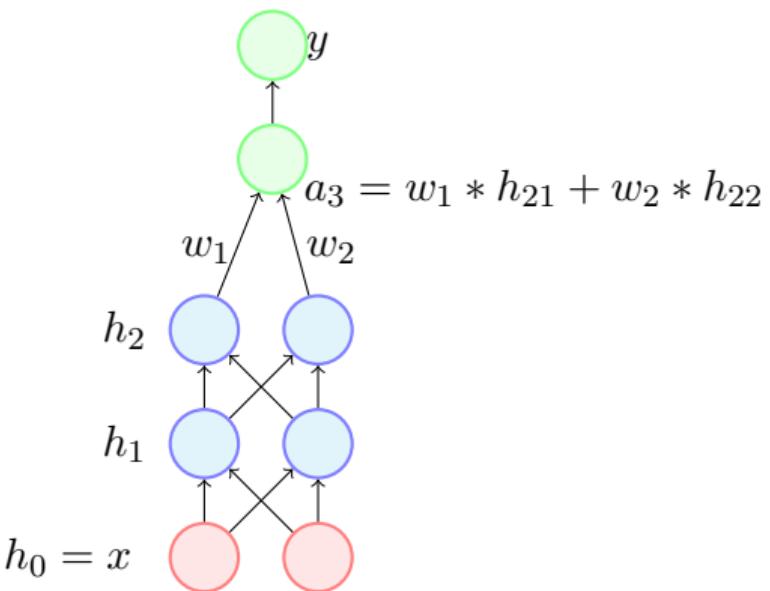
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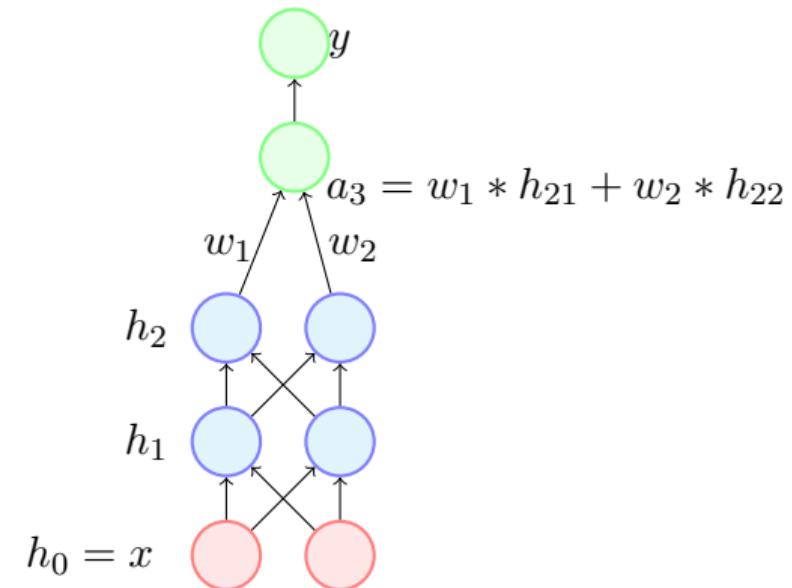
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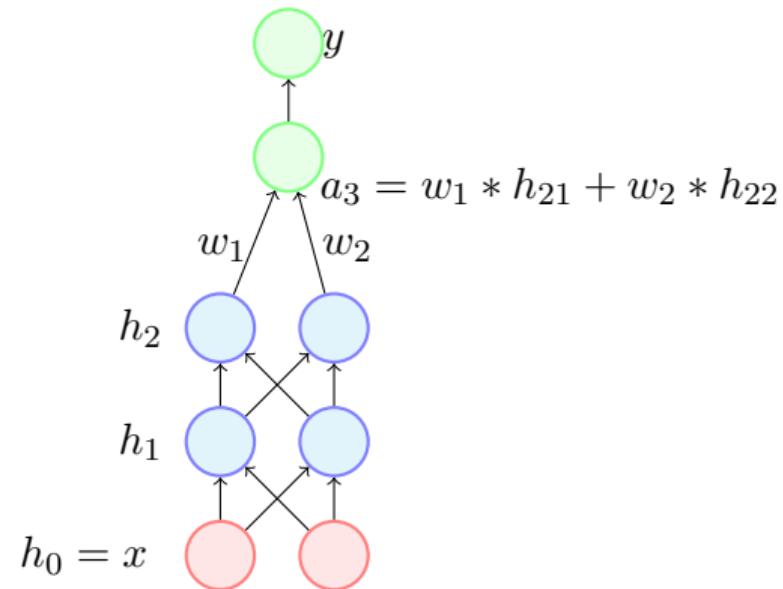
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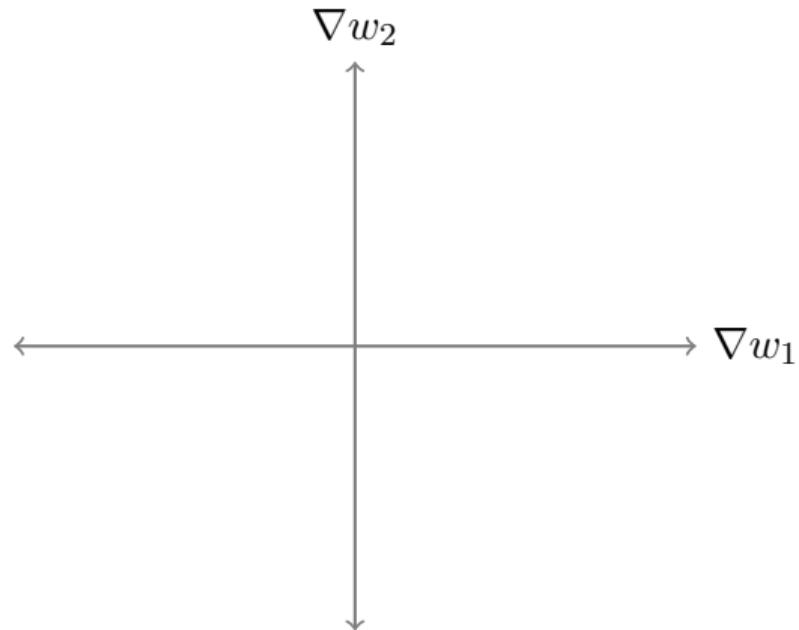
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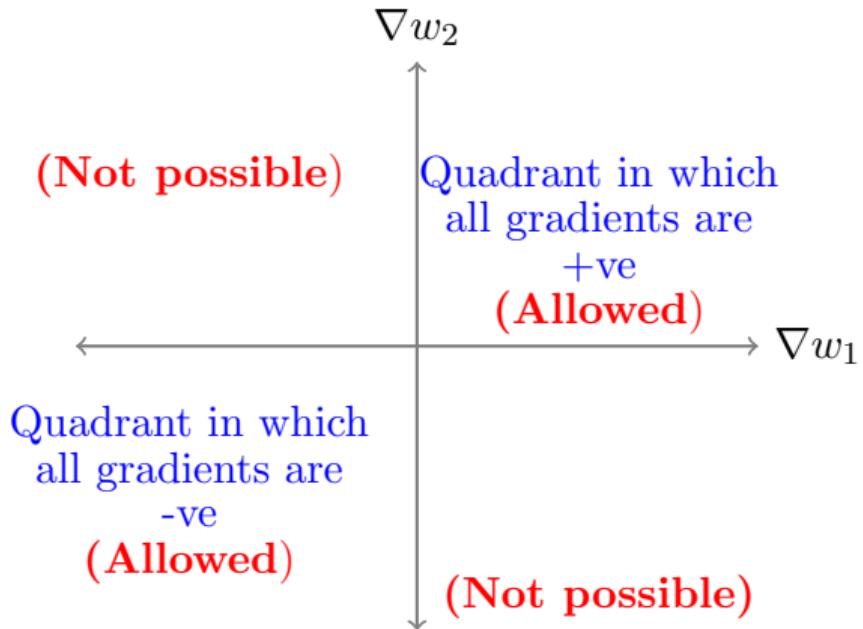


- Essentially, either all the gradients at a layer are positive or all the gradients at a layer are negative

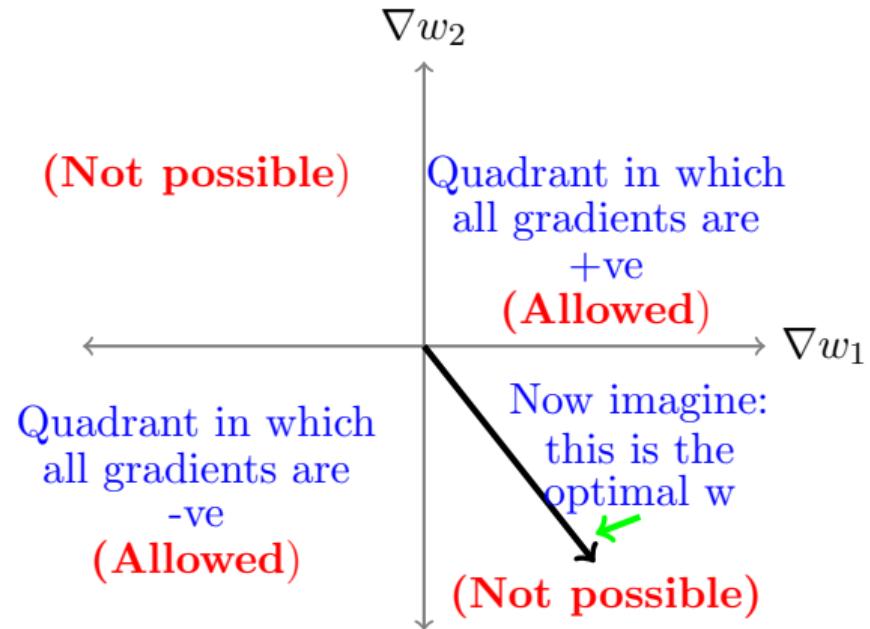
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- Sigmoids are not zero centered
- This restricts the possible update directions



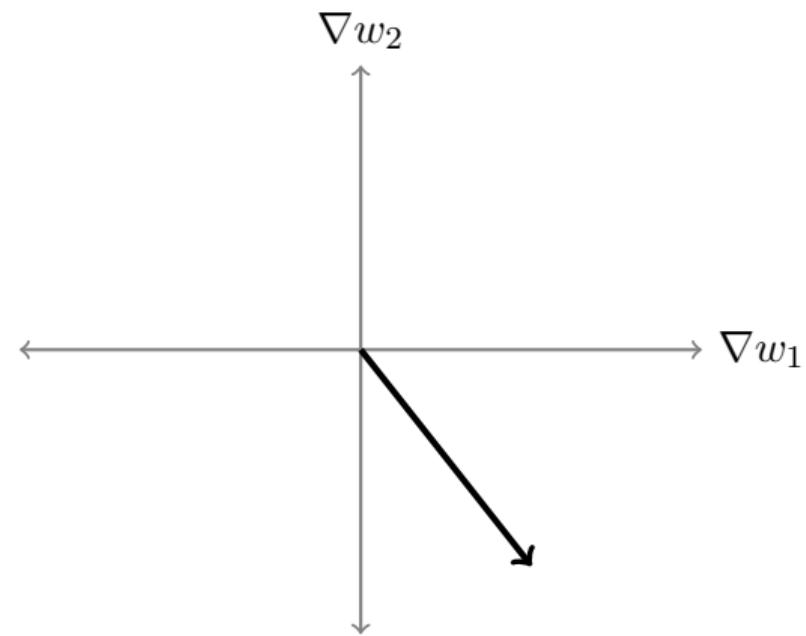
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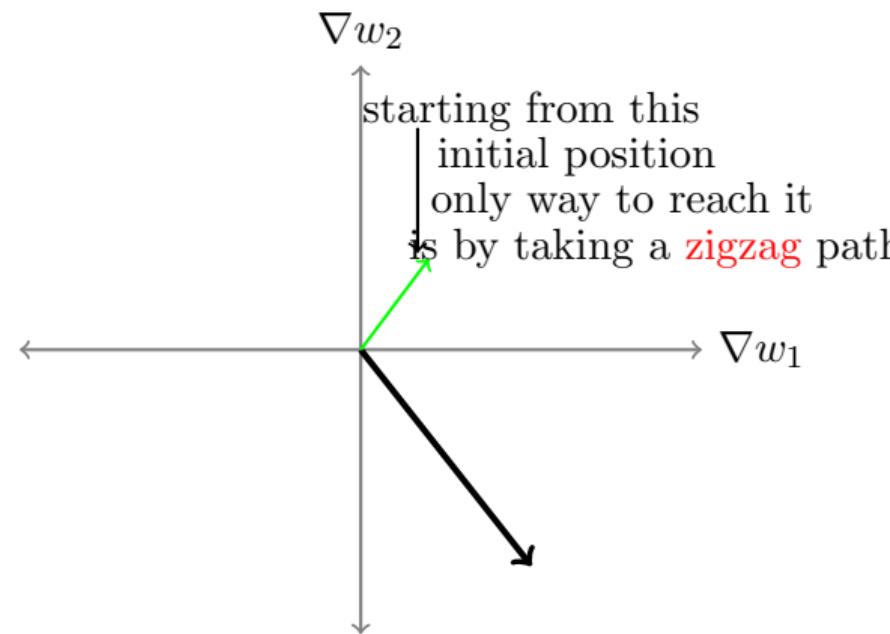
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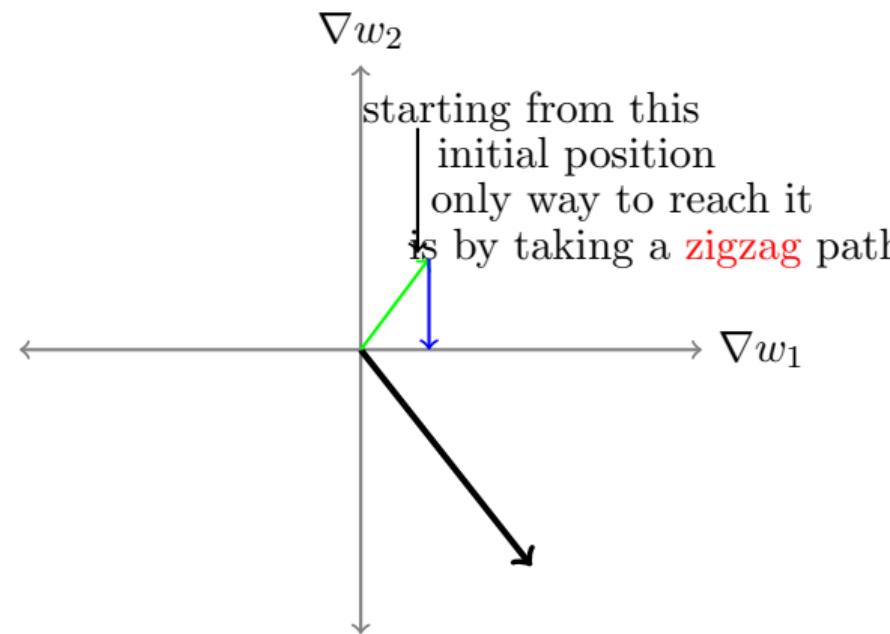
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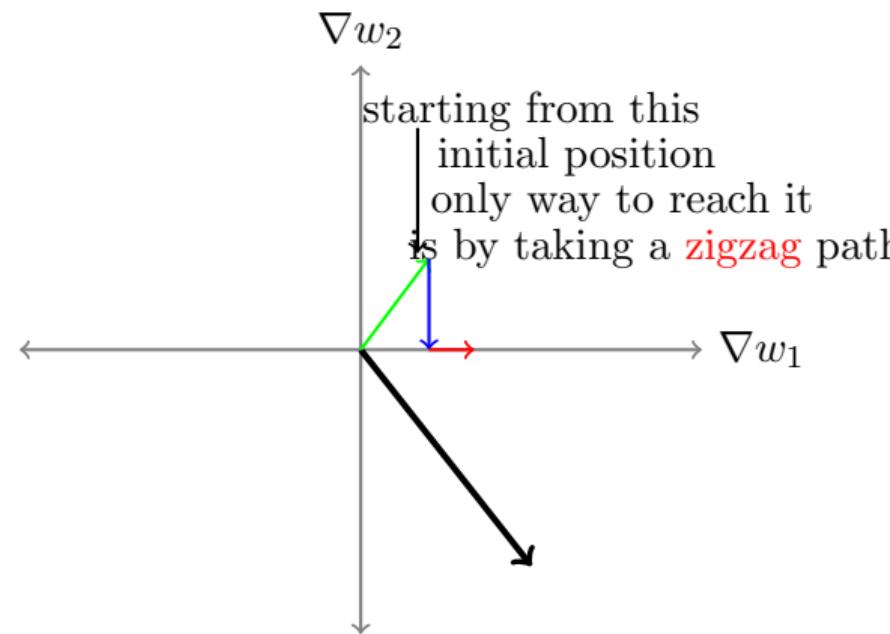
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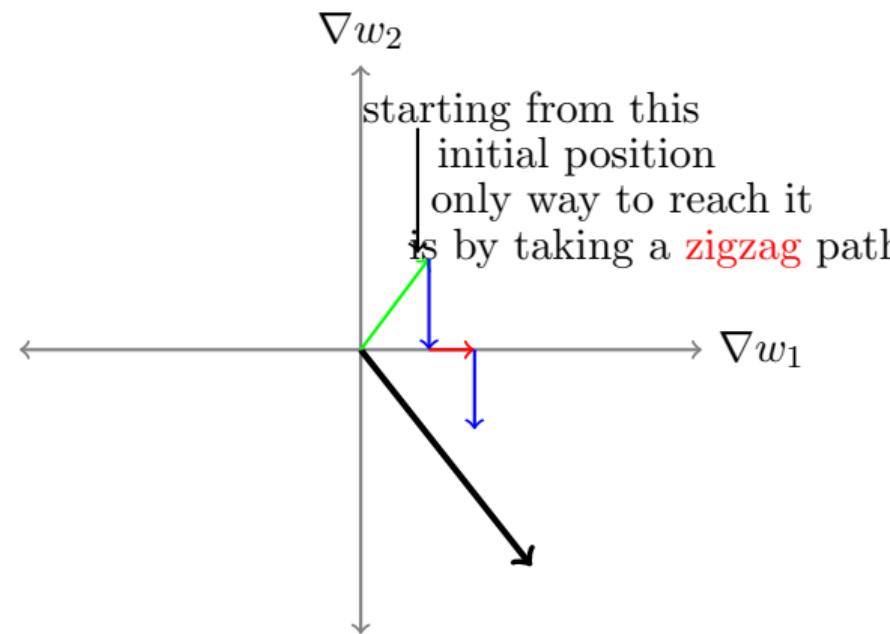
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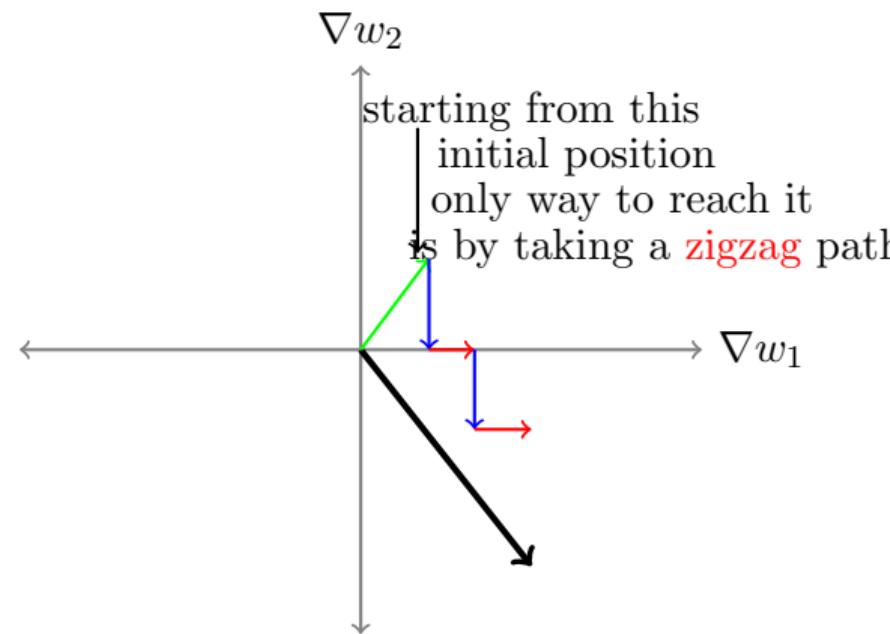
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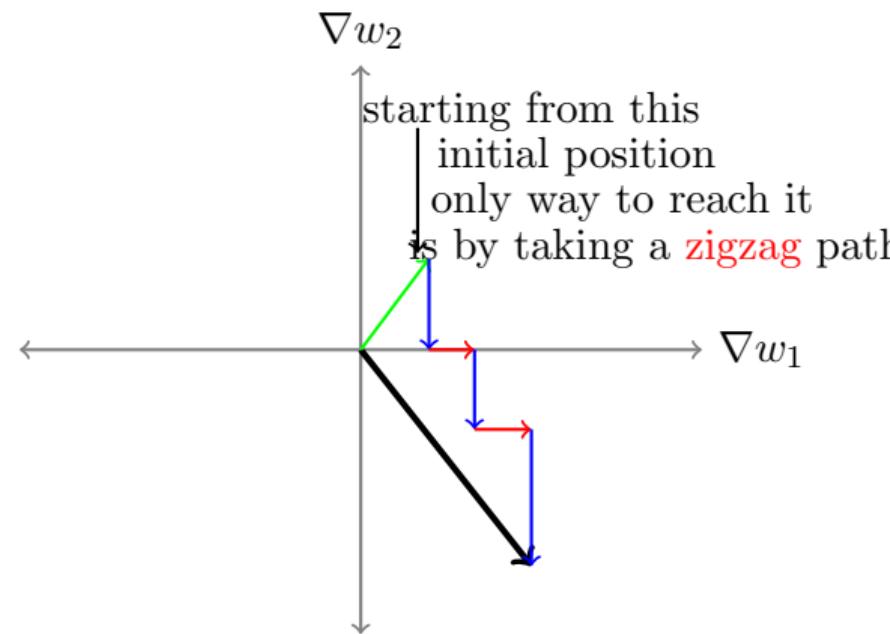
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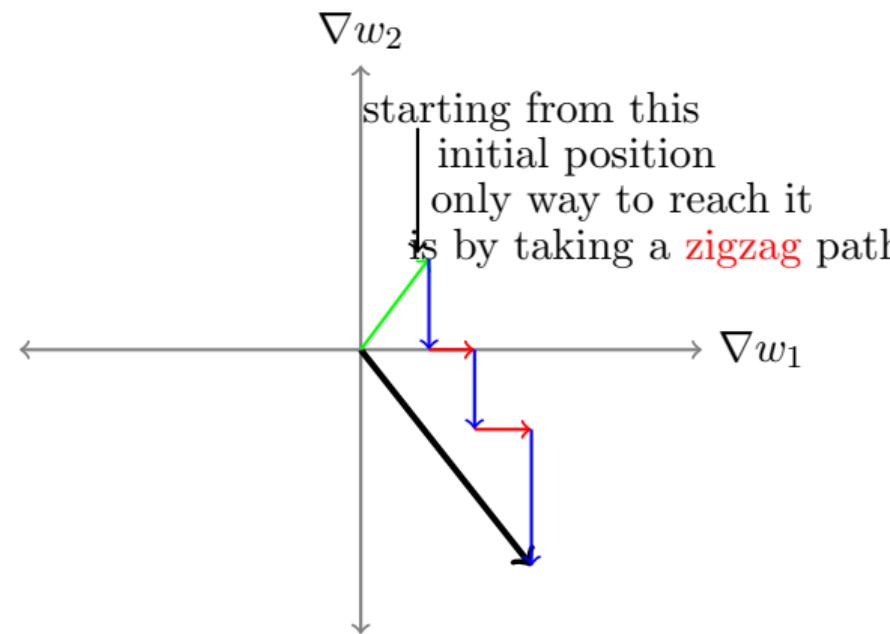
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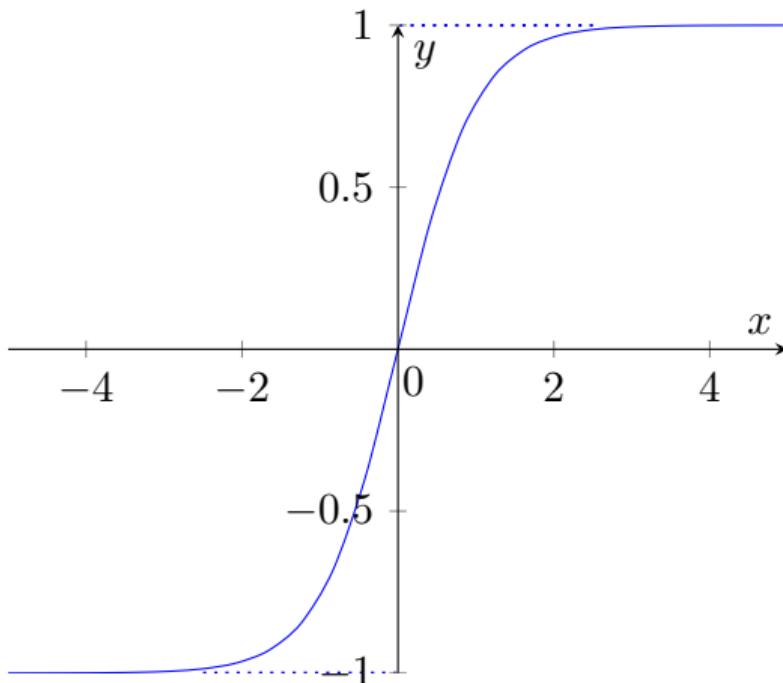
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- Saturated neurons cause the gradient to vanish
- Sigmoids are not zero centered
- And lastly, sigmoids are computationally expensive (because of  $\exp(x)$ )



