CSA0666 Analytical Questions - 1 GIRIJA.B 192311044

1. Solve following recurrence relations.

a) $x(n) = x(n-1) + 5$ for $n>1$ $x(1)=0$

$x(1)=0$

$x(n) = x(n-1)+5$

$n=2$: $x(2) = x(1)+5 = 0+5 = 5$

$n=3$: $x(3) = x(2)+5 = 5+5=10$

$n=4$: $x(4) = x(3)+5 = 10+5 = 15$

$n=5$: $x(5) = x(4)+5 = 15+5 = 20.$

$\Rightarrow x(n) = x(n-1)+5$
$= x(1) + 5(n-1)$
$= 0 + 5(n-1)$
$= 5(n-1)$

Now, $\boxed{x(n) = 5(n-1)}$

b) $x(n) = 3x(n-1)$ for $n>1$, $x(1)=4$

$x(1)=4$, $x(n) = 3x(n-1)$

$n=2$: $x(2) = 3x(1) = 3\times4 = 12$

$n=3$: $x(3) = 3x(2) = 3\times12 = 36$

$n=4$: $x(4) = 3x(3) = 3\times36 = 108$

$n=5$: $x(5) = 3x(4) = 108\times3 = 324$

Now,

$x(n) = 3x(n-1)$

$x(n) = 3^{n-1}\cdot x(1)$

$= 3^{n-1}\cdot 4$

$\boxed{x(n) = 4\cdot 3^{n-1}}$

c) $x(n) = x(n/2) + n$ for $n > 1$ $x(1) = 1$ (Solve for $n = 2^k$)

$x(n) = x(n/2) + n \quad x(1) = 1 \quad n = 2^k$

$n = 2$:

$x(2) = x(2/2) + 2 = x(1) + 2 = 1 + 2 = 3$

$n = 4$: $x(4) = x(4/2) + 4 = x(2) + 4 = 3 + 4 = 7$

$n = 8$: $x(8) = x(8/2) + 8 = x(4) + 8 = 7 + 8 = 15$

Here,

$x(2^1) = 3 = 2^2 - 1$

$x(2^2) = 7 = 2^3 - 1$

$x(2^3) = 15 = 2^4 - 1$

So, $\boxed{x(2^k) = 2^{k+1} - 1}$

d) $x(n) = x(n/3) + 1$ for $n > 1$ $x(1) = 1$ (Solve for $n = 3^k$)

$n = 3$: $x(3) = x(3/3) + 1 \Rightarrow x(1) + 1 = 2$

$n = 9$: $x(9) = x(9/3) + 1 \Rightarrow x(3) + 1 = 2 + 1 = 3$

$n = 27$: $x(27) = x(27/3) + 1 \Rightarrow x(9) + 1 = 3 + 1 = 4$

Now, $n = 3^k$.