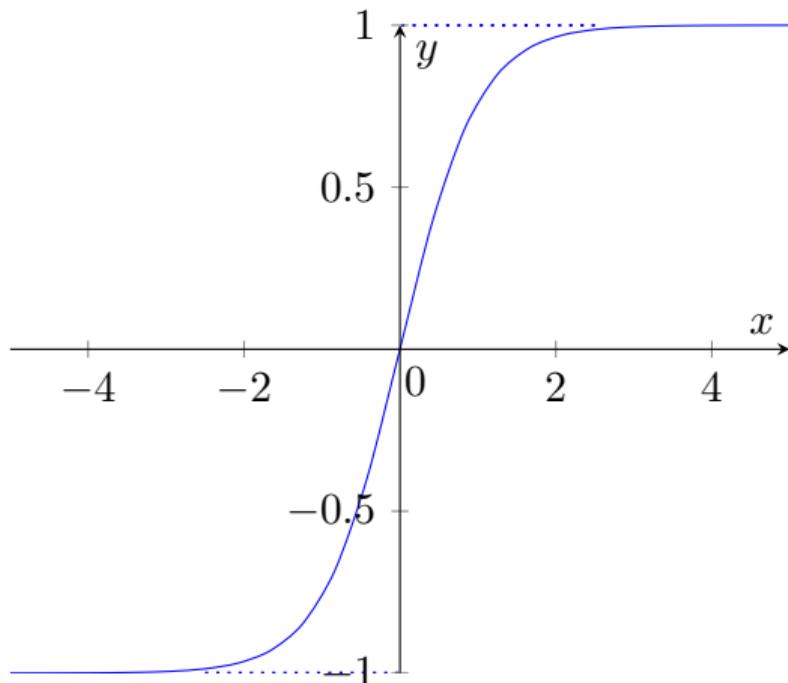
$\tanh(x)$



- Compresses all its inputs to the range [-1,1]

$$f(x) = \tanh(x)$$

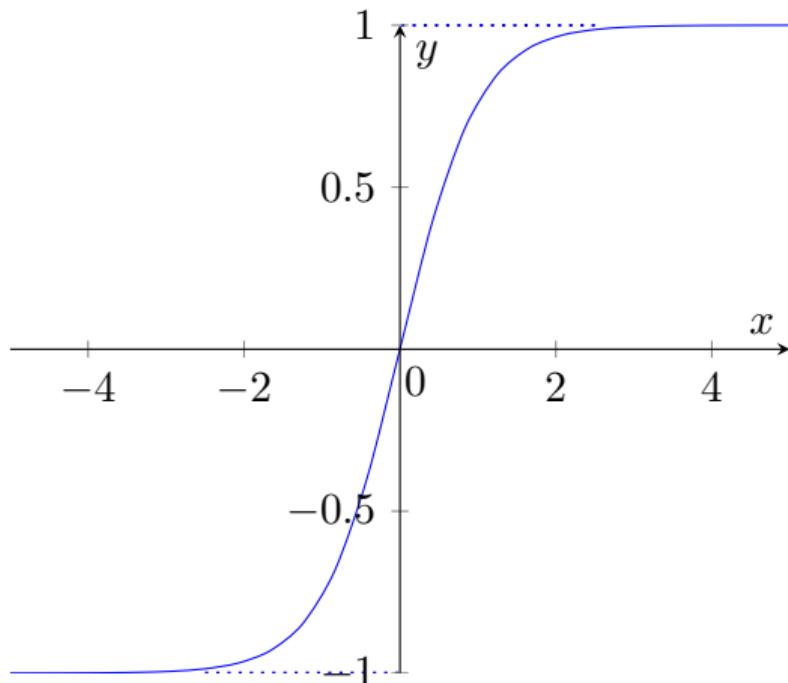
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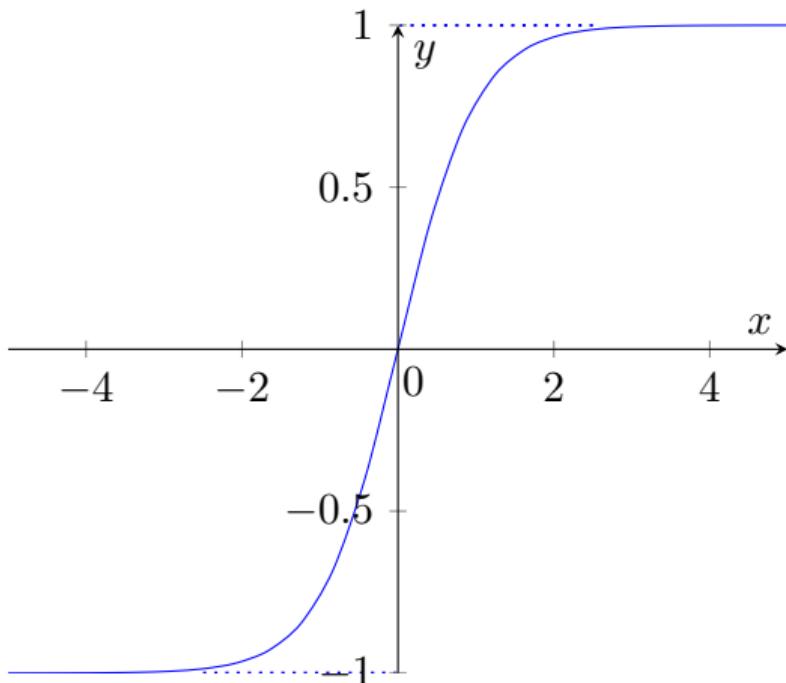
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- What is the derivative of this function?

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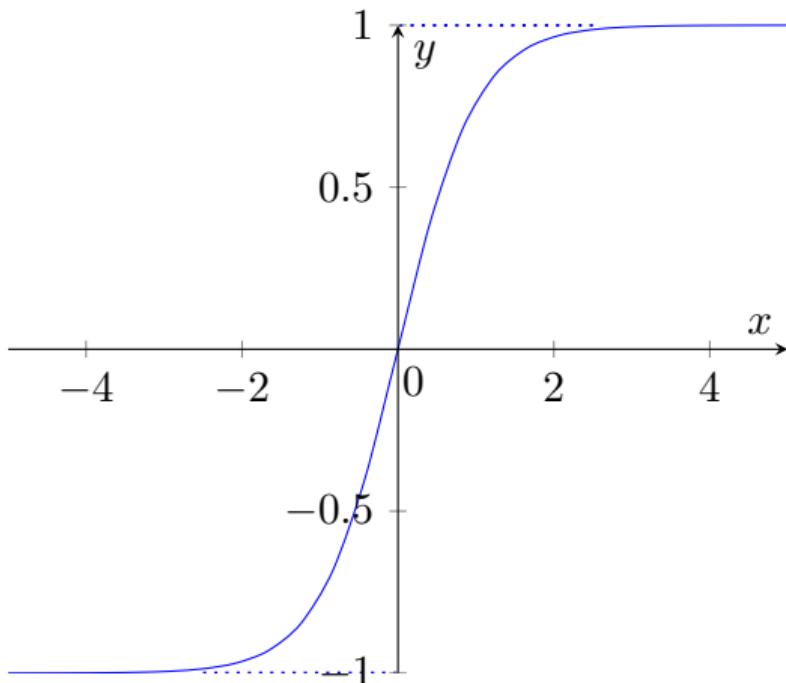


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$$\frac{\partial \tanh(x)}{\partial x} = (1 - \tanh^2(x))$$

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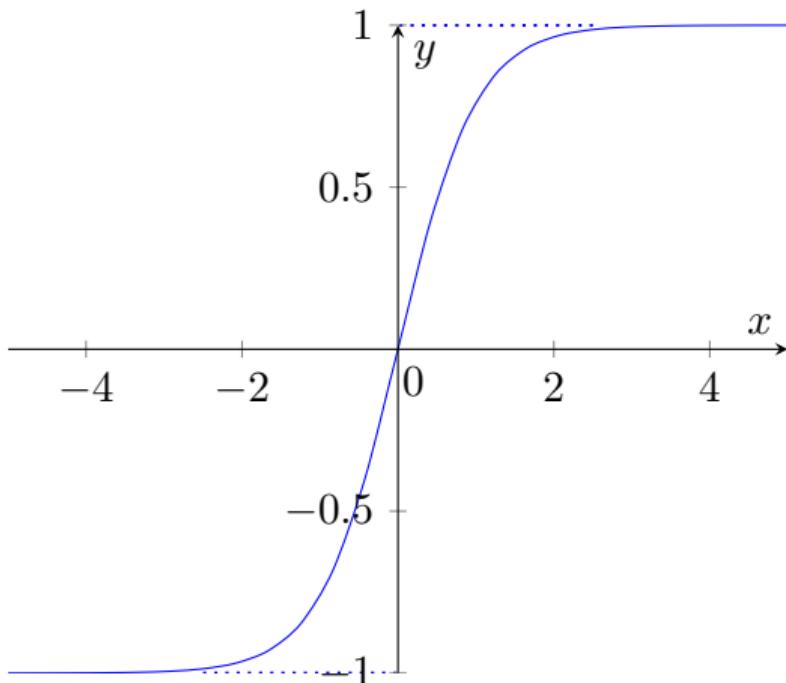
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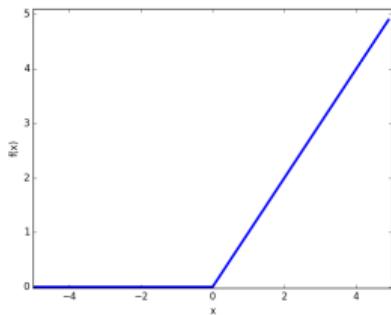
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- The gradient still vanishes at saturation
- Also computationally expensive

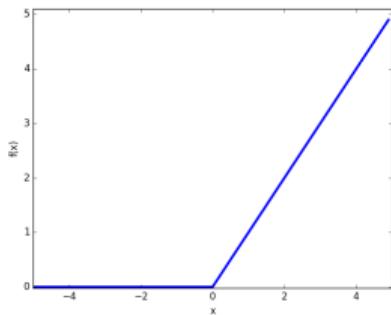
# ReLU



- Is this a non-linear function?

$$f(x) = \max(0, x)$$

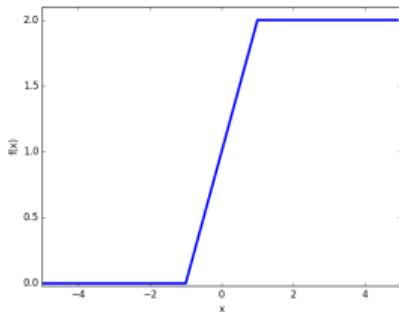
# ReLU



- Is this a non-linear function?
- Indeed it is!

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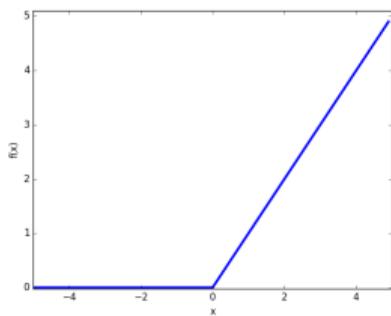
## ReLU



- Is this a non-linear function?
- Indeed it is!
- In fact we can combine two ReLU units to recover a piecewise linear approximation of the sigmoid function

$$f(x) = \max(0, x + 1) - \max(0, x - 1)$$

# ReLU



$$f(x) = \max(0, x)$$

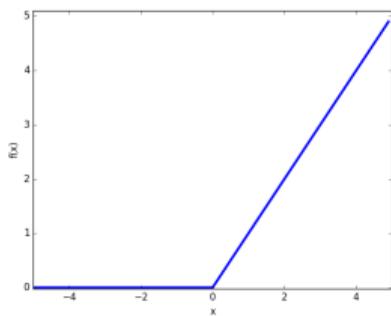
## Advantages of ReLU

- Does not saturate in the positive region

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<sup>1</sup>ImageNet Classification with Deep Convolutional Neural Networks- Alex Krizhevsky Ilya Sutskever, Geoffrey E. Hinton, 2012

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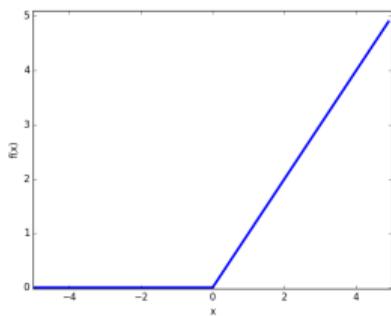
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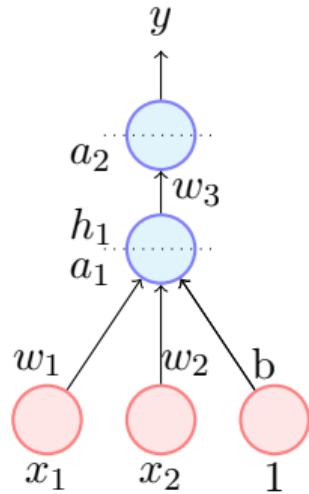
## Advantages of ReLU

- Does not saturate in the positive region
- Computationally efficient
- In practice converges much faster than *sigmoid/tanh*<sup>1</sup>

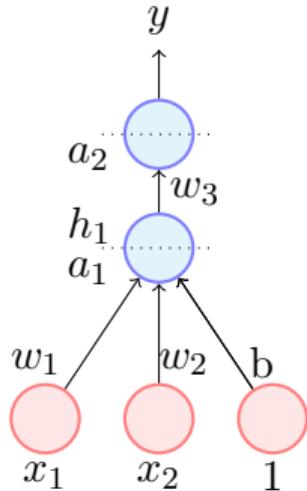
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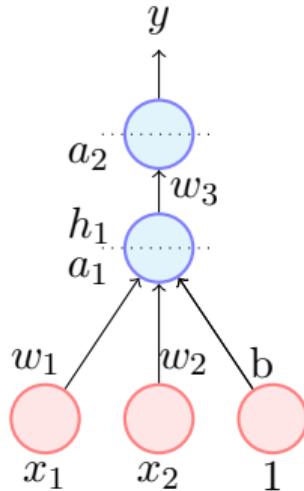


- In practice there is a caveat
- Let's see what is the derivative of  $\text{ReLU}(x)$



$$\begin{aligned}\frac{\partial \text{ReLU}(x)}{\partial x} &= 0 \quad \text{if } x < 0 \\ &= 1 \quad \text{if } x > 0\end{aligned}$$

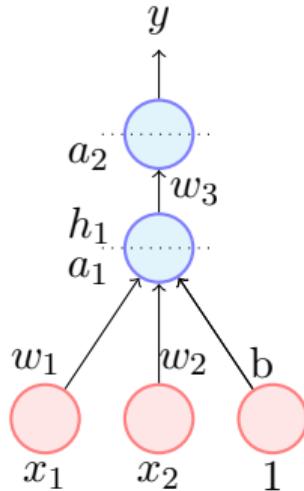
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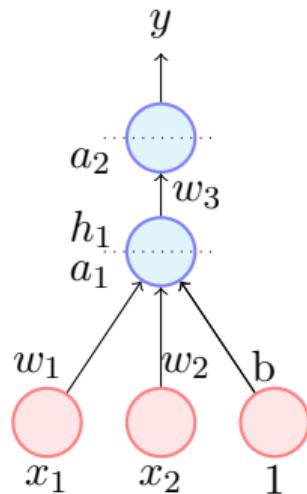
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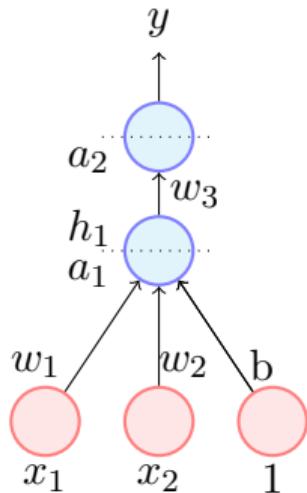
- Now consider the given network
- What would happen if at some point a large gradient causes the bias  $b$  to be updated to a large negative value?

$$w_1x_1 + w_2x_2 + b < 0 \quad [if \quad b << 0]$$



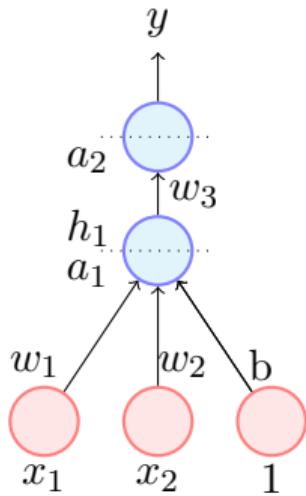
- The neuron would output 0 [dead neuron]

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- The neuron would output 0 [dead neuron]
- Not only would the output be 0 but during backpropagation even the gradient  $\frac{\partial h_1}{\partial a_1}$  would be zero

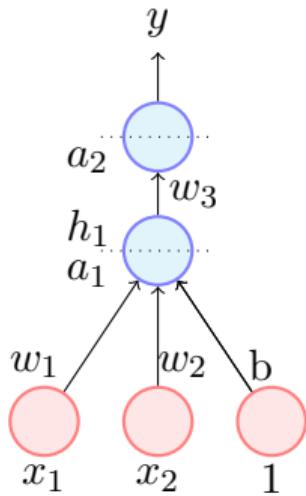
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- The weights  $w_1$ ,  $w_2$  and  $b$  will not get updated  
[ $\therefore$  there will be a zero term in the chain rule]

$$\nabla_{w_1} = \frac{\partial \mathcal{L}(\theta)}{\partial y} \cdot \frac{\partial y}{\partial a_2} \cdot \frac{\partial a_2}{\partial h_1} \cdot \frac{\partial h_1}{\partial a_1} \cdot \frac{\partial a_1}{\partial w_1}$$

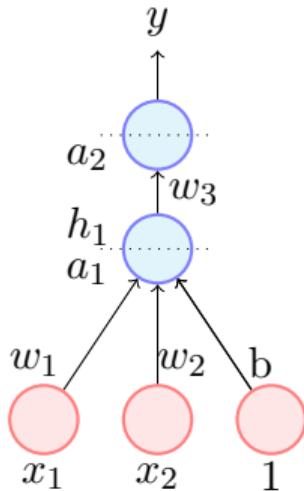
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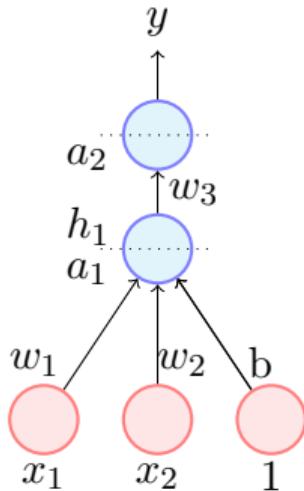
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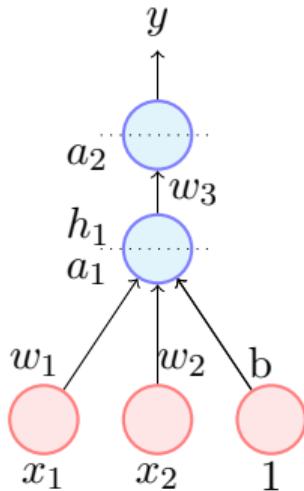
- The neuron will now stay dead forever!!



- In practice a large fraction of ReLU units can die if the learning rate is set too high



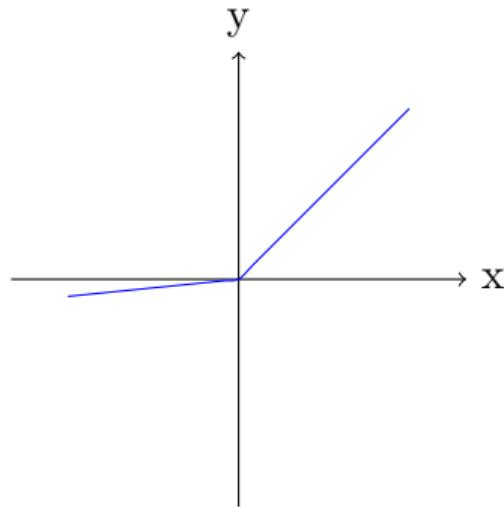
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- In practice a large fraction of ReLU units can die if the learning rate is set too high
- It is advised to initialize the bias to a positive value (0.01)
- Use other variants of ReLU (as we will soon see)

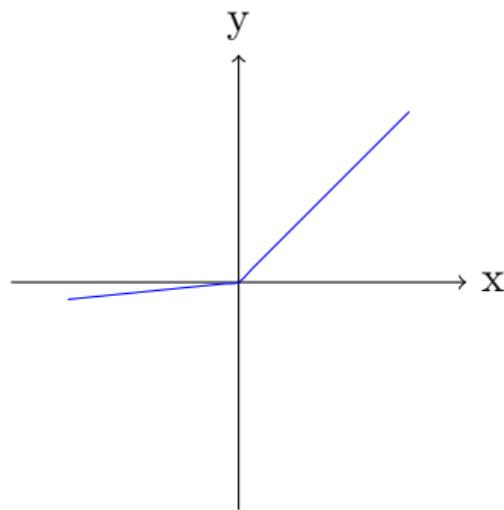
- No saturation

## Leaky ReLU



$$f(x) = \max(0.01x, x)$$

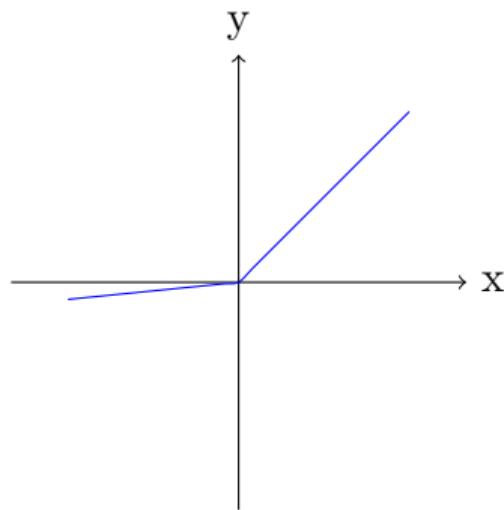
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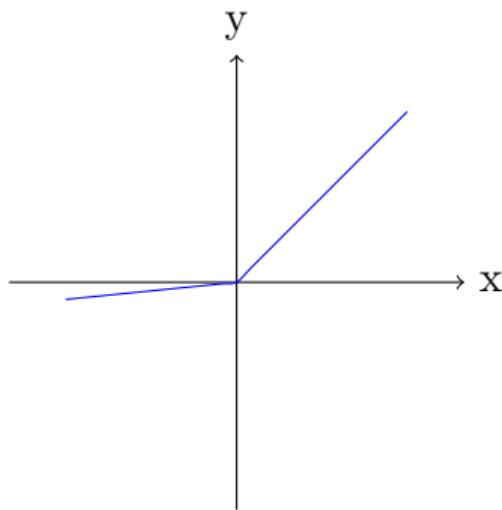
## Leaky ReLU



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- Computationally efficient

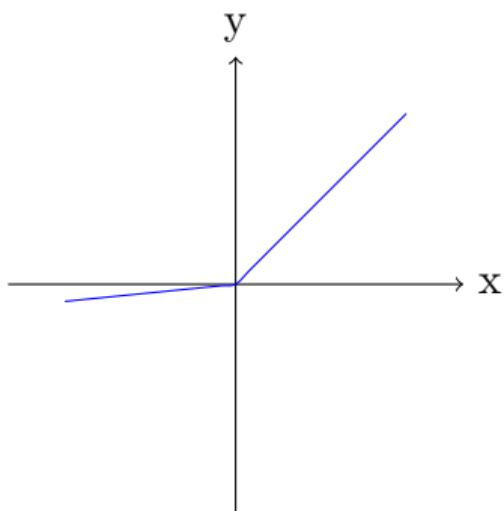
## Leaky ReLU



$$f(x) = \max(0.01x, x)$$

- No saturation
- Will not die ( $0.01x$  ensures that at least a small gradient will flow through)
- Computationally efficient
- Close to zero centered outputs

## Leaky ReLU



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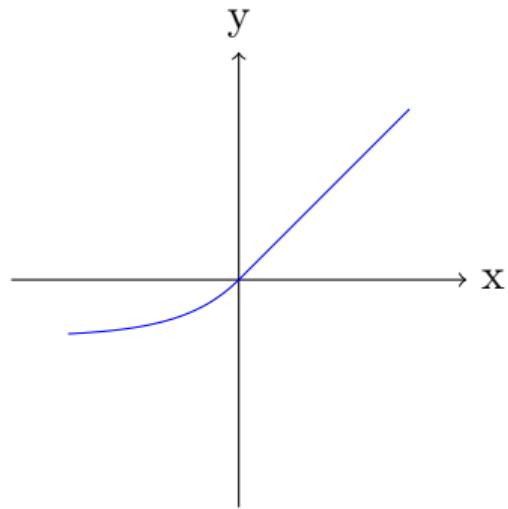
## Parametric ReLU

$$f(x) = \max(\alpha x, x)$$

$\alpha$  is a parameter of the model

$\alpha$  will get updated during backpropagation

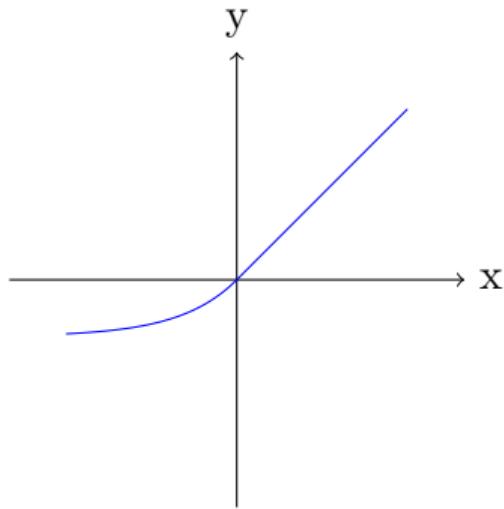
## Exponential Linear Unit



- All benefits of ReLU

$$\begin{aligned}f(x) &= x \quad \text{if } x > 0 \\&= ae^x - 1 \quad \text{if } x \leq 0\end{aligned}$$

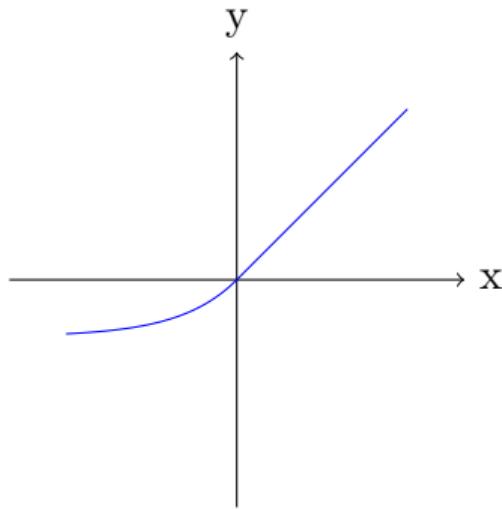
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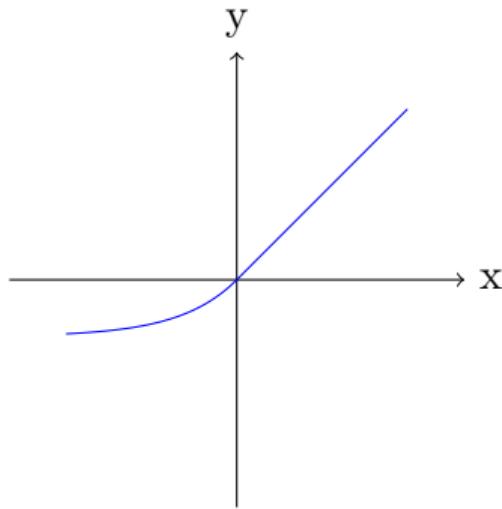
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## Exponential Linear Unit



- All benefits of ReLU
- $ae^x - 1$  ensures that at least a small gradient will flow through
- Close to zero centered outputs
- Expensive (requires computation of  $\exp(x)$ )

$$\begin{aligned}f(x) &= x \quad \text{if } x > 0 \\&= ae^x - 1 \quad \text{if } x \leq 0\end{aligned}$$