$x(3^1) = 2 = 1 + 1$

$x(3^2) = 3 = 2 + 1$

$x(3^3) = 4 = 3 + 1$

$\boxed{x(3^k) = k + 1}$

2. Evaluate following recurrences completely.

(i) $T(n) = T(n/2)+1$ where $n=2^k$ for all $k \geq 0$

Given,

$$T(n) = T(n/2)+1 \qquad n = 2^k$$

Substitute $n=2^k$

$$T(2^k) = T(2^k/2)+1 = T(2^{k-1})+1$$

Now,

$$T(2^{k-1}) = T\left(\frac{2^{k-1}}{2}\right)+1 = T(2^{k-2})+1$$

Again,

$$T(2^{k-2}) = T(2^{k-2}/2)+1 = T(2^{k-3})+1$$

$$\vdots$$

$$T(2^1) = T(2^0)+1$$

Now,

$$n = 2^k \Rightarrow k = \log_2 n$$

$$T(2^k) = T(2^{k-1})+1 = T(2^{k-2})+1+1 = \dots = T(2^0)+k.$$

Since,

$$2^0 = 1, \; T(2^0) = T(1) \qquad T(1) = 1$$

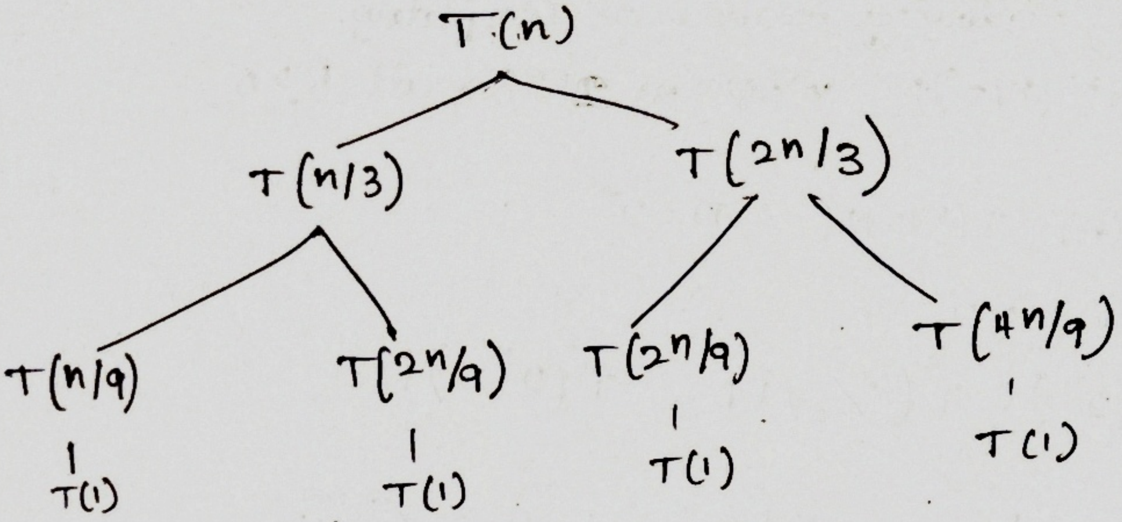
$$T(2^k) = 1+k. \qquad \boxed{O(\log n)}$$

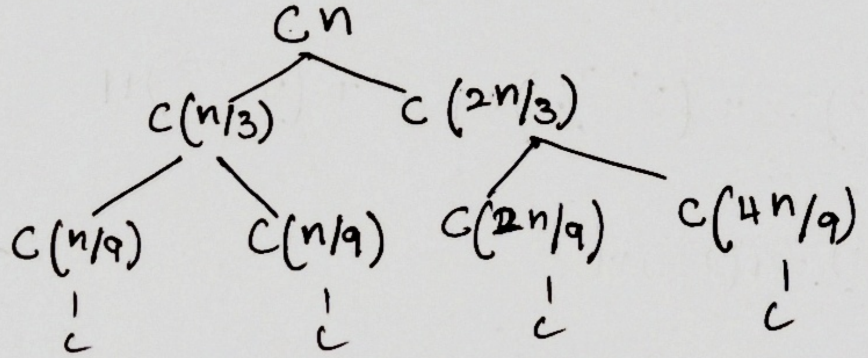
$$\boxed{T(n) = 1+\log_2 n}$$

(ii) $T(n) = T(n/3) + T(2n/3) + cn$.

We use Recursion tree method,

$$T(n) = T(n/3) + T(2n/3) + cn$$





length $= \log_3 n$ (divided by 3)

$T(n) = cn \log_3 n \Rightarrow \omega(n \log n)$

3. Consider following algorithm.

min1(A[0....n-1])
if n=1 return A[0] — 1
Else temp = min1[A[0...n-2])
if temp ≤ A[n-1] return temp
else
return A[n-1] — n-1

a) what does this algorithm compute?

b) setup a recurrence relation for algorithms basic operations count and solve it.

a) This algorithm computes minimum element in an array A of size n.

If $i<n$, A[i] is smaller than all elements, then, A[j], $j=i+1$ to $n-1$, then it returns A [i]. It also returns the leftmost mimimal element.

b) main comparison occurs during recursion.

So, $T(n)=T(n-1)+1$, when $n>1$ (one comparison at every step except, $n=1$)

$T(1)=0$ (no compare when $n=1$).

$$T(n)=T(1)+(n-1)*1$$
$$=0+(n-1)$$
$$=n-1$$

$\therefore$ Time Complexity = $O(n)$.

4. Analyze order of growth

(i) $F(n)=2n^2+5$ and $g(n)=7n$ use $\Omega(g(n))$ notation.

Given,

$F(n)\geq c.g(n)$

$F(n)=2n^2+5$

$c.g(n)=7n$

$n=1$ $F(1)=2(1)^2+5=7$

$g(1)=7$

$n=2$

$F(2)=2(2)^2+5$

$=8+5=13$

$g(2)=7\times2=14$

$n=3$

$F(3)=2(3)^2+5$

$=18+5$

$=23$

$g(3)=21$

$n=1$, $7=7$

$n=2$, $13=14$

$n=3$, $23=21$

$n\geq3$, $F(n)\geq g(n).c$

$F(n)$ is always greater than or equal to $c*g(n)$ when, n value is greater or equal to 3.

$$\therefore F(n) = \Omega(g(n)).$$

$F(n)$ is ~~at~~ grows more than $g(n)$ from below Asymptotically.