## Maxout Neuron

- Generalizes ReLU and Leaky ReLU

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

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$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

- Generalizes ReLU and Leaky ReLU
- No saturation! No death!
- Doubles the number of parameters

## Things to Remember

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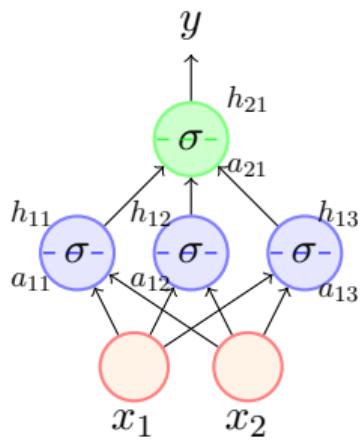
## Things to Remember

- Sigmoids are bad
- ReLU is more or less the standard unit for Convolutional Neural Networks
- Can explore Leaky ReLU/Maxout/ELU
- tanh sigmoids are still used in LSTMs/RNNs (we will see more on this later)

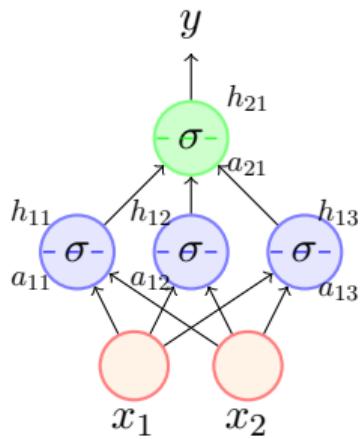
## Module 9.4 : Better initialization strategies

## Deep Learning has evolved

- Better optimization algorithms
- Better regularization methods
- Better activation functions
- **Better weight initialization strategies**

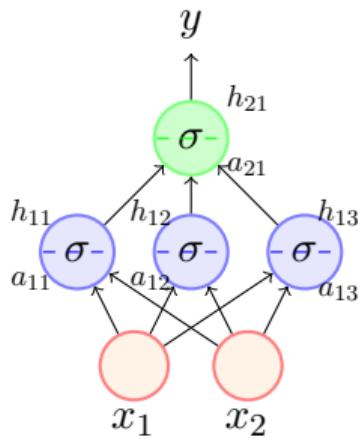


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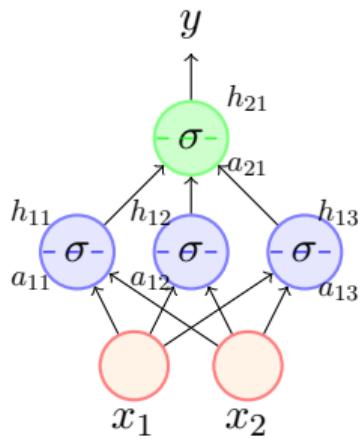
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- What happens if we initialize all weights to 0?

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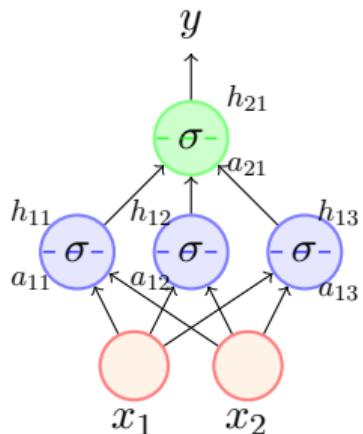


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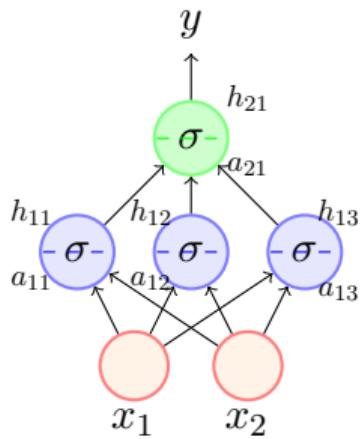
- What happens if we initialize all weights to 0?
- All neurons in layer 1 will get the same activation

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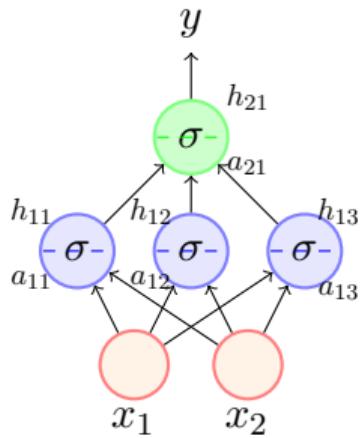
- Now what will happen during back propagation?

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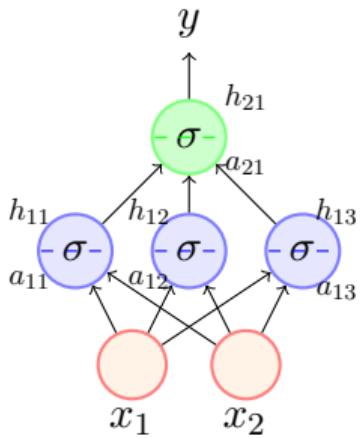
$$\nabla w_{11} = \frac{\partial \mathcal{L}(\mathbf{w})}{\partial y} \cdot \frac{\partial y}{\partial h_{11}} \cdot \frac{\partial h_{11}}{\partial a_{11}} \cdot x_1$$

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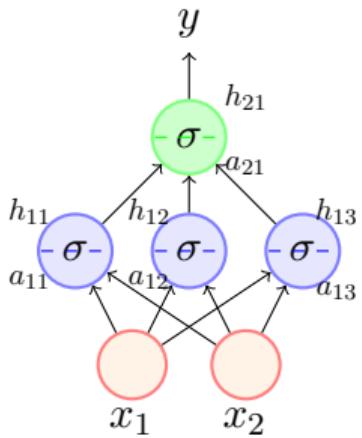
$$\nabla w_{21} = \frac{\partial \mathcal{L}(\mathbf{w})}{\partial y} \cdot \frac{\partial y}{\partial h_{12}} \cdot \frac{\partial h_{12}}{\partial a_{12}} \cdot x_1$$

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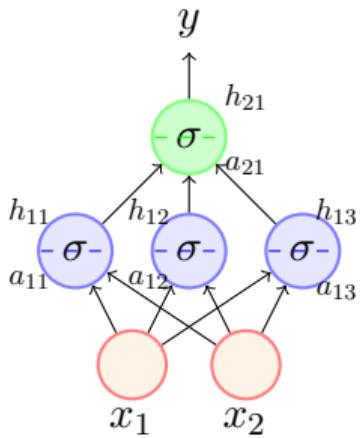
but  $h_{11} = h_{12}$

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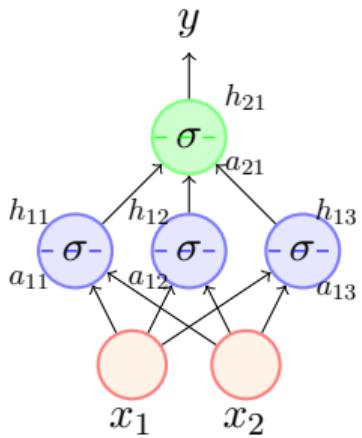
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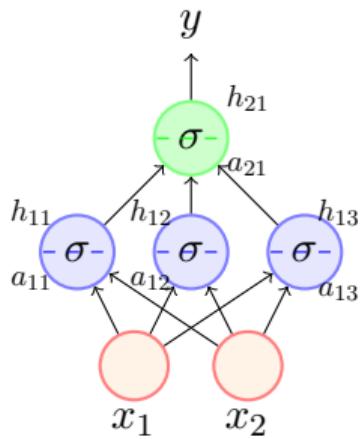
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$$\therefore \nabla w_{11} = \nabla w_{21}$$



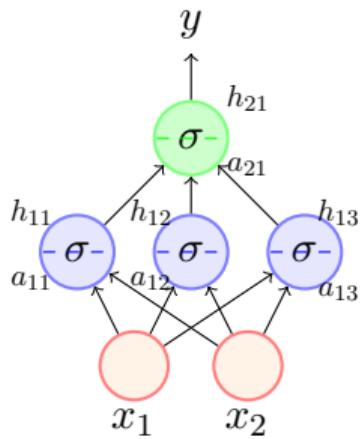
- Hence both the weights will get the same update and remain equal

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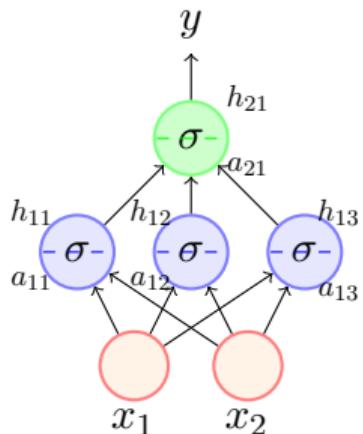
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- Infact this symmetry will never break during training

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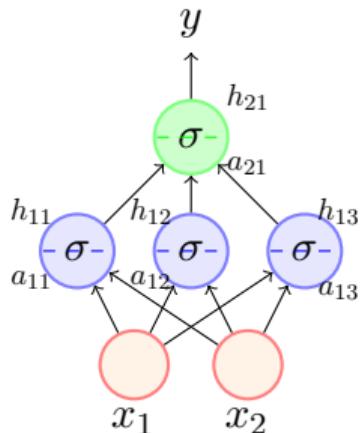
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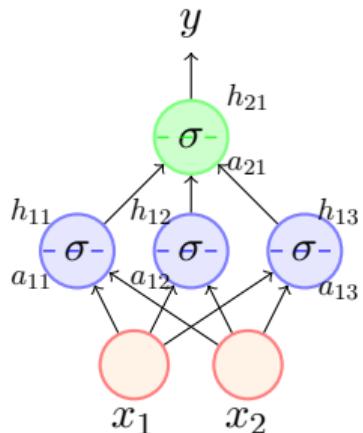
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- And for all weights in layer 2 (infact, work out the math and convince yourself that all the weights in this layer will remain equal )



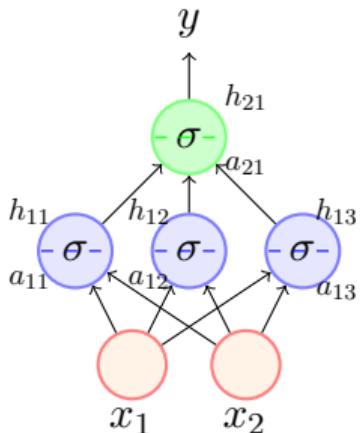
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- And for all weights in layer 2 (infact, work out the math and convince yourself that all the weights in this layer will remain equal )
- This is known as the **symmetry breaking problem**
- This will happen if all the weights in a network are initialized to the **same value**

```
D = np.random.randn(1000,500)
hidden_layer_sizes = [500]*10
nonlinearities = ['tanh']*len(hidden_layer_sizes)

act = {'relu':lambda x: np.maximum(0, x), 'tanh': lambda x: np.tanh(x),
       'sigmoid':lambda x: 1/ (1 + np.exp(-x))}
Hs = {}

for i in xrange(len(hidden_layer_sizes)):
    X = D if i == 0 else Hs[i-1]
    fan_in = X.shape[1]
    fan_out = hidden_layer_sizes[i]
    W = np.random.randn(fan_in, fan_out) * 0.01

    H = np.dot(X, W)
    H = act[nonlinearities[i]](H)
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```

We will now consider a feedforward network with:

```

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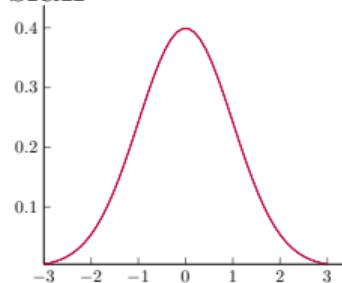
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We will now consider a feedforward network with:

- input: 1000 points, each  $\in R^{500}$
- input data is drawn from unit Gaussian



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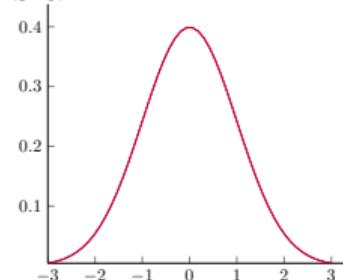
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- the network has 5 layers

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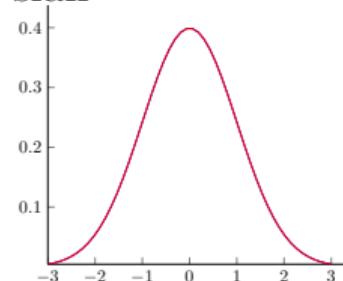
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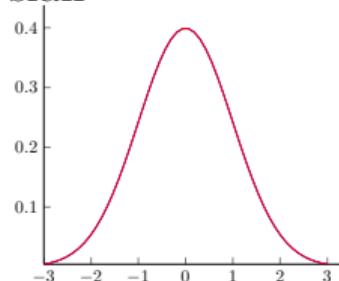
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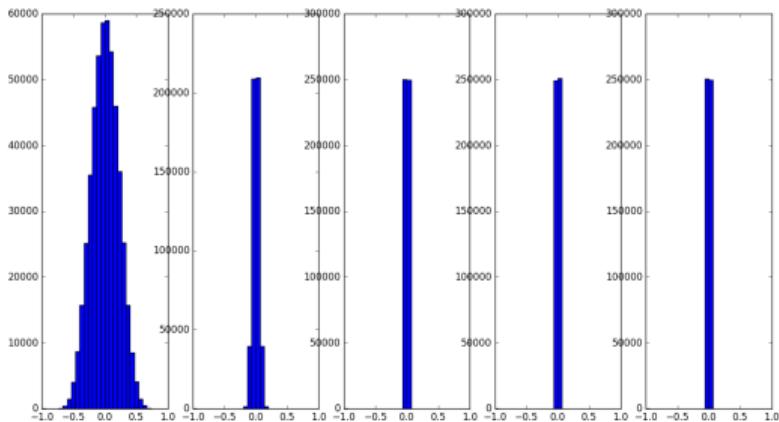
- the network has 5 layers
- each layer has 500 neurons
- we will run forward propagation on this network with different weight initializations

```
W = np.random.randn(fan_in, fan_out) * 0.01
```

- Let's try to initialize the weights to small random numbers

```
W = np.random.randn(fan_in, fan_out) * 0.01
```

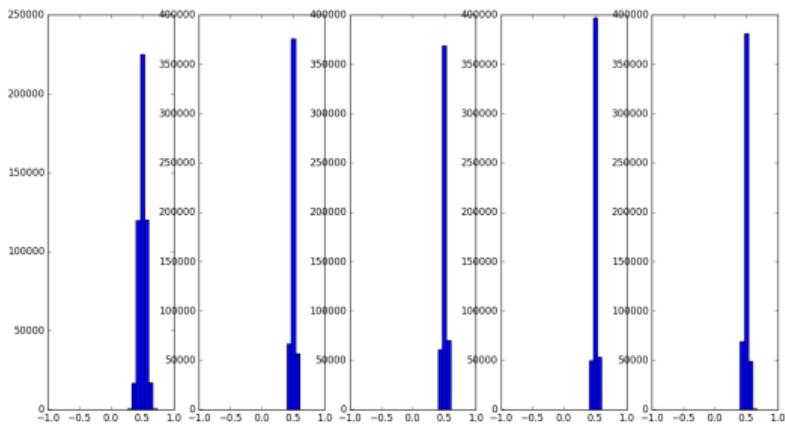
- Let's try to initialize the weights to small random numbers
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tanh activation functions

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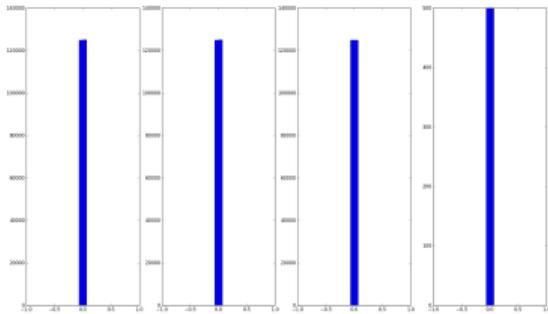


sigmoid activation functions

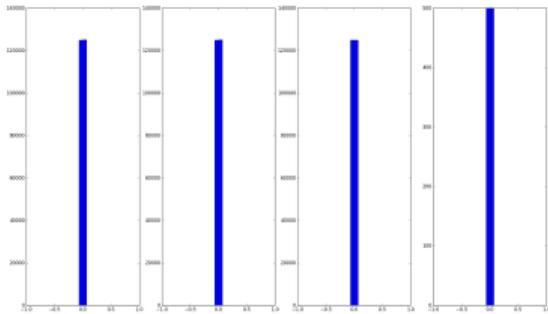
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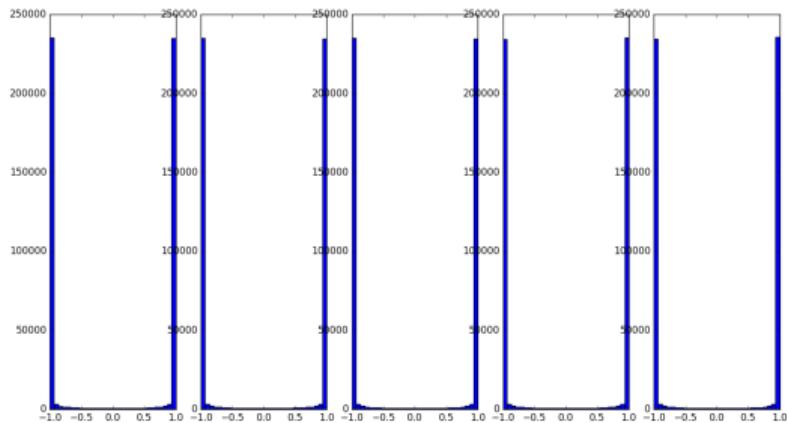
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- Recall that  $\nabla w_1$  is proportional to the activation passing through it
- If all the activations in a layer are very close to 0, what will happen to the gradient of the weights connecting this layer to the next layer?
- They will all be close to 0 (vanishing gradient problem)

```
W = np.random.randn(fan_in, fan_out)
```

- Let us try to initialize the weights to large random numbers

```
W = np.random.randn(fan_in, fan_out)
```

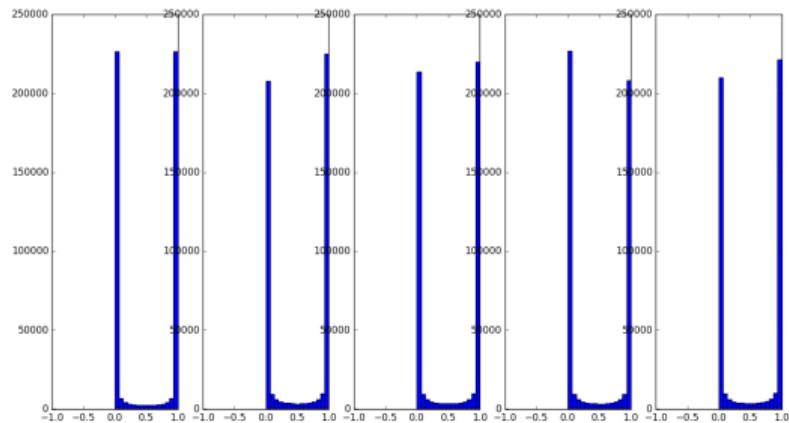
- Let us try to initialize the weights to large random numbers



tanh activation with large weights

```
W = np.random.randn(fan_in, fan_out)
```

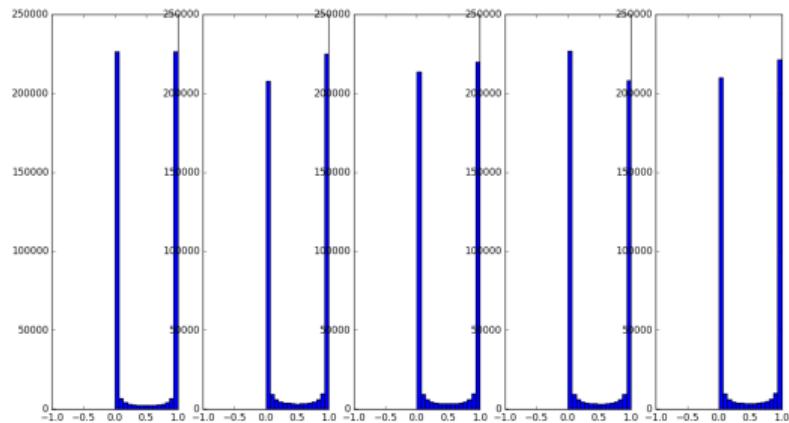
- Let us try to initialize the weights to large random numbers



sigmoid activations with large weights

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W = np.random.randn(fan_in, fan_out)
```

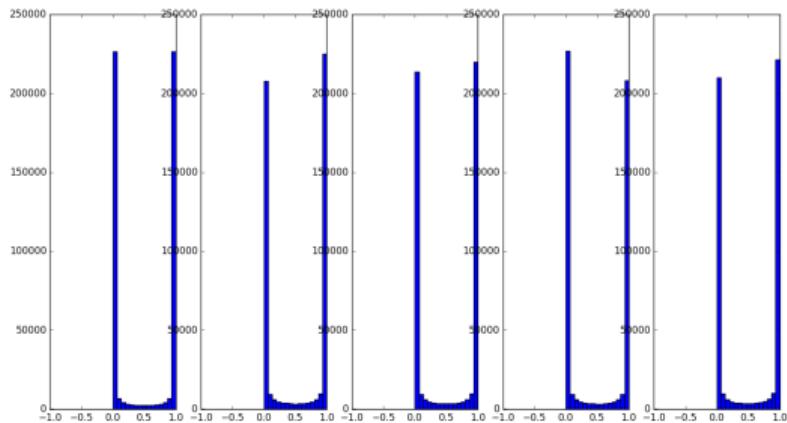
- Let us try to initialize the weights to large random numbers
- Most activations have saturated



sigmoid activations with large weights

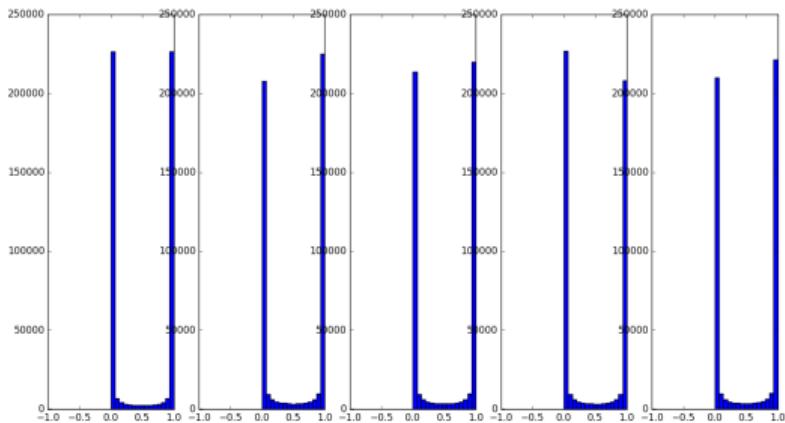
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- What happens to the gradients at saturation?



sigmoid activations with large weights

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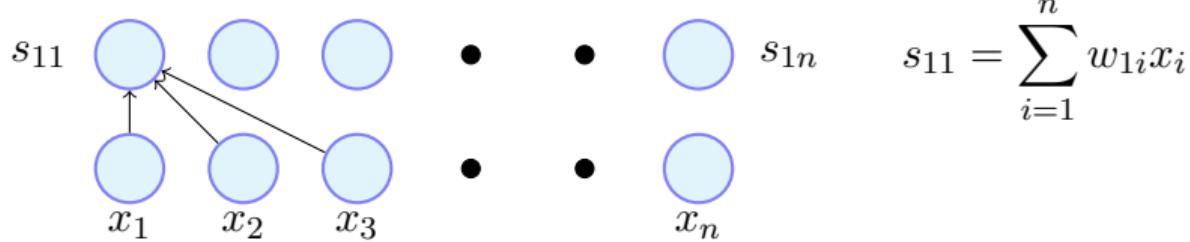


sigmoid activations with large weights

- Let us try to initialize the weights to large random numbers
- Most activations have saturated
- What happens to the gradients at saturation?
- They will all be close to 0 (vanishing gradient problem)

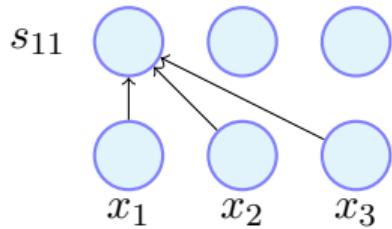
- Let us try to arrive at a more principled way of initializing weights

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$$s_{11} = \sum_{i=1}^n w_{1i} x_i$$

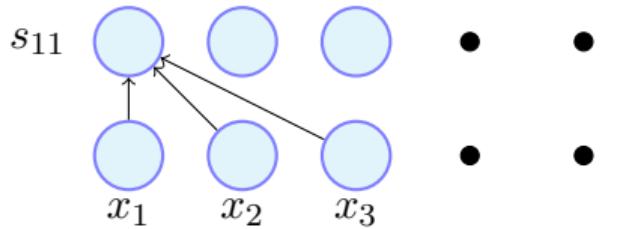
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$$s_{11} = \sum_{i=1}^n w_{1i} x_i$$

$$Var(s_{11}) = Var\left(\sum_{i=1}^n w_{1i} x_i\right) = \sum_{i=1}^n Var(w_{1i} x_i)$$

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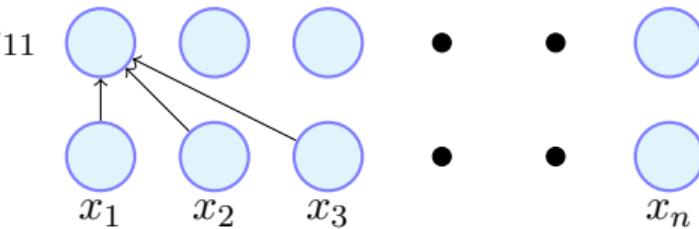


$$s_{1n} = \sum_{i=1}^n w_{1i}x_i$$

$$Var(s_{11}) = Var\left(\sum_{i=1}^n w_{1i}x_i\right) = \sum_{i=1}^n Var(w_{1i}x_i)$$

$$= \sum_{i=1}^n [(E[w_{1i}])^2 Var(x_i)]$$

$$+ (E[x_i])^2 Var(w_{1i}) + Var(x_i)Var(w_{1i})]$$



- [Assuming 0 Mean inputs and weights]

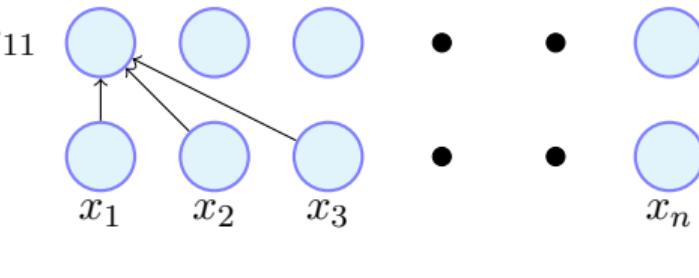
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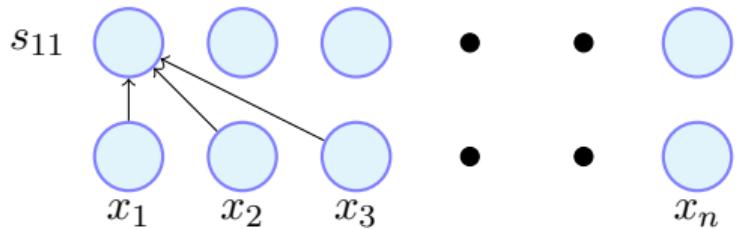
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- [Assuming 0 Mean inputs and weights]
- [Assuming  $Var(x_i) = Var(x) \forall i$ ]
- [Assuming  $Var(w_{1i}) = Var(w) \forall i$ ]

$$s_{11} = \sum_{i=1}^n w_{1i} x_i$$

$$Var(s_{11}) = Var\left(\sum_{i=1}^n w_{1i} x_i\right) = \sum_{i=1}^n Var(w_{1i} x_i)$$

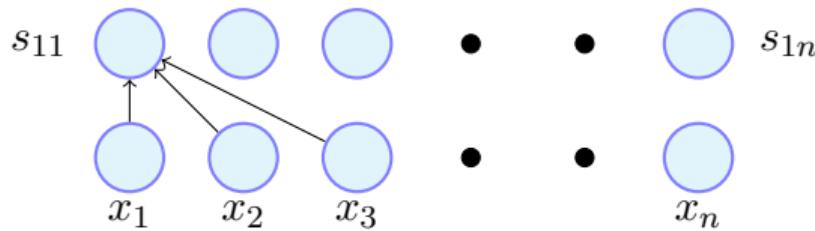
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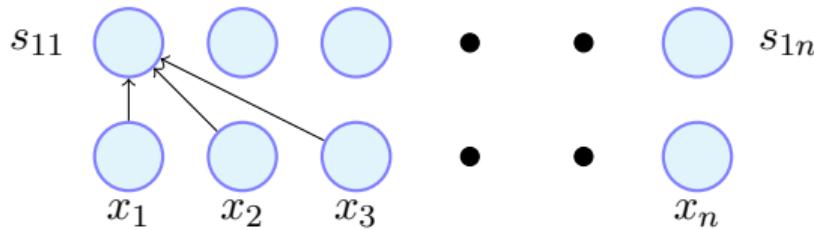
$$= (nVar(w))(Var(x))$$

- In general,



$$Var(S_{1i}) = (nVar(w))(Var(x))$$

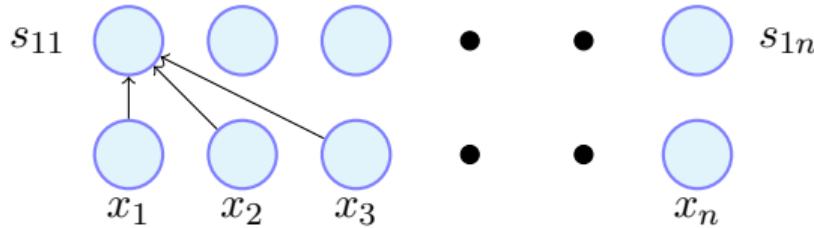
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?

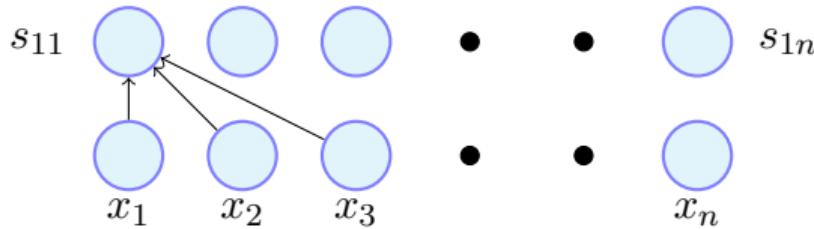
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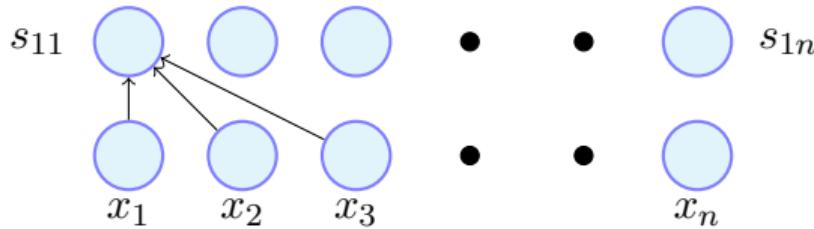
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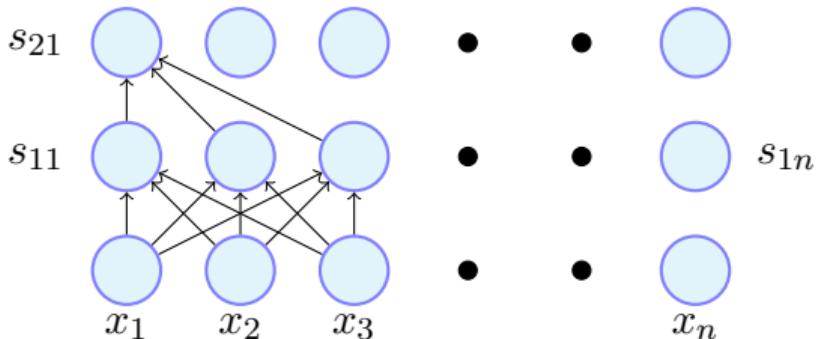
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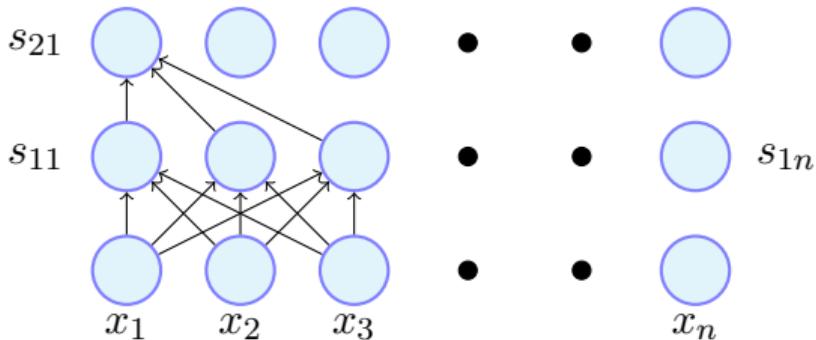
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- Let us see what happens if we add one more layer

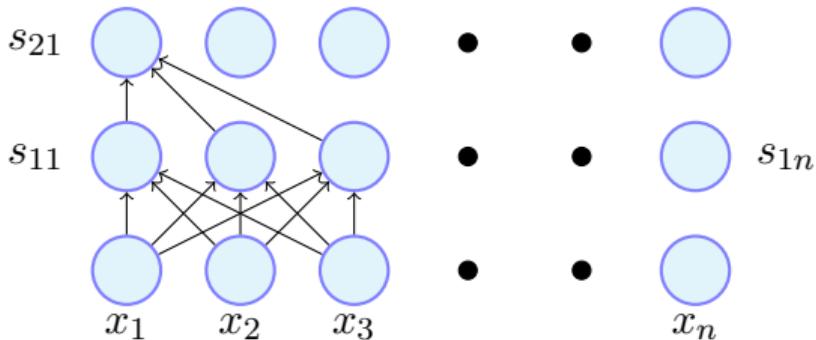


- Let us see what happens if we add one more layer
- Using the same procedure as above we will arrive at



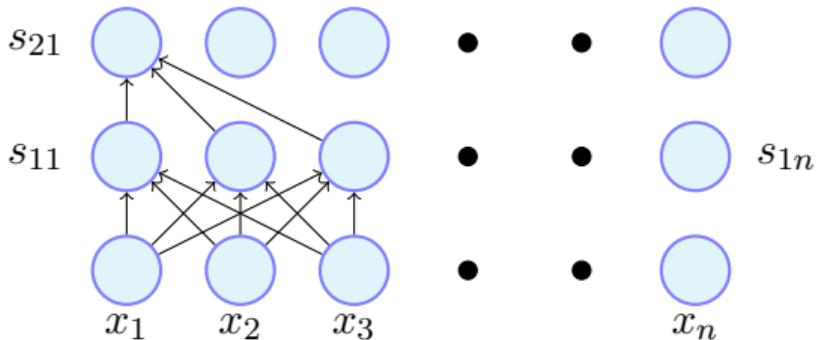
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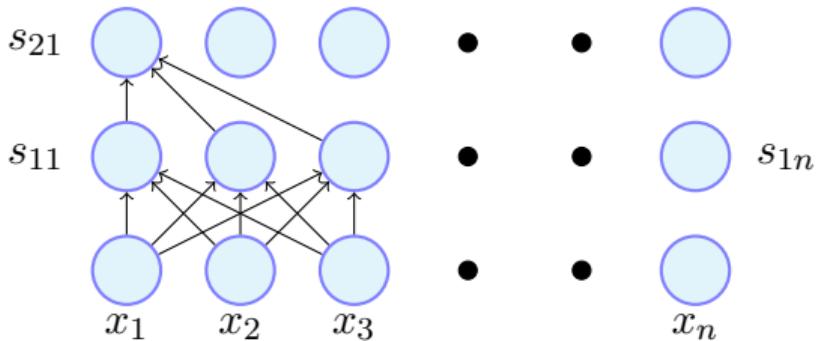
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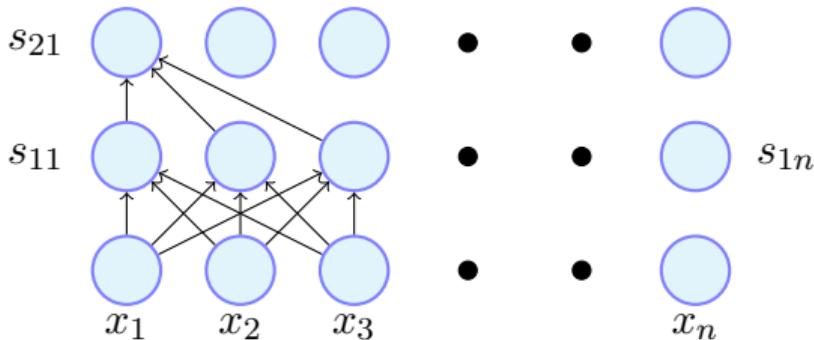


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$$Var(s_{21}) \propto [nVar(w_2)][nVar(w_1)]Var(x)$$



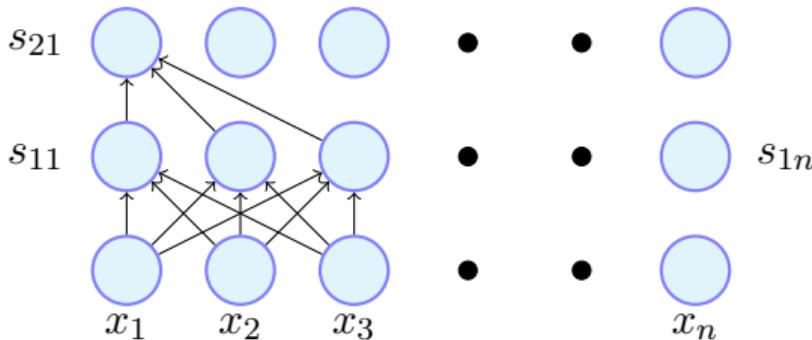
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$$\propto [nVar(w)]^2Var(x)$$

Assuming weights across all layers  
have the same variance

- In general,

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$$Var(s_{ki}) = [nVar(w)]^k Var(x)$$

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- To ensure that variance in the output of any layer does not blow up or shrink we want:

$$Var(az) = a^2(Var(z))$$

$$nVar(w) = 1$$

- If we draw the weights from a unit Gaussian and scale them by  $\frac{1}{\sqrt{n}}$  then, we have :

$$\begin{aligned} nVar(w) &= nVar\left(\frac{z}{\sqrt{n}}\right) \\ &= n * \frac{1}{n} Var(z) = 1 \end{aligned}$$

- In general,

$$Var(s_{ki}) = [nVar(w)]^k Var(x)$$

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$$Var(az) = a^2(Var(z))$$

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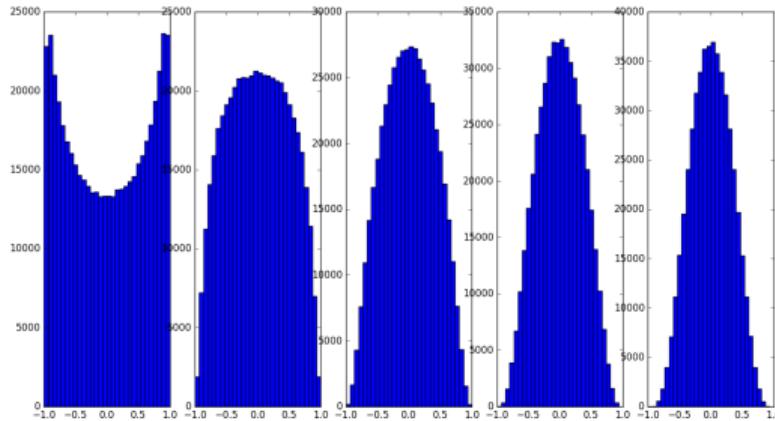
$$= n * \frac{1}{n} Var(z) = 1 \leftarrow (\textcolor{red}{Unit Gaussian})$$

```
W = np.random.randn(fan_in, fan_out) / sqrt(fan_in)
```

- Let's see what happens if we use this initialization

```
W = np.random.randn(fan_in, fan_out) / sqrt(fan_in)
```

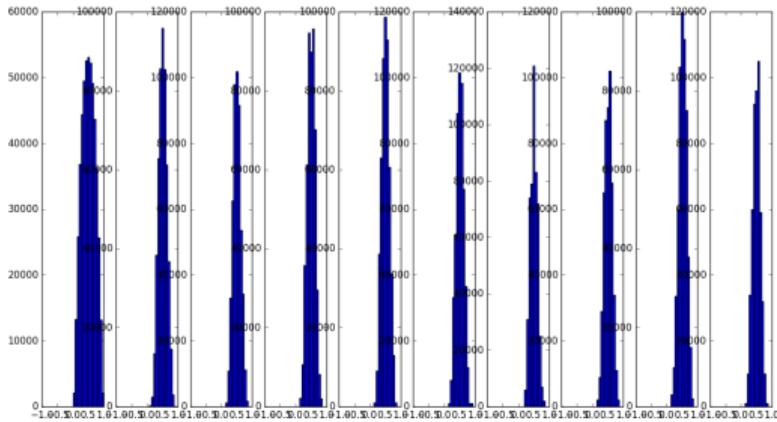
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tanh activation

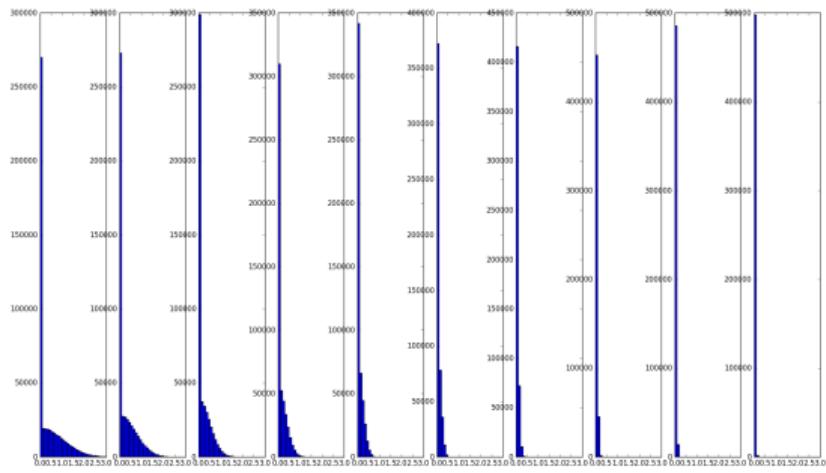
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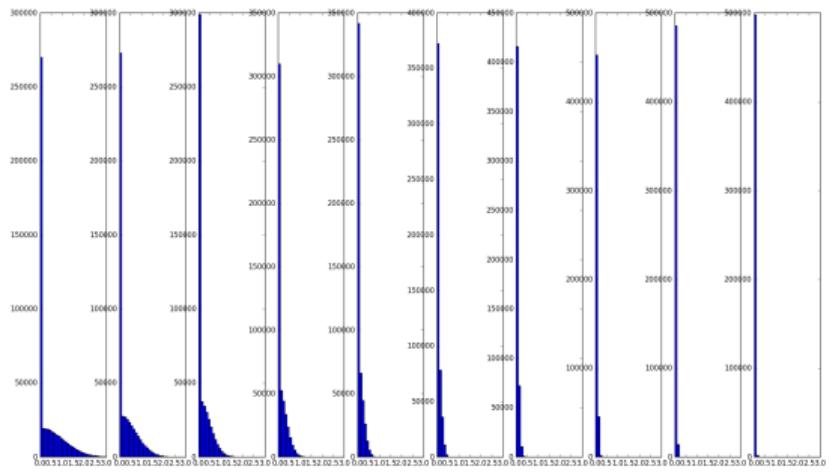


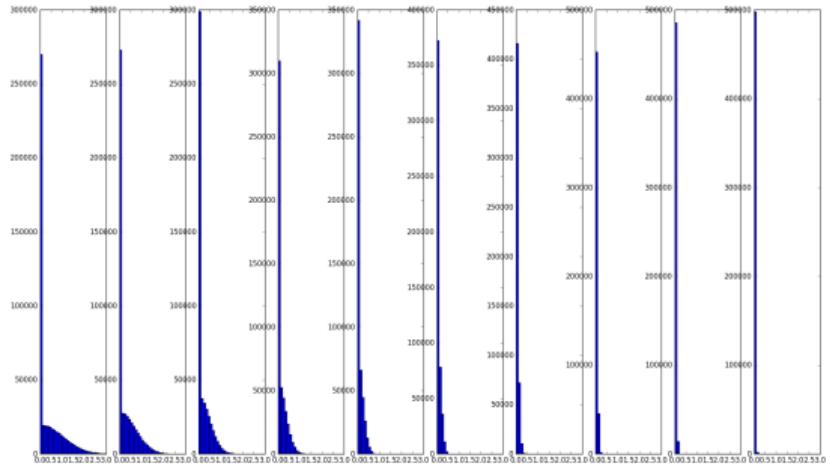
sigmoid activations

- However this does not work for ReLU neurons

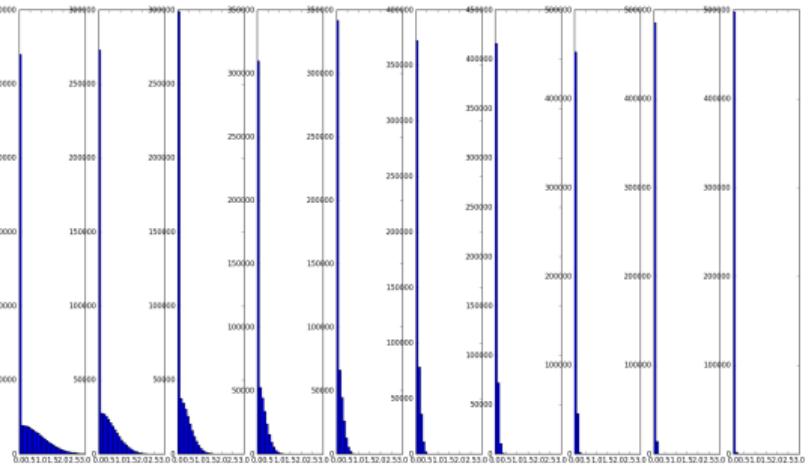


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- Why ?





- However this does not work for ReLU neurons
- Why ?
- Intuition: *He et.al.* argue that a factor of 2 is needed when dealing with ReLU Neurons



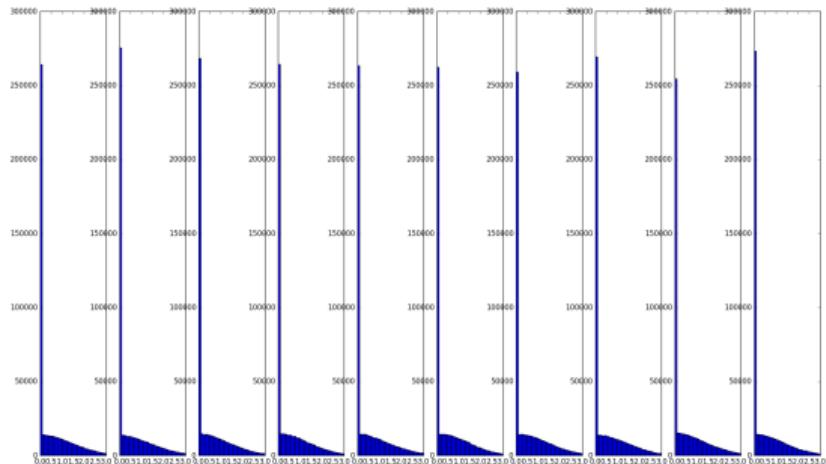
- However this does not work for ReLU neurons
- Why ?
- Intuition: *He et.al.* argue that a factor of 2 is needed when dealing with ReLU Neurons
- Intuitively this happens because the range of ReLU neurons is restricted only to the positive half of the space

```
W = np.random.randn(fan_in, fan_out) / sqrt(fan_in/2)
```

- Indeed when we account for this factor of 2 we see better performance

```
W = np.random.randn(fan_in, fan_out) / sqrt(fan_in/2)
```

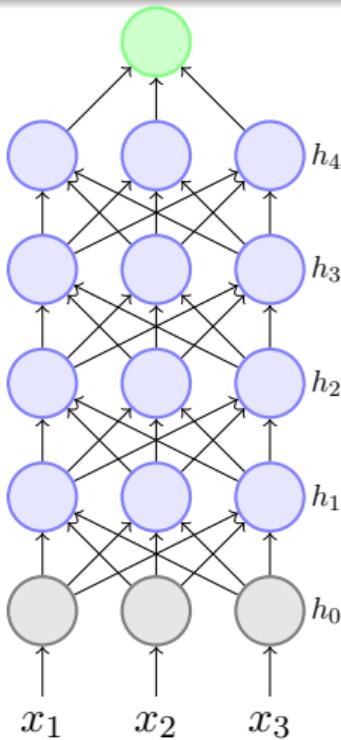
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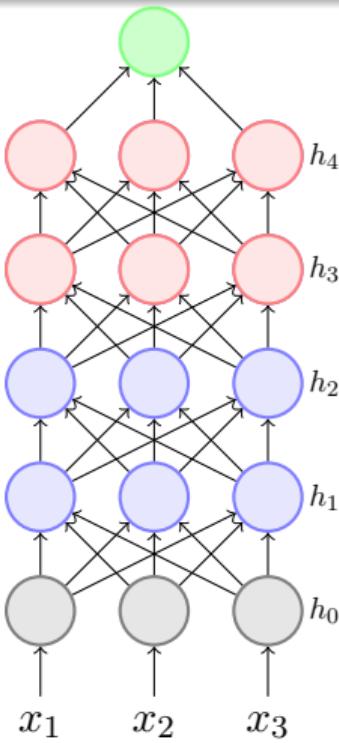
## Module 9.5 : Batch Normalization

We will now see a method called batch normalization which allows us to be less careful about initialization

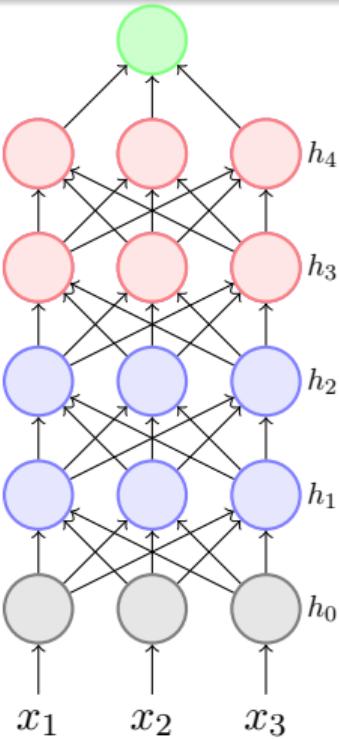
- To understand the intuition behind Batch Normalization let us consider a deep network



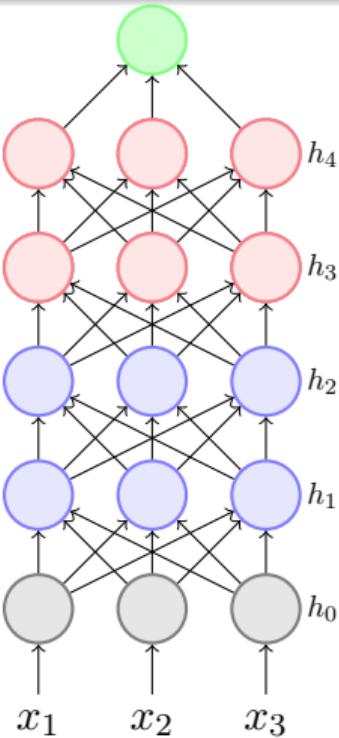
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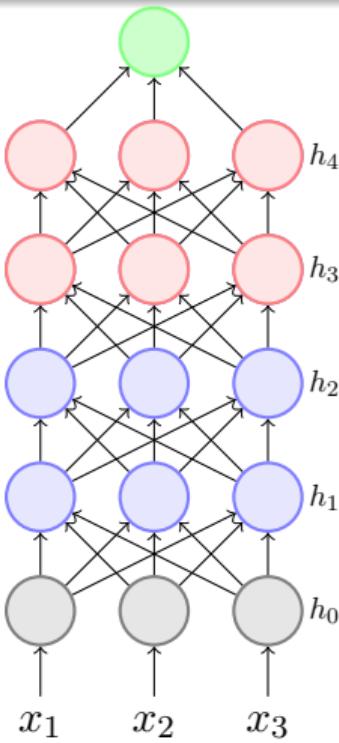
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- Let us focus on the learning process for the weights between these two layers



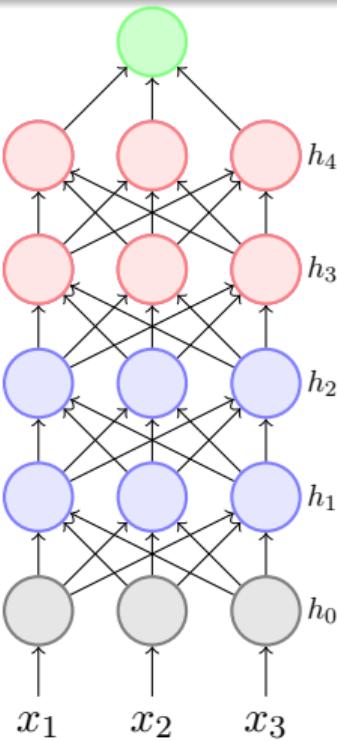
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- Typically we use mini-batch algorithms



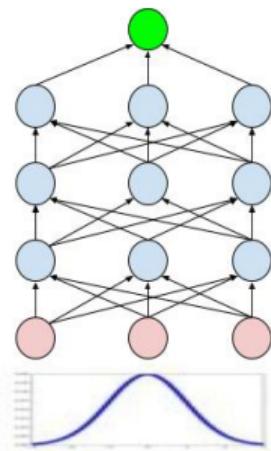
- To understand the intuition behind Batch Normalization let us consider a deep network
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- Typically we use mini-batch algorithms
- What would happen if there is a constant change in the distribution of  $h_3$



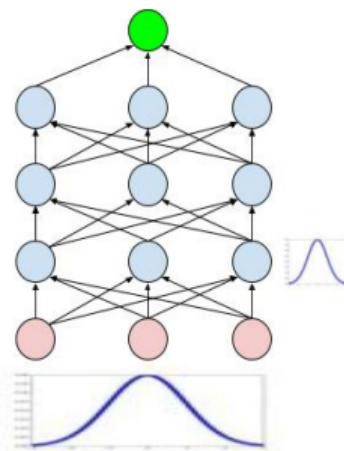
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- What would happen if there is a constant change in the distribution of  $h_3$
- In other words what would happen if across mini-batches the distribution of  $h_3$  keeps changing



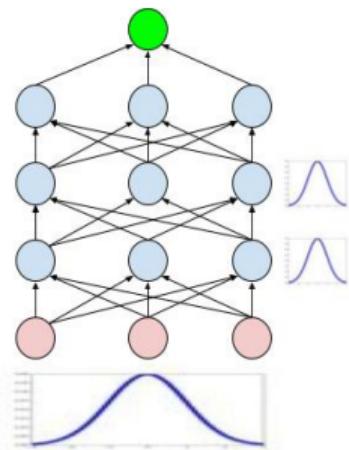
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- Typically we use mini-batch algorithms
- What would happen if there is a constant change in the distribution of  $h_3$
- In other words what would happen if across mini-batches the distribution of  $h_3$  keeps changing
- Would the learning process be easy or hard?



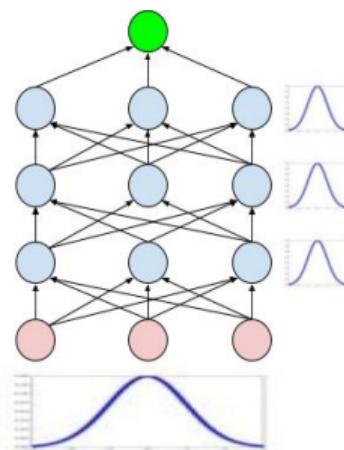
- It would help if the pre-activations at each layer were unit gaussians



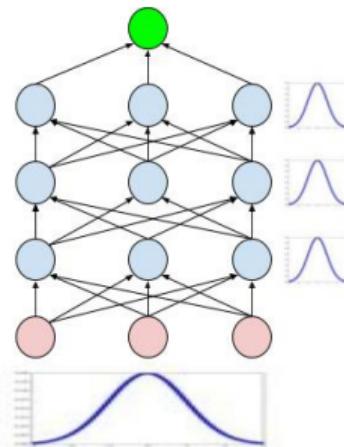
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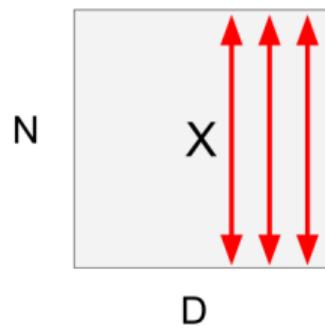


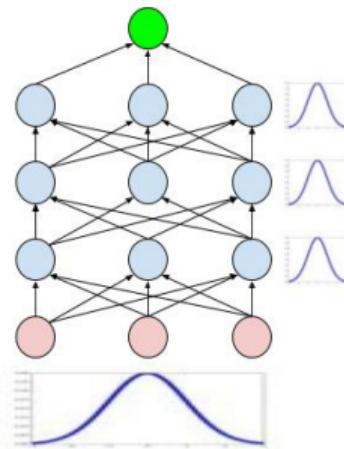
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- It would help if the pre-activations at each layer were unit gaussians
- Why not explicitly ensure this by standardizing the pre-activation ?

$$\hat{s}_{ik} = \frac{s_{ik} - E[s_{ik}]}{\sqrt{var(s_{ik})}}$$

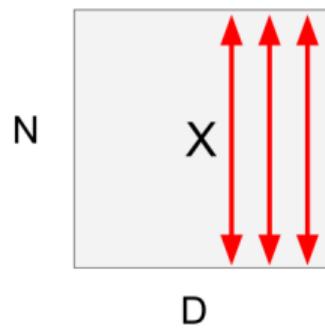


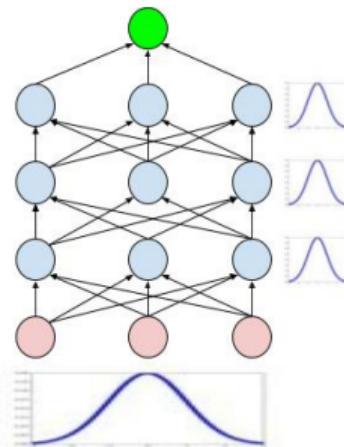


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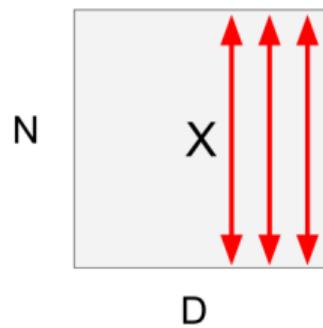


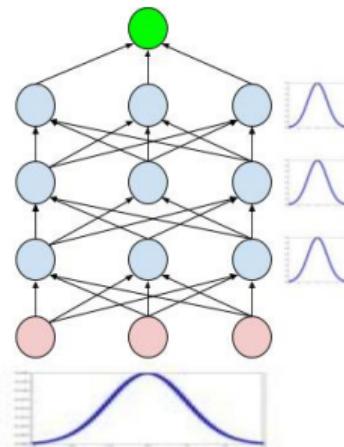


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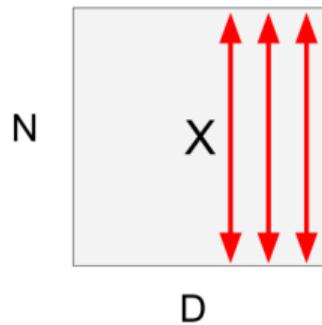




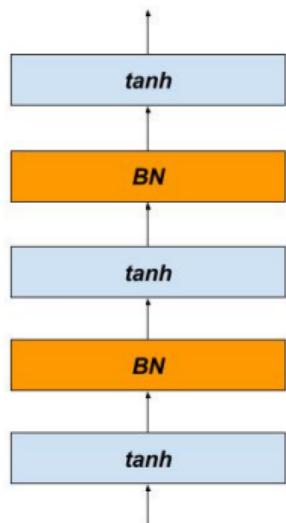
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$$\hat{s}_{ik} = \frac{s_{ik} - E[s_{ik}]}{\sqrt{\text{var}(s_{ik})}}$$

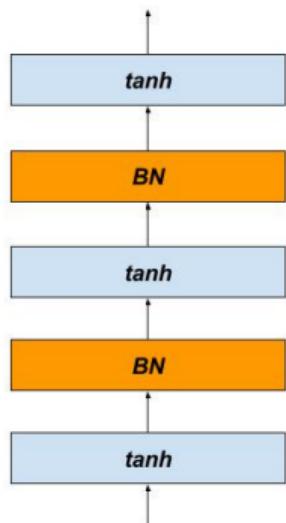
- But how do we compute  $E[s_{ik}]$  and  $\text{Var}[s_{ik}]$ ?
- We compute it from a mini-batch
- Thus we are explicitly ensuring that the distribution of the inputs at different layers does not change across batches



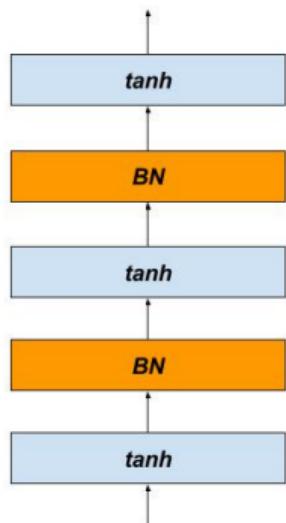
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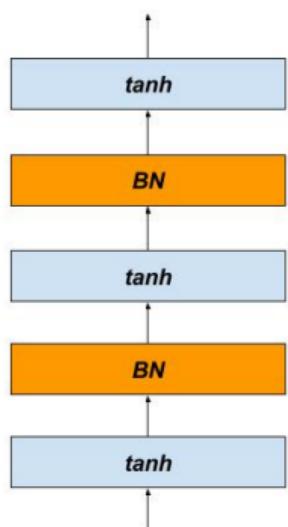


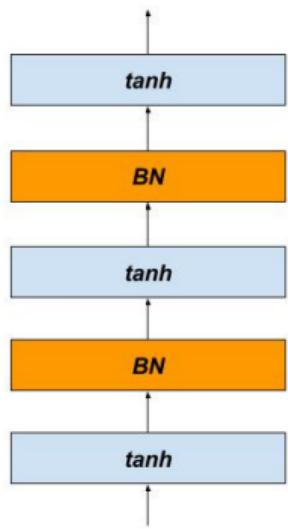
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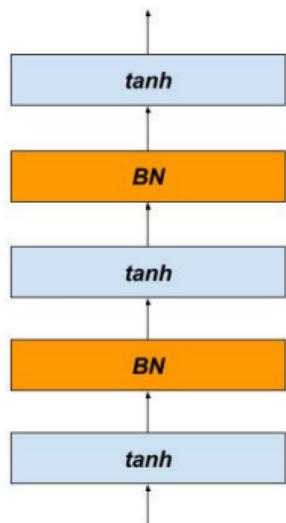
- This is what the deep network will look like with Batch Normalization
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- Hence we can backpropagate through this layer

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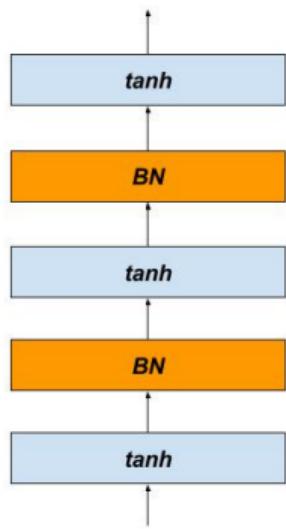


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$$y^{(k)} = \gamma^k \hat{s}_{ik} + \beta^{(k)}$$



$\gamma^k$  and  $\beta^k$  are additional parameters of the network.

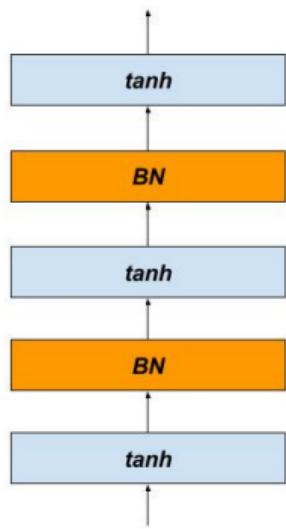
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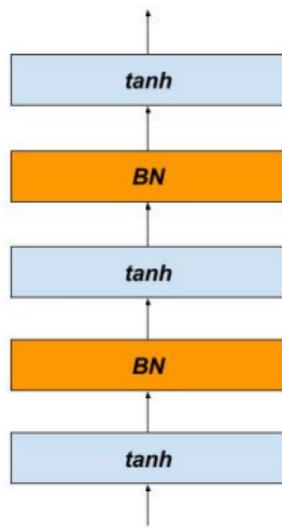
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- We will recover  $s_{ik}$



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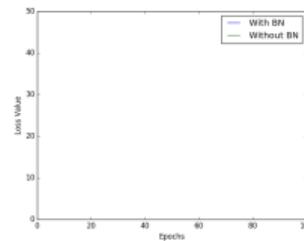
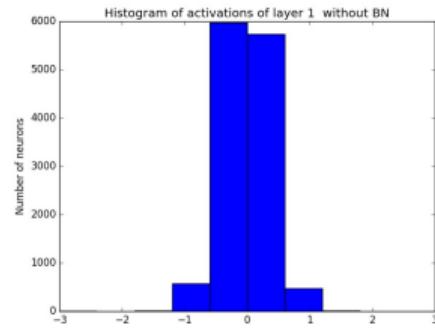
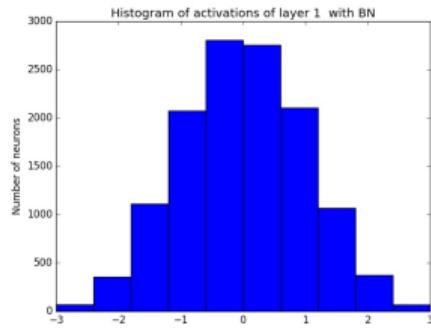
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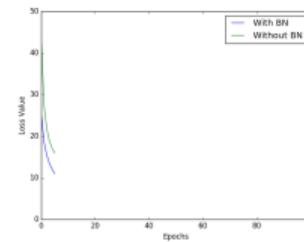
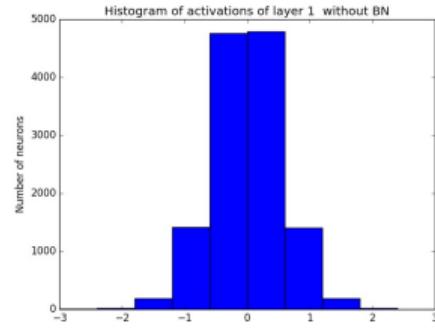
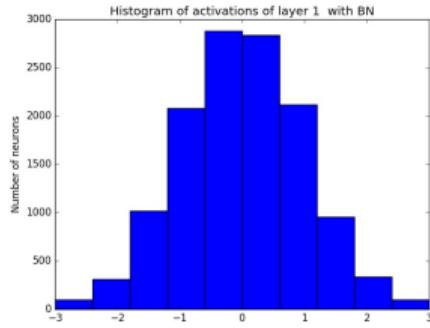
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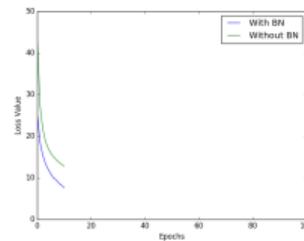
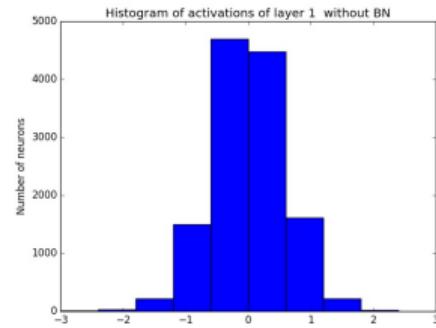
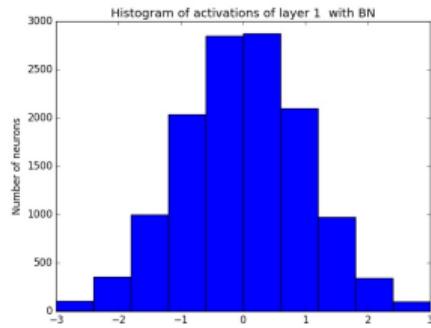
$$\beta^k = E[x^k]$$

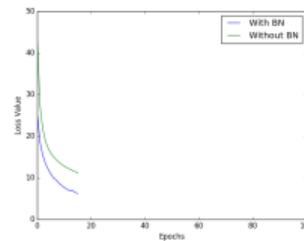
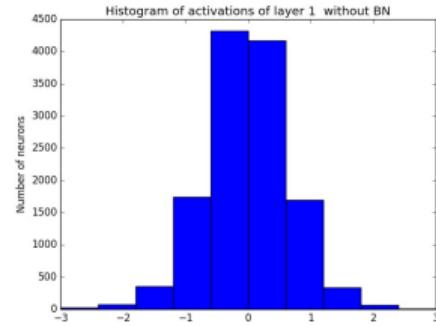
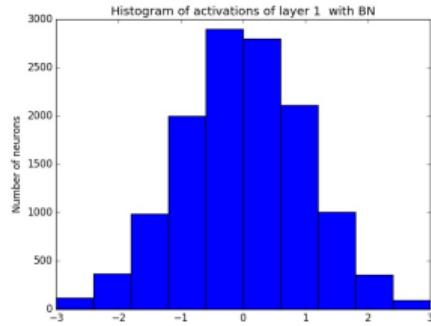
- We will recover  $s_{ik}$
- In other words by adjusting these additional parameters the network can learn to recover  $s_{ik}$  if that is more favourable

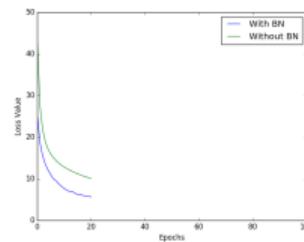
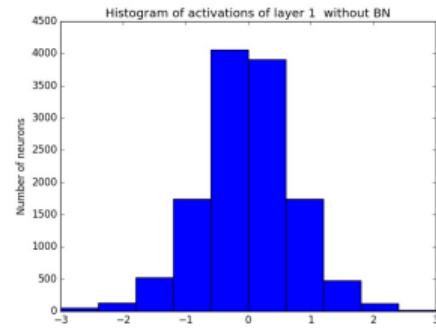
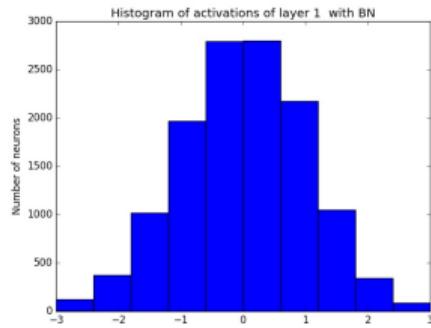
We will now compare the performance with and without batch normalization on  
MNIST data using 2 layers....

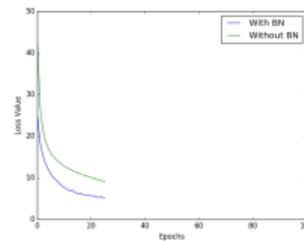
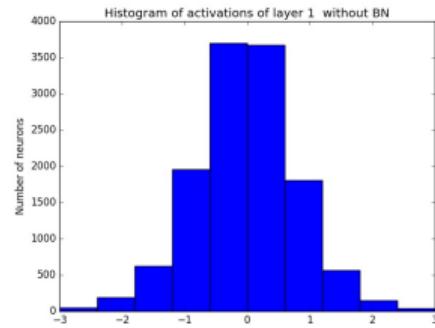
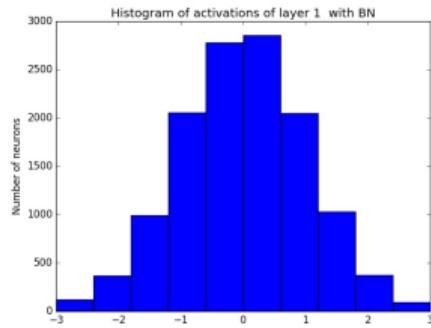


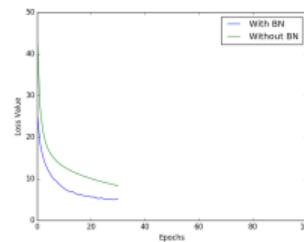
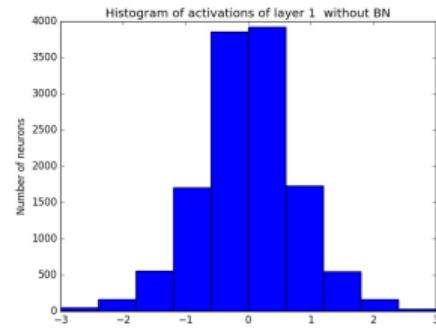
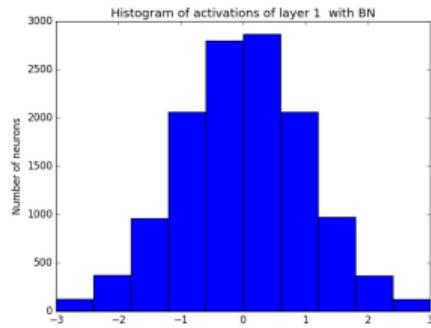


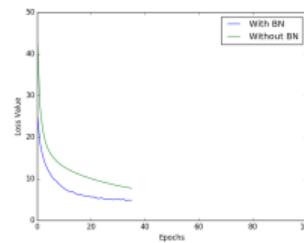
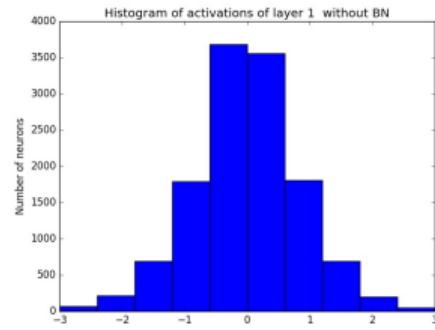
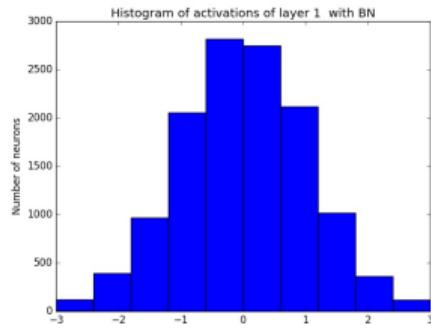


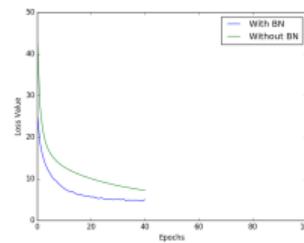
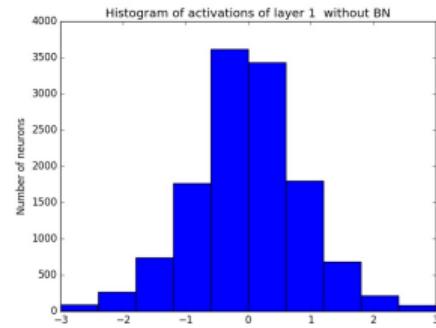
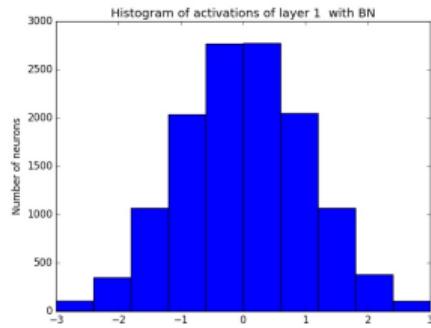


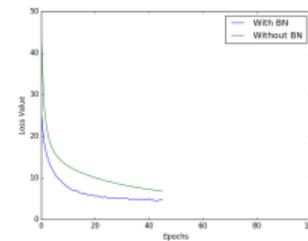
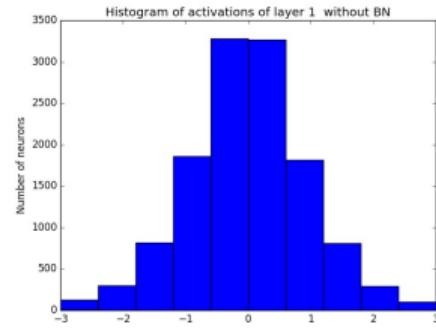
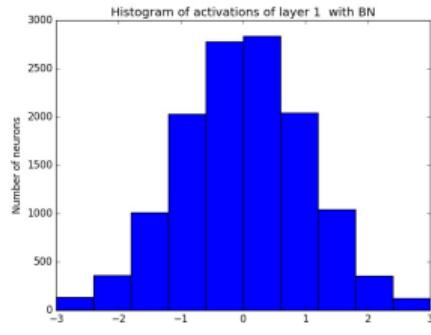


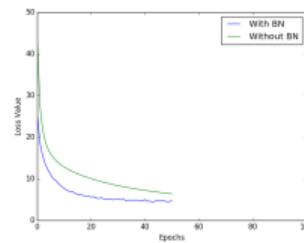
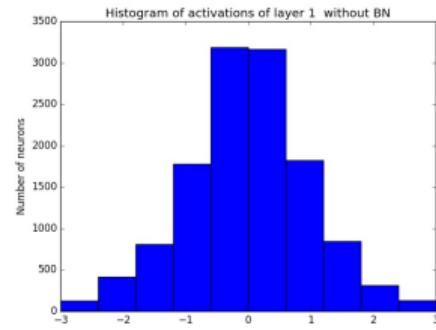
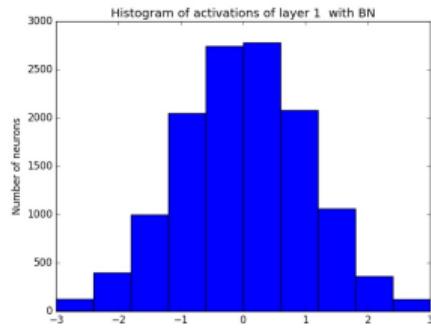


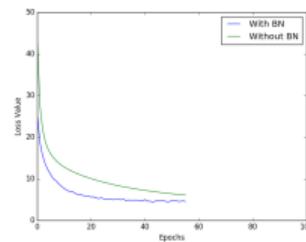
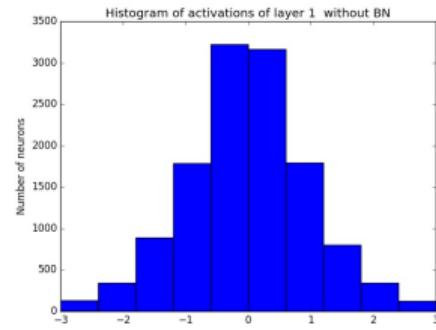
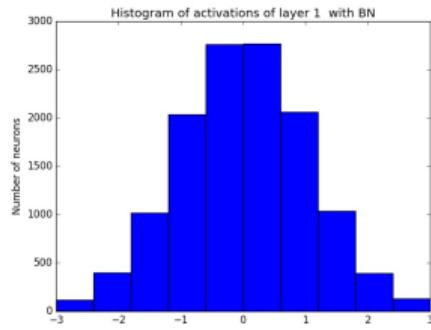


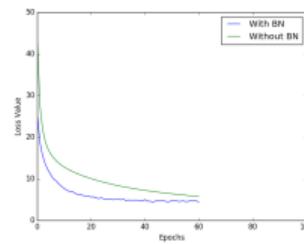
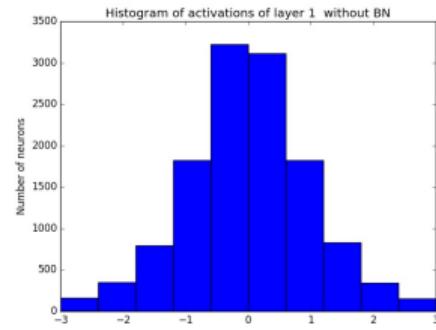
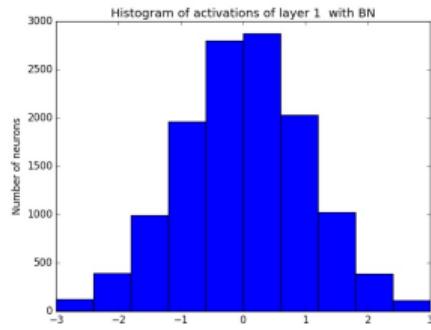


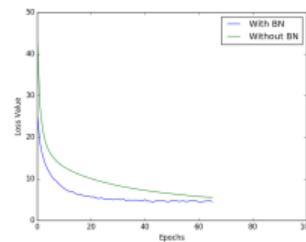
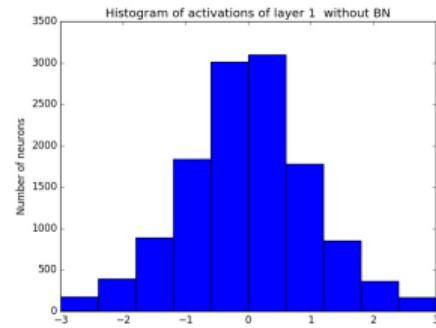
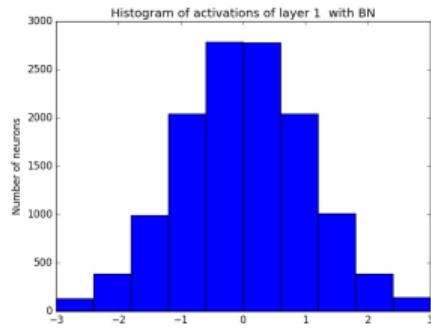


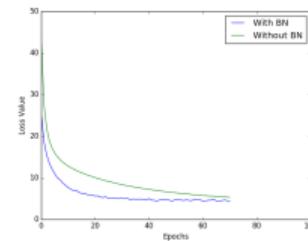
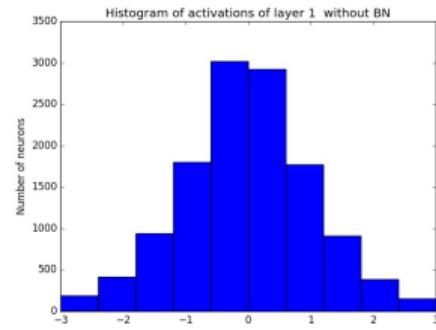
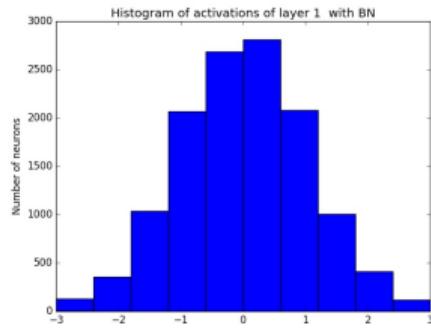


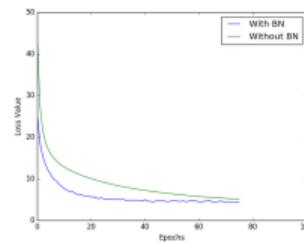
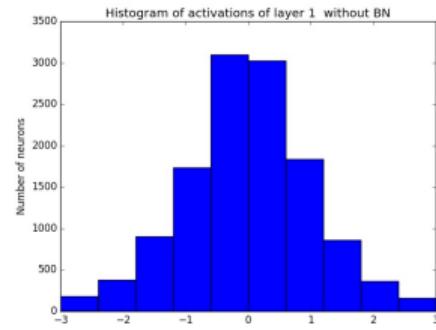
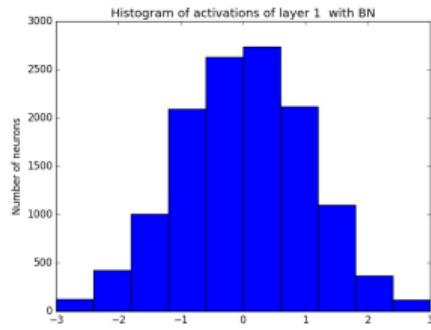


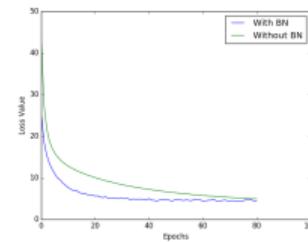
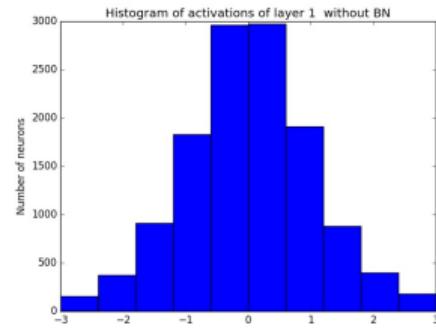
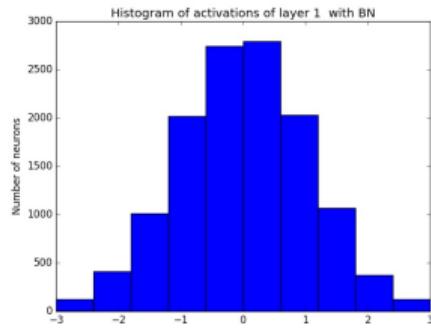


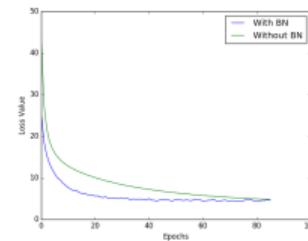
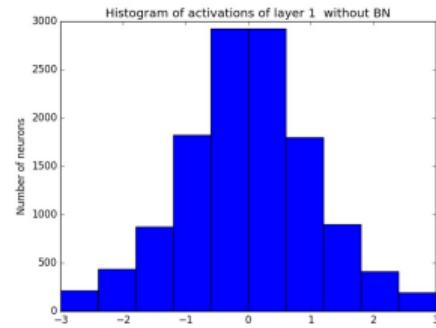
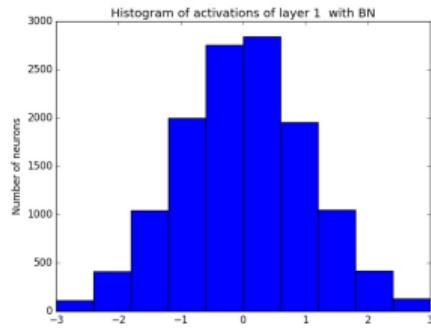


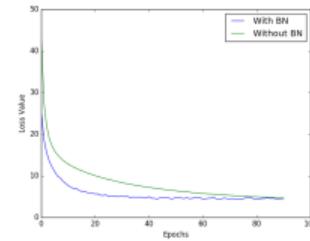
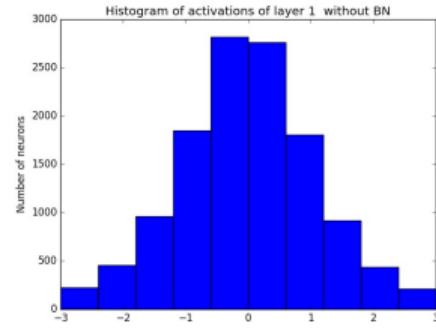
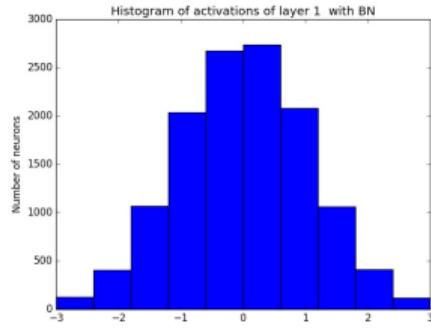


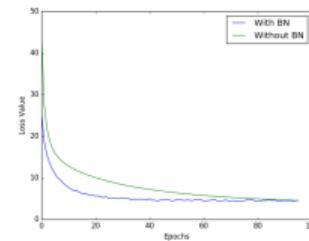
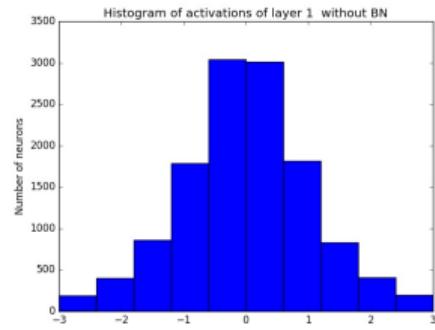
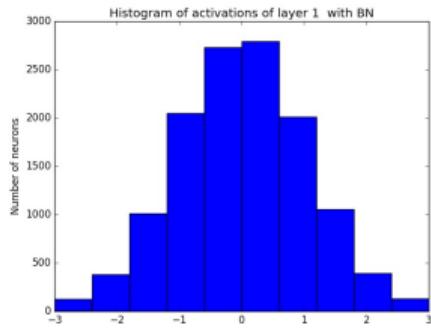












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- Data driven initialization methods

